Efficiency of sports betting markets

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IASE, Gijon (may 2008)

Abstract Sports and betting have grown together to become multi billion dollar industries. The advent of the internet has globalized the betting market even more, with a few clicks it is possible to put a wager on events far away. Betting odds are also routinely used as indicators of winning probabilities or indicators of relative strengths. This assumes that betting markets process information efficiently. In this talk, we test this hypothesis for two types of soccer betting markets, and the market for tennis bets.

Plan

- Where do we stand (literature review)?
  - NFL
  - Economic theory
  - Soccer
- Regression-based tests
- Three applications (soccer, tennis, and again, soccer)

Information efficiency (1)

A market in which prices always ‘fully’ reflect available information is called ‘efficient’. (Fama, 1970)

A capital market is said to be efficient if it fully and correctly reflects all relevant information in determining prices. Formally, the market is said to be efficient with respect to some information set . . . if security prices would be unaffected by revealing that information to all participants. Moreover, efficiency with respect to an information set . . . implies that it is impossible to make economic profits by trading on the basis [of that information set]. (Malkiel, 1992)

Information efficiency (2)

> A perfect market for a stock is one in which there are no profits to be made by people who have no special information about the company, and in which it is difficult even for people who do have special information to make profits, because the price adjusts so rapidly as the information becomes available . . . Thus we would like to see *randomness* in the prices of successive transactions, rather than great continuity. . . If the price is going to move up, it should move up all at once, rather than in a series of small steps. (Black, 1971)

Terminology (1)

(Wikipedia) 'In probability theory and statistics the odds in favour of an event or a proposition are the quantity $p/(1-p)$, where p is the probability of the event or proposition. . . In gambling, the "odds" on display are not the chances that the event will occur, but are the amounts that the bookmaker will pay. However, in a free market under perfect information, the odds of paying tend to drift to the true probability odds.'

Terminology (2)

(Wikipedia) 'Taking an event with a 1 in 5 probability of occurring (i.e. 0.2 or 20%), then the odds are $0.2/(1-0.2) = 0.2/0.8 = 0.25$. If you bet 1 at fair odds and the event occurred, you would receive back 4 plus your original 1 stake. This would be presented in fractional odds of 4 to 1 against (written as 4 : 1 or 4/1).'

Decimal odds ($Odds$): total payout by bookmaker (including stake), so 5.0 in this case.

Fair odds (no expected profit, no expected loss):

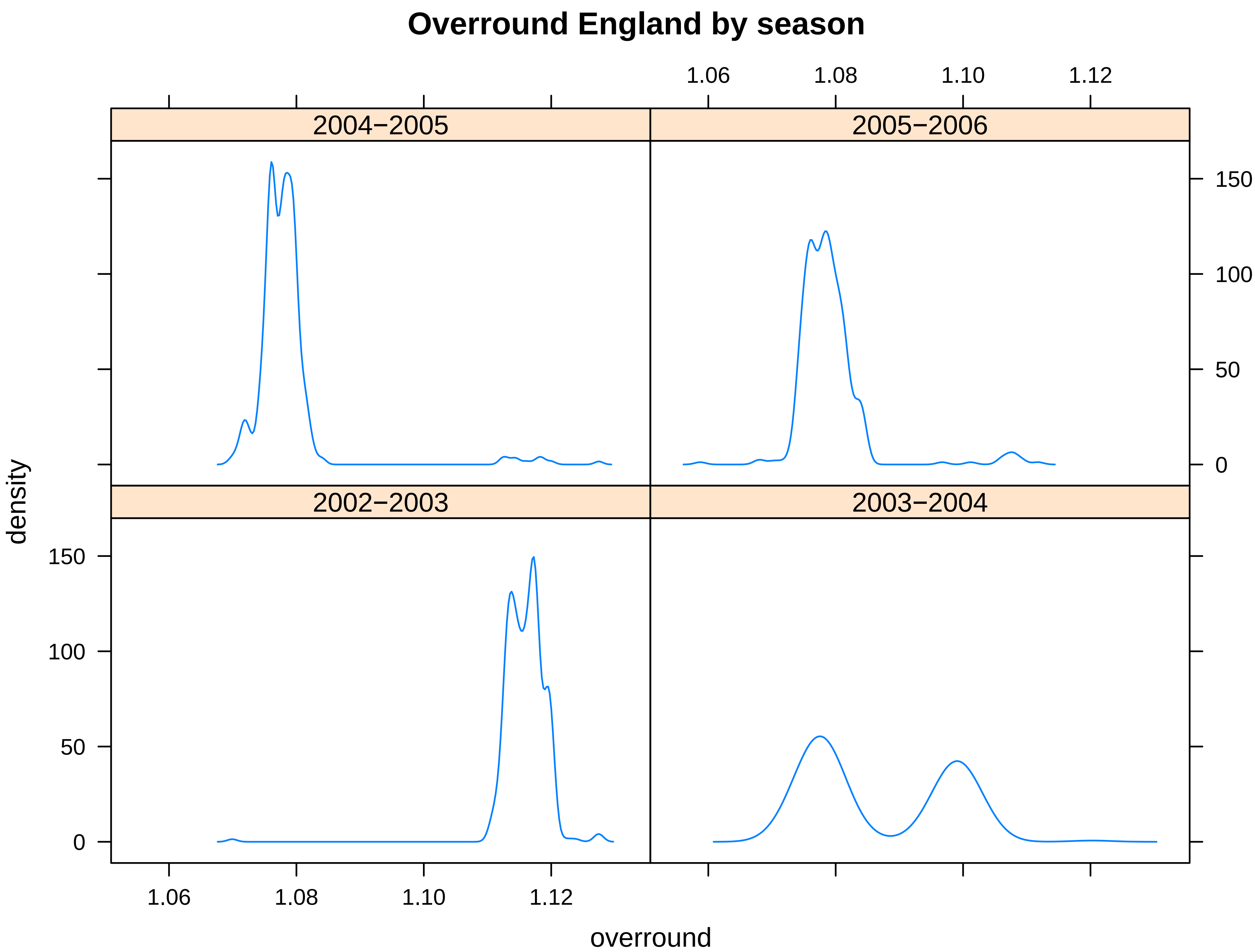
$$\Pr(\text{event}) \times Odds = 1.$$

Terminology (3)

(Wikipedia) ‘Vigorish, or simply ”vig”, or ”juice”, is the amount charged by a bookmaker for his services. The term is Yiddish slang originating from the Russian word for ”winnings,” vyigrysh. The concept is also known as the overround.’

Example: Final Roland Garros (tennis), `Bet365.com` paid 1.16 if Henin-Hardene were to win, and 5 if Ivanovic would win. 0.862 vs. 0.20, overround 0.062.

Example: RKC Waalwijk-Ajax, `Bet365.com` paid 5, 3.2, 1.66 for a home win, draw, away win, corresponding to probabilities 0.2, 0.31, 0.60, overround 0.11.


Overround England by season
2004−2005
2005−2006
2002−2003
2003−2004
density
overround
1.06
1.08
1.10
1.12
0
50
100
150

Zuber et al., JPE 1985

Point spread betting, 11 for 10 rule

- $PS_{it} = \beta_0 + \beta_1 VL_{it} + \epsilon_{it}$
- VL initially posted by bookmaker, later adjusted
- EMH: $\beta_0 = 0$, $\beta_1 = 1$
- 1983 NFL, 13 out of 16 weeks EHM is not rejected
- 15 out of 16 weeks $\beta_0 = 0, \beta_1 = 0$ not rejected

Zuber et al., JPE 1985 (cont'd)

- $PS_{it} = \beta' x_{it} + \epsilon_{it}$
- Each variable: HT $-$ AT
- yards rushed, yards passed, number of wins, fumbles, interceptions, number of rookies
- 'The results are gratifyingly neat and clear.' (p. 803)
- $R^2 = 0.733$ (first eight weeks) or $R^2 = 0.727$ (entire 18 weeks)

Zuber et al., JPE 1985 (cont'd)

- Estimate model up to $t-1$, PPS_{it}
- Value bet: $|PPS_{it} - VL_{it}| > \lambda$, $\lambda = 0.5, 1, 2, 3$
- $\lambda = 0.5$, bet U$ 110, grand total U$ 2920
- Gains not increasing in λ
- Single NFL season
- Expoitable opportunity?

Sauer et al., JPE 1988

- ‘In particular, there is a stronger weak test.’ (p. 207)
- Not all implications of efficiency are used
- Pooling all 16 weeks: $\beta_0 = 0$, $\beta_1 = 1$ $F = 0.25$, $\beta_0 = 0$, $\beta_1 = 0$, $F = 12.2$
- $PS_{it} - VL_{it} = \beta'(x_{it}^h - x_{it}^a) + \epsilon_{it}$
- $\beta = 0$, $F = 0.55$

Sauer et al., JPE 1988 (cont'd)

- Estimate model up to $t-1$, PPS_{it}

- Value bet: $|PPS_{it} - VL_{it}| > \lambda$, $\lambda = 0.5, 1, 2, 3$

- Strategy is highly profitable in 1983 (U$ 1380), but unsuccessful in 1984: U$ -2920.

Dare and MacDonald, JFE 1996

- $PS_{it} = \beta_0 + \beta_1 VL_{it} + \epsilon_{it}$, home team minus away team

- Alternatively: favorite first, so that $VL_{it} \geq 0$

- Only five possibilities: FH-UA, UH-FA, HP-AP, FN-UN, PN

- Restrictions on regression

- 1980-1985: $\beta_0 = 0$, $\beta_1 = 1$ not rejected, only pick-em games and Superbowl

Gray and Gray, JFE 1996

- Model $\Pr(PS_{it} > VL_{it}) = \Phi(\beta_0 + \beta_1 HOME_i + \beta_2 FAV_i)$,
- $\beta_2 < 0$, favorites are less likely to beat the spread
- Recent past performance: momentum/contrarian investment literature, and applied psychology and economics literature on streaks
- Team that is doing well is more likely to beat the spread
- Bet on home team underdog, 4% net return

Boulier et al, App. Ec. 2006

- $PS_{it} = \beta_0 + \beta_1 VL_{it} + \epsilon_{it}$
- $PS_{it} = \beta_0 + \beta_1 VL_{it} + \gamma' z_{it} + \epsilon_{it}$
- NYT Power Score, different surface, dome
- NFL 1994-2000
- $\beta_0 > 0$, not exploitable, all other tests nonsignificant

Shin, Ec. J.. 1992

- Betting as a vehicle to study asset pricing with insider trading
- Contingent claims market, Arrow-Debreu securities
- Market exists for a short time, outcome is acknowledged
- Anecdotal evidence of insider trading
- Favorite-longshot bias: favorites win too often (compared to odds), and longshots don't win often enough.

Odds

- Fair odds:

$$\text{Pr(event)} \times Odds = 1.$$

  or

$$\text{Pr(event)} = 1/Odds.$$

- Do these odds reflect all available information?

- Model based tests and betting based tests

Pope and Peel, Economica 1989

- Football matches in the UK, 1981/82 season, 1290 games, four bookmakers
- Development sample 1055 games, 225 holdout
- Some bets have positive return
- $HW = \beta_0 + \beta_1 (1/Odds) + \epsilon_{ij}$
- $\beta_1 < 1$, not significant

Pope and Peel, Economica 1989 (cont'd)

- S1: bet if logit probability exceeds implied probability, returns on holdout sample lower than on development sample
- S2: use info from experts
- Some bets have positive return
- Draws: odds do not have all information

Goddard and Asimakopoulos, J. Int. Forecasting 2004

- Look for profitable betting strategies
- Forecast football results
- Poisson type models vs. discrete choice
- $y^*_{ij} = \theta_i - \theta_j + \epsilon_{ij}$

Goddard and Asimakopoulos, J. Int. Forecasting 2004 (cont'd)

- θ_i parametrized:
  - Form
  - Significance of a game
  - Elimination of FA Cup
  - Distance
- Model development: 1989-1998, forecast 1999, etc

Goddard and Asimakopoulos, J. Int. Forecasting 2004 (cont'd)

- $HW = \beta_0 + \beta_1(1/Odds) + \beta_2(\widehat{\pi} - (1/Odds)) + \epsilon_{ij}$

- $\beta_0 = 0, \beta_1 = 1, \beta_2 = 0$ not rejected

- Bet on highest probability event: positive return at start and end of season

- $HW = \beta_0 + \beta_1(1/Odds) + \beta_2(\widehat{\pi} - (1/Odds)) + \epsilon_{ij} + \beta_2(\pi - \widehat{\pi})$

Goddard and Thomas, J. Int. Sp. Fin. 2006

- Results modeled by ordered probit model, based on FIFA rankings and match results from previous tournaments
- Evaluation of EURO 2004 by simulation
- Prospects of Greece were not underestimated
- Home advantage of Portugal is underestimated
- Individual games: not profitable

Bettings based tests (1)

- Rating measures difference of quality between two teams
- Stefani, Norman and Clark, Koning
- Step 1: estimate rating
- Step 2: estimate relation between rating and result
- Step 3: identify value bets

Betting based tests (2)

- Quality Q_{it} is number of goals scored minus number of goals conceded during the last k games

- Varies by team and over time

- Quality difference: $Q_{it} - Q_{jt}$

- Estimate $\Pr(HW) = G(Q_{it} - Q_{jt})$

- Bet if $\Pr(HW) > 1/Odds$, expected value $\Pr(HW) \times Odds > 1$

Model based tests

$$
\begin{aligned}
\Pr(HW) = 1/Odds &= \frac{1}{1 + Odds - 1} \\
&= \frac{1}{1 + \exp(\log(Odds - 1))} \\
&= \frac{1}{1 + \exp(-\beta_0 - \beta_1 \log(Odds - 1))}.
\end{aligned}
$$

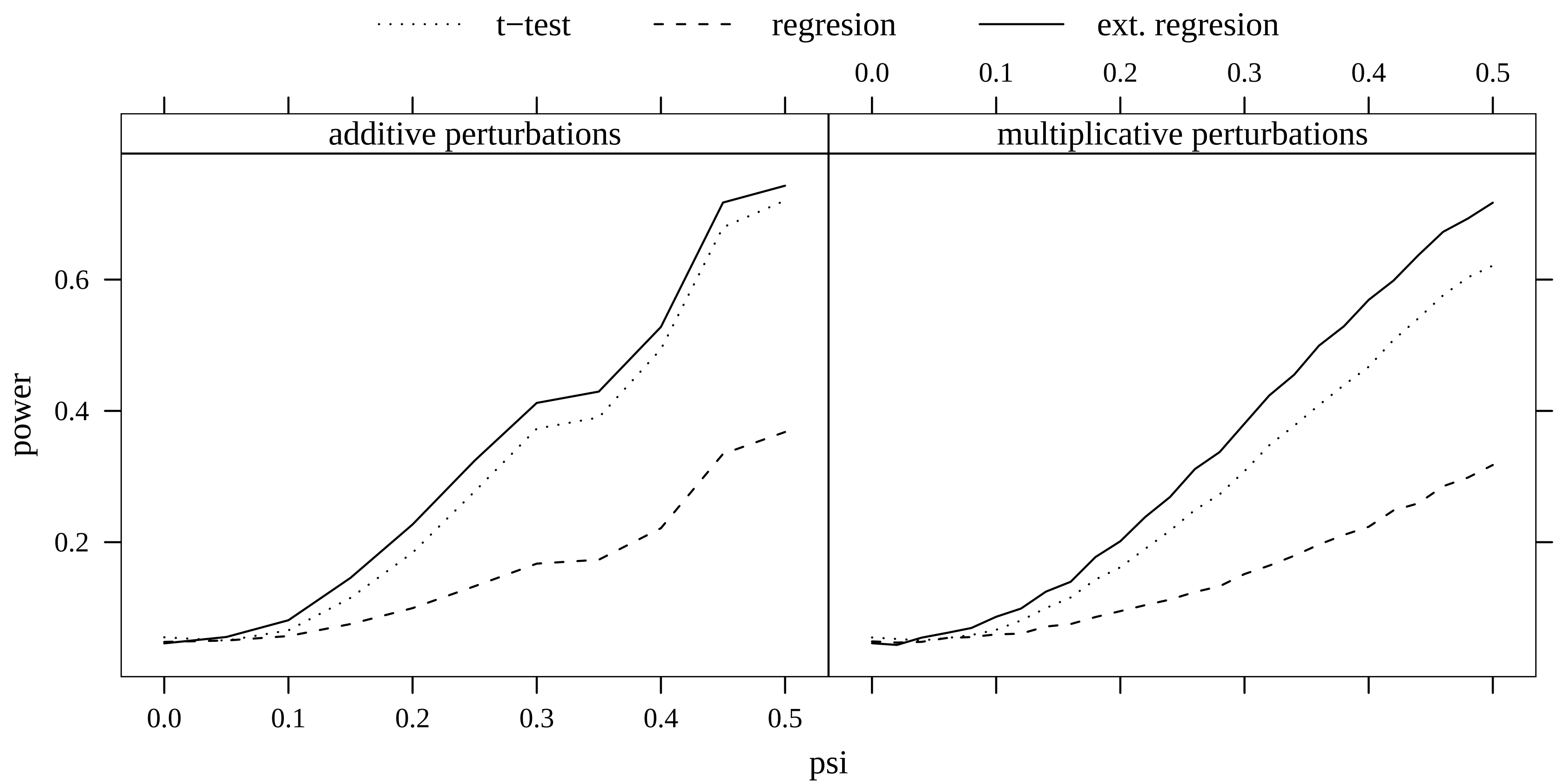
Estimate

$$
\Pr(HW) = \frac{1}{1 + \exp(-\beta_0 - \beta_1 \log(Odds - 1) - \gamma' z_{ijt})}
$$

and test $\beta_0 = 0$, $\beta_1 = -1$ and $\gamma = 0$.

Simulation experiment

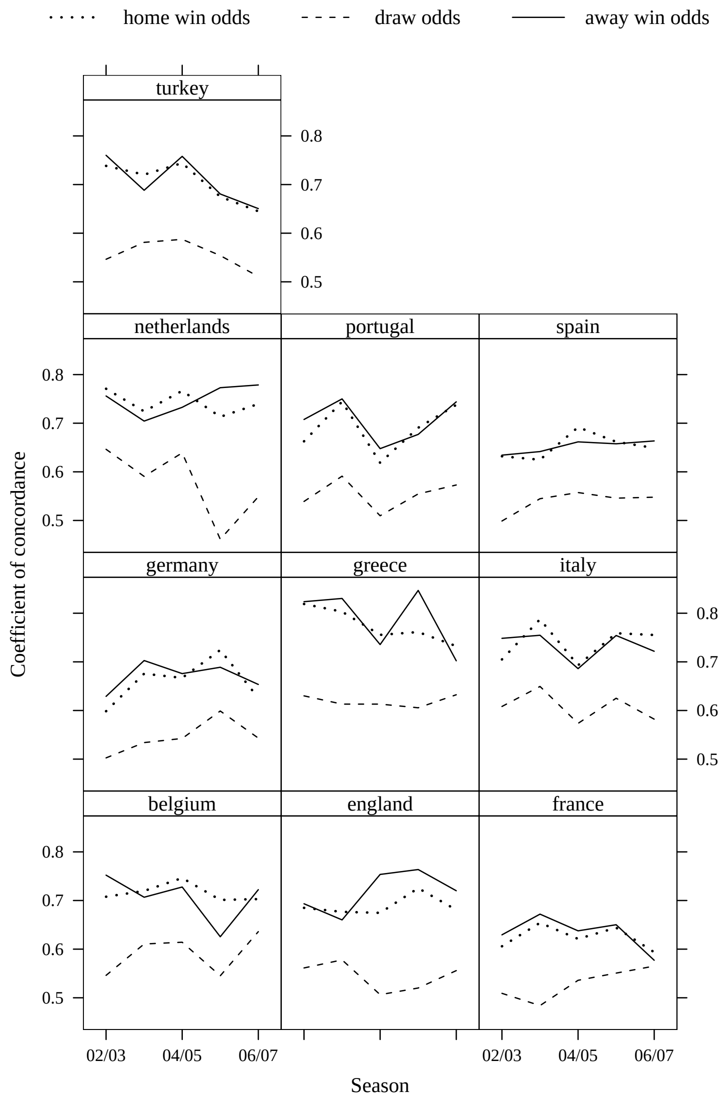
- posted odds, Netherlands, 2005-2006, 306 games, 76 affected
- additive: $1/Odds + 4\psi U$
- multiplicative: $1/Odds(1+\psi)$
- 10000 seasons simulated, test statistics

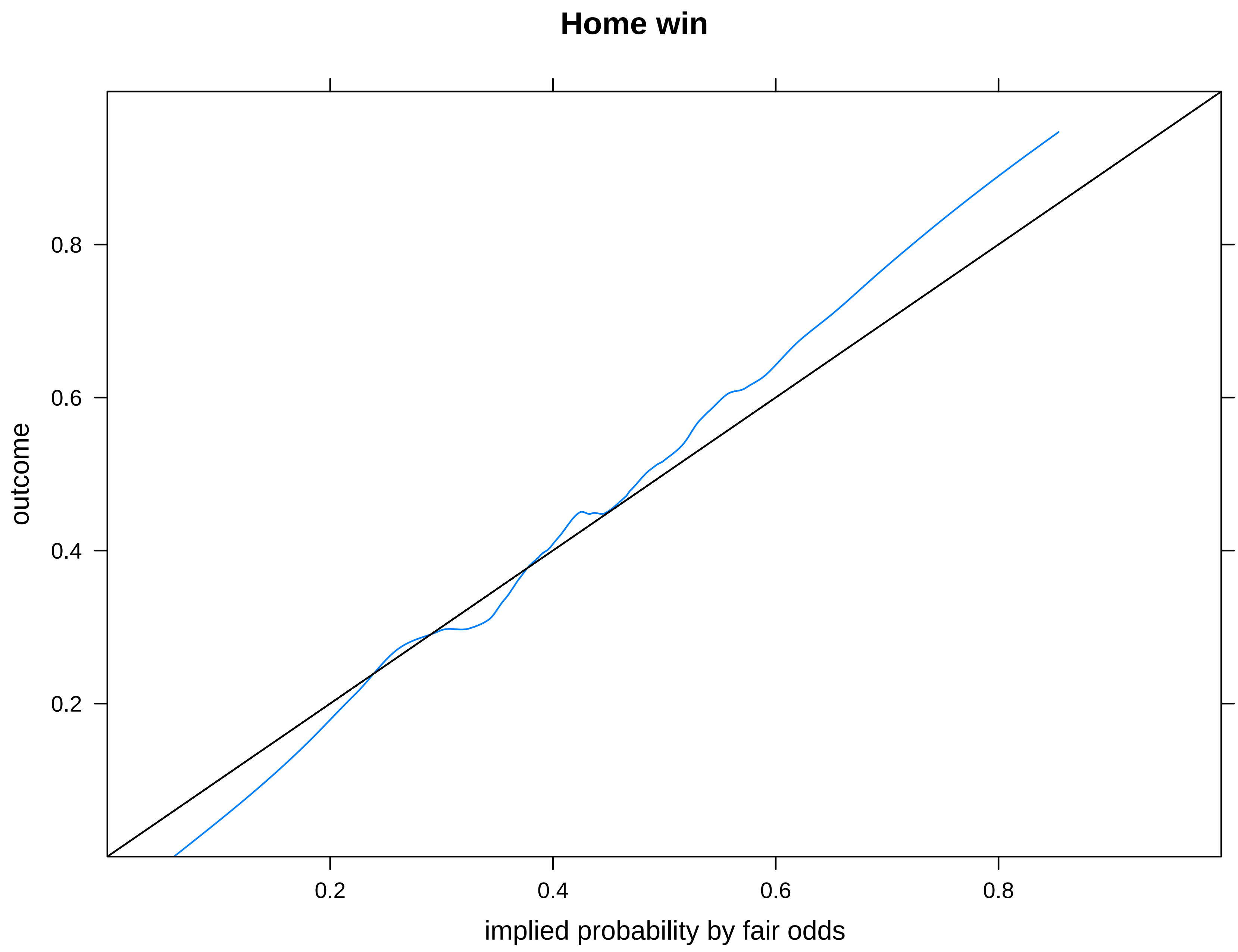


t−test
regresion
ext. regresion
additive perturbations
multiplicative perturbations
power
psi
0.2
0.4
0.6
0.0
0.1
0.2
0.3
0.4
0.5

Soccer

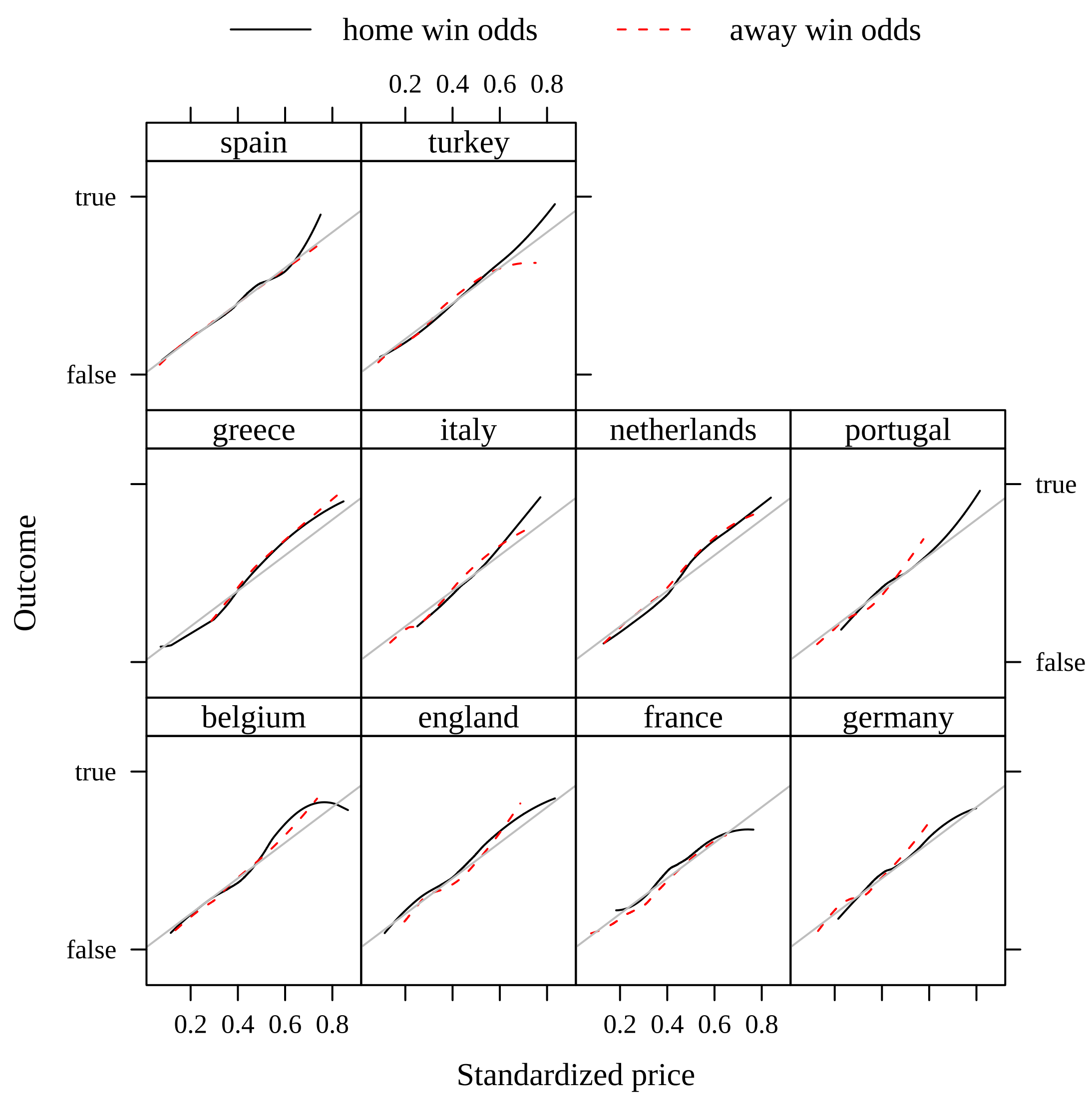
- Results on soccer competitions in different countries
- Key variables: home goals, away goals, date, average home win (since 2001-2002)
- Additional variables: match characteristics as number of bookings, attendance, referee (not for all countries)
- Data used: from 2002-2003 onwards
- Odds are posted two days before the game


home win odds
draw odds
away win odds
turkey
netherlands
portugal
spain
germany
greece
italy
belgium
england
france
0.8
0.7
0.6
0.5
Coefficient of concordance
02/03
04/05
06/07
Season


Home win
outcome
0.2
0.4
0.6
0.8
implied probability by fair odds

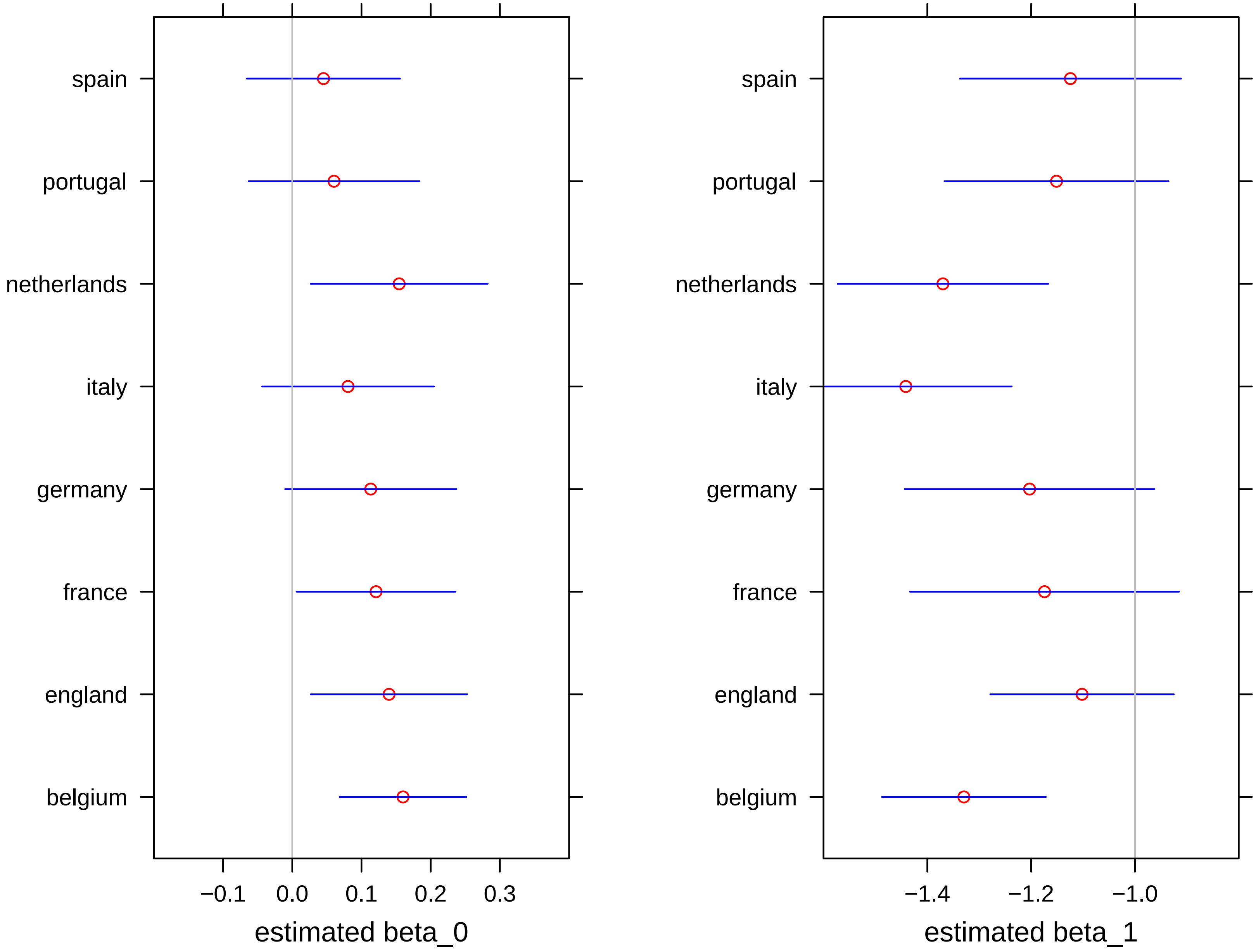
Win probability and odds (3)

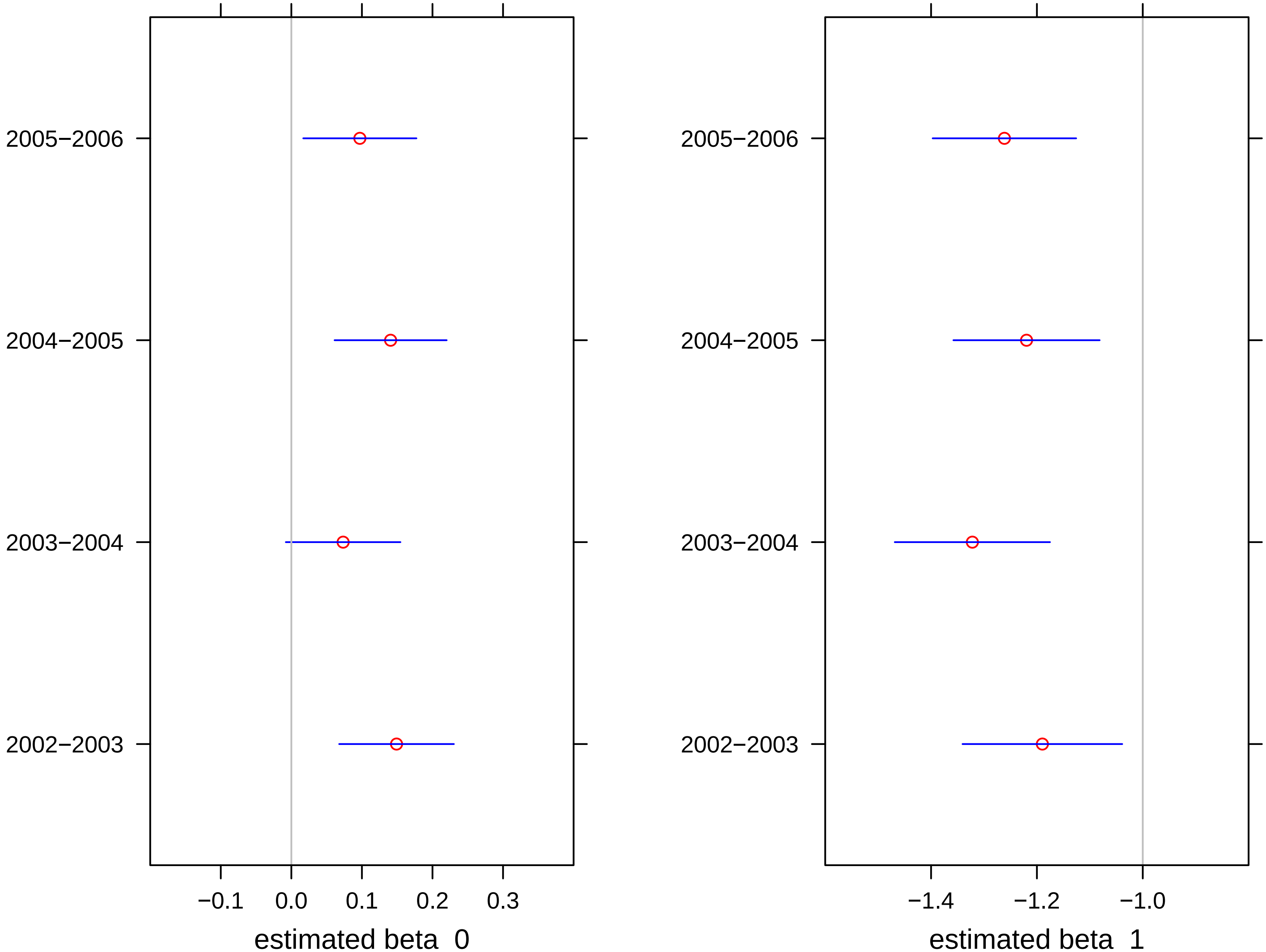
```
lrm(formula = ftr == "H" ~ log(av.odds.h - 1), data = soccer.data3)


Frequencies of Responses
FALSE  TRUE
 8540  7597

      Obs           C          Dxy        Gamma        R2        Brier
    16137       0.698        0.397        0.398     0.165        0.218

          Coef    S.E.    Wald Z P
Intercept -0.1640 0.01689  -9.71 0
av.odds.h -1.0838 0.02634 -41.15 0
```

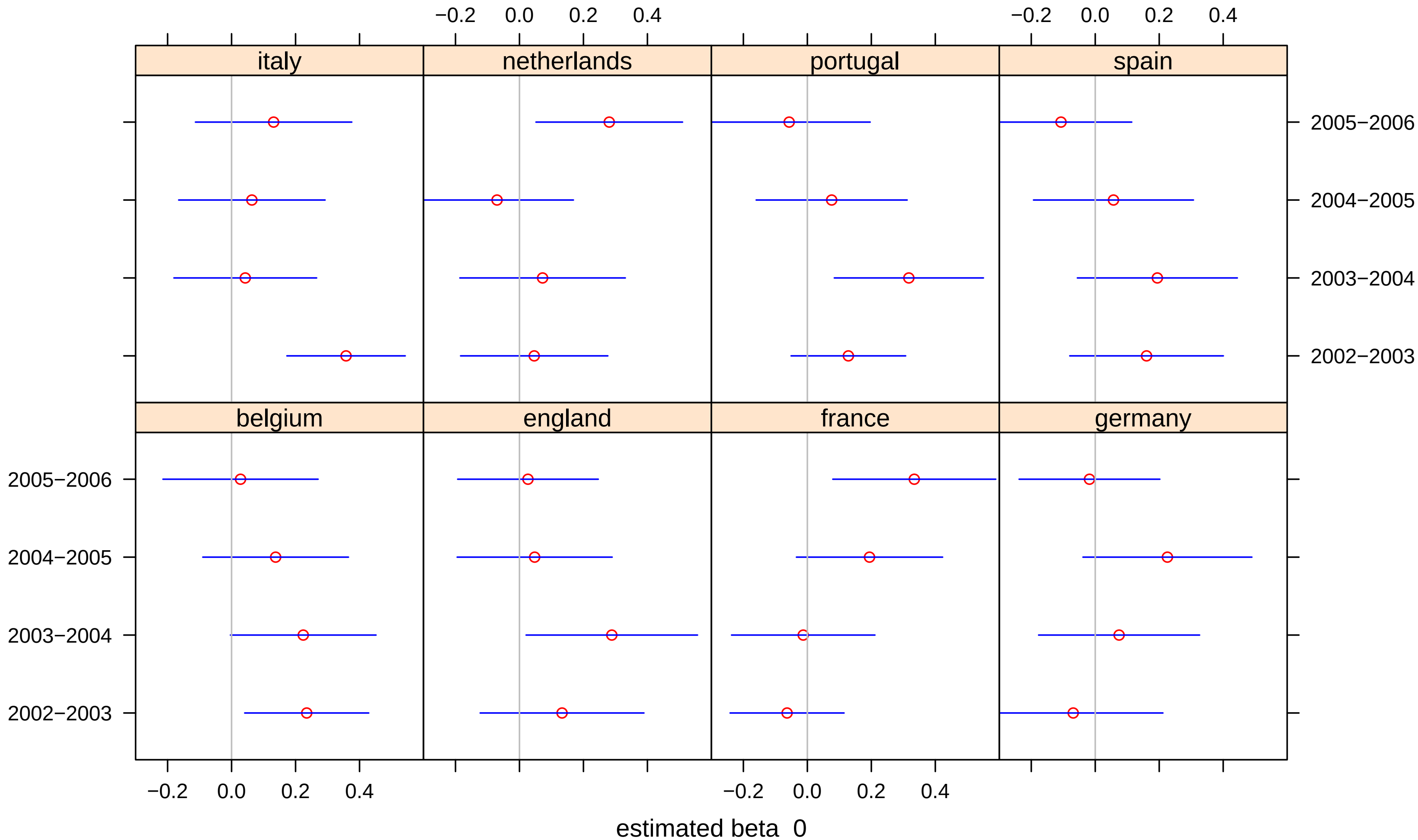

home win odds
away win odds
spain
turkey
greece
italy
netherlands
portugal
belgium
england
france
germany
true
false
0.2
0.4
0.6
0.8
Outcome
Standardized price

Win probability and odds (4)

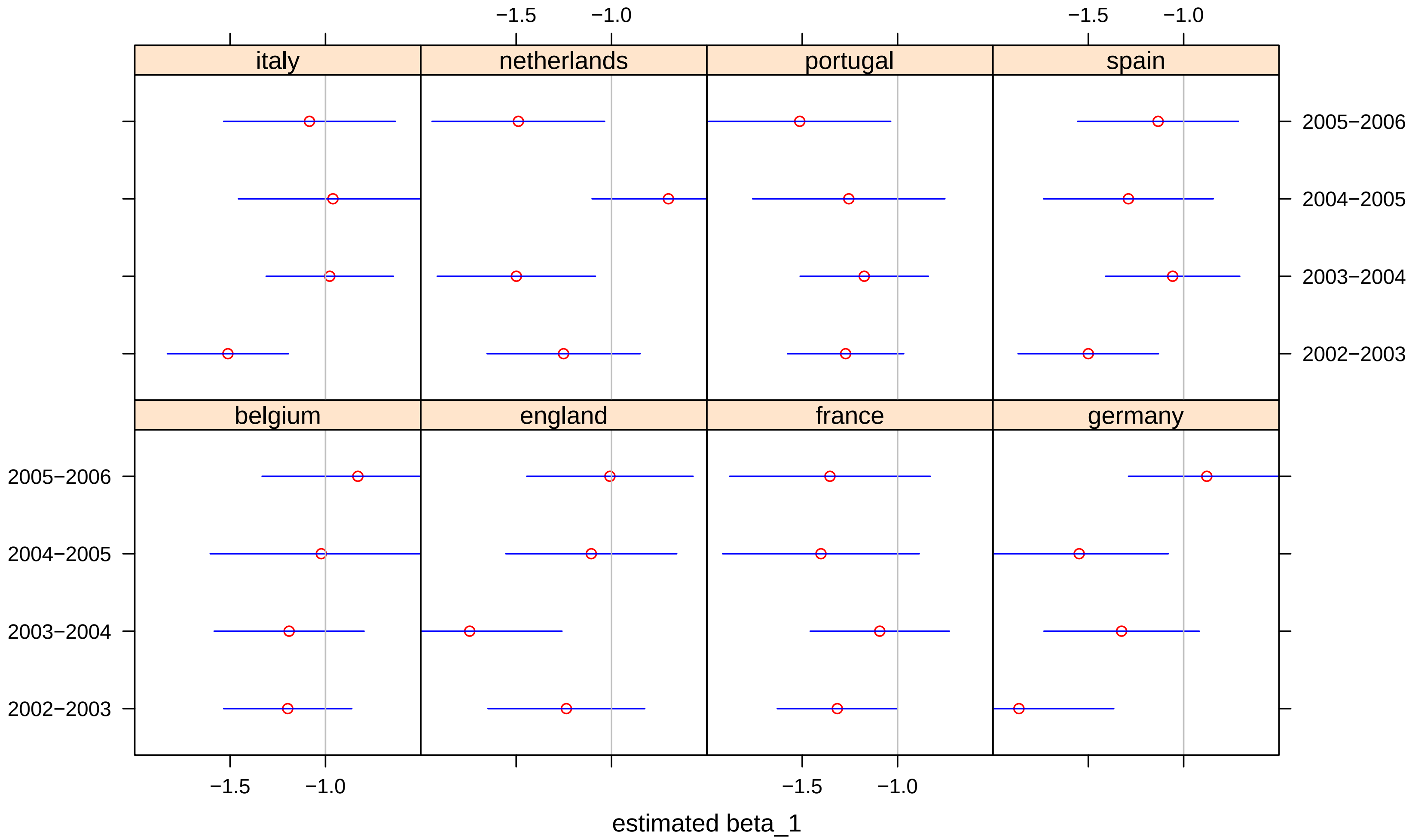
```
lrm(formula = ftr == "H" ~ log(b365h.f - 1), data = england)

Frequencies of Responses
FALSE  TRUE
  801   719

             Coef    S.E.    Wald Z P
Intercept  0.1399 0.05766   2.43 0.0153
b365h.f   -1.1019 0.09020 -12.22 0.0000
```


spain
portugal
netherlands
italy
germany
france
england
belgium
−0.1
0.0
0.1
0.2
0.3
estimated beta_0
spain
portugal
netherlands
italy
germany
france
england
belgium
−1.4
−1.2
−1.0
estimated beta_1


2005−2006
2004−2005
2003−2004
2002−2003
−0.1
0.0
0.1
0.2
0.3
estimated beta_0
−1.4
−1.2
−1.0
estimated beta_1



italy
netherlands
portugal
spain
belgium
england
france
germany
2005−2006
2004−2005
2003−2004
2002−2003
−0.2
0.0
0.2
0.4
estimated beta_0



italy
netherlands
portugal
spain
belgium
england
france
germany
2005−2006
2004−2005
2003−2004
2002−2003
−1.5
−1.0
estimated beta_1

Additional variables

- Financial variables

- Match characteristics
  - points
  - goals scored
  - moment in season
  - promotion/relegation last season
  - position

Tennis (1)

- Men's tennis: 2002-2007, women's tennis: 2007

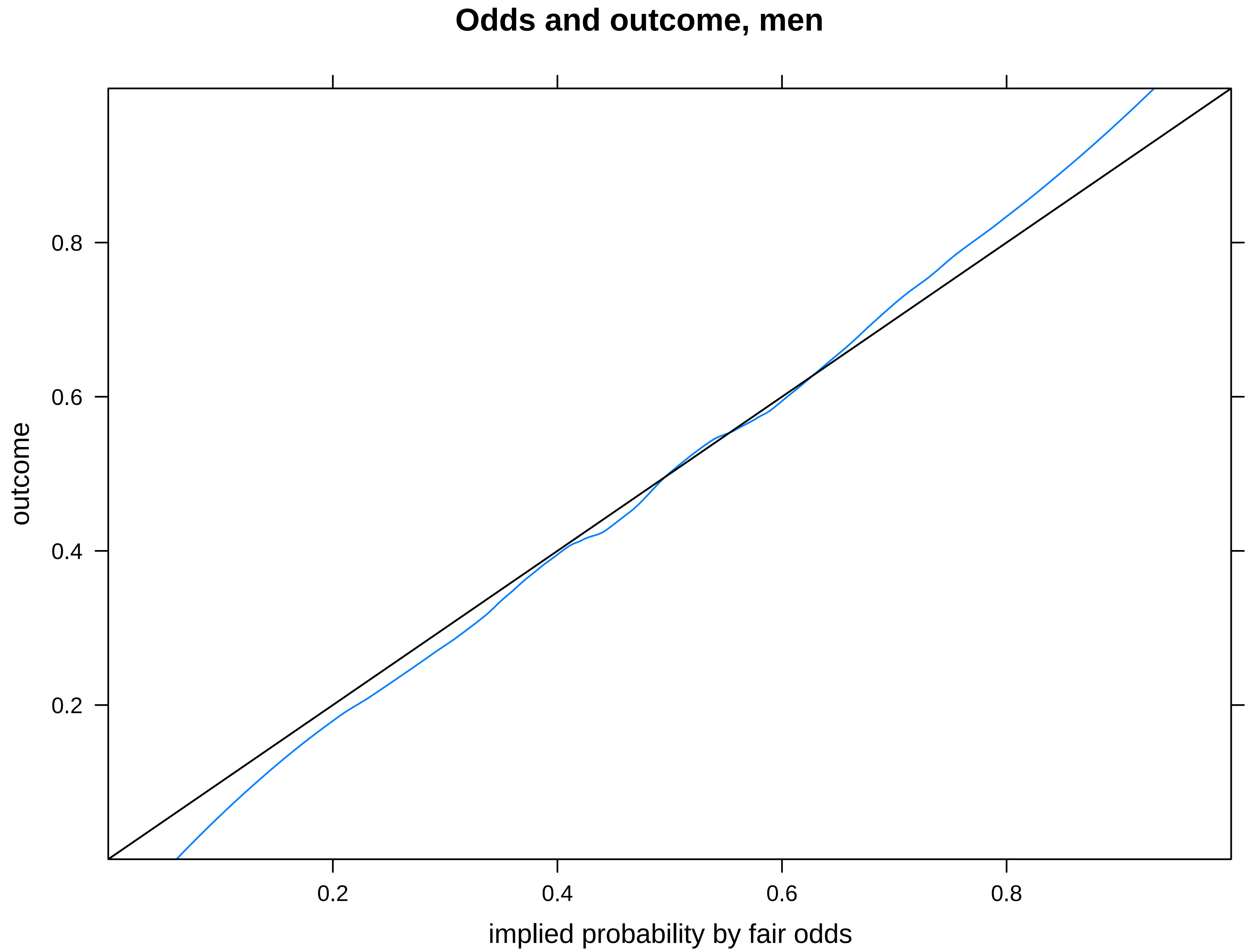
- Ranking is important

- Other information: court (indoor/outdoor), surface, ranking, round, best-of-three or best-of-five

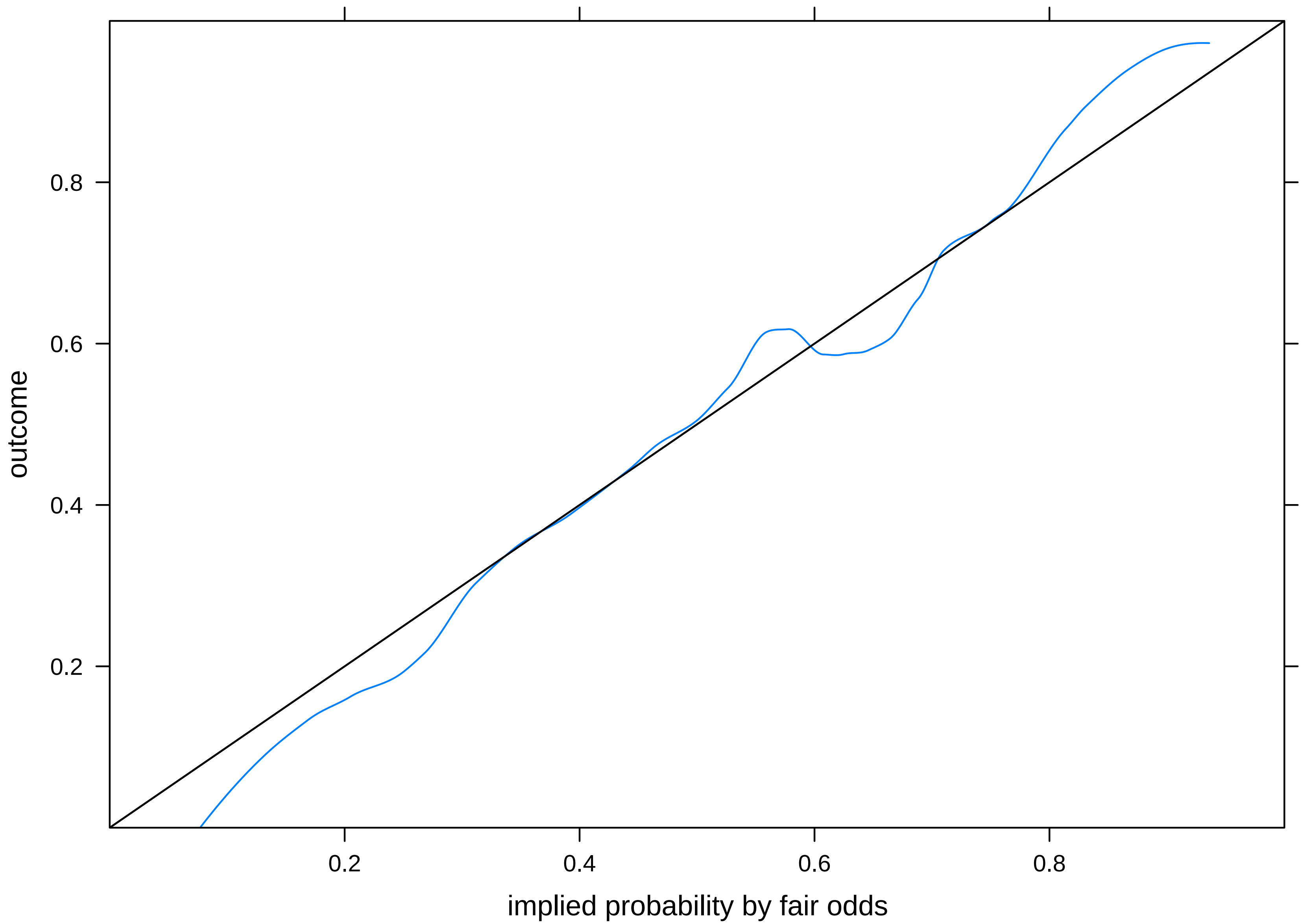
- $\Lambda(\beta' x) = 1 - \Lambda(-\beta' x)$

Tennis (2)

```
lrm(formula = Result ~ log(B365.F - 1), data = men.tennis)

          Coef      S.E.    Wald Z P
Intercept -0.005686 0.02060  -0.28 0.7825
B365.F    -1.147360 0.02550 -44.99 0.0000

lrm(formula = Result ~ log(B365.F - 1), data = women.tennis)
          Coef     S.E.    Wald Z P
Intercept -0.01855 0.06717  -0.28 0.7824
B365.F    -1.20002 0.07460 -16.09 0.0000
```

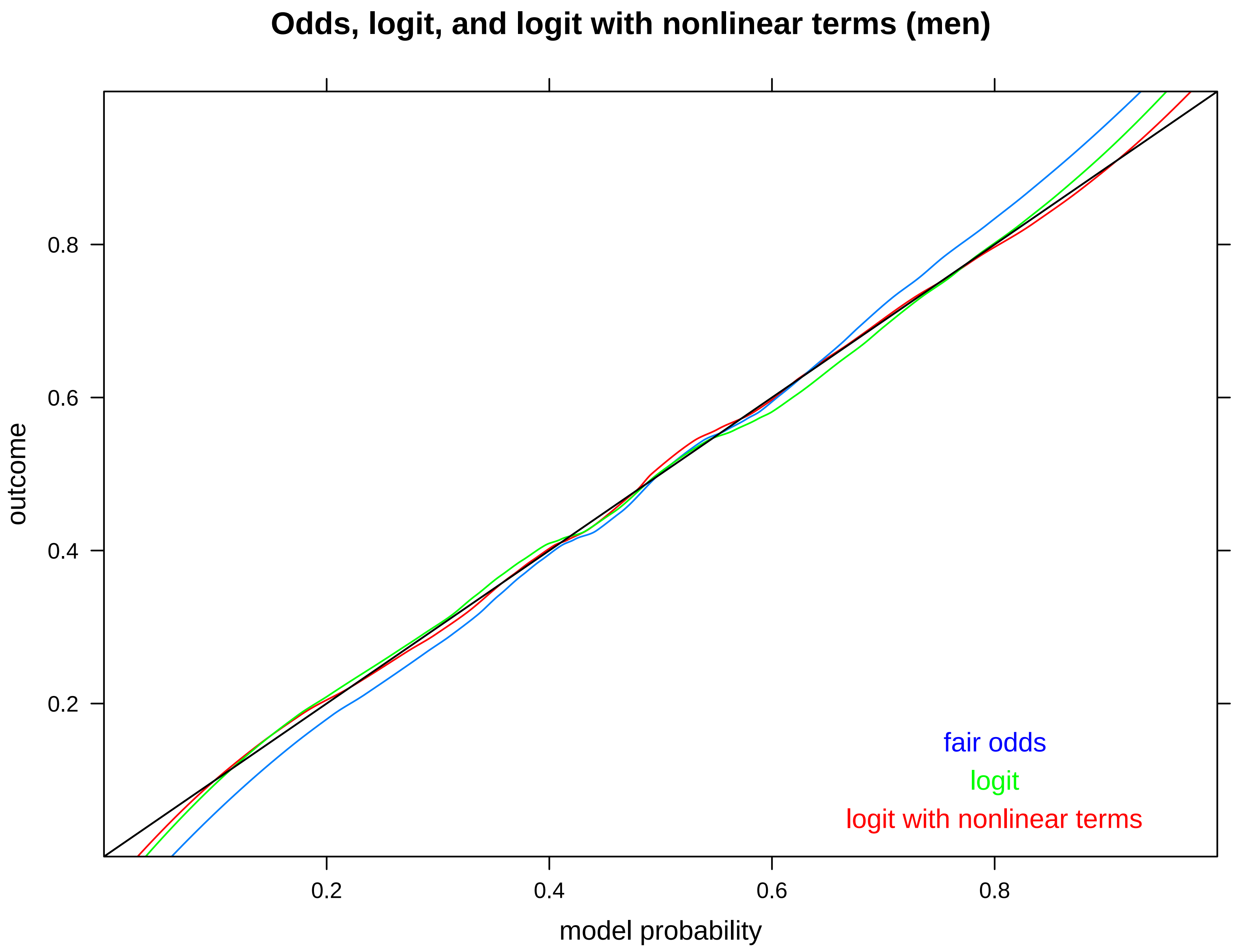

Odds and outcome, men
outcome
0.2
0.4
0.6
0.8
0.2
0.4
0.6
0.8
implied probability by fair odds

Odds and outcome, women



Tennis (3)

```
> anova(lrm(Result~rcs(log(B365.F-1),4),data=men.tennis))
                Wald Statistics          Response: Result

 Factor     Chi-Square d.f. P
 B365.F     1952.22    3    <.0001
  Nonlinear    8.45    2    0.0146
 TOTAL      1952.22    3    <.0001
```

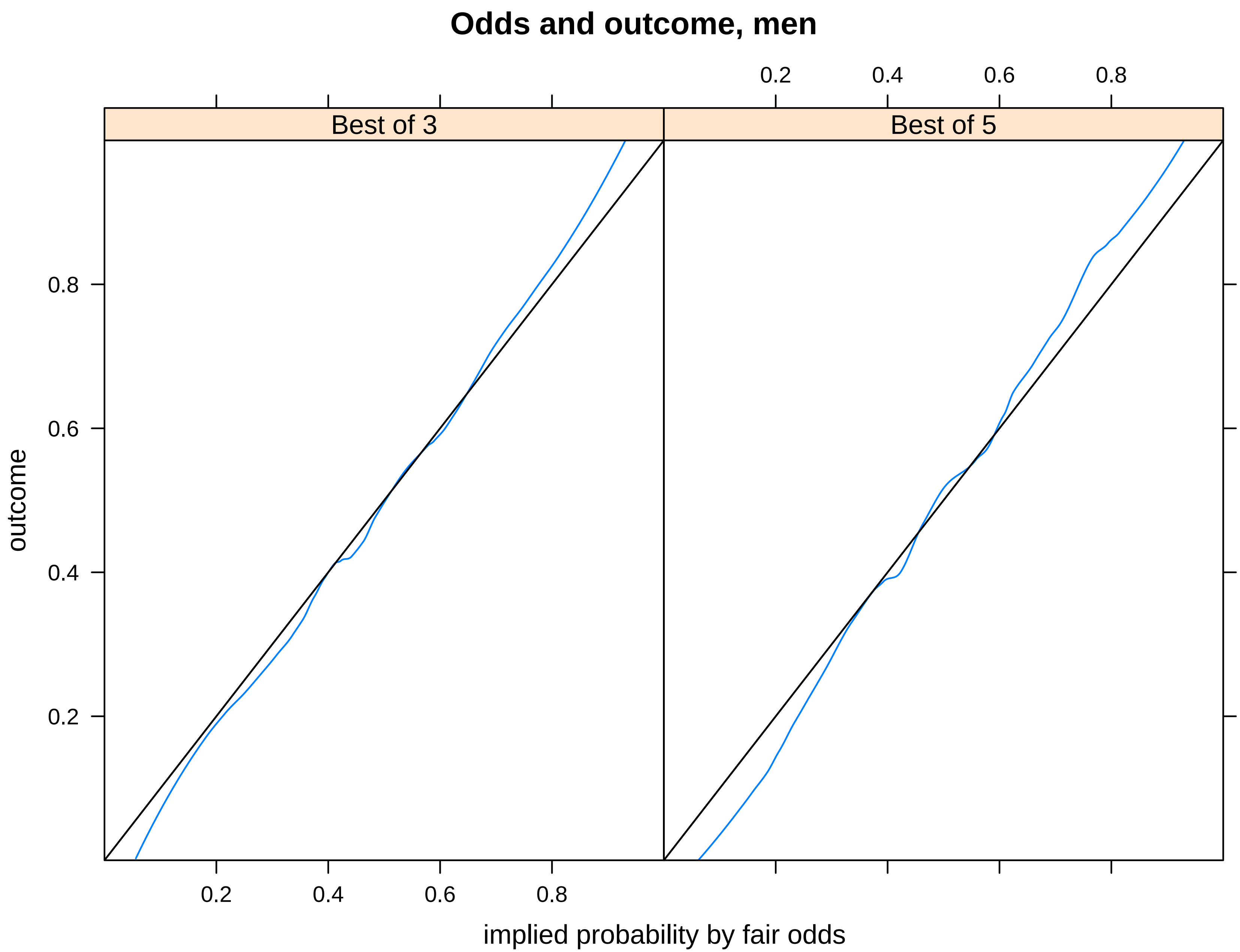


Odds, logit, and logit with nonlinear terms (men)
0.2
0.4
0.6
0.8
outcome
model probability
fair odds
logit
logit with nonlinear terms

Tennis (4)

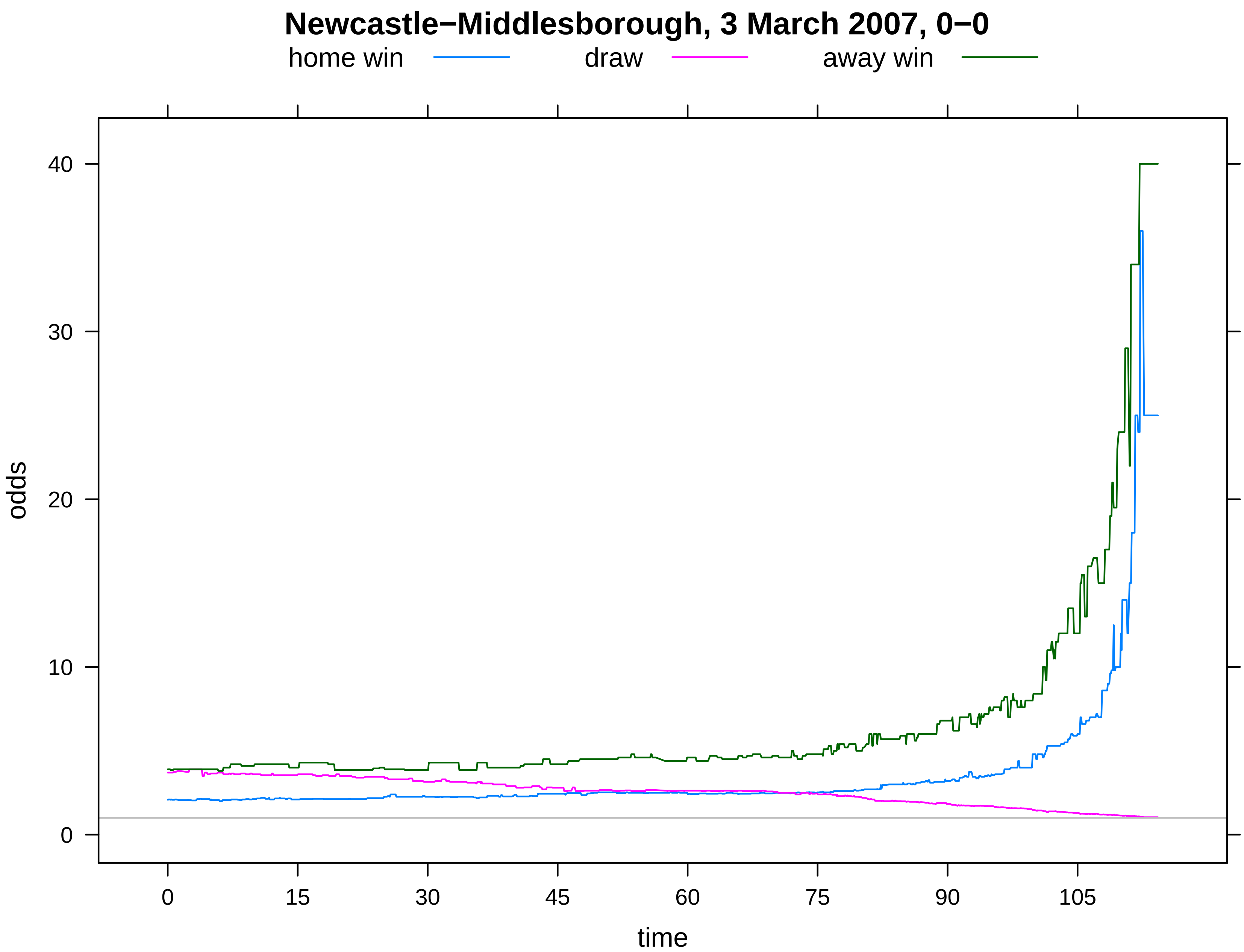
Court: not significant Rank: not significant

```
> anova(lrm(Result~Best.of*log(B365.F-1),data=men.tennis))
                Wald Statistics          Response: Result

 Factor                               Chi-Square d.f. P
 Best.of  (Factor+Higher                    8.87    2    0.0119
  All Interactions                          8.56    1    0.0034
 B365.F  (Factor+Higher                  2004.80    2    <.0001
  All Interactions                          8.56    1    0.0034
 Best.of * B365.F  (Factor                  8.56    1    0.0034
 TOTAL                                   2004.83    3    <.0001
```


Odds and outcome, men
Best of 3
Best of 5
0.2
0.4
0.6
0.8
outcome
implied probability by fair odds

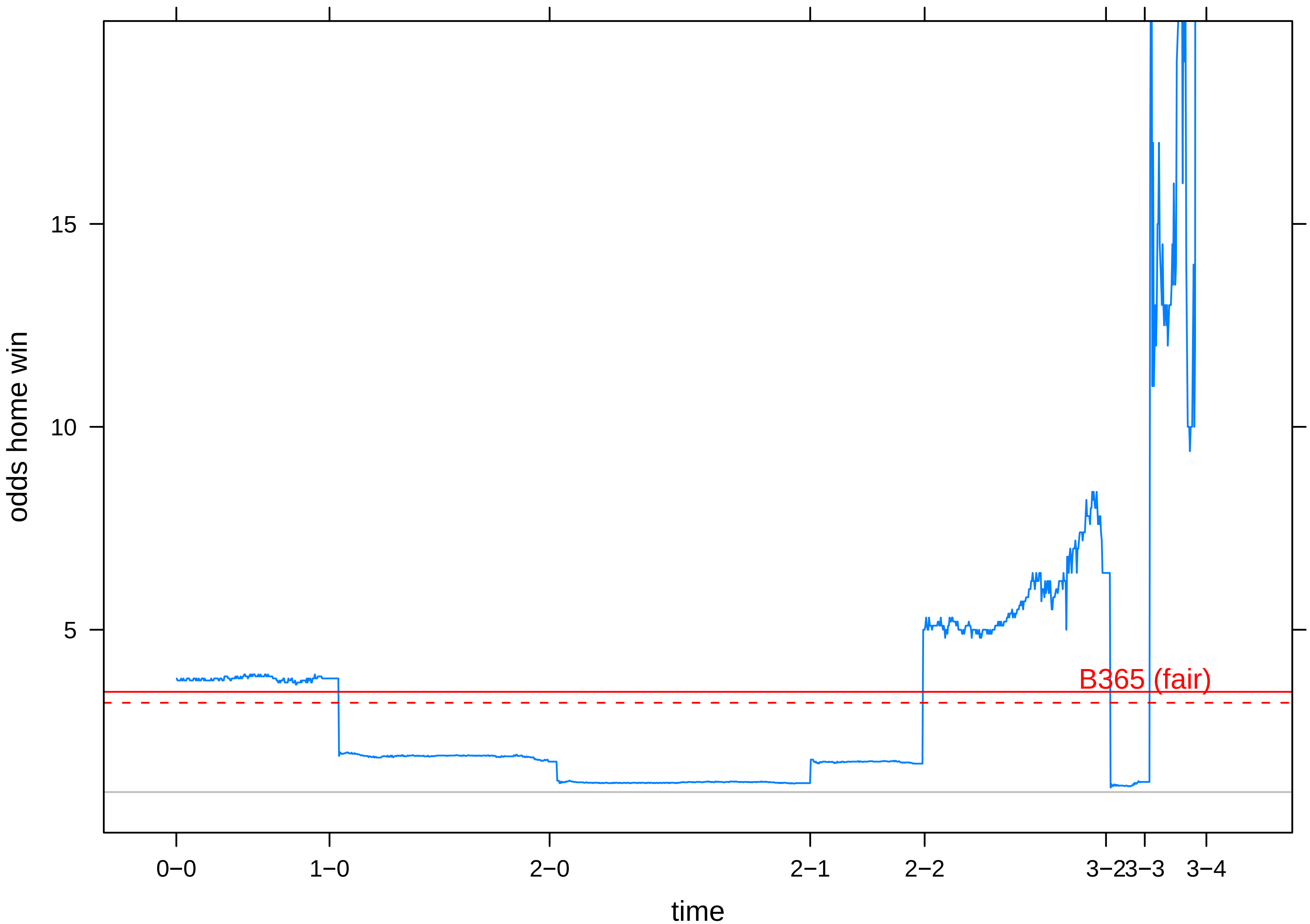
Betting exchange (1)

- Continuous trading
- 1% commission
- 1 minute trading stop
- data set



Newcastle−Middlesborough, 3 March 2007, 0−0
home win
draw
away win
odds
time
0
10
20
30
40
0
15
30
45
60
75
90
105

West Ham United−Tottenham Hotspur, 4 March 2007, 3−4



Information efficiency (2)

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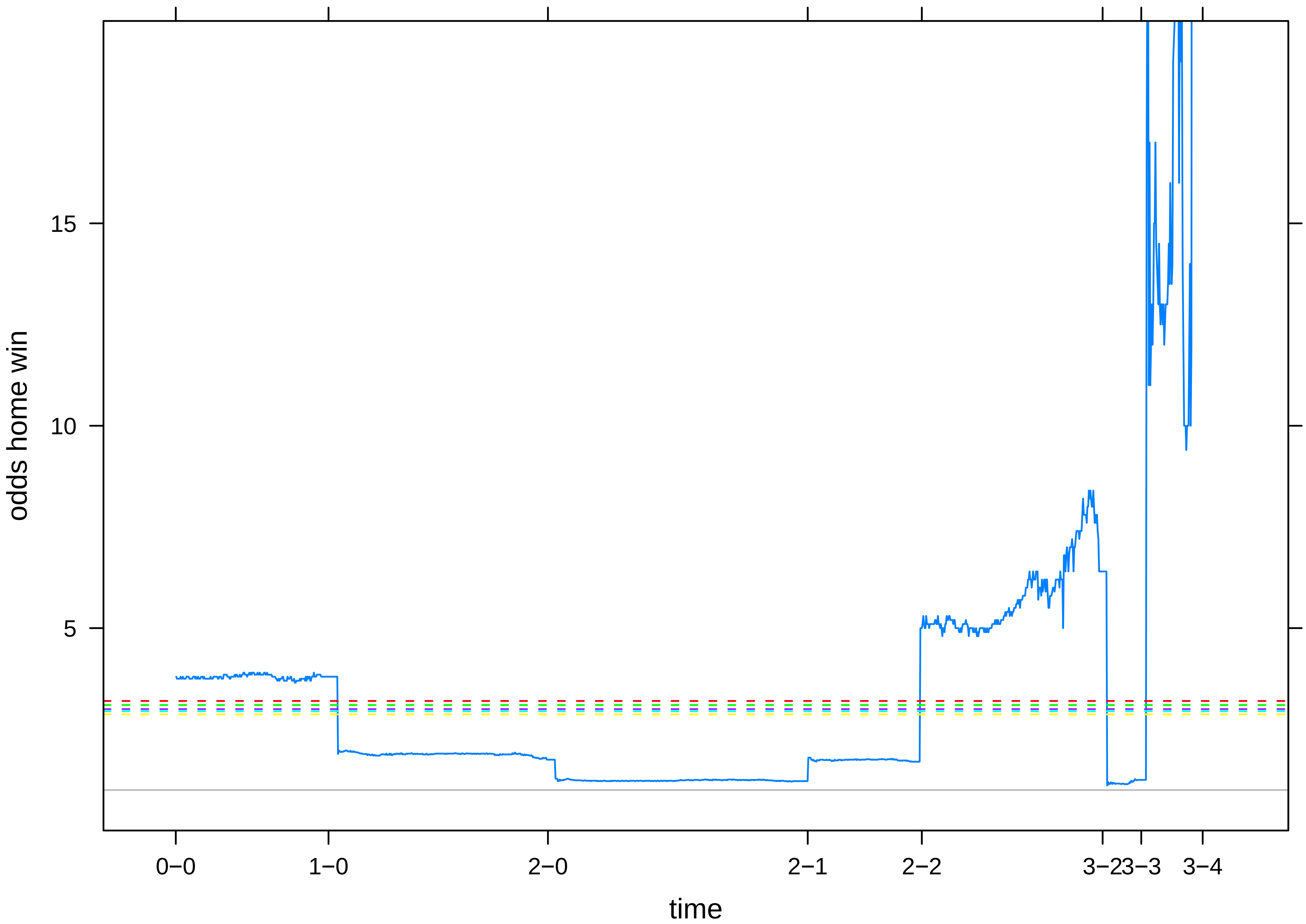
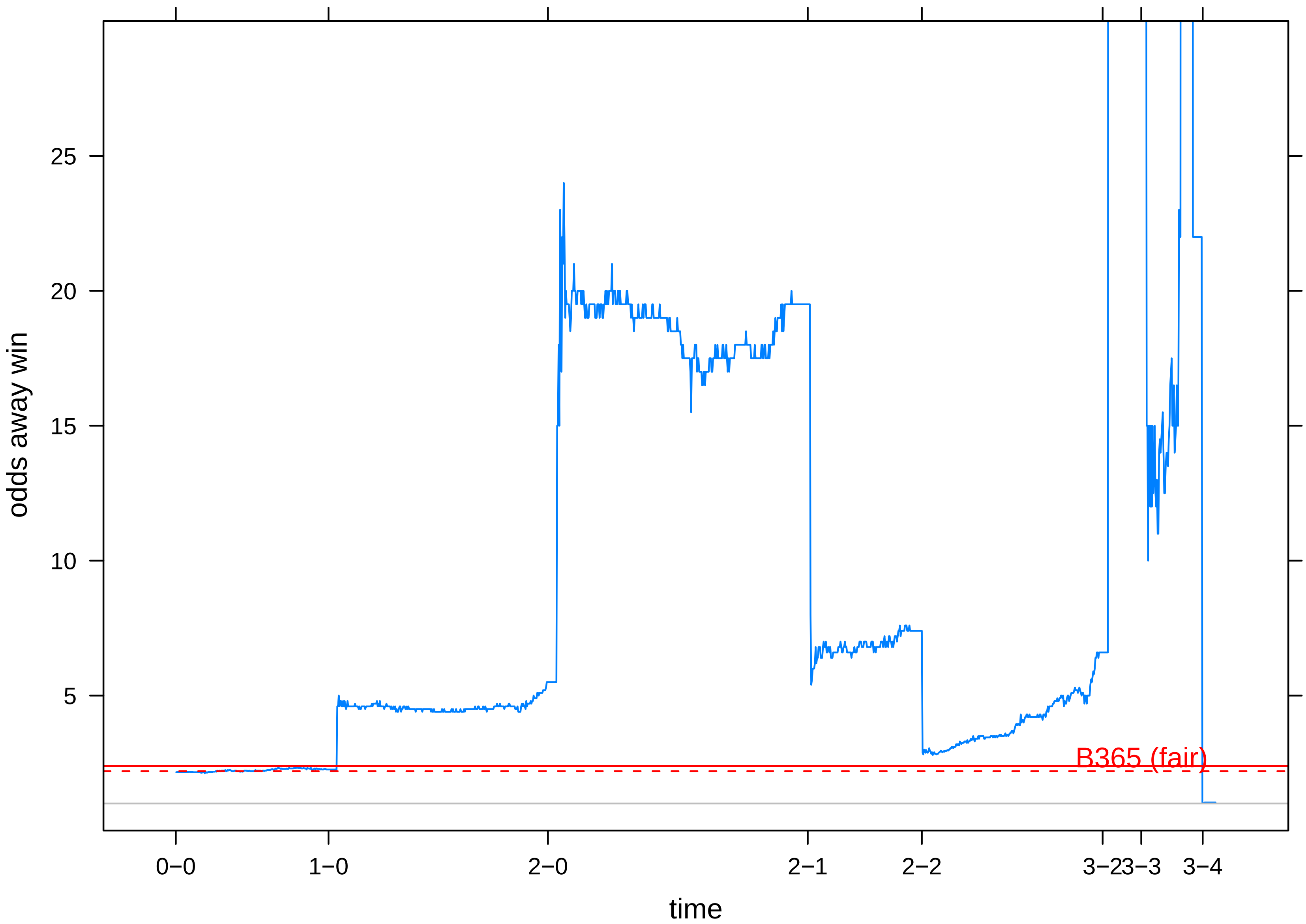
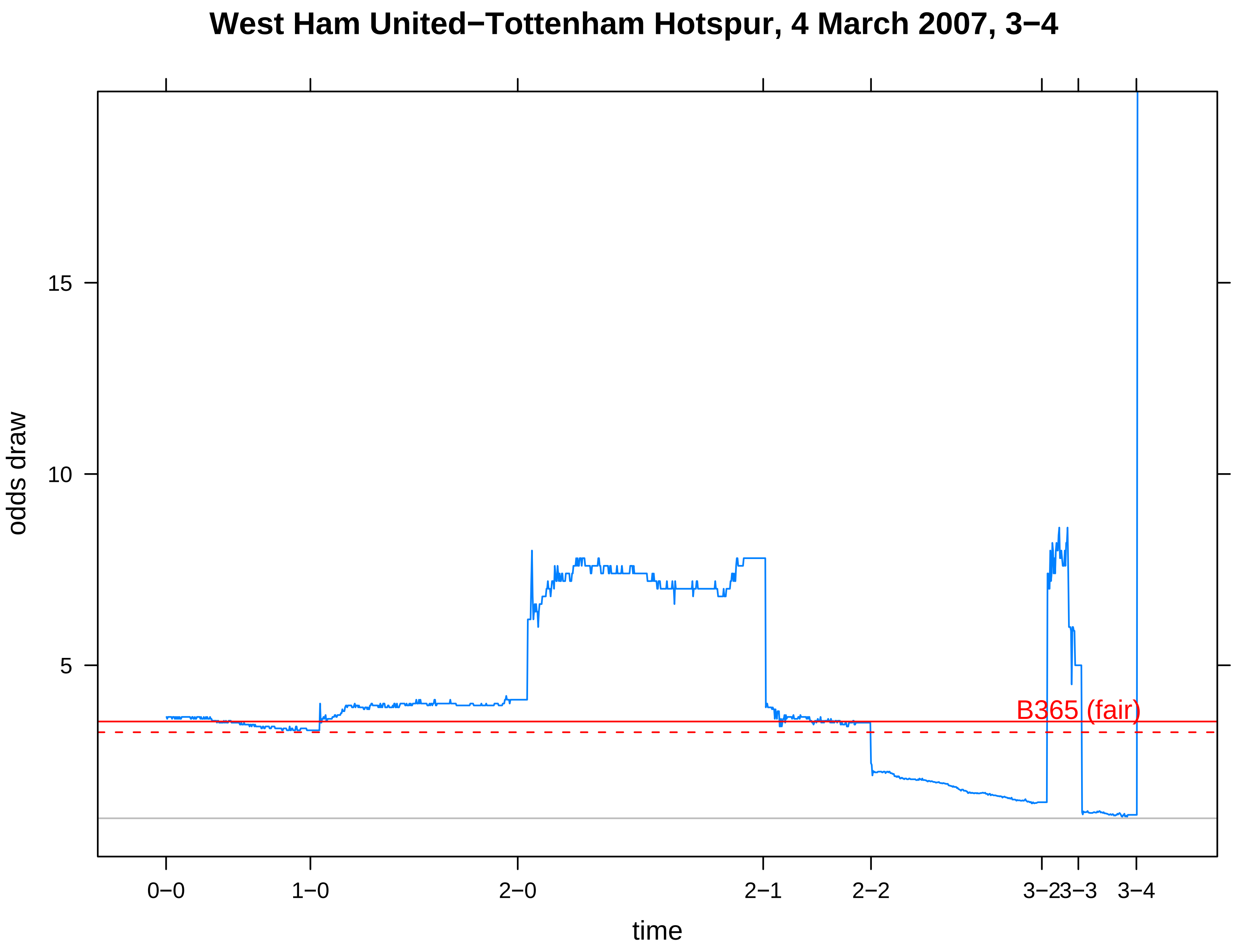
West Ham United−Tottenham Hotspur, 4 March 2007, 3−4



West Ham United−Tottenham Hotspur, 4 March 2007, 3−4




West Ham United−Tottenham Hotspur, 4 March 2007, 3−4
odds draw
15
10
5
B365 (fair)
0−0
1−0
2−0
2−1
2−2
3−2
3−3
3−4
time

Betting exchange (2)

First minute odds:

```
lrm(formula = team1w ~ log(team1lpm - 1), data =
 be.data.inplay[min1,])

Frequencies of Responses
 0  1
56 49

       C
      0.738

          Coef     S.E.   Wald Z P
Intercept  0.05319 0.2205  0.24  0.8094
team1lpm  -1.17135 0.3041 -3.85  0.0001
```

Betting exchange (3) 46th minute odds:

```
lrm(formula = team1w ~ log(team1lpm - 1), data =
 be.data.inplay[min46,])

Frequencies of Responses
 0  1
56 49
       Obs          C
       105      0.885

          Coef   S.E.   Wald Z P
Intercept -0.086 0.2627 -0.33  0.7434
team1lpm  -1.337 0.2729 -4.90  0.0000
```

Betting exchange (4)

After first goal in game:

```
lrm(formula = team1.wins ~ log(av.new.odds - 1), data = goals[e,
    ])
            Coef    S.E.   Wald Z P
Intercept   -0.1927 0.2106 -0.91  0.3602
av.new.odds -0.9907 0.1425 -6.95  0.0000

lrm(formula = team1.wins ~ log(first.new.odds - 1), data = goals[e,
    ])
               Coef    S.E.   Wald Z P
Intercept      -0.2079 0.2051 -1.01  0.3108
first.new.odds -0.9541 0.1389 -6.87  0.0000
```

Betting exchange (5)

- Interaction with goals being scored by home team or away team are not significant

- Not enough information to assess effect of red card (Ridder et al, 1991)

- Trading volumes

Conclusions

- Logit based tests are useful
- Not all markets efficient
- Increasing efficiency over time?
- Extensions of efficiency research to ‘smaller’ markets useful
- Betting exchanges offer new insights