Scattering in strong backgrounds and why you should care

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work with Bogna, Casali, Cristofoli, Ilderton, Klisch, MacLeod, Mason, Nekovar, Sharma & Tourkine

Scattering amplitudes 101

How do we usually compute *scattering amplitudes*?

Perturbation theory (few exceptions)

- QFT: turn Feynman diagram crank
- String theory: worldsheet correlators of vertex operators

Additional (almost implicit) assumption:

- expanding around a *trivial* field configuration

Strong-field scattering

Suppose we consider scattering in a *non-trivial* (asymp. flat) field configuration:

- Background a fixed solution to classical (non-linear) equations of motion
- Treated *non-perturbatively* $\leftrightarrow$ 'strong' background field
- $\Rightarrow$ use background field theory [Furry, DeWitt, 't Hooft, Abbott,...]
- Scattering quantum perturbations on strong background encodes back-reaction/depletion effects

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- Non-linear regime of QED; high-energy, heavy ion collisions; black holes/gravitational waves

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However, strong-field scattering is a **hard** problem

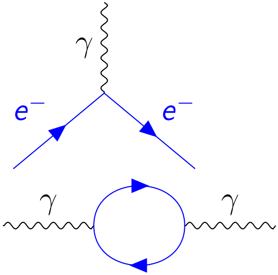
- Background-coupled Feynman rules a nightmare, String worldsheet CFT no longer free
- Functional d.o.f. in background $\rightarrow$ no rational functions
- Non-pert. effects: memory, tails
- S-matrix may not exist (e.g., black hole backgrounds)

...but still interesting

Non-pert. backgrounds induce new physics!

E.g., single photon emission
or photon helicity flip in strong-field QED

- underpin detection targets at current/upcoming experiments (ELI, FACET-II, LUXE)



State-of-the-art

Despite study for ~ 100 years, precision frontiers of strong-field QFT are low:

- QED in plane wave background $\rightarrow$ 4-point tree [Baier-Katkov-Strakhovenko, Ritus,...] , 2-point 1-loop [Toll, Ritus]
- QCD in plane wave background $\rightarrow$ 4-point tree [TA-Casali-Mason-Nekovar] , 2-point 1-loop [TA-Ilderton]
- GR in plane wave background $\rightarrow$ 3-point tree [TA-Casali-Mason-Nekovar]

Roughly LO/NLO precision around background

Stark contrast

...with $N^{\infty}LO$ information in a trivial background:

all-multiplicity tree- and loop-level formulae for gluon/graviton scattering

[Parke-Taylor, Witten, Roiban-Spradlin-Volovich, Hodges, Cachazo-Skinner, Cachazo-He-Yuan, TA-Casali-Skinner, Geyer-Mason-Monteiro-Tourkine,...]

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A countably **infinite** precision gap in even the simplest strong backgrounds!

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Of course not!

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Try to convince you that:

- strong-field scattering encodes wealth of physical information
  - all-order data for classical physical observables

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Try to convince you that:

- strong-field scattering encodes wealth of physical information
  - ▷ all-order data for classical physical observables
- in some (non-constant!) backgrounds, can be computed to *arbitrary multiplicity*
  - ▷ using string-theoretic methods

Basics

What exactly do we mean by a strong-field amplitudes?

Denote fields by $\mathcal{F}$, classical action $S[\mathcal{F}]$

- let Φ be exact solution to classical e.o.m.s – the *background*.
- evaluate action on $S[\Phi + \phi]$, discard all terms less than $O(\phi^2)$

$\rightarrow$ obtain background field action $S[\Phi; \phi]$

governs fluctuations ϕ on background Φ

Tree-level strong-field amplitudes: $\{\phi_1, \ldots, \phi_n\}$ solutions to free, background-coupled eqs with appropriate bndry conds. Define:

$$\varphi_0^{[n]} := \sum_{i=1}^{n} \varepsilon_i \, \phi_i$$

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Strong-field, *n*-point tree amplitude:

$$\mathcal{M}_n^{(0)} := \left. \frac{\delta^n S\left[\Phi; \varphi^{[n]}_{\max\{0,n-3\}}\right]}{\delta\varepsilon_1 \cdots \delta\varepsilon_n} \right|_{\varepsilon_1 = \cdots = \varepsilon_n = 0}$$

Upshot

Strong-field amps = multi-linear piece of background field action

[Schwinger, Boulware-Brown, Arafeva-Faddeev-Slavnov, Abbott-Grisaru-Schaefer, Jevicki-Lee, Rosly-Selivanov, Costello]

- 'perturbiner' definition extremely robust
- coincides w/ S-matrix when it exists
- when it doesn't, still encodes expected dynamical content of scattering
- higher loops: use ℓ-loop effective action

What does it mean to compute a strong-field amp?

In general, amplitudes look like:

$$\mathcal{M}_n^{(0)} = \int \underbrace{\mathrm{d}\mu_n}_{\text{measure}} \; \overbrace{\mathcal{I}_n^{(0)}}^{\text{integrand}} \; \underbrace{\mathcal{V}_n}_{\text{wavefunctions}}$$

- in trivial background, integral gives momentum conservation
- in general strong fields, *cannot* perform integrals analytically
- 'compute strong-field amp' $\rightsquigarrow$ determine $\mathrm{d}\mu_n$, $\mathcal{I}_n^{(0)}$, $\mathcal{V}_n$ analytically

Example:

Photon emission in plane wave ('non-linear Compton scattering')

$$A^{\mathrm{bac}} = -a_{\perp}(x^{-})\, x^{\perp}\, \mathrm{d}x^{-}\,, \qquad a_{\infty} := \int_{-\infty}^{+\infty} \mathrm{d}x^{-}\, a_{\perp}(x^{-})$$

$$\mathcal{M}_3^{(0)}(p \to p' + k) = e\, \delta^3_{+,\perp}(p' + k - p + e\, a_{\infty}) \int\limits_{-\infty}^{+\infty} \mathrm{d}x^{-}$$

$$\times\ \epsilon(k) \cdot P(x^{-}) \exp\left[\mathrm{i} \int^{x^{-}} \mathrm{d}s\, \frac{k \cdot P(s)}{(p-k)_+}\right],$$

for $P_{\mu} := p_{\mu} - e\, \delta^{\perp}_{\mu}\, a_{\perp} + \frac{\delta^{-}_{\mu}}{2p_{+}}(2ep \cdot a - e^2\, a^2)$

All-order physics

Non-perturbative background $\rightarrow$ *infinite* order in coupling when expanded

Even at low precision/multiplicity!

Example

Eikonal approximation: resummation of small angle $2 \to 2$ scattering

$$\mathcal{M}_{\text{eik}} \leftrightarrow \mathcal{M}_2^{(0)} \text{ in strong background sourced by the other particle at large impact parameter}$$

- Old idea: formulated for special cases long ago ['t Hooft, Amati-Ciafaloni-Veneziano, Jackiw-Kabat-Ortiz, Kabat-Ortiz]
- Now fully covariant, for all stationary backgrounds [TA-Cristofoli-Tourkine]
  - reproduces known eik. amps, detects cases which fail, computes *new* $\mathcal{M}_{\text{eik}}$
  - holomorphic/stringy factorization needed to evaluate integrals

Surprising application

What is the massless (ultraboosted) limit of the Kerr metric?

Lack of clarity in literature, many contradictory claims

[Ferrari-Pendenza, Balasin-Nachbagauer, Griffiths-Podolsky, Barrabes-Hogan, Frolov-Israel-Zelnikov,...]

More precisely...

Is there an *interesting* (i.e., long-range spin effects) massless limit of Kerr in the class

$$\mathrm{d}s^2 = \mathrm{d}s^2_{\mathbb{M}} + G\,\delta(x^-)\,f(x^\perp)\,(\mathrm{d}x^-)^2?$$

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Spoiler alert: No...but there is one with interesting finite-size effects at short distances

Relation between $\mathcal{M}_2^{(0)}$ and $\mathcal{M}_{\rm eik}$ implies

$$f(x^{\perp}) \sim \int \mathrm{d}^2 q_{\perp}\, \mathrm{e}^{\mathrm{i}\, q_{\perp}\, x^{\perp}}\, M_4^{(0)}$$

$M_4^{(0)}$ Born scattering between scalar probe and metric source

This gives a two way street:

1. Pick a metric, read off 4-point amplitude, see if it makes sense
2. Pick a 4-point amplitude, look at associated metric

Result

Ultraboosting Kerr metric directly $\rightarrow$ spin effects vanish

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Ultraboost *source* of Kerr [Israel, Balasin-Nachbagauer]

$$f(x^\perp) = 8\,\log(\mu\, r) - 4\,\Theta(a-r)\left[2\,\log\left(\frac{r}{a+\sqrt{a^2-r^2}}\right) + \frac{\sqrt{a^2-r^2}}{a}\right]$$

No spin effects at large impact parameter...

but *incredibly* simple Born amplitude:

$$G\,\frac{s^2}{8\,t}\left(\frac{\sin(a\sqrt{-t})}{a\sqrt{-t}} + \cos(a\sqrt{-t})\right)$$

Classical observables

More generally...

Direct relationship between scattering amps. and classical observables (scattering angle, waveform) in trivial background

[Amati-Ciafaloni-Veneziano, Kosower-O'Connell-Maybee,..., Lee-Lee-Mazumdar]

Example: LO classical impulse in 2-body scattering

$$\Delta p^\mu \sim \lim_{\hbar \to 0} \hbar^2 \int \mathrm{d}^4\bar{q}\, \delta(\bar{q}\cdot p)\, \delta(\bar{q}\cdot P) \\ \times \mathrm{e}^{-\mathrm{i}\, b\cdot\bar{q}}\, \bar{q}^\mu\, \bar{M}_4^{(0)}(p, P, p+\hbar\bar{q}, P-\hbar\bar{q})$$

Simple idea

Also true for observables/scattering *in strong backgrounds*

[TA-Cristofoli-Ilderton]

Classical observables for probe particle $\leftrightarrow$ self-force expansion

[Cutler-Kennefick-Poisson, Barack, Poisson-Pound-Vega, Harte-Taylor-Flanagan,...]

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Basic ingredients: massive particle states

$$|\Psi\rangle = \int \mathrm{d}\Phi(p)\, \phi(p)\, \mathrm{e}^{\mathrm{i}\, p\cdot b/\hbar}\, |p\rangle$$

$\mathrm{d}\Phi(p)$ Lorentz-invariant on-shell measure, $\phi(p)$ wavepacket with classical limit

time-evolution through strong-field S-matrix

Waveform

First 'self-force' contribution to classical waveform at $\mathscr{I}^+$:

[TA-Cristofoli-Ilderton-Klisch]

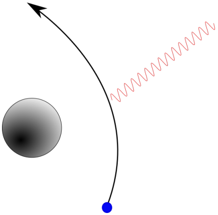
$$W_{\mu\nu\rho\sigma}(u, z, \bar{z}) = -\frac{\kappa}{2\pi^2\sqrt{\hbar}} \,\mathrm{Im} \int_0^\infty \mathrm{d}\omega \, \mathrm{e}^{-\mathrm{i}\omega u} \, k_{[\mu}\varepsilon_{\nu]} \, k_{[\sigma}\varepsilon_{\rho]} \, \alpha(k)$$

for

$$\alpha(k) := \int \mathrm{d}\Phi(p') \, \langle \Psi | \mathcal{S}^\dagger | p' \rangle \, \langle p', k | \mathcal{S} | \Psi \rangle$$

Upshot

1SF waveform controlled by tree-level 2-point and 3-point amps on background



Physically interesting!

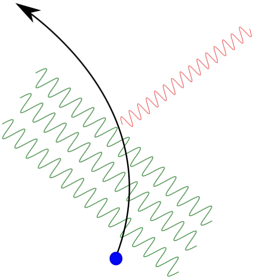
- for BH background, leading waveform for 2-body problem in probe limit
- provides access to high-precision post-Minkowskian (PM) results [Damour]

Plane wave background

Computing 3-point amp. in BH hard (not impossible!)
Instead use *plane wave* spacetimes:

$$\mathrm{d}s^2 = 2\mathrm{d}x^- \, \mathrm{d}x^+ - \mathrm{d}x^a \, \mathrm{d}x^a - H_{ab}(x^-) \, x^a x^b \, (\mathrm{d}x^-)^2$$

$H_{ab}(x^-)$ traceless and compactly supported



Still very interesting

- self-force in radiative grav. field
- classical waveforms not known
- approximates/constrains result in *any* spacetime via Penrose limits [TA-Cristofoli-Ilderton-Klisch, to appear]
- we already know the required amps! [TA-Ilderton]

Result

Schematically,

$$W_{\mu\nu\rho\sigma}(u, z, \bar{z}) = -\frac{\kappa^2}{\pi}\, \hat{x}_{[\mu}\, \hat{x}_{[\sigma} \int_{-\infty}^{\infty} \mathrm{d}y\, \delta(u - \hat{x} \cdot X(y))\, \mathcal{T}_{\rho]\nu]}(\hat{x}, y)$$

$\hat{x}^\mu = (1, \hat{\mathbf{x}})$, $X^\mu(y) =$ geodesic orbit, $\mathcal{T}_{\rho\nu}$ determined by $\mathcal{M}_3^{(0)}$

All-orders in background (i.e., in κ under $H_{ab} \to \kappa\, H_{ab}$)

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Specific example: $H_{ab} = \kappa\,\delta(x^-)\,\mathrm{diag}(\lambda,-\lambda)$

$$\boxed{W_{+1+1}(u,\theta=\pi) = -\frac{\kappa^2\,p_+}{4\pi^2\sqrt{8}}\,\frac{\partial^2}{\partial u^2}\left(\frac{\nu\log(\nu+\sqrt{\nu^2-1})}{\sqrt{\nu^2-1}}\right)}$$

for $\nu := \kappa\,\lambda\,\sqrt{2\frac{p_+^2}{m^2}}\,|u|$

So what?

Could never hope to reproduce this result in a trivial background

$\rightarrow$ an *infinite* amount of PM information!

- only requires ‘classical’ part of amplitude
- similar story for other classical observables in PW backgrounds
- possible to attack the BH problem directly...stay tuned

  [TA-Cristofoli-Ilderton-Klisch, to appear]

So, even low mult. strong-field amps carry lots of classical info...

...but what about full quantum amps and higher multiplicity?

- recall huge precision gap compared to trivial background
- main objects of interest in most applications

Quick aside

Note: high-multiplicity scattering in strong backgrounds a serious problem!

- more external states $\Rightarrow$ more powers of small coupling
- but also more insertions of background-dressed wavefunctions and propagators
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High mult. can dominate low mult. in a strong background

Basic question:

Can we compute high-multiplicity scattering amplitudes in (any) strong field QFT?

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YES!

Chiral strong fields

What we can do: *all-multiplicity* tree-level scattering of gluons/gravitons in large class of *self-dual* gauge fields/spacetimes

[TA-Mason-Sharma, TA-Bogna-Mason-Sharma to appear]

Ingredients:

- twistor theory [Penrose, Ward, Atiyah-Hitchin-Singer,...]
- chiral 2d CFTs/string theories [Berkovits-Witten, Skinner, TA-Mason-Sharma]

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Today: gluon scattering in Cartan-valued SD backgrounds

The backgrounds

Self-dual radiative gauge fields:

- complex, asymp. flat Yang-Mills field in 4d Minkowski spacetime
- determined by characteristic data $\tilde{\mathcal{A}}(u, z, \bar{z})$ on $\mathscr{I}$ $\Leftrightarrow$ field strength is *self-dual*

$$F|_{\mathscr{I}^+} = \partial_u \tilde{\mathcal{A}}\, \mathrm{d}u \wedge \mathrm{d}\bar{z}$$

For simplicity, assume $\tilde{\mathcal{A}}$ valued in Cartan of gauge group

External states

Gluons, characterised by:

- asymptotic null momenta $k_{\alpha\dot{\alpha}} = \kappa_\alpha \, \tilde{\kappa}_{\dot{\alpha}}$
- Colour T^a and 'charge' $[\tilde{\mathcal{A}}, \mathsf{T}^a] = e\, \tilde{\mathcal{A}}\, \mathsf{T}^a$
- helicity ± 1

Gluons move through the SD background, get *dressed*:
$\tilde{\kappa}_{\dot{\alpha}} \to \tilde{K}_{\dot{\alpha}}(x)$, x-dependence explicitly controlled by $\tilde{\mathcal{A}}$

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Whole setup has an elegant description via twistor string theory

Twistor string theory

Chiral, heterotic 2d CFT governing holomorphic maps $\mathbb{CP}^1 \hookrightarrow \mathbb{PT} \subset \mathbb{CP}^3$ [Witten, Berkovits,...]

Correlators compute scattering amps of gluons in trivial background

SD rad. gauge field $\leftrightarrow$ holomorphic bundles $E \rightarrow \mathbb{PT}$ [Ward]

$\Rightarrow$ couples to worldsheet current algebra, e.g.,

$$\frac{1}{2\pi}\int_\Sigma \rho^m\,\bar{\partial}\rho^m \quad \rightarrow \quad \frac{1}{2\pi}\int_\Sigma \rho^m\,\bar{D}|_\Sigma \rho^m$$

for $\bar{D}|_\Sigma$ (0,1)-partial connection on E pulled back to worldsheet, obeys $\bar{D}^2 = 0$

Upshot:

Conjecture: gluon scattering in SD rad. gauge field $=$ correlators in background-coupled twistor string

This is a workable problem!

- Twistor string remains anomaly free (holomorphicity of gauge field) [Mason-Skinner]
- Worldsheet CFT is free – current algebra OPEs dressed by holo. frames of E

Result

Example: MHV scattering, gluons r, s negative helicity
Colour-ordered partial amplitude:

$$\frac{\langle r\, s\rangle^4}{\langle 1\, 2\rangle\, \langle 2\, 3\rangle \cdots \langle n\, 1\rangle} \int \mathrm{d}^4 x \, \exp\left[\sum_{i=1}^{n} \mathrm{i}\, k_i \cdot x + e_i\, g(x, \kappa_i)\right]$$

where

$$g(x, z) := \frac{1}{2\pi \mathrm{i}} \int_{\mathbb{CP}^1} \frac{\mathrm{d}z' \wedge \mathrm{d}\bar{z}'}{z - z'}\, \tilde{A}(x, z')$$

Dramatically simpler that expected from background field expansion

More generally...

With these techniques:

- get candidate formulae for *all* tree-level amps of gauge theory and gravity in *all* SD rad. backgrounds
- can prove MHV formulae; other configs pass multiple tests
- can identify KLT kernel/double copy for SD rad. backgrounds [TA-Cristofoli-Klisch, to appear]
- extend to scattering in SD charge/black hole backgrounds [TA-Bogna-Mason-Sharma, to appear]

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Strong-field scattering:

- playground where perturbative & non-perturbative interact
- crying out for new approaches
- encodes all-order information for classical observables (scattering angle, waveform, etc.)
- novel methods provide route to attack higher-precision

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