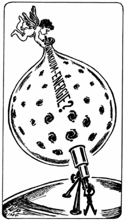
Aspects of De Sitter Spacetimes



Dionysios Anninos

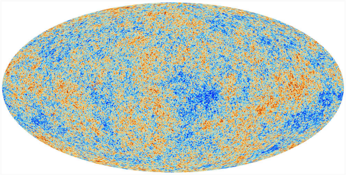
King's College London

The Royal Society

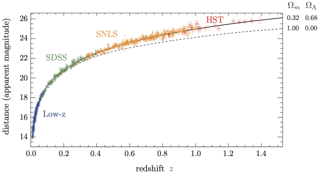
Eurostrings, Gijón-Xixón, April 24th 2023

Shape of the Large Scale Universe

Cosmological & astrophysical observations indicate our Universe experiences epochs of accelerated expansion



Planck Collaboration



Left Cosmic microwave background (CMB) inhomogeneities (2003 – onwards)

Right Type Ia supernova measurements (1998 – onwards)

Quantum Features

Quantum effects play a role in both quasi de Sitter epochs

$$\Psi$$

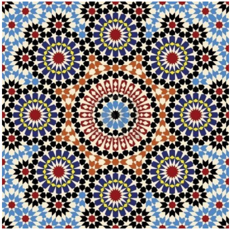
Inflationary hypothesis Quantum fluctuations of light fields, including gravitational field, are responsible for refined patterns in the CMB

Guth-Linde-Albrecht,Steinhart-Starobinsky–Mukhanov,Chibishov-Pi,Hawking-...

Dark energy Quantum contributions to $\Lambda \rightsquigarrow$ cosmological constant problem

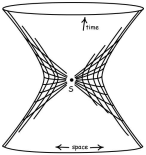
Nernst-...-Lenz-...-Zeldovich-...-Weinberg-...

Geometry & Symmetries



Global Geometry

$$\boxed{\frac{ds^2}{\ell^2} = -dT^2 + \cosh^2 T d\Omega_d}$$



- No natural notion of spatial/null infinity or S-matrix.
- Isometries $= SO(d+1,1)$.
- At $\mathcal{I}^+$ $SO(d+1,1)$ acts as the conformal group on S^d

Eternal Inflation & Conformal Field Theory

Eternal Inflation has symmetries of 3d Euclidean CFT

-Hull-...-Strominger-Witten-...-Maldacena-...

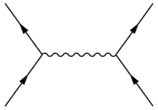
- Late time profiles of field operators transform as conformal operators

- At the free level, associate profiles to unitary irreps of $SO(d+1,1)$

Harish Chandra-...-Hirai-...-Higuchi-...- Basile,Bekaert,Boulanger-...-Letsios-...

- Correlations at $\mathcal{I}^+$ satisfy CFT Ward identities

...-Pimentel,Maldacena-...-Bzowski,McFadden,Skenderis-...



- **4pt functions** tree-level & h.s. exchange...

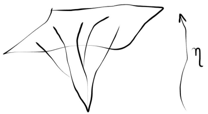
...-Mata,Raju,Trivedi-...-D.A.,Denef,Monten,Sun-...-ArkaniHamed,Baumann,Joyce,Pueyo,Lee,Pimentel-Sleight,Taronna-Bonifacio,Goodhew,Joyce,Pajer,Stefanyszyn-...

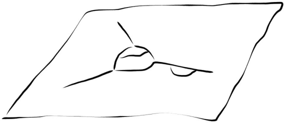
Hartle-Hawking & Conformal Field Theory

Late-time Hartle-Hawking wavefunction governed by 3d CFT

...-Maldacena-...-Hartle,Hawking,Hertog-...-D.A.,Anous,Freedman,Konstantinidis-...-Chakraborty,Chakravarty,Godel,Paul,Raju-

$$\overset{?}{\Longrightarrow} \quad \boxed{\Psi[g] = Z_{\mathrm{CFT}}[g]}$$





BULK CALCULATION:
$f(\mathbf{k}_i) = \int \frac{d\eta}{\eta^4} \prod_i G(\mathbf{k}_i, \eta)$

BOUNDARY CALCULATION:
$f(\mathbf{k}_i) = \langle \prod_i \mathcal{O}(\mathbf{k}_i) \rangle_{\mathrm{CFT}}$

Applications to cosmology ...-Leigh,Larsen,van der Schaar-...-McFadden,Skenderis-...

DS/CFT?

A complete dS/CFT conjecte should likely address:

- Gauge invariant observables at $\mathcal{I}^+$ & measure for $|\Psi[g]|^2$
- Relatedly – Hilbert space and microscopic operator content at $\mathcal{I}^+$

- Unitarity* & locality $\leftrightsquigarrow$ reality & locality properties of CFT
- Relatedly – Analogue/replacement of state-operator correspondence?

* **Cosmological bootstrap developments**:

...-Sengor,Skordis-Anous,Skulte-Benincasa-ArkaniHamed,Baumann,Joyce,Pueyo,Lee,Pimentel-Sleight,Taronna-Goodhew,Jazayeri,Pajer-Penedones,Hogervorst,SalehiVasiri-Gorbenko,Komatsu-...

- Recent progress in 'finite time' $\Psi[g]$ and T^2 deformations

AraujoRegado,Khan,Wall-...

- Issues can be concretely addressed in exotic higher-spin theories

...-D.A.,Hartman,Strominger-...-D.A.,Denef,Harlow-D.A.,Denef,Monten,Sun-...Neiman-...-Strominger,Cotler-...

Cosmological Event Horizon

De Sitter's Static Universe

Λ dominated world $\rightsquigarrow$ observers surrounded by **cosmological event horizon**

This is the cosmological no hair conjecture

Hawking,Moss-Wald-...

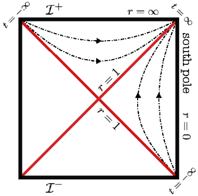
Physics classically **inaccessible** beyond the de Sitter horizon

De Sitter Horizon Thermodynamics

STATIC GEOMETRY: $$ds^2 = -\left(1 - \frac{r^2}{\ell^2}\right) dt^2 + \left(1 - \frac{r^2}{\ell^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Temperature: $\mathrm{T} = \frac{\hbar c}{2\pi\ell}$

Figari, Höegh Krohn, Nappi-Gibbons, Hawking-...



Entropy: $\mathcal{S}_0 = \frac{A_{\text{horizon}}}{4G} = \frac{3\pi c^3}{\Lambda\hbar G}$

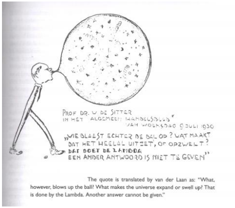
Bekenstein-Hawking...-Gibbons, Hawking-...-Banks, Fischler-Parikh, Verlinde-Susskind-...-Dong, Silverstein, Torroba-...

In our world $\mathcal{S}_0 \sim 10^{122} \gg S_{\text{BH}} \sim 10^{100} \gg S_{\text{matter}} \sim 10^{90}$

Gibbons & Hawking, more concretely, postulate the **quantum** entropy

$$\boxed{\mathcal{S} \equiv \log \mathcal{Z}_{\text{grav}} = \log \int \mathcal{D}g \; e^{-S_E[g_{ij},\Lambda]}}$$

for fields on a **sphere** topology



$\mathcal{S}$ is **diffeomorphism & field redefinition invariant** quantity

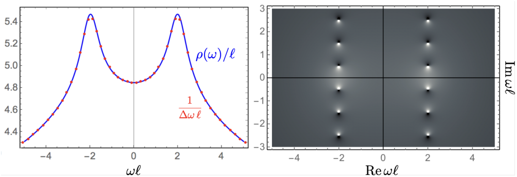
Matter Contributions

Thermal gas for arbitrary species content

$$\boxed{\log \mathcal{Z}_{\text{matter}} = \sum_{\text{species}} \int_{\mathbb{R}^+} d\omega \rho(\omega) \log \frac{e^{-\frac{\beta\omega}{2}}}{1 - e^{-\beta\omega}}}$$

where $\rho(\omega) = \rho_{\text{bulk}}(\omega) - \rho_{\text{edge}}(\omega)$ is group theoretically fixed

D.A.,Denef,Law,Sun-...



$\rho_{\text{edge}}(\omega)$ stem from edge modes (for $s \geq 1$) localised at the horizon

Buidovich,Polikarpov-. . . -Kabat-. . . -Donnelly,Wall-...

Macroscopic Precision Formulae

3+1 dimensional gravity – one loop result on S^4

$$\boxed{\mathcal{S} = \mathcal{S}_0 - 5\log\mathcal{S}_0 - \left(\frac{331}{90} + \frac{8}{3}\right)_{\text{b+e}} \log\frac{\ell^2}{\ell_{\text{ref}}^2} + 15.3655 + \ldots}$$

...-Chritensen,Duff-...-Volkov,Wipf-...-D.A.,Denef,Law,Sun-...-Benedetti,Casini-David,Mukherjee-...

2+1 dimensional gravity – all loop result on S^3

$$\boxed{\mathcal{S}_{\text{3d}} = \mathcal{S}_0 - \log\frac{\mathcal{S}_0}{4\pi} + 2\log\sinh\frac{4\pi^2}{\mathcal{S}_0}}$$

Witten-...-Carlip-Guadagini,Tomassini-...-Castro,Lashkari,Maloney-...-Gukov,Mariño,Putrov-...-D.A.,Denef,Law,Sun-D.A.,Harris-...

Lower dimensional methods

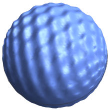
2D Quantum Gravity

General structure Gibbons-Hawking path-integral + 2d CFT

$$\log \mathcal{Z}_{\text{grav}} = \vartheta + \left(\frac{24}{\left(\sqrt{c-1} - \sqrt{c-25}\right)^2} - 1 \right) \log \frac{1}{\Lambda} + f(c)$$

Knizhnik,Polyakov,Zamolodchikov-...-Distler,Kawai-David-...

For $c \to +\infty$ (timelike Liouville) one can tame geometric fluctuations



Euclidean dS_2 semiclassical saddle + systematic all loop conjecture for $f(c)$

Polchinski-...-Giribet-...-D.A.,Bautista,Be**atrix M**ühlmann-Mühlmann-...

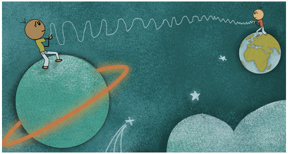
Entanglement Entropy

In the $c \to +\infty$ limit, we have expansion

$$\boxed{\log \mathcal{Z}_{\text{grav}}[\Lambda, c] \approx \vartheta + \left(\frac{c}{6} - \frac{19}{6} + \ldots\right) \log \frac{1}{\Lambda} + \frac{c}{6} \log c + \ldots}$$

Logarithmic term $\rightsquigarrow$ 2d CFT thermal/entanglement entropy

Holzhey,Larsen,Wilczek-...-Casini,Huerta-Cardy,Calabrese-...



Sub-leading terms $\rightsquigarrow$ entanglement from gravity fluctuations

D.A.,Mühlmann

Developments in Near-DS$_2$ JT Gravity

- Sharp expressions for Hartle-Hawking WF as Schwarzian

Maldacena,Turiaci,Yang-Cotler,Jensen,Maloney-...

$$\boxed{\Psi_{\text{HH}} = \mathcal{Z}_{\text{Schwarzian}}[L, \phi_b] = \left(\frac{\phi_b}{L}\right)^{3/2} e^{-i2\phi_b L + i\frac{4\pi^2\phi_b}{L} + \frac{\mathcal{S}_0}{2}} \ .}$$

- 2d static patch with Dirichlet wall & near-Nariai geometry

Svesko,Verheijden,Verlinde,Visser-D.A.,Harris-Jacobson,Visser-...

Developments in 3D Gravity with $\Lambda > 0$

- Novel solutions of black holes in dS_3 for modified Einstein's equation

$$\boxed{\frac{ds^2}{\ell^2} = -\left(1 - r^2 - \frac{\mu}{r}\right) dt^2 + \left(1 - r^2 - \frac{\mu}{r}\right)^{-1} dr^2 + r^2 d\varphi^2}$$

Emparan,Pedraza,Svesko,Tomasevic,Visser

- Novel Chern-Simons methods for quantum matter contributions to $\mathcal{Z}_{\text{matter}}$

$$\boxed{\log \mathcal{Z}_{\text{scalar}} = i \int_{\mathcal{C}} \frac{d\alpha}{\alpha} \frac{\cos \alpha/2}{\sin \alpha/2} \, \mathrm{Tr}_j e^{\frac{\alpha}{2\pi} \oint A_L} \mathrm{Tr}_j e^{\frac{\alpha}{2\pi} \oint A_R}}$$

Castro,Coman,Fliss,Zukowski

- New ideas on dS_3/CFT_2 and WZW models

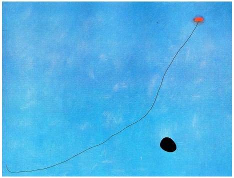
Hikida,Nishioka,Takayanagi,Taki

Embedding in A Larger Framework

Worldline/Worldtube Perspective

de Sitter has no spatial boundary. Observations along timelike **worldline**?

...-D.A.,Hartnoll,Hofman-...-Witten-...



Naïvely problematic in gravity. Nonetheless, large N quantum mechanical models encode emergent space, e.g. BFSS, $\mathrm{AdS}_2/\mathrm{CFT}_1$

...-Polyakov...-Das,Jevicki-...-Banks,Fischler,Shenker,Susskind-. . .

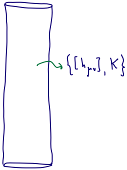
De Sitter Observer & Timelike Surfaces

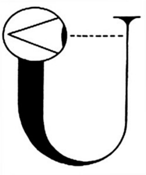
Decorate static patch with ‘observer’ to compute **von Neumann entropy** for resulting type II_1 algebra with maximally entropic state.

Chandrasekaran,Longo,Penington,Witten-. . . , building from: Leutheusser,Liu

Well-posedness for manifolds with timelike boundaries $\rightsquigarrow$ fixing conformal class of induced metric + trace of extrinsic curvature.

Anderson-An,Anderson-Witten-Strominger,Bredberg-D.A.,Anous,Bredberg,Ng-...-D.A.,Galante,Mühlmann-...





Worldtubes as holographic surfaces in static patch?

York-...-Banks,Fischler-Susskind-...-D.A.,Anous,Bredberg,Ng-...-Jacobson,Banihashemi-...-Blacker,Hartnoll

Dynamical Features of De Sitter Horizon

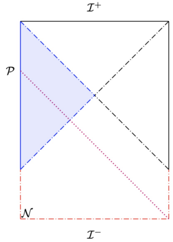
- Energy pulse $\rightsquigarrow$ Horizon 'opens-up' more vertical Penrose diagram

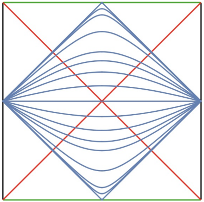
Gao,Wald-...-D.A.,Hofman,Galante-...

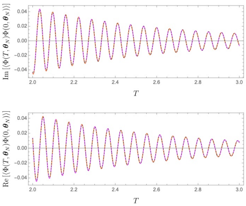
- No (real) geodesics connecting spacelike separated points

...-Galante,Chapman,Kramer-D.A.,Galante,Mühlmann-Chapman,Galante,Harris,Sheorey,Vegh-Aalsma,Faruk,van der Schaar,Visser,de Witte-...

- Oscillatory correlations for heavy particles & suppressed decay for light fields







Timelike Surfaces in DS_3

- Generalisations of $T\bar{T}$-deformations $\rightsquigarrow$ dS_3 physics with timelike boundaries

Coleman,Mazenc,Shyam,Silverstein,Soni-Shyam-...

- Opens up avenue to implement $\mathrm{AdS}_3/\mathrm{CFT}_2$ methods for dS_3 static patch

$$\boxed{\mathsf{E}_{\text{Brown-York}}[\ell/\ell_{Pl}, r_c] = \mathsf{E}_{T\bar{T}}[c, \lambda]}$$

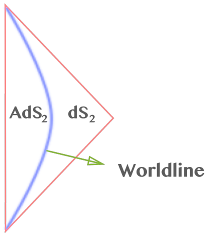
building from: Smirnov,Zamolodchikov-Cavaglia,Negro,Szecsenyi,Tateo-...-McGough,Mezei,Verlinde-...

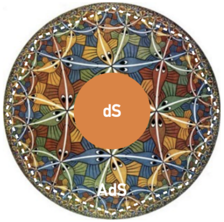
- Matching of logarithmic correction to Gibbons-Hawking entropy

Embed DS$_2$ into AdS$_2$/CFT$_1$ Framework

Simple model: Embed dS$_2$ in asymptotically AdS$_2$ world (deformed JT gravity)

D.A.,Hofman-...-Ecker,Grumiller,McNees building from: Guth-Farhi-...-Freivogel,Hubeny,Maloney,Myers,Rangamani,Shenker-...-





Probe de Sitter horizon with **sharp observables** anchored at AdS$_2$ boundary.

Toward A Microscopic Construction

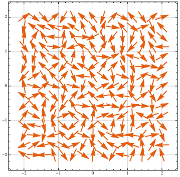
Progress in AdS_2/CT_1 duality $\rightsquigarrow$ microscopic model

Sachdev,Ye-Kitaev-Rosenhaus,Polchinski-Maldacena,Stanford-...

dS as IR regime of SYK + relevant $\cancel{SL(2,\mathbb{R})}$ deformation

$$\boxed{H = j_{i_1,\ldots,i_q}\psi_{i_1}\ldots\psi_{i_q} + \lambda\, j_{i_1,\ldots,i_{q'}}\psi_{i_1}\ldots\psi_{i_{q'}}}$$

j are random variables & $i = 1, 2, \ldots, N$



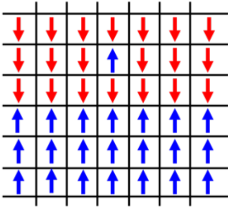
Analytically & numerically tractable **thermal renormalisation group flows**

...-D.A.,Galante,Sheorey-...

Outlook

Ising Model for Quantum De Sitter?

Guided by holographic paradigm & Euclidean methodology, **idealised models** & **concrete calculables** for quantum de Sitter space are beginning to surface



Constraints on Microphysical Toy Models

- Universal features of $\mathcal{Z}_{\text{grav}}$

Reminiscent of Sen's logarithmic BPS black hole corrections. Perhaps 3d/2d gravity loop expansions for $\mathcal{Z}_{\text{grav}}$ can be reproduced by a microphysical model?

Coleman,Mazenc,Shyam,Silverstein,Soni-Mühlmann-Hertog,Bobev,Hong-...

- SYK inspired models

Some similar features between AdS_2 & dS_2, suggesting SYK as a model. Basic challenge is to distinguish black hole/Rindler like features from dS features, e.g. radially shrinking horizon size...

D.A.,Hofman-...-D.A.,Galante,Sheorey-... also: -Susskind-Susskind,Lin-...

- Spectral constraints for $\mathcal{Z}_{\text{grav}}$

Analogy is Klebanov-Giombi sum of one-loop AdS higher spin tower reproducing dual $O(N)$ vector model. Negative edge modes produce novel source of cancelation. Engineer perturbative dS spectra exhibiting UV cancelations? SUSY inspired spectra?

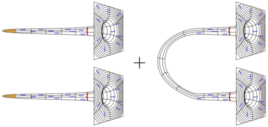
D.A.,BenettiGenolini,Mühlmann-D.A.,RiosFukelman-...

Quantum Information in Quantum Cosmology?

- Topology & wormholes in de Sitter inconclusive

...-Chen,Gorbenko,Maldacena-...

$$\mathrm{Tr}\,\rho_R^2 \quad = \quad$$



$$= \quad e^{-S_{\text{Hawking}}/\hbar} \quad + \quad e^{-S_{\text{Wormhole}}/\hbar}$$

- Absence of classical maximin surfaces

...-Shaghoulian-Levine,Shaghoulian-...-Bousso,Penington-

- Coupling to open/non-gravitational system may help clarify issues

...-Hartman,Jiang,Shaghoulian-Aalsma,AguilarGuitierrez,Sybesma-AguilarGuitierrez,ChatwinDavies,Hertog,PinzaniFokeeva,Robinson-KamesKing,Verheijden,Verlinde-...

Precision Cosmology

As we enter the era of precision **experimental** cosmology, we are also prompted into an era of precision **theoretical** cosmology

Quantitative data at **quantum level** bring us closer to clarifying quantities such as **de Sitter entropy**

Microphysical degrees of freedom of exponentially expanding spacetimes & dynamical properties are, likely, crucial to understand

No clear **consolidation/synthesis** of ideas yet, but common threads emerging

Muchas Gracias!

Review of Reviews

- Strominger, Spradlin, Volovich, "Les Houches Lectures on de Sitter Space" hep-th/0110007
- Witten, "Quantum gravity in de Sitter space" hep-th/0106109
- Bousso, "TASI Lectures on the Cosmological Constant" 0708.4231
- Anninos, "de Sitter Musings" 1205.3855
- Galante, "Modave Lectures on de Sitter space & Holography" 2304.xxxxx