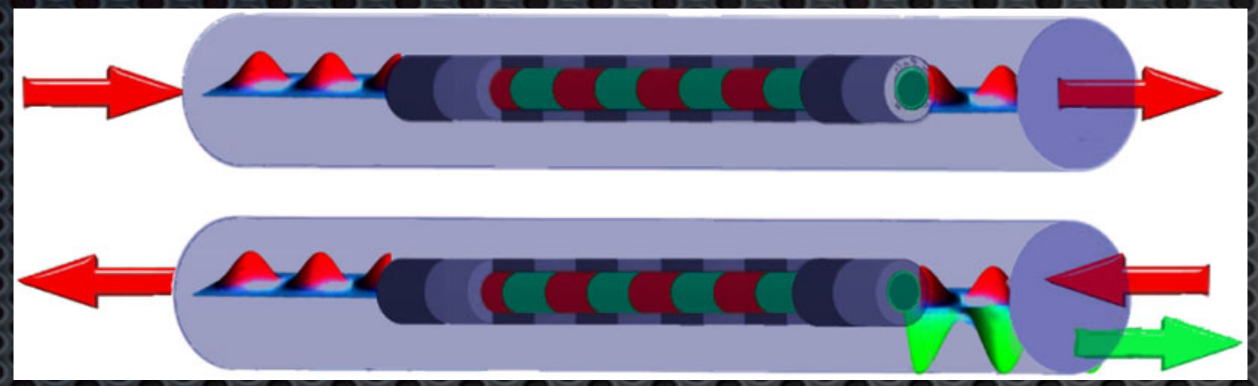
Non-Hermitian Holography

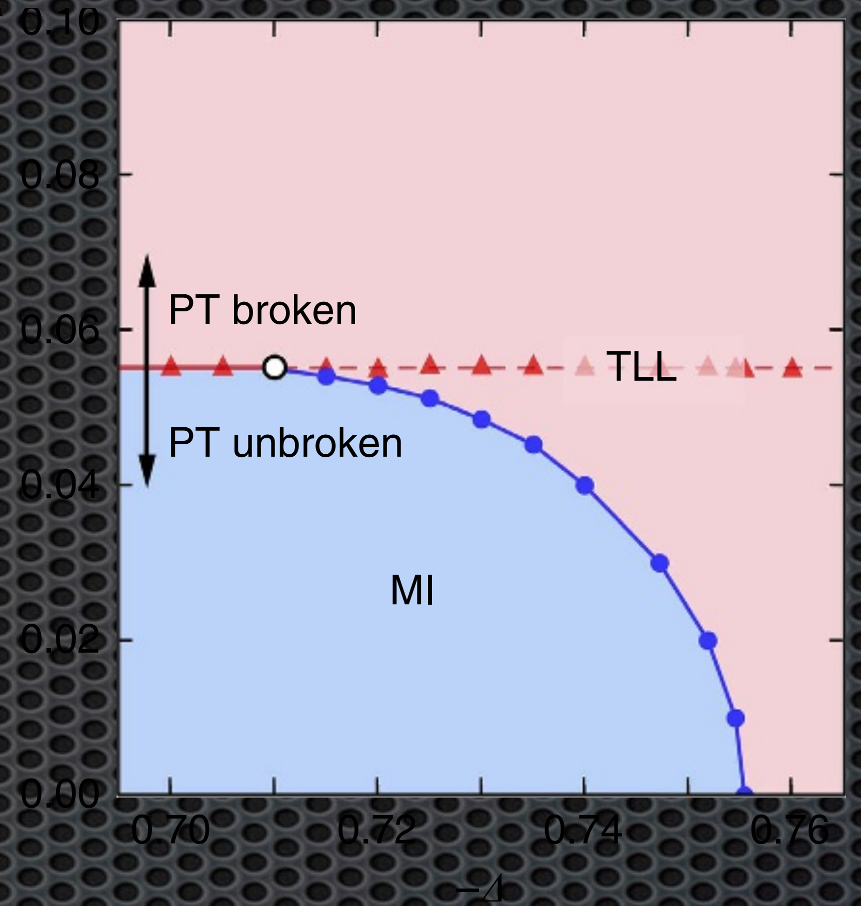
Daniel Areán, Xixón, April 2023

Based on 1912.06647 w/ K. Landsteiner and I. Salazar; and in progress w/ D. García Fariña

Non-Hermitian Holography. Outlook

- Non-Hermitian PT-symmetric QM





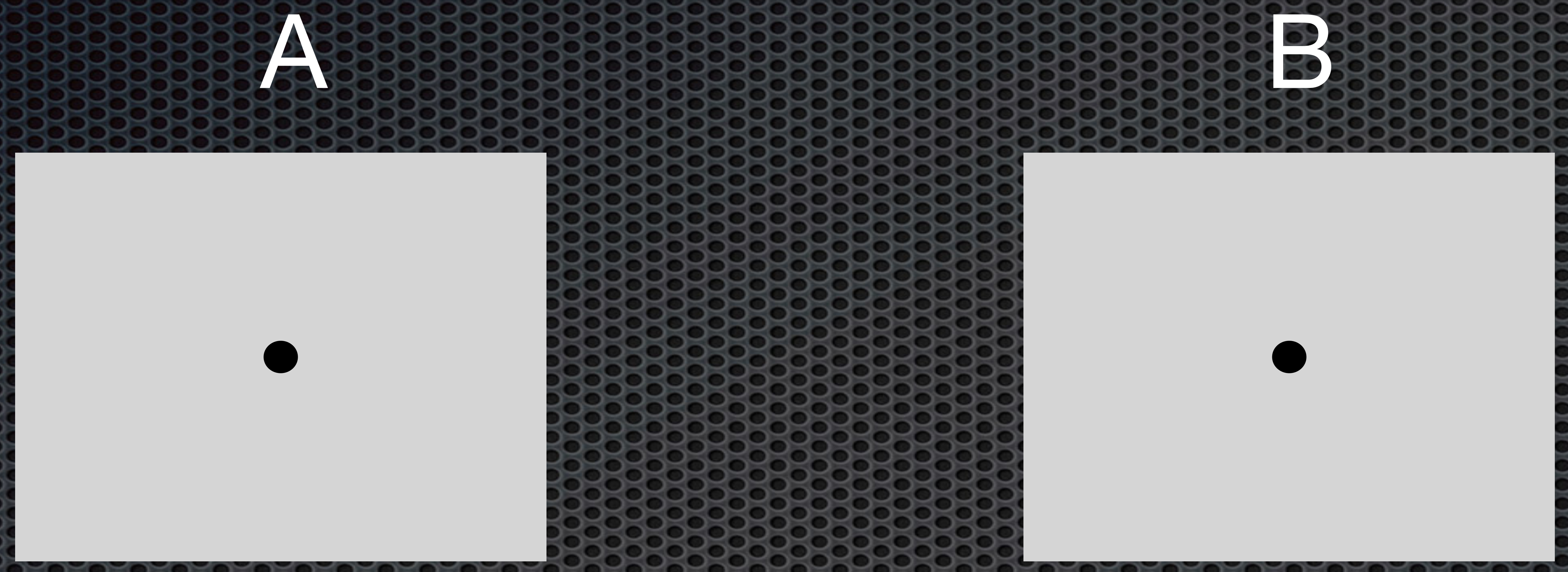
applications in optics, many body systems (cold atoms), quantum simulation, …

- non-Hermitian PT-symmetric holography

- Modulated (non-Hermitian) configurations

PT-symmetric QM. 2 state Hamiltonian

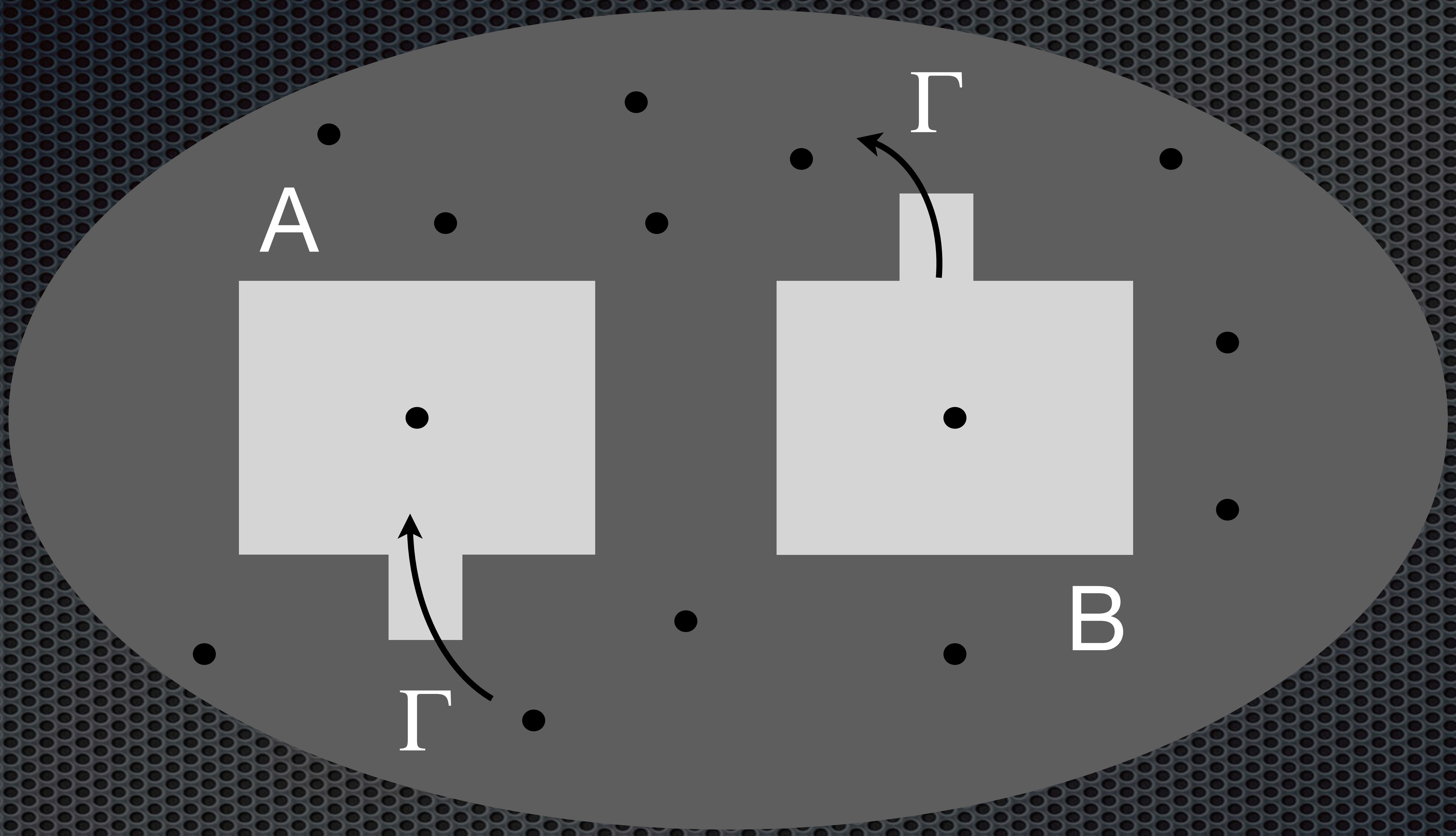
[Bender’97]



$$H = \begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix}$$

$$\psi_{A,B} = e^{-iEt}$$

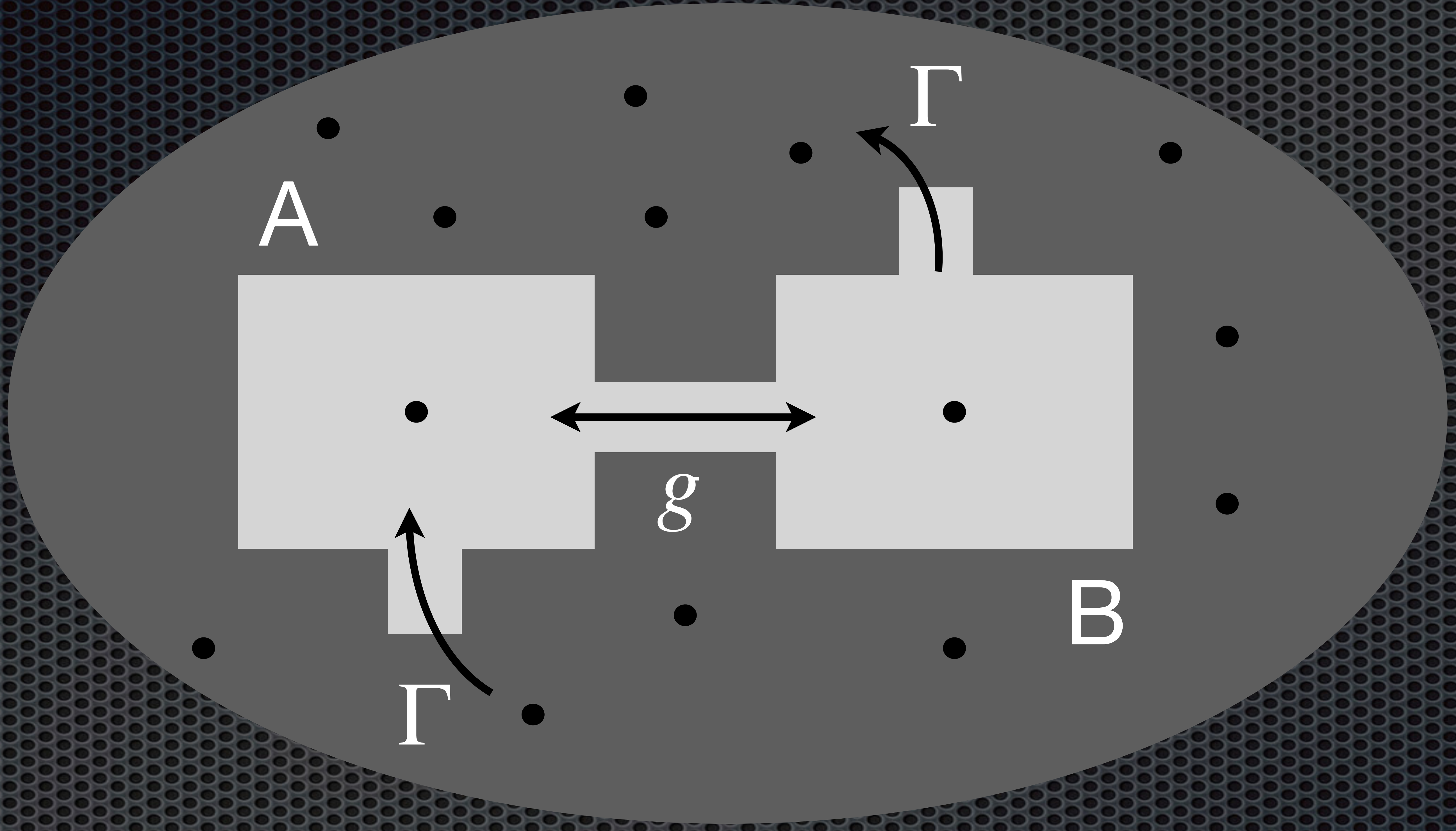
PT-symmetric QM. 2 state Hamiltonian



$$H = \begin{pmatrix} E - i\Gamma & 0 \\ 0 & E + i\Gamma \end{pmatrix}$$

$$\psi_{A,B} = e^{-(iE \pm \Gamma)\, t}$$

PT-symmetric QM. 2 state Hamiltonian



$$H = \begin{pmatrix} E - i\Gamma & g \\ g & E + i\Gamma \end{pmatrix}$$

Eigenvalues

$$\lambda_{\pm} = E \pm \sqrt{g^2 - \Gamma^2}$$

PT-symmetric QM

$$H = \begin{pmatrix} E - i\Gamma & g \\ g & E + i\Gamma \end{pmatrix}$$

$$\lambda_{\pm} = E \pm \sqrt{g^2 - \Gamma^2}$$

PT-symmetric phase: $|g| > \Gamma$

PT-broken phase: $|g| < \Gamma$

Exceptional point: $g = \Gamma$

PT-symmetric QM

$$T(H) = \begin{pmatrix} E + i\Gamma & g \\ g & E - i\Gamma \end{pmatrix} \qquad P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$P\,T(H)\,P = H$$

PT-symmetric phase: $|g| > \Gamma$

PT-broken phase: $|g| < \Gamma$

PT-symmetric QM. Pseudo-hermitian H

[Mostafazadeh'02, '03, '20; Fring '22]

$$[H, PT] = 0\,, \quad PT\,|\psi\rangle = e^{i\phi}\,|\psi\rangle\,, \quad PT\,\lambda\,|\psi\rangle = \lambda^*\,|\psi\rangle$$

[➥ $(PT)^2 = 1$]

H is pseudo-Hermitian (it has real eigenvalues)

Dyson map: $\eta : H \longrightarrow h = \eta H \eta^{-1}\,, \quad h = h^\dagger$

➥ metric $\rho = \eta^\dagger \eta \;\Leftrightarrow\; \rho\, H^\dagger = H$

$$\langle \psi | \tilde{\psi} \rangle_\rho \equiv \langle \psi | \rho\, \tilde{\psi} \rangle \Rightarrow \langle \psi | H \tilde{\psi} \rangle_\rho = \langle H \psi | \tilde{\psi} \rangle_\rho$$

PT-symmetric QM. Pseudo-hermitian H

$$H_2(\vec{g}) = E\,\mathbb{I} + \vec{g}\cdot\vec{\sigma} \Longleftrightarrow H_2'(\vec{g}') = D(\overrightarrow{\alpha})^\dagger H_2(\vec{g})\, D(\overrightarrow{\alpha})$$

$SU(2)$ rotation $D(\overrightarrow{\alpha}) = e^{i\vec{\alpha}\cdot\vec{\sigma}/2}$ $\Rightarrow$ Dyson map: $\eta(\vec{\beta}) = D(-i\vec{\beta})$

E.g. $\vec{g} = (g',0,0)$, $\overrightarrow{\alpha} = (0,i\beta,0) \Longrightarrow H_{\rm nH} = E + g'(\cosh\beta\sigma_1 + i\sinh\beta\sigma_3)$

Eigenvalues $\lambda_\pm = E \pm g'\sqrt{\cosh^2\beta - \sinh^2\beta}$

Exceptional point $\begin{cases} \beta \to \infty \\ g' \to e^{-\beta}\tilde{g} \end{cases} \Rightarrow$

PT-symmetric QM. Dirac fermions

Mass term: $\mathscr{L}_{\text{mass}} = M\left[\psi^\dagger\gamma^0(1-\gamma^5)\psi\right] + \bar{M}\left[\psi^\dagger\gamma^0(1+\gamma^5)\psi\right]$

$\mathscr{L}_{\text{mass}} = \mathscr{L}^\dagger_{\text{mass}} \leftrightarrow M = \bar{M}^*$. Define $M = \frac{1}{2}(m_r + i\,m_i)$

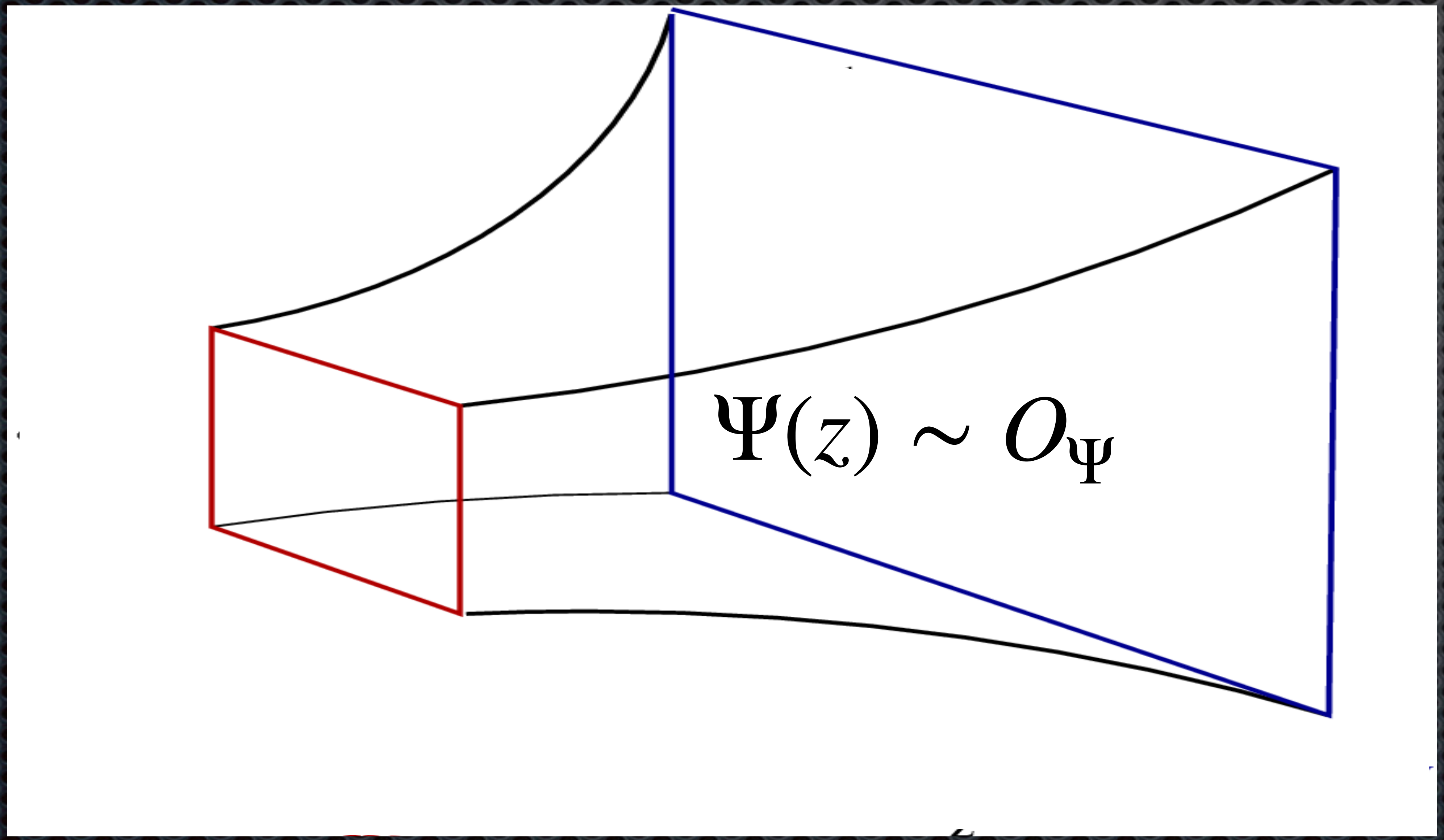
Non-Hermitian 'rotation': $M \to e^{i\alpha}M\,,\ \bar{M} \to e^{-i\alpha}\bar{M}$ w/ $\alpha = i\beta$

➥$\mathscr{L}_{\text{mass}} \to \mathscr{L}_{\text{nH}} = m_r \cosh\beta\,\psi^\dagger\gamma^0\psi - m_i \sinh\beta\,\psi^\dagger\gamma^0\gamma^5\psi$

Holography: complex operator $\mathcal{O} \sim \Psi(r)$ where $\Psi\,|_{\text{boundary}} = M$

Non-Hermitian Holography

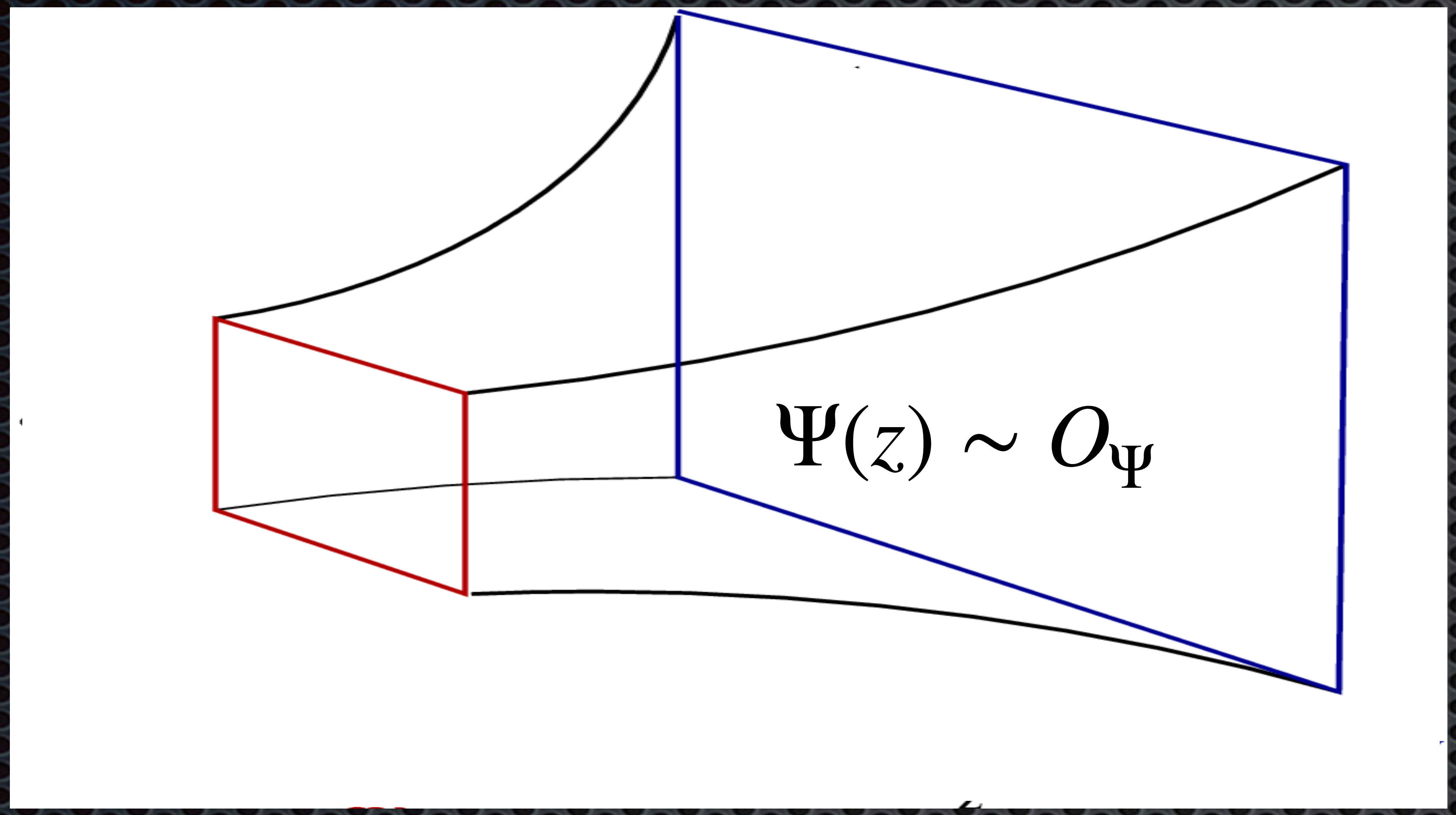
Complex scalar field Ψ in $\sim$ AdS geometry: $\Psi|_{\text{boundary}} = M$,



$$\sim \mathscr{L} = \ldots + \bar{M}\langle O \rangle + M\langle O^\dagger \rangle$$

Non-Hermitian Holography

Complex scalar field Ψ in $\sim \mathrm{AdS}$



Dyson map: $U(1)$ w/ imaginary phase

$$\Psi \sim M \rightarrow \Psi \sim M\, e^{-\theta}$$

$$\bar{\Psi} \sim M \rightarrow \Psi \sim M\, e^{\theta}$$

$$S = \int d^{d+1}x\sqrt{-g}\left[R - 2\Lambda + F^2 + \frac{1}{2}g^{ab}\left(D_a\Psi\,\bar{D}_b\bar{\Psi} + a \leftrightarrow b\right) - m^2\,\bar{\Psi}\Psi + v\,(\bar{\Psi}\Psi)^2\right]$$

[Gubser&Rocha'08]

Holography w/ non-Hermitian boundary conditions

$$S = \int d^{d+1}x\sqrt{-g}\left[R - 2\Lambda + F^2 + \frac{1}{2}g^{ab}\left(D_a\Psi\,\bar{D}_b\bar{\Psi} + a \leftrightarrow b\right) - m^2\,\bar{\Psi}\Psi + v\,(\bar{\Psi}\Psi)^2\right]$$

UV ($r \sim 0$) boundary conditions ($m^2 = -2$)

$$\Psi = M\,r + vr^2 + o(r^3)$$
$$\bar{\Psi} = M\,r + vr^2 + o(r^3)$$

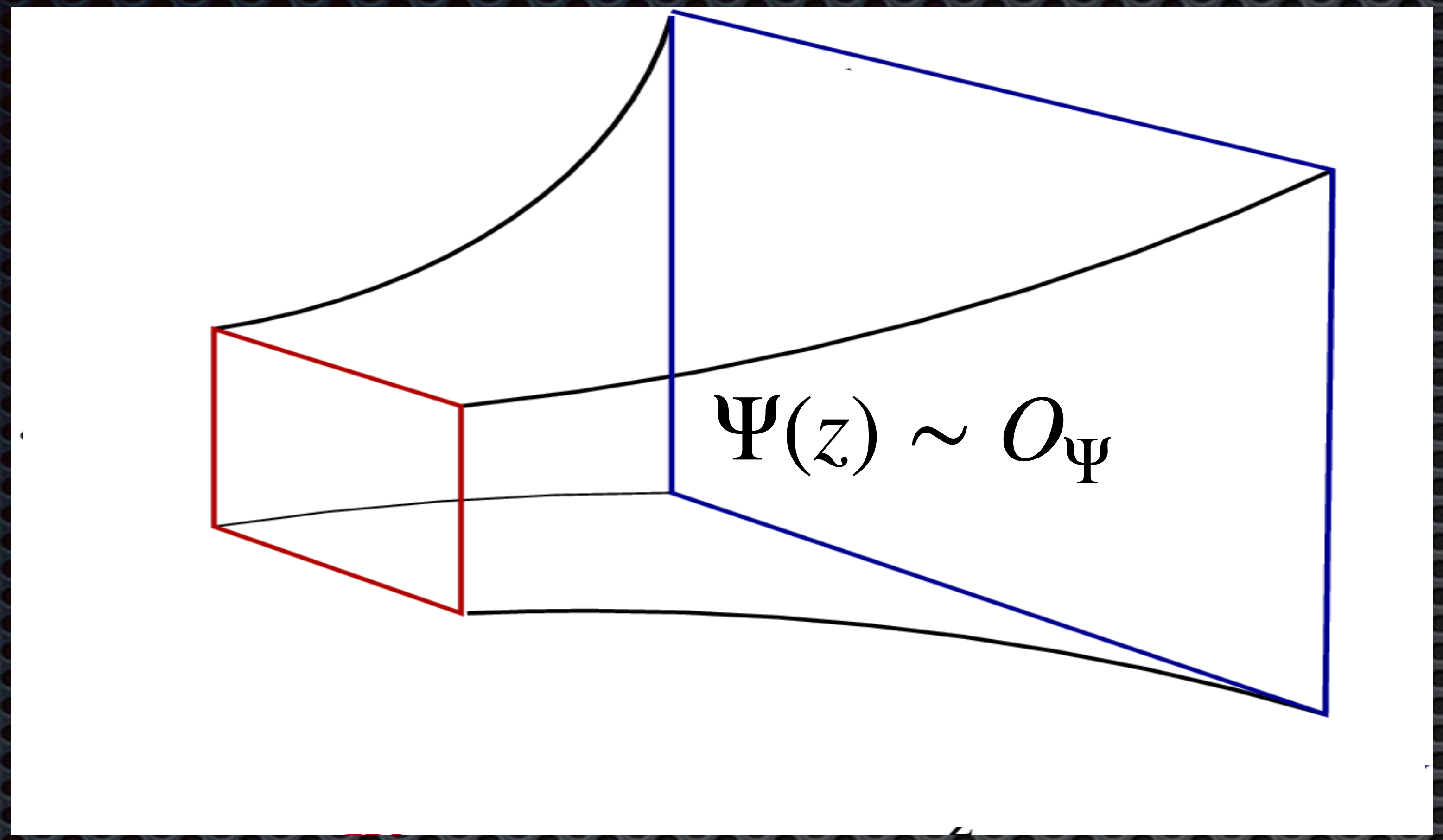
→

$$\Psi_{\text{boundary}} \to e^{i\alpha}\,\Psi_{\text{boundary}}$$
$$\bar{\Psi}_{\text{boundary}} \to e^{-i\alpha}\,\Psi_{\text{boundary}}$$

Holographic Dyson map: $\alpha = i\beta$

Holography w/ non-Hermitian boundary conditions

UV $(r \sim 0)$ non-Hermitian boundary conditions $(m^2 = -2)$



$$\Psi = M\, e^{-\beta}\, r + \ldots = (1 - \xi)\, s\, r + \ldots$$

$$\bar{\Psi} = M\, e^{\beta}\, r + \ldots = (1 + \xi)\, s\, r + \ldots$$

$$e^{2\beta} = \frac{1 + \xi}{1 - \xi}$$

$$M = \sqrt{1 - \xi^2}$$

Pseudo-Hermitian regimen: $\xi < 1$

Exceptional point: $\xi = 1$

non-Hermitian regime: $\xi > 1$

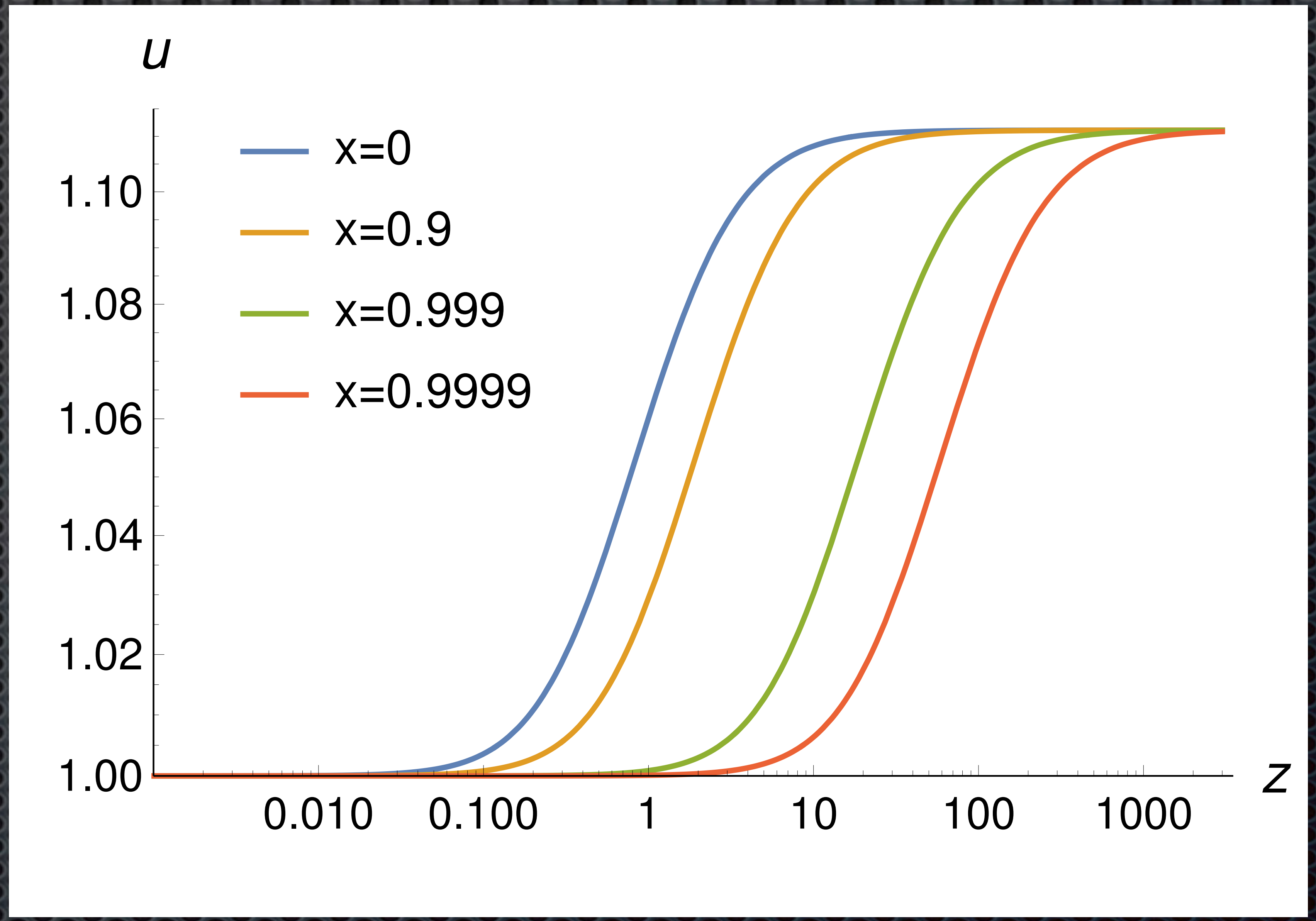
Non-Hermitian Holography [1912.06647]

Pseudo-Hermitian domain walls: $\xi < 1$

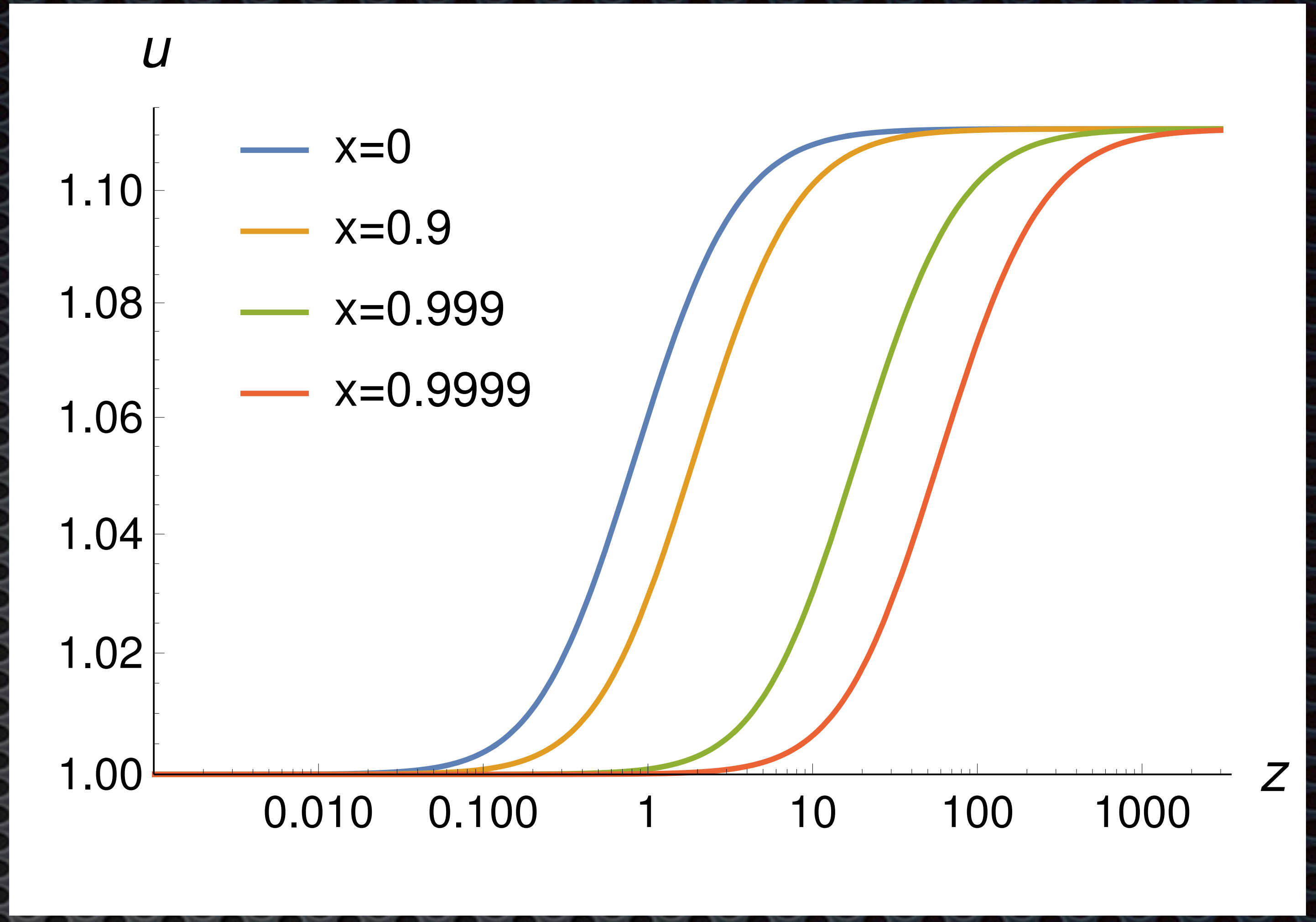
$T = 0$ AdS_4 to AdS_4 domain walls

$$\Psi \sim s\,(1-\xi)\,r$$

$$\bar{\Psi} \sim s\,(1+\xi)\,r$$



Non-Hermitian Holography [1912.06647]



Holographic dyson map $(\xi < 1)$

Bulk action invariant under

$$\Psi(r) \to e^{\beta}\Psi(r)$$
$$\bar{\Psi}(r) \to e^{-\beta}\Psi(r)$$

Pseudo-Hermitian

$$\Psi \sim s(1-\xi)r$$
$$\bar{\Psi} \sim s(1+\xi)r$$

$\sim$

Hermitian

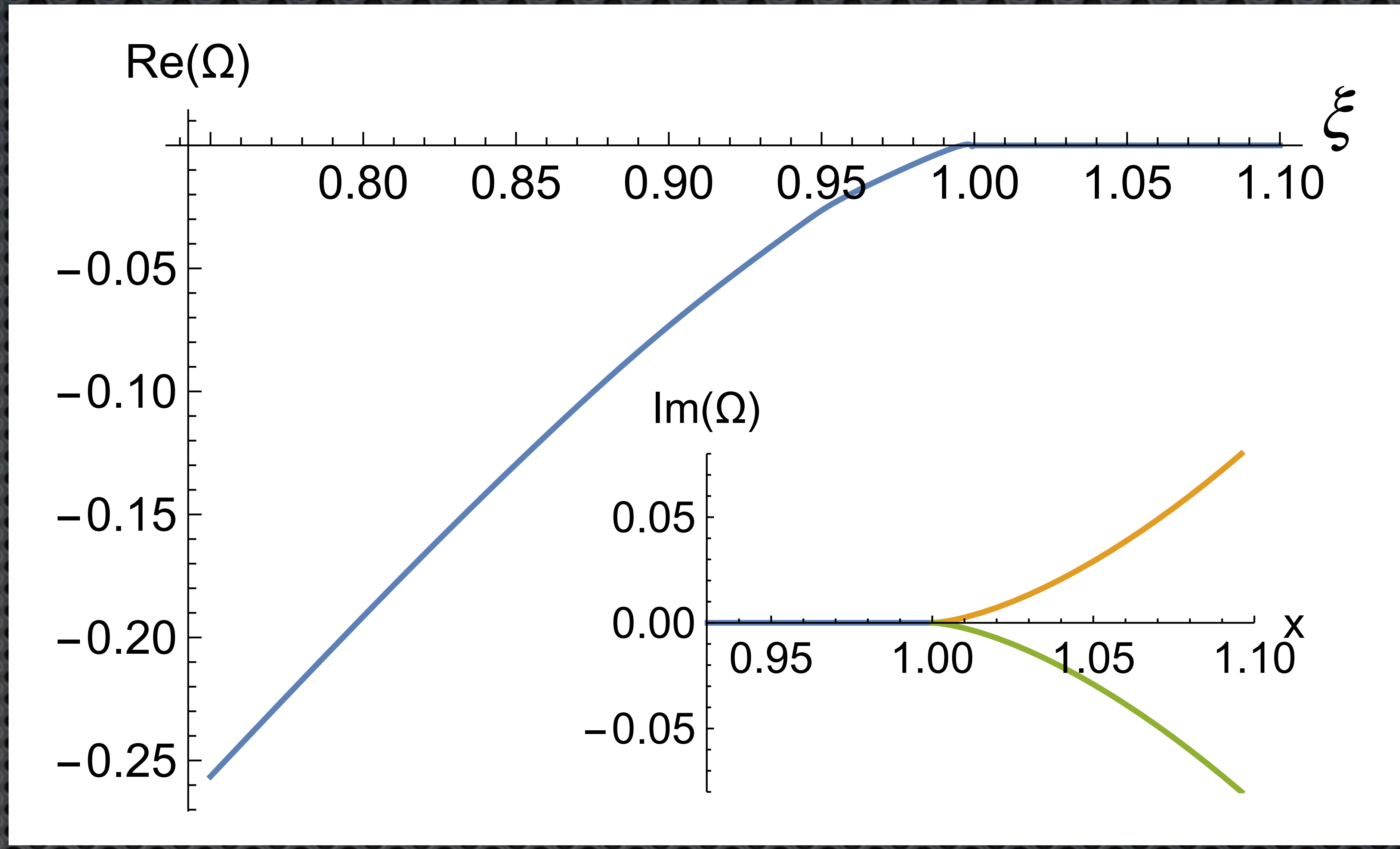
$$\Psi \sim s\sqrt{1-\xi^2}\,r$$
$$\bar{\Psi} \sim s\sqrt{1-\xi^2}\,r$$

Non-Hermitian Holography [1912.06647]

$$\Psi \sim s\,(1-\xi)\,r$$

$$\bar{\Psi} \sim s\,(1+\xi)\,r$$

Pseudo Hermitian for $\xi < 1$



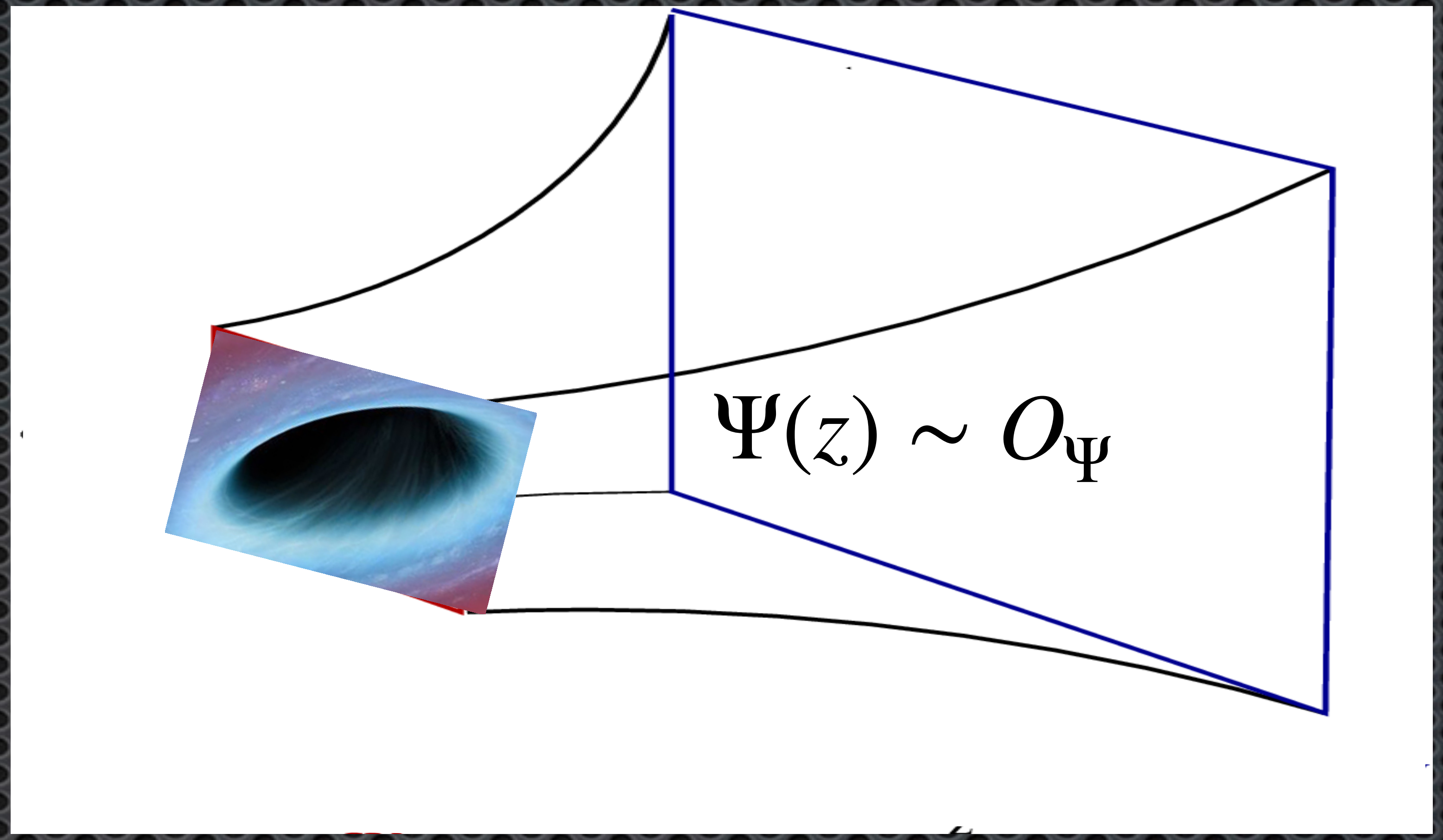
PT-symmetric phase: $\xi < 1 \quad \sim \quad \xi = 0$

PT-broken phase: $\xi > 1 \;\Rightarrow\;$ complex backgrounds

Non-Hermitian Holography [1912.06647]

Finite Temperature

Black brane geometry $\Longrightarrow T/s \neq 0$



$$\Psi \sim s\,(1-\xi)\,r$$

$$\bar{\Psi} \sim s\,(1+\xi)\,r$$

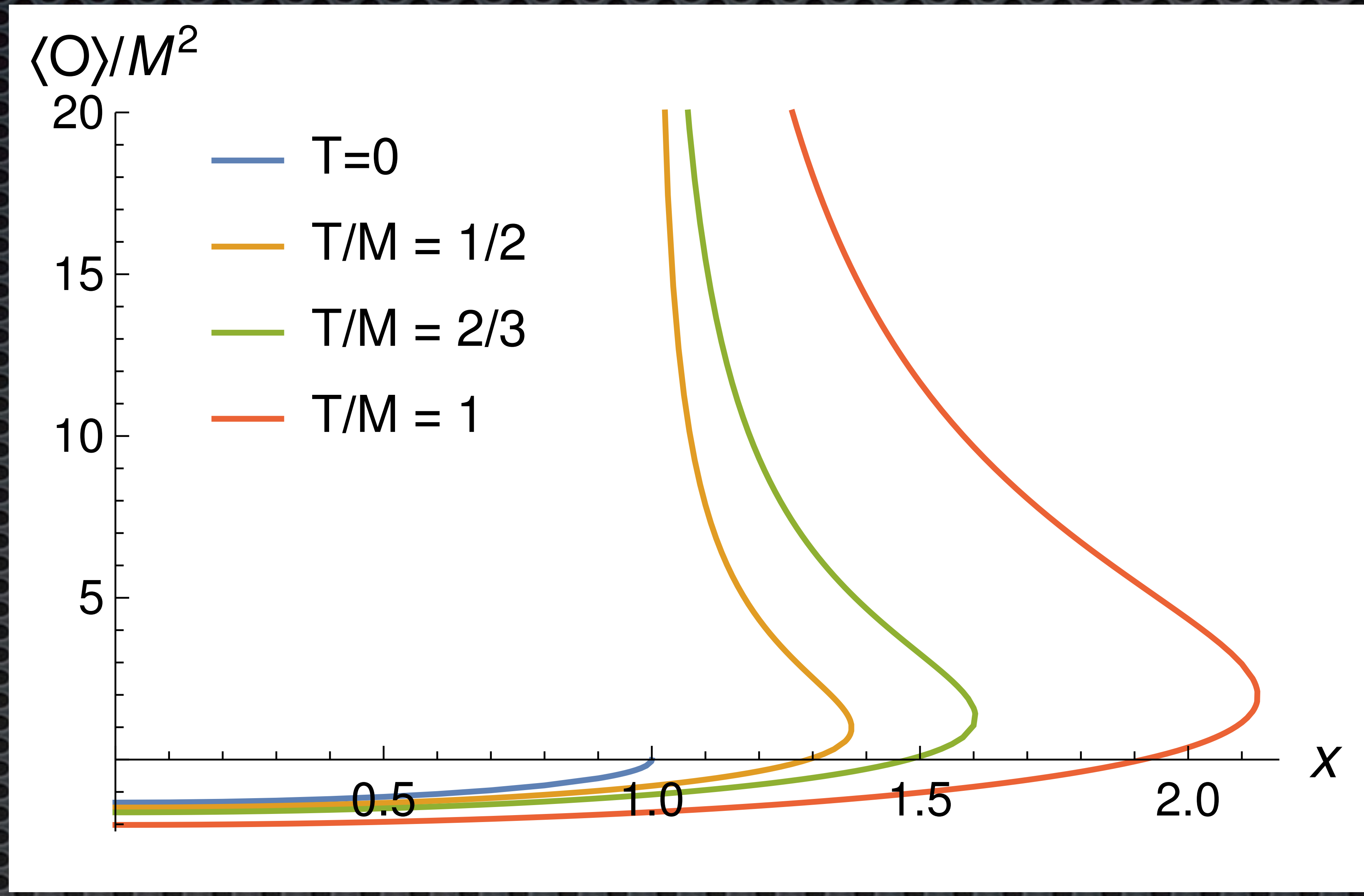
Real geometries up to $x_c > 1$

(unstable solutions)

Non-Hermitian Holography [1912.06647] Finite Temperature

$$\Psi \sim s\,(1-\xi)\,r$$

$$\bar{\Psi} \sim s\,(1+\xi)\,r$$



Real geometries up to $x_c > 1$

$\xi < 1$

Pseudo-Hermitian phase

$\xi = 1$

Exceptional point: AdS_4

$1 < \xi < \xi_c$

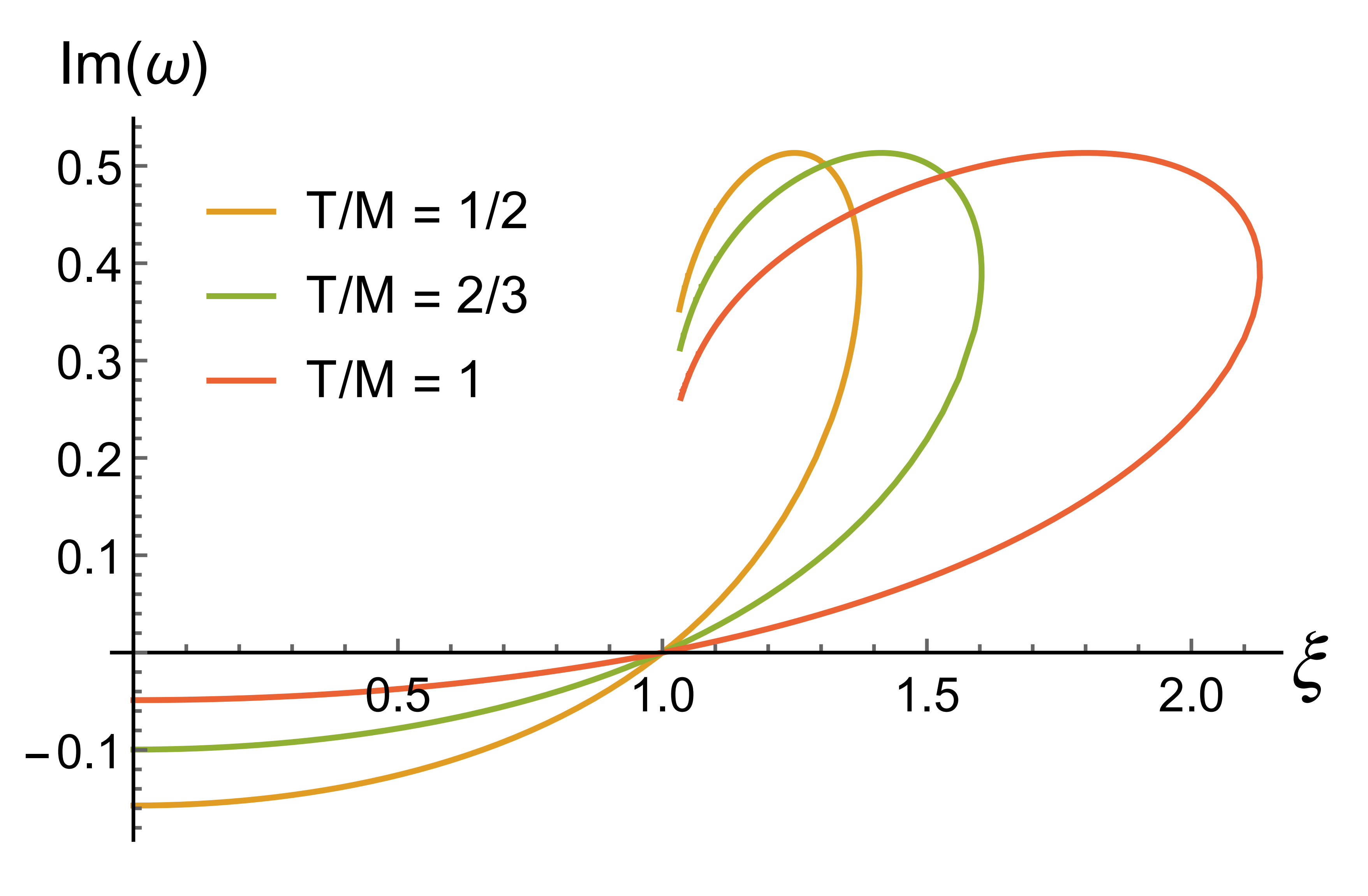
Two new (unstable) solutions

[see also Xian et al'23]

Non-Hermitian Holography [1912.06647] Finite Temperature

$$\Psi \sim s\,(1-\xi)\,r$$

$$\bar{\Psi} \sim s\,(1+\xi)\,r$$



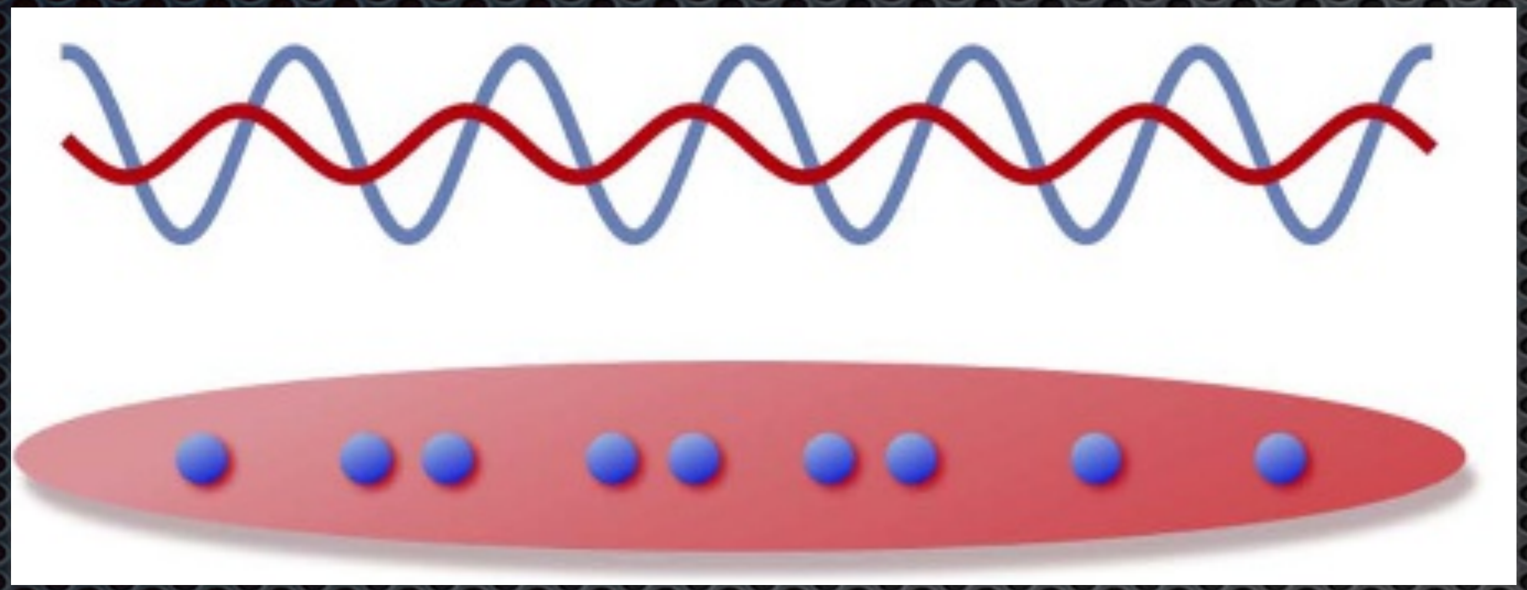
Real geometries up to $x_c > 1$

$$1 < \xi < \xi_c \implies \mathrm{Im}(\omega_{\mathrm{QNM}}) > 0$$

Non-Hermitian inhomogeneous holography

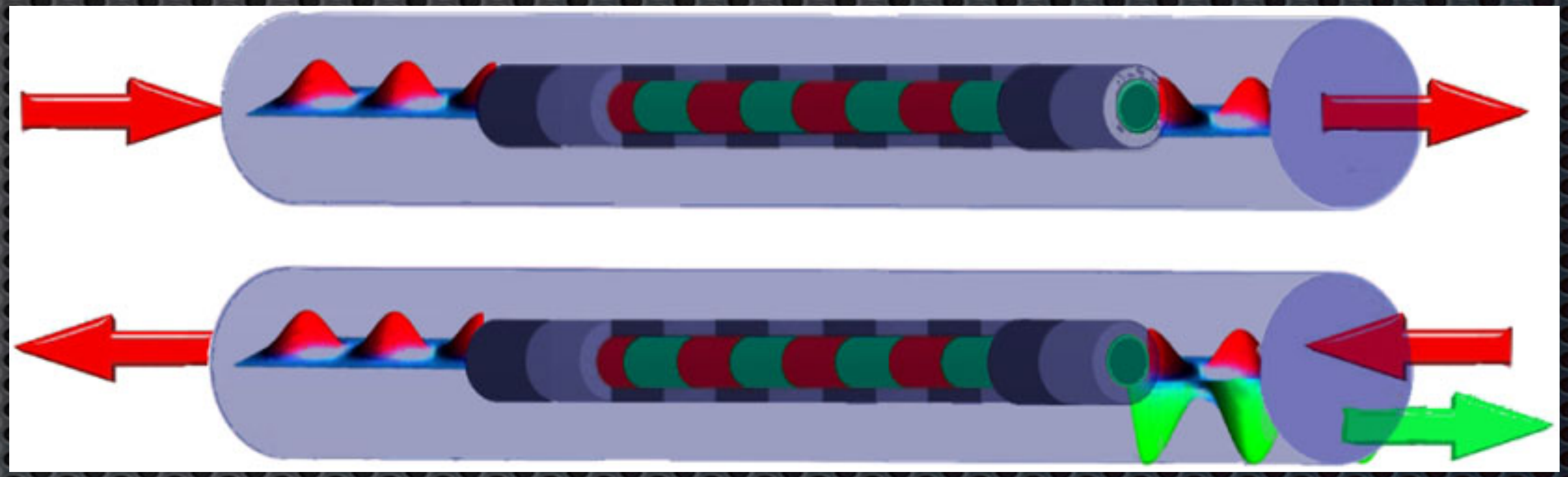
Space dependent non-Hermitian deformation

[Ashida et al '17]



New PT-breaking quantum critical points

[Lin et al '11]



PT-symmetric induced invisibility

➡ Non-Hermitian modulated BCs

$$\Psi \sim s\,(1-\xi(x))\,r$$

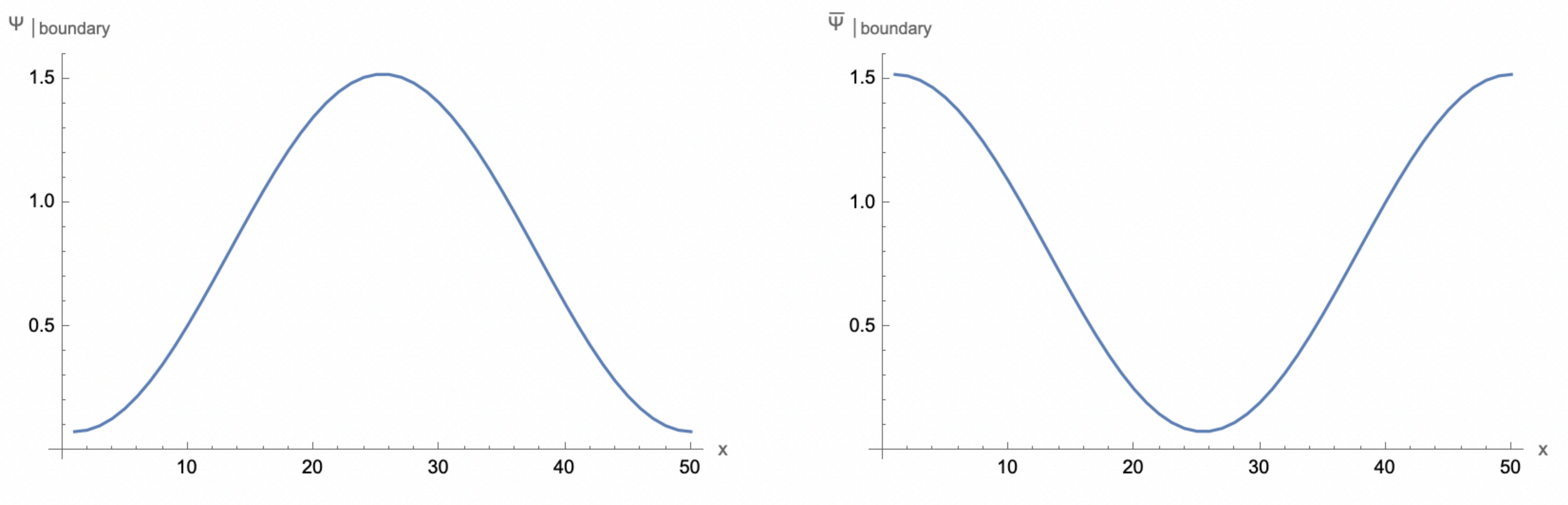
$$\bar{\Psi} \sim s\,(1+\xi(x))\,r$$

Non-Hermitian inhomogeneous holography

Space dependent non-Hermitian deformation

$$\Psi \sim s\,(1 - \xi(x))\,z$$

$$\bar{\Psi} \sim s\,(1 + \xi(x))\,z$$

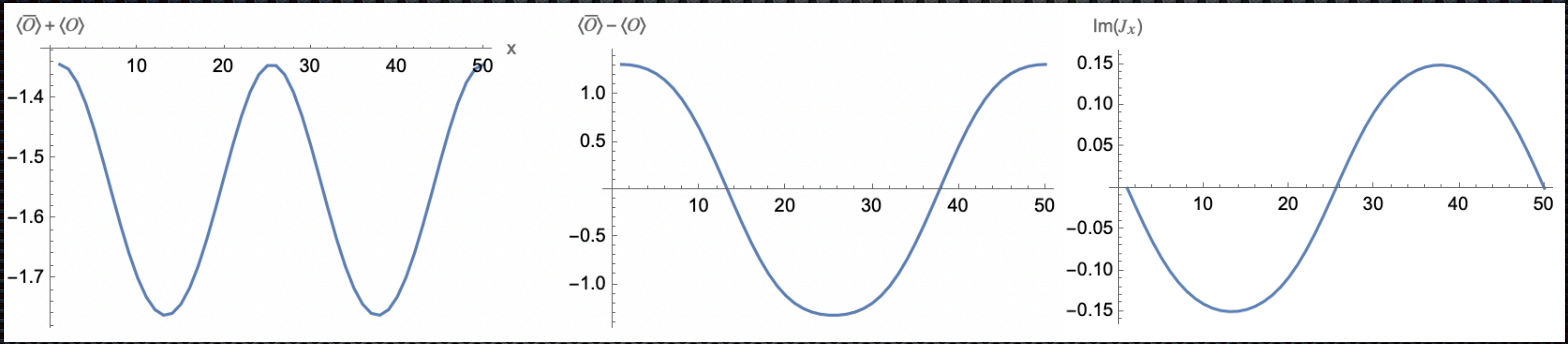


in asymptotically AdS black brane geometry ($T/s \approx 0.30$)

[see also Landsteiner&Morales-Tejera'22]

Non-Hermitian inhomogeneous holography

$$\Psi \sim s\,(1 - \xi(x))\,,\;\; \bar{\Psi} \sim s\,(1 + \xi(x)) \qquad \Longrightarrow \qquad \langle \mathrm{Im}\,(J_x) \rangle \neq 0$$



$T/s \approx 0.30$

Real geometries dual to non-Hermitian states

Non-Hermitian inhomogeneous holography

PT-symmetric (pseudo-Hermitian) phase: gauging the Dyson map

[see also Chernodub&Millington'97]

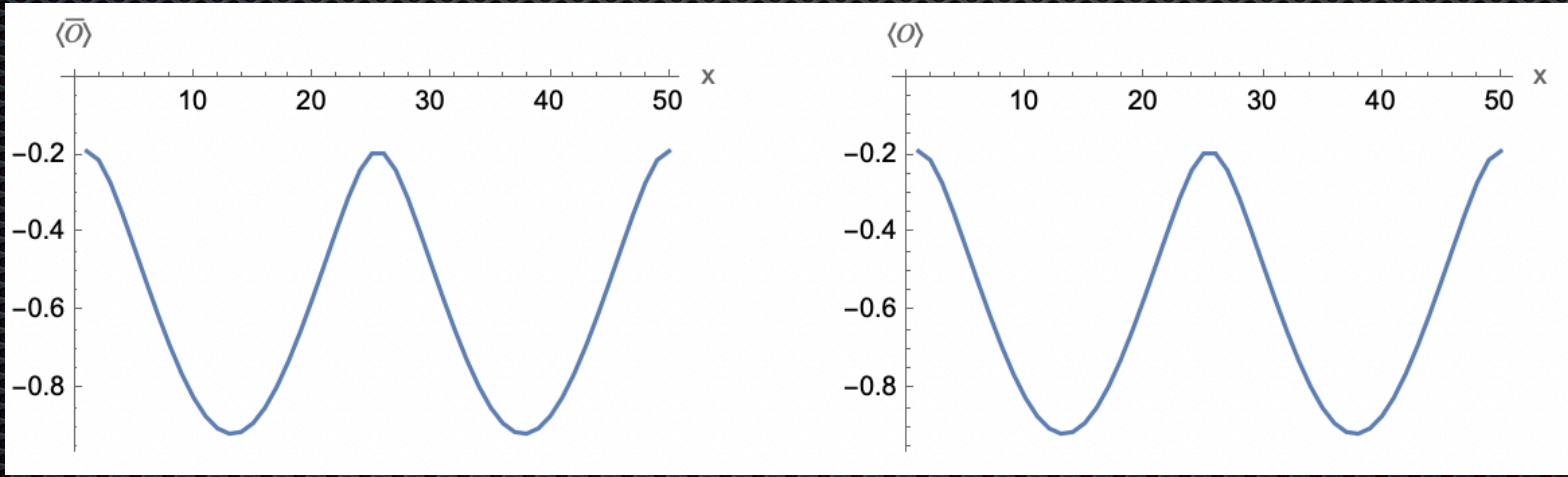
PT-symmetric		Hermitian
$\Psi \sim s\,(1-\xi(x))\,z$	$\sim$	$\Psi \sim s\sqrt{1-\xi(x)^2}\,z$
$\bar{\Psi} \sim s\,(1+\xi(x))\,z$		$\bar{\Psi} \sim s\sqrt{1-\xi(x)^2}\,z$

$$A_x = -\frac{i}{2}\partial_x \log\left(\frac{1-\xi(x)}{1+\xi(x)}\right)$$

Non-Hermitian inhomogeneous holography

PT-symmetric (pseudo-Hermitian) phase: gauging the Dyson map

$$\Psi \sim s\,(1-\xi(x)),\ \bar{\Psi} \sim s\,(1+\xi(x)),\ A_x = -\frac{i}{2}\partial_x \log\left(\frac{1-\xi(x)}{1+\xi(x)}\right)$$

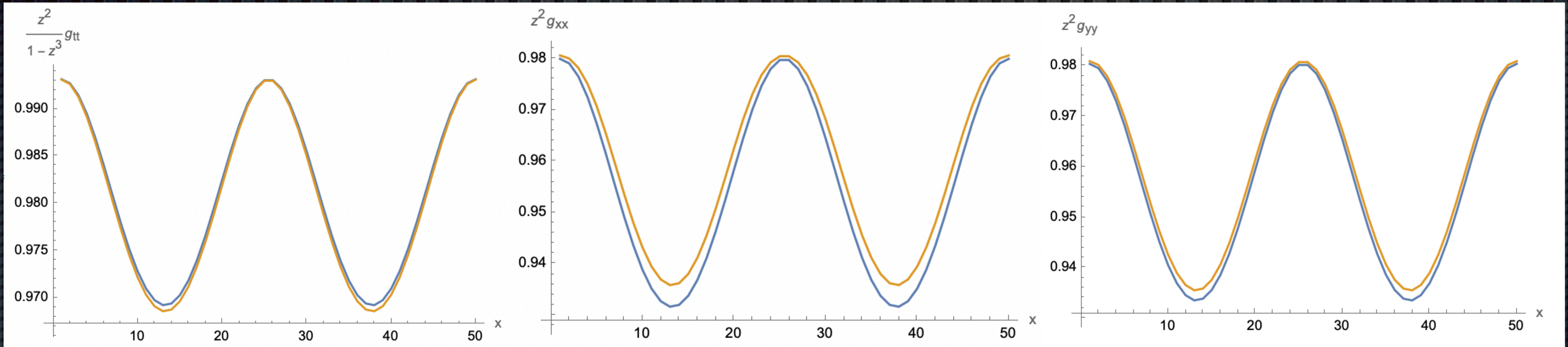


$\langle J_x \rangle = 0$

$T/s \approx 0.30$

Non-Hermitian inhomogeneous holography

non-Hermitian $\langle \mathrm{Im}(J_x) \rangle \neq 0$ vs PT-symmetric geometry



$T/s \approx 0.30$

Non-hermitian holography

Overview & To do

- (Minimal) Holographic model of nH PT-symmetric theories
- Exhibits PT-symmetric and PT-broken phases
- Modulated pseudo-Hermitian and non-Hermitian solutions
- QNMs (stability) and transport
- Finite charge density and spontaneous symmetry breaking
- …