A CFT perspective on AdS amplitudes

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Main Goal

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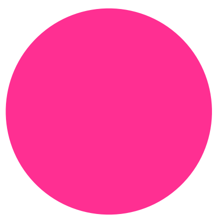
Correlators in CFTs



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Amplitudes in AdS

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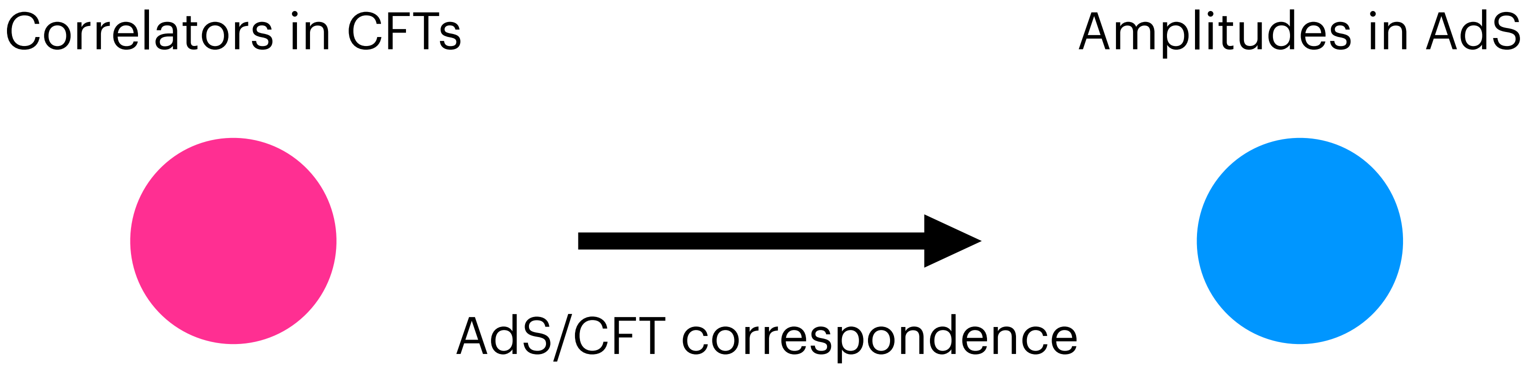
Correlators in CFTs
Amplitudes in AdS
AdS/CFT correspondence

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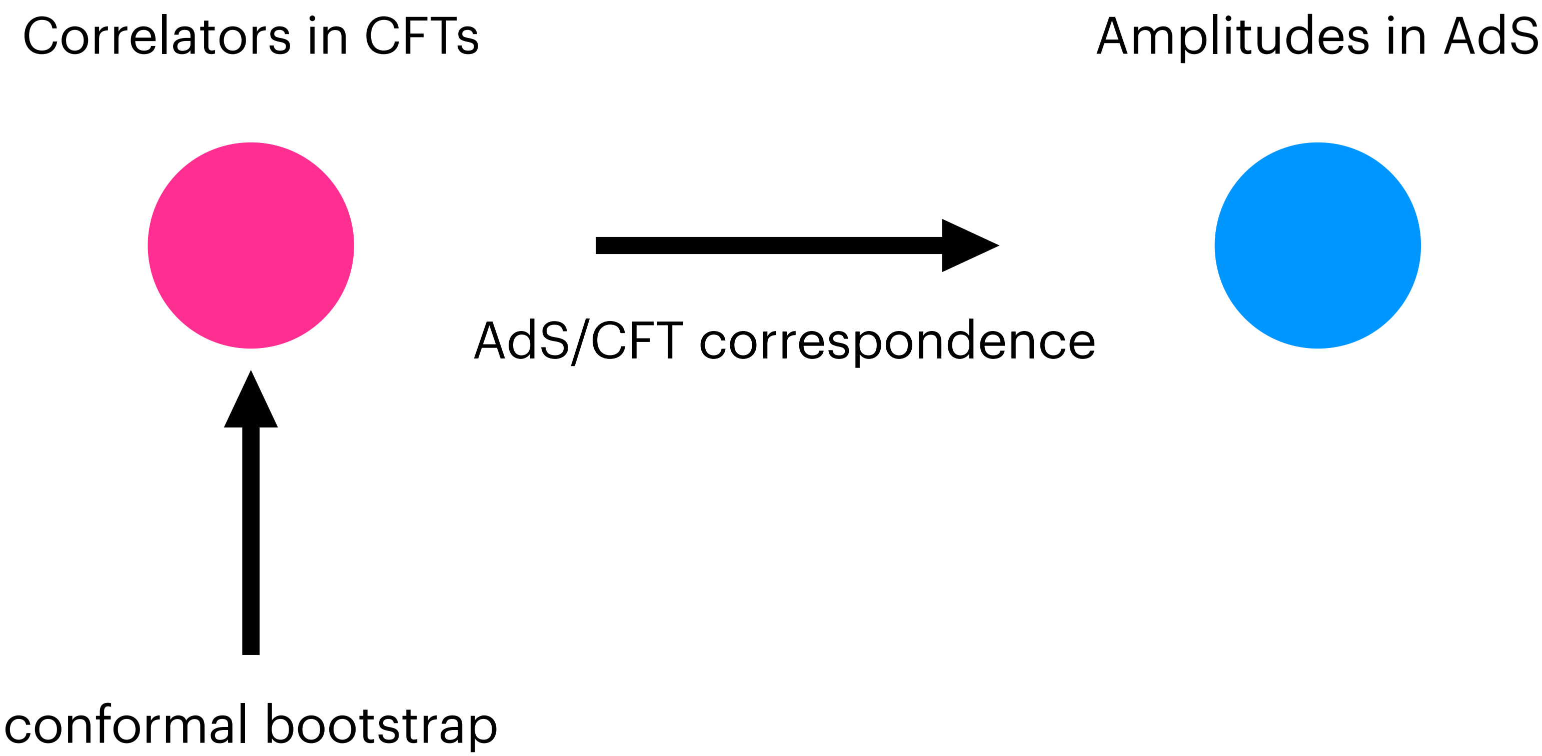
Correlators in CFTs
Amplitudes in AdS
AdS/CFT correspondence
conformal bootstrap

Why?

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Correlators of holographic CFTs

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Scattering amplitudes on curved spaces

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Correlators of holographic CFTs

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CFT data for unprotected operators

Setup

Weakly coupled regime in the bulk is **supergravity** and corresponds to large central charge and string length to zero.

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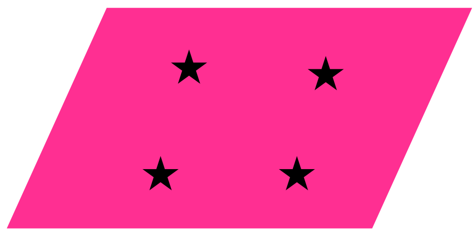

$\mathbb{R}^{d-1,1} = \partial \mathrm{AdS}_{d+1}$

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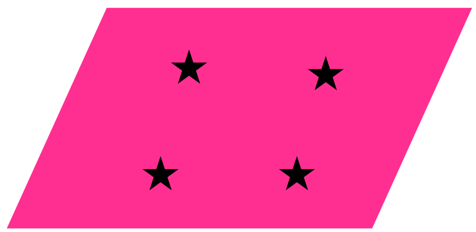

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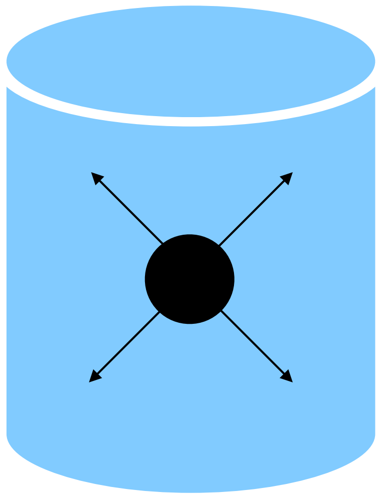

$\mathrm{AdS}_{d+1} \times \mathrm{S}^q$

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AdS/CFT correspondence

CFT

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4 dimensional $\mathcal{N} = 4$ Super Yang Mills with SU(N) gauge group and SU(4) R-symmetry

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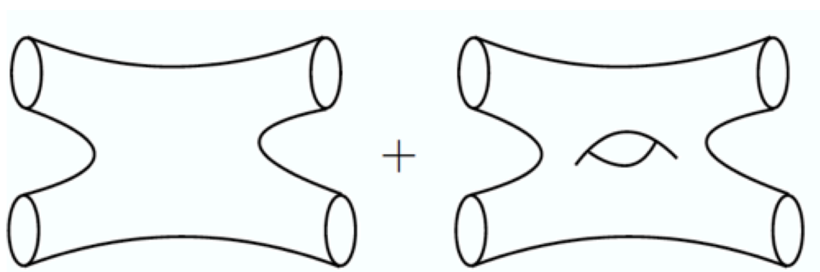 $+$

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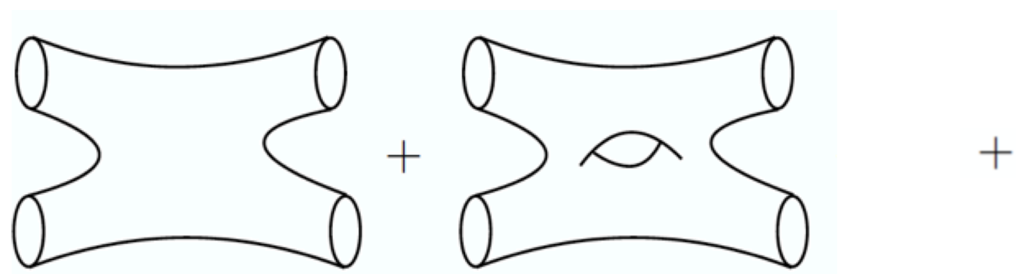
Genus expansion



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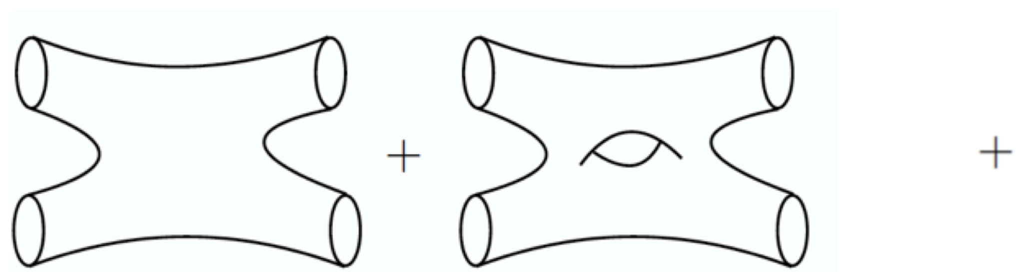

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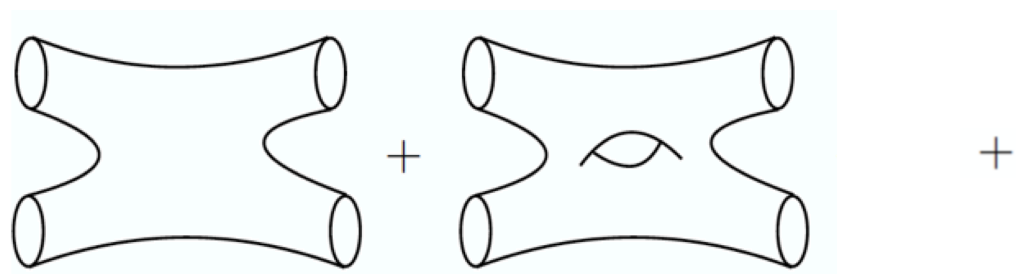

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$p = 2$	Graviton
$p \geq 3$	Kaluza Klein modes

Expansion

$$\langle \mathcal{O}_2(x_1)\mathcal{O}_2(x_2)\mathcal{O}_2(x_3)\mathcal{O}_2(x_4)\rangle = \frac{\mathcal{G}(u,v)}{x_{12}^4 x_{34}^4}$$

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Idea

Understand how to use the symmetries of the CFT (conformal symmetry, super symmetry, integrability....) to construct higher order correlators.

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Plan of the talk

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Other results and open problems

Method

Let us go back to the correlator, conformal symmetry fixes the four point function as

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$$\mathcal{G}(u,v,\sigma,\tau) = \sum_{\Delta,\ell,r} c^{2(r)}_{\Delta,\ell}\, g^{(r)}_{\Delta,\ell}(u,v)\, Y^{(r)}(\sigma,\tau)$$

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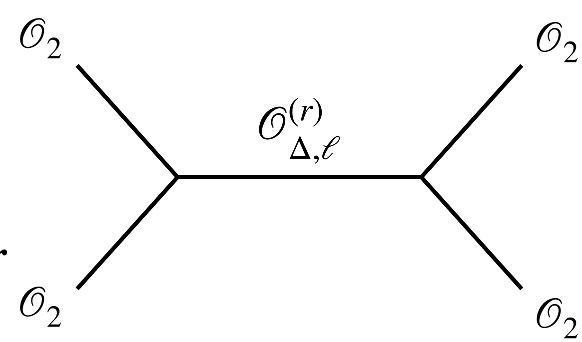

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Supersymmetry

Superconformal Ward Identities let us achieve two goals:

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Beem, Rastelli, van Rees 2010

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$\mathcal{G}(u, v, \sigma, \tau) \rightarrow \mathcal{G}^{short}(u, v)$ depend on N

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$$v^2\mathcal{G}^{short}(u,v) - u^2\mathcal{G}^{short}(v,u) + u^2 - v^2 = -\frac{u-v}{c} + v^2\mathcal{H}(u,v) + u^2\mathcal{H}(v,u)$$

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3) the function $\mathcal{H}(u,v)$ is decomposable in terms of superconformal blocks

$$\mathcal{H}(u,v) = \sum_{\Delta,\ell} a_{\Delta,\ell}\, g^{s}_{\Delta,\ell}(u,v) = \sum_{\Delta,\ell} a_{\Delta,\ell}\, u^{\frac{\Delta-\ell-4}{2}} g_{\Delta+4,\ell}(u,v)$$

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$$\Delta = \Delta^{(0)} + \frac{1}{N^2}\gamma^{(1)} + \frac{1}{N^4}\gamma^{(2)} + \cdots$$

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$\lambda \to \infty$

Double trace operarors

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Correlators

$$\mathcal{H}(u,v) = \sum_{\Delta,\ell} a_{\Delta,\ell} u^{\frac{\Delta-\ell-4}{2}} g_{\Delta+4,\ell}(u,v)$$

We would like to focus on the supergravity regime, which means that we need to expand all the ingredients in large N and λ $\quad c \sim N^2$

$$\boxed{\mathcal{H}(u,v)} = \mathcal{H}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{H}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{H}^{(2)}(u,v) + \cdots$$

$$\boxed{\Delta_{ST} \to \lambda^{1/4}}$$

$$\boxed{\Delta} = \Delta^{(0)} + \frac{1}{N^2}\gamma^{(1)} + \frac{1}{N^4}\gamma^{(2)} + \cdots$$

$\lambda \to \infty$

Double trace operarors

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Leading term

The leading term is given by the disconnected diagram and by doing the OPE decomposition it is possible to see that only double trace operators contribute to this term.

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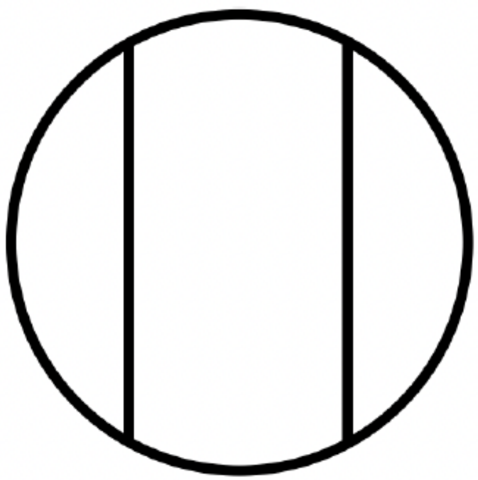

$$\Delta^{(0)} = 4 + 2n + \ell$$

Double traces: $\mathcal{O}_2 \, \Box^n \, \partial_\ell \mathcal{O}_2$

Inversion Formula

It is possible to write a relation that invert the OPE allowing us to reconstruct the correlator by knowing only its singularities as $v \to 0$ or $\bar{z} \to 1$

Caron Huot 2017

Simmons-Duffin Stanford Witten 2017

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analytic continuation around $\bar{z} \to 1$

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kernel

double discontinuity

$$c_{\Delta,\ell} \xrightarrow[\Delta \to \Delta_k]{} \frac{a_{\Delta_k,\ell}}{\Delta - \Delta_k}$$

has poles at the dimension of the exchanged operator with residue the square of the three point function

Caron Huot 2017

Simmons-Duffin Stanford Witten 2017

Tree Level

We expand at leading order N^{-2} and we get

D'Hoker, Freedman, Mathur, Matusis, Rastelli 1999

Arutyunov Frolov 2000

Alday, Caron Huot 2018

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$$\mathcal{H}^{(1)}(u,v) = \sum_{n,\ell} u^{2+n} \left(a^{(1)}_{n,\ell} + \frac{1}{2} a^{(0)}_{n,\ell} \gamma^{(1)}_{n,\ell} \left(\log u + \frac{\partial}{\partial n} \right) \right) g_{4+2n+\ell,\ell}(u,v)$$

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$$v^2 \mathcal{G}^{short}(u,v) - u^2 \mathcal{G}^{short}(v,u) + u^2 - v^2 = -\frac{u-v}{c} + v^2 \mathcal{H}(u,v) + u^2 \mathcal{H}(v,u)$$

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Caveat:

$$\mathscr{H}^{(1)}(u,v) = \sum_{n,\ell} u^{2+n}\left(a_{n,\ell}^{(1)} + \frac{1}{2}a_{n,\ell}^{(0)}\gamma_{n,\ell}^{(1)}\left(\log u + \frac{\partial}{\partial n}\right)\right) g_{4+2n+\ell,\ell}(u,v)$$

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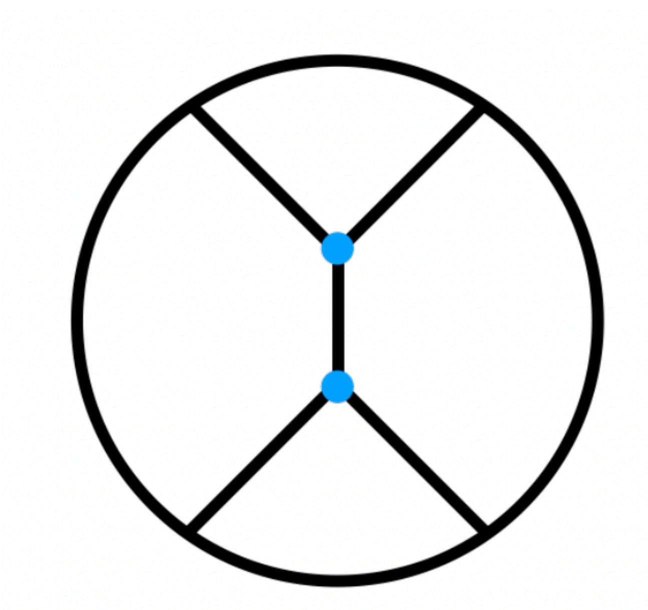

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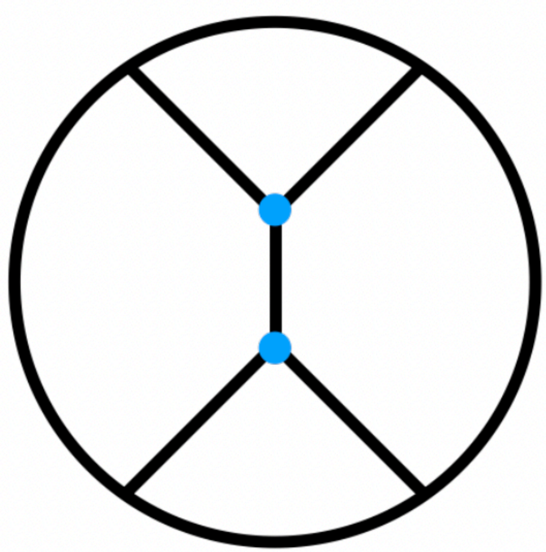

completely fixed by the knowledge of the protected part of the correlator and the leading order data

One Loop

At one loop the situation is different, mainly for two reasons:

Aprile, Drummond, Heslop, Paul 2017

Alday, AB 2017

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completely specified by tree level data!

Aprile, Drummond, Heslop, Paul 2017

Alday, AB 2017

Mixing

Caveat: mixing between different operators with the same bare dimension and quantum numbers.

$$\sum_{n,\ell} a^{(0)}_{n,\ell} \qquad \sum_{n,\ell} a^{(0)}_{n,\ell}\gamma^{(1)}_{n,\ell} \qquad \sum_{n,\ell} a^{(0)}_{n,\ell}\left(\gamma^{(1)}_{n,\ell}\right)^2$$

Alday, Zhou 2019 2020

Aprile, Drummond, Heslop, Paul 2017 2018 2019

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This mixing can be solved by considering all the four point functions of the type

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Alday, Zhou 2019 2020

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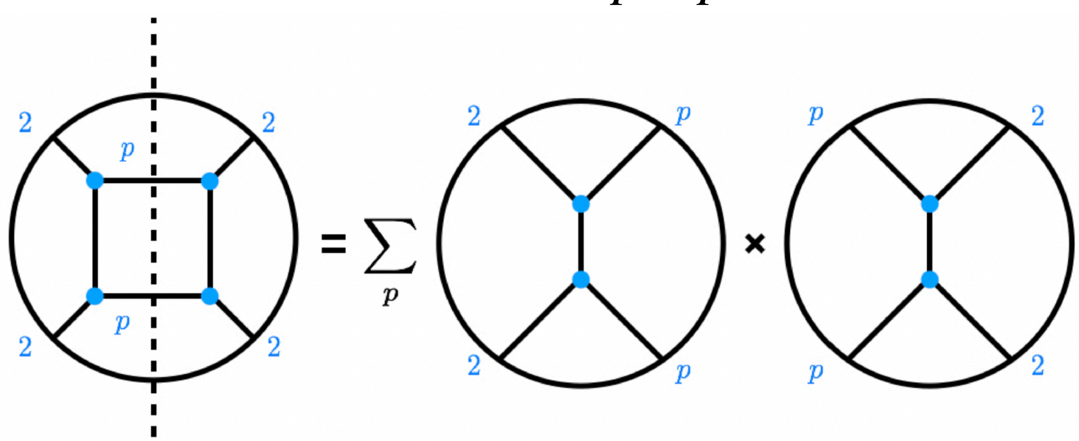

Alday, Zhou 2019 2020

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All Loops

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2) There are further mixing problems to take into account and it becomes unfeasible.

However...

Drummond, Paul 2022

Huang, Ye Yuan 2021

Caron-Huot, Trinh 2018
Aprile, Drummond, Heslop, Paul 2018
AB, Fardelli, Georgoudis 2020

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- Two loops: OPE reasoning + educated ansatz for the $\mathscr{H}^{(3)}(u, v)$

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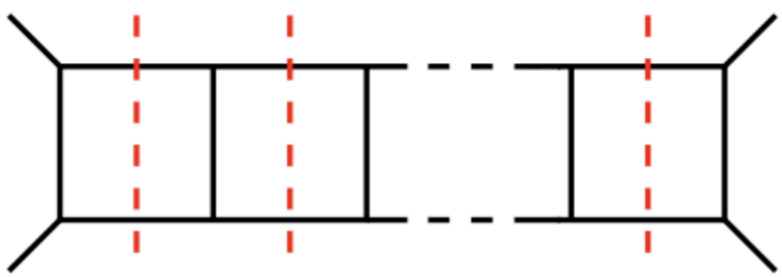

s-channel consecutive cuts

Caron-Huot, Trinh 2018
Aprile, Drummond, Heslop, Paul 2018
AB, Fardelli, Georgoudis 2020

Stringy corrections

How do we take care of the α' (or λ) expansion?

Aprile, Drummond, Paul, Santagata 2021
Drummond, Paul, Santagata 2020
Drummond, Glew, Paul 2020
Aprile, Drummond, Glew, Santagata 2022
Alday, AB, Perlmutter 2018
Drummond, Paul 2019
Drummond, Nandan, Paul, Rigatos 2019

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$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{G}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{G}^{(2)}(u,v) + \dots$$

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The procedure to bootstrap higher loops is similar to the leading terms, but there is a disruptive difference!

Aprile, Drummond, Paul, Santagata 2021
Drummond, Paul, Santagata 2020
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Stringy corrections

We can always add **crossing symmetric** solution to our $\mathscr{H}^{(1)}(u, v)$

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Why?

Stringy corrections

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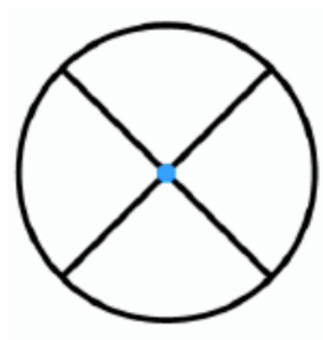
- they have support only for finitely many spins.

CAVEAT: Since crossing symmetry and the inversion formula do not give any useful information, the coefficient in front of such solutions is completely arbitrary.



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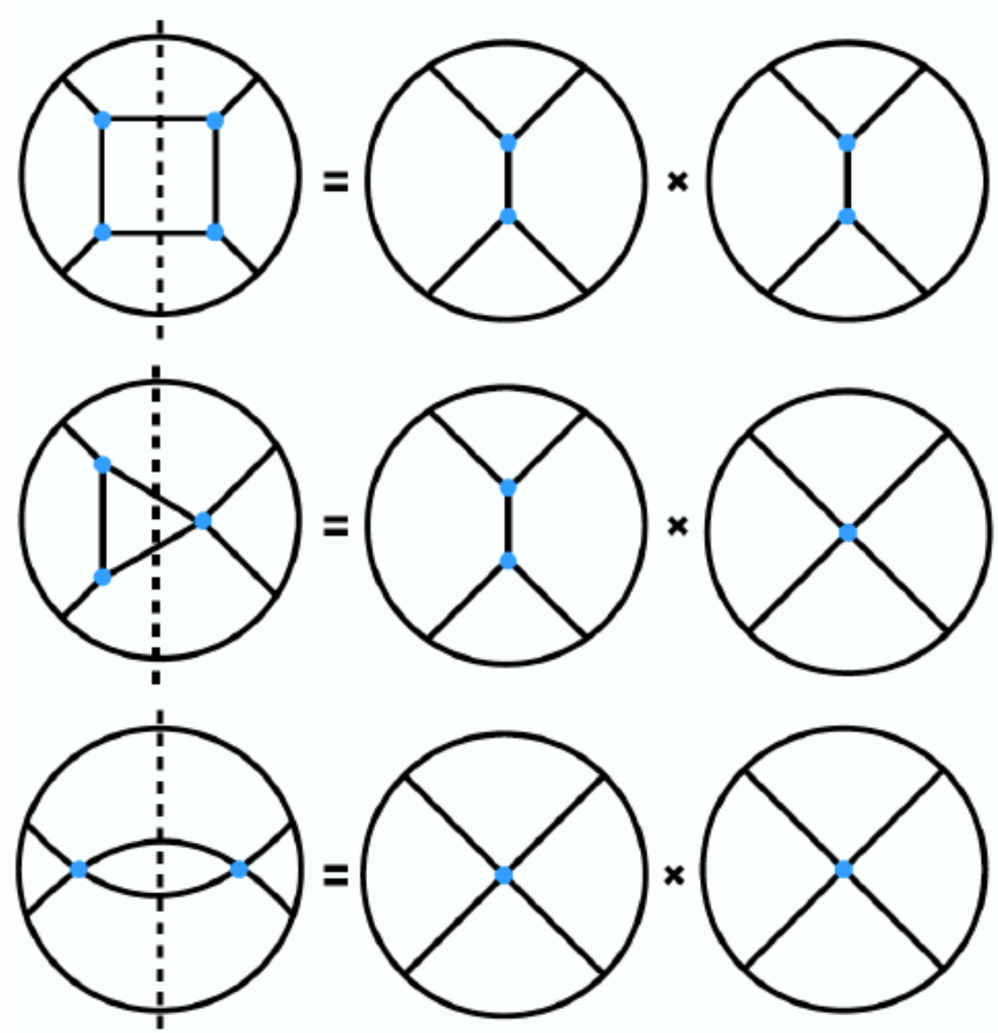

$\lambda^{-3/2}$

λ^{-3}

Fixing coefficients

- Agreement with the flat space limit

$$\frac{\Gamma[-\frac{\alpha' s}{4}]\Gamma[-\frac{\alpha' t}{4}]\Gamma[-\frac{\alpha' u}{4}]}{\Gamma[1+\frac{\alpha' s}{4}]\Gamma[1+\frac{\alpha' t}{4}]\Gamma[1+\frac{\alpha' u}{4}]}$$

- Supersymmetric localization

Binder, Chester, Pufu, Yang
Chester

Abl, Heslop, Lipstein

- Bound on chaos + integrability + single valued

Alday, Hansen, Silva

structure of the Virasoro Shapiro
amplitude in curved space!

see Tobias Hansen's talk!

Gluon amplitudes

To consider gluons, we need to add D-branes

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4d $\mathcal{N} = 2$

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Scalar superconformal primary, half- BPS operator of $\Delta = 2$

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2) proliferation of colour structures

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One loop gluon amplitudes: Alday, AB, Zhou 2021

Two loops gluon amplitudes: Huang, Wang, Yuan, Zhou 2023

Double copy:

Zhou 2021

AB, Fardelli, Manenti, Zhou 2022

Drummond, Glew, Santagata 2022

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how to relate amplitudes of gluons with amplitudes of graviton in curved AdS space.

Zhou 2021

AB, Fardelli, Manenti, Zhou 2022

Drummond, Glew, Santagata 2022

Some other results

- Kaluza Klein modes

  Aprile, Drummond, Heslop, Paul 2017 …
  Alday, Zhou 2020
  Rastelli, Zhou 2016, 2017

- Higher point functions

  Goncalves, Pereira, Zhou 2019
  Goncalves, Meneghelli, Pereira, Vilas Boas, Zhou 2023

- Higher trace operators

  AB, Fardelli, Manenti 2022
  Ma, Zhou 2022

- Other backgrounds

  Ceplak, Giusto, Huges, Russo 2021

Open problems

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Resum the large N series?
e.g. eikonal

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Caron-Huot, Coronado, Trinh, Zahraee, 2022

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Vertices in string amplitudes
for higher genus

Thank you!