Generalized Symmetries for Generalized Gravitons

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Ministerio de Ciencia e Innovación
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Based on:

- [Benedetti, PB, Magán]
  arXiv:2304.XXXXX

Outline

1. Generalized symmetries and region algebras
2. Generalized symmetries of linearized gravity
    2.1. Einstein gravity in $D = 4$
    ⋆ **2.2. Einstein gravity in $D \geq 5$**
    ⋆ **2.3. Higher-curvature gravities**
3. Conclusions and plans

1. Generalized symmetries and region algebras

Region algebras in QFT

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$$\mathcal{A}_{\text{add}}(R') \subseteq (\mathcal{A}_{\text{add}}(R))' \quad [\text{causality}] \quad \text{which implies} \quad \mathcal{A}_{\text{add}}(R) \subseteq \mathcal{A}_{\max}(R)$$

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- From von Neumann's double commutant theorem it is easy to prove that

$$\mathcal{A}_{\rm max}(R') = \mathcal{A}_{\rm add}(R') \vee \{b\}$$

where $\{b\}$ are non-locally generated operators in the causal complement R'.

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- The charged operators are supported on $(p-1)$-dimensional manifolds

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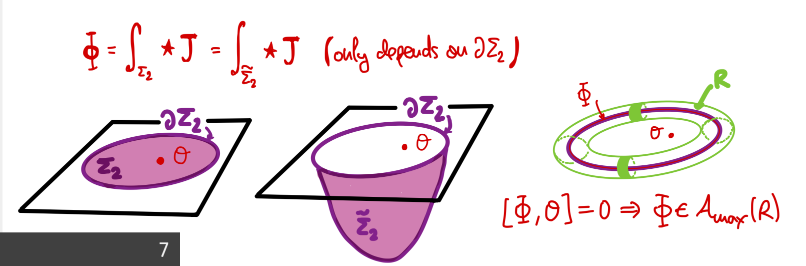

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  $\Rightarrow$ Generalized symmetries always come in pairs

- Furthermore, if $e^{ig\Phi}$ is charged under a continuous non-compact symmetry group, there exists a $(D-p)$-form current $\tilde{J}$ such that $d\star\tilde{J}=0$ and $\star\tilde{J}\neq dG$ with G a physical field such that [Benedetti, Casini, Magan]

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2. Generalized symmetries of linearized gravity

Motivations

- Conserved charges in gravity

  [(Iyer, Lee), Wald; Komar; Bondi, Metzner, Sachs; Arnowitt, Deser, Misner; Regge, Teitelboim...]

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- Explicit realizations of the dual-pairs principle

  [Benedetti, Casini, Magan]

2.1. Einstein gravity in $D = 4$

Linearized Einstein gravity

Linearized perturbations on Minkowski spacetime

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,, \quad ||h_{\mu\nu}|| \ll 1\,, \quad h_{[\mu\nu]} = 0\,, \quad h \equiv \eta^{\mu\nu} h_{\mu\nu}$$

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- The Einstein gravity action reduces to the Fierz-Pauli one

$$S_{\text{EH}} = \frac{1}{16\pi G}\int \mathrm{d}^D x \sqrt{|g|} R \quad \Rightarrow \quad S_{\text{FP}} = \frac{1}{16\pi G}\int \mathrm{d}^D x \left[\left(1 + \frac{h}{2}\right) R^{(1)} + R^{(2)}\right] .$$

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- Gauge symmetry-like invariance ($\Leftrightarrow$ linearized diffeomorphisms)

$$h_{\mu\nu} \to h_{\mu\nu} + 2\partial_{(\mu}\xi_{\nu)}$$

We look for p-form conserved currents for this theory

Linearized Einstein gravity

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- The Riemann tensor is the generator of the gauge-invariant algebra
- Our currents should be formed from contractions of $\{R_{\mu\nu\rho\sigma}, \eta_{\mu\nu}, \varepsilon_{\mu_1\dots\mu_D}\}$
- It is useful and illuminating to use the dual Riemann tensor

$$R^{*}_{\mu_1\dots\mu_{D-2}\alpha\beta} \equiv \frac{1}{2}\,\varepsilon_{\mu_1\dots\mu_{D-2}\lambda\sigma}\,R^{\lambda\sigma}{}_{\alpha\beta}\,.$$

Linearized Einstein gravity

The on-shell curvatures satisfy a series of properties

$R_{\mu\nu\alpha\beta} = -R_{\nu\mu\alpha\beta} = -R_{\mu\nu\beta\alpha}$	[Skew Symmetry]
$R_{\mu\nu\alpha\beta} = R_{\alpha\beta\mu\nu}$	[Interchange Symmetry]
$\eta^{\mu\alpha} R_{\mu\nu\alpha\beta} = 0$	[Einstein Equation]
$\varepsilon^{\mu_1\cdots\mu_{D-3}\alpha\beta\gamma} R_{\alpha\beta\gamma\nu} = 0$	[1st Bianchi identity]
$\varepsilon^{\mu_1\cdots\mu_{D-3}\alpha\beta\gamma} \partial_\alpha R_{\beta\gamma\mu\nu} = 0$	[2nd Bianchi identity]
$\partial^\mu R_{\mu\nu\alpha\beta} = 0$	[Einstein Equation]

$R^*_{\mu_1\mu_2\cdots\mu_{D-2}\alpha\beta} = -R^*_{\mu_2\mu_1\cdots\mu_{D-2}\alpha\beta} = \dots$	[Levi-Civita skew symmetry]
$R^*_{\mu_1\mu_2\cdots\mu_{D-2}\alpha\beta} = -R^*_{\mu_1\mu_2\cdots\mu_{D-2}\beta\alpha}$	[Riemann skew symmetry]
$\eta^{\gamma\alpha} R^*_{\gamma\mu_1\cdots\mu_{D-3}\alpha\beta} = 0$	[1st Bianchi identity]
$\varepsilon^{\mu_1\mu_2\cdots\mu_{D-1}\beta} R^*_{\mu_1\mu_2\cdots\mu_{D-1}\alpha} = 0$	[Einstein Equation]
$\varepsilon^{\mu_1\mu_2\cdots\mu_{D-1}\beta} R^*_{\alpha\mu_1\mu_2\cdots\mu_{D-1}} = 0$	[Einstein Equation]
$\partial^\gamma R^*_{\gamma\mu_1\cdots\mu_{D-3}\alpha\beta} = 0$	[2nd Bianchi identity]
$\partial^\beta R^*_{\mu_1\cdots\mu_{D-2}\alpha\beta} = 0$	[Riemman conservation]
$\varepsilon^{\mu_1\mu_2\cdots\mu_{D-1}\gamma} \partial_{\mu_1} R^*_{\mu_2\mu_3\cdots\mu_{D-1}\alpha\beta} = 0$	[Riemman conservation]
$\varepsilon^{\nu_1\nu_2\cdots\nu_{D-3}\alpha\beta\gamma} \partial_\gamma R^*_{\mu_1\mu_2\cdots\mu_{D-2}\alpha\beta} = 0$	[2nd Bianchi identity]

Generalized symmetries for $D = 4$ Einstein gravitons

In $D = 4$, one finds the following conserved two-forms

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$$B_{\mu\nu} \equiv R_{\mu\nu\alpha\beta}\, (x^{\alpha} b^{\beta} - x^{\beta} b^{\alpha})\,, \qquad \text{[4 independent]}$$

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- This makes a total of 20 independent conserved two-forms in $D = 4$.

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- This makes a total of 20 independent conserved two-forms in $D = 4$.
- Integrating these charges on $\Sigma_2 \Leftrightarrow$ non-locally generated flux operators on ring-like regions $\Leftrightarrow$ violations of duality for regions with non-trivial π_1.

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⋆ 2.2. Einstein gravity in $D \geq 5$

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  - Some of the tilded and some of the untilded are no longer conserved

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B_{\mu\nu} &\equiv R_{\mu\nu\alpha\beta}\,(x^\alpha b^\beta - x^\beta b^\alpha)\,, && [D] \\
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- The dual-pairs principle would suggest that $D(D+1)(D+2)/6$ dual currents should exist...

Generalized symmetries in $D \geq 5$ Einstein gravity

- However, only $\{\tilde{A}, \tilde{B}\}$ are conserved: $\mathrm{d} \star \tilde{A} = \mathrm{d} \star \tilde{B} = 0$,

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\tilde{B}_{\mu_1\mu_2\ldots\mu_{D-2}} &\equiv R^{*}_{\mu_1\mu_2\ldots\mu_{D-2}\alpha\beta}\, (x^{\alpha}\tilde{b}^{\beta} - x^{\beta}\tilde{b}^{\alpha})\,, && [D]
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  which does not vanish for $D \geq 5$ ...
- This would be fine if we did not know about the dual-pairs principle...
- Either we are missing tilded currents, or some of the untilded ones in fact become exact in $D \geq 5$

- One can try to construct modified versions of $\tilde{C}, \tilde{D}$ exploiting the failure of the Bianchi identity.

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- This is precisely the case! It turns out that the $\{A, C\}$ currents become exact for $D \geq 5$

$$\star A = \mathrm{d} \star \mathcal{A}\,,$$
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$$\mathcal{A}_{\mu\nu\rho} \sim -R^*_{\mu\nu\rho\alpha_1\ldots\alpha_{D-3}}\, \tilde{a}^{\alpha_1\ldots\alpha_{D-3}\sigma}\, x_\sigma\,,$$
$$\mathcal{C}_{\mu\nu\rho} \sim R^*_{\mu\nu\rho\alpha_1\ldots\alpha_{D-3}} \left(\frac{1}{2}\, \tilde{c}^{\alpha_1\ldots\alpha_{D-3}}\, x^2 + \frac{\eta^{\alpha_1\ldots\alpha_{D-3}}_{\beta_1\ldots\beta_{D-3}}}{(D-4)!}\, c^{\beta_1\ldots\beta_{D-4}\sigma}\, x^{\beta_{D-3}}\, x_\sigma \right)\,.$$

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These relations are not true in $D = 4$ ($\mathcal{A}, \mathcal{C}$ are not skew-symmetric differential forms in that case).

Generalized symmetries for $D \geq 5$ Einstein gravitons

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- Hence, the dual-pairs principle prevails. We have $D(D+1)/2$ conserved 2-form currents and $D(D+1)/2$ conserved $(D-2)$-form currents for a total of $D(D+1)$ generalized symmetries
- The right set of conserved currents is $\{B, D, \tilde{A}, \tilde{B}\}$,

$$
\begin{aligned}
B_{\mu\nu} &\equiv R_{\mu\nu\alpha\beta}\left(x^\alpha b^\beta - x^\beta b^\alpha\right), && [D]\\
D_{\mu\nu} &\equiv R_{\mu\nu\alpha\beta}\left(x^\alpha d^{\beta\gamma}x^\gamma - x^\beta d^{\alpha\gamma}x^\gamma + \frac{1}{2}d^{\alpha\beta}x^2\right), && [D(D-1)/2]\\
\tilde{A}_{\mu_1\mu_2\ldots\mu_{D-2}} &\equiv R^*_{\mu_1\mu_2\ldots\mu_{D-2}\alpha\beta}\,\tilde{a}^{\alpha\beta}, && [D(D-1)/2]\\
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\end{aligned}
$$

Generalized symmetries for $D \geq 5$ Einstein gravitons

- Hence, the dual-pairs principle prevails. We have $D(D+1)/2$ conserved 2-form currents and $D(D+1)/2$ conserved $(D-2)$-form currents for a total of $D(D+1)$ generalized symmetries
- The right set of conserved currents is $\{B, D, \tilde{A}, \tilde{B}\}$,

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\end{aligned}$$

- The associated generalized charges read

$$\Phi = \int_{\Sigma_{D-2}} \star\left(B + D\right), \quad \Psi = \int_{\Sigma_2} \star\left(\tilde{A} + \tilde{B}\right).$$

3. Conclusions and plans

Guided by the dual-pairs principle, we have constructed new conserved p-form currents for linearized Einstein gravity in general dimensions. The corresponding fluxes are associated to operators which violate duality in regions with non-trivial π_1 and $\pi_{(D-3)}$ (rings and their complements).

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- Break symmetries. Fractons?

WHAT?

SCHOOL ON ENTANGLEMENT IN QFT

WHERE?

ICC UNIVERSITY OF BARCELONA

WHEN?

JUNE 19 - JUNE 23

WHO ARE THE LECTURERS?

- HORACIO CASINI
- STEFAN HOLLANDS
- VERONIKA HUBENY
- SERGEY SOLODUKHIN

⋆ 2.3. Higher-curvature gravities

Generalization ⇒ linearization of higher-curvature gravities $\mathcal{L}(R_{\mu\nu\rho\sigma}, g^{\mu\nu})$.

- For a Minkowski background, the most general theory with non-trivial linearized equations involves a general quadratic modification of the Einstein-Hilbert term. The modified FP action reads

$$S_{\mathrm{FP}} + \frac{1}{16\pi G}\int \mathrm{d}^D x \left[\alpha_1 R^2_{(1)} + \alpha_2 R^{(1)}_{\mu\nu} R^{\mu\nu}_{(1)} + \alpha_3 R^{(1)}_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma}_{(1)}\right]$$

- The linearized equations read [PB, Cano, Min, Visser]

$$\left(1 - \frac{\partial^2}{m_g^2}\right) R_{\mu\nu} - \Delta_{\mu\nu} R = 0\,, \quad \text{where} \quad \Delta_{\mu\nu} \equiv \frac{1}{2}\eta_{\mu\nu}\left[1 - \frac{\partial^2}{m_g^2}\right] + \frac{(D-2)(m_g^2 - m_s^2)}{2(D-1)m_s^2 m_g^2}\left[\partial_\mu\partial_\nu - \eta_{\mu\nu}\partial^2\right]\,,$$

where we defined

$$\alpha_1 \equiv \frac{(D-2)m_g^2 + D m_s^2}{4(D-1)m_s^2\, m_g^2} + \alpha_3\,, \qquad \alpha_2 \equiv -\frac{1}{m_g^2} - 4\alpha_3\,,$$

- Metric perturbation ⇔ usual transverse graviton + spin-0 massive mode + spin-2 massive mode: $\partial^2 h^T_{\mu\nu} = 0\,, \quad (\partial^2 - m_s^2)\phi = 0\,, \quad (\partial^2 - m_g^2)h^M_{\mu\nu} = 0.$

Let us focus on $D = 4$.

- Some of the tilded currents are identical to the Einstein gravity ones, and they remain conserved, namely, $\mathrm{d}\star\tilde{A} = \mathrm{d}\star\tilde{B} = 0$

$$\tilde{A}_{\mu_1\mu_2\ldots\mu_{D-2}} \equiv R^{*}_{\mu_1\mu_2\ldots\mu_{D-2}\alpha\beta}\,\tilde{a}^{\alpha\beta}\,, \quad [6] \qquad \tilde{B}_{\mu_1\mu_2\ldots\mu_{D-2}} \equiv R^{*}_{\mu_1\mu_2\ldots\mu_{D-2}\alpha\beta}\,(x^{\alpha}\tilde{b}^{\beta} - x^{\beta}\tilde{b}^{\alpha})\,, \quad [4]$$

- Natural to expect 10 additional untilded charges. However, the Riemann tensor is neither traceless nor divergenceless anymore... Modified Riemann tensor

$$J_{\mu\nu\alpha\beta} \equiv \left[1 - \frac{\partial^2}{m_g^2}\right] R_{\mu\nu\alpha\beta} + \Delta_{\mu\beta}R_{\nu\alpha} - \Delta_{\mu\alpha}R_{\nu\beta} + \Delta_{\nu\alpha}R_{\mu\beta} - \Delta_{\nu\beta}R_{\mu\alpha}\,,$$

shares symmetries of Riemann and divergenceless. With this: $\mathrm{d}\star A = \mathrm{d}\star B = 0$,

$$A_{\mu\nu} = J_{\mu\nu\alpha\beta}\,a^{\alpha\beta}\,, \quad [6] \qquad C_{\mu\nu} = J_{\mu\nu\alpha\beta}\,c^{\alpha\beta\gamma}x_{\gamma}\,, \quad [4]$$

- These makes again a total of 20 conserved currents.

$$Q = \int_{\Sigma_2} (\tilde{A} + \tilde{B} + A + C)$$