Extremal Kerr entropy in higher-derivative gravities

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based on arXiv:2303.13286 w/ **Marina David**

Eurostrings 2023 - Gijón
April 24 2023

Higher-derivative corrections: EFT of quantum gravity

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Corrections to **black hole entropy** as a probe for quantum gravity (Alejandro's talk)

$$\text{EFT} \quad S = \frac{A}{4G} + \ldots \quad \longleftrightarrow \quad \text{QG origin} \quad S = \log N_{\text{micro}}$$

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In this talk: corrections to the entropy of the **Kerr black hole**

Corrections to Kerr BH entropy:

Introduction

Corrections to Kerr BH entropy: only known at **first order** Reall, Santos '19

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We focus on **extremal black holes** $\longrightarrow$ near-horizon geometries Sen '05; Astefanesei, Goldstein, Jena, Sen, Trivedi '06, ...

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Goals

- Corrections to the extremal Kerr entropy beyond leading order
- Rigorous computation of the BH angular momentum

Charges from Komar integrals

$$I = \int d^D x \sqrt{|g|} \mathcal{L}\left(g^{\mu\nu}, R_{\mu\nu\alpha\beta}\right)$$

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$$\mathcal{E}_{\mu\nu} = R_{\mu}{}^{\sigma\alpha\beta} P_{\nu\sigma\alpha\beta} - \frac{1}{2}\mathcal{L} g_{\mu\nu} + 2\nabla^{\alpha}\nabla^{\beta} P_{\mu\alpha\nu\beta}\,, \quad P_{\mu\nu\alpha\beta} = \frac{\partial \mathcal{L}}{\partial R^{\mu\nu\alpha\beta}}$$

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Given a Killing vector ξ we have a **Noether current** Iyer, Wald '94

$$\boldsymbol{J}_{\xi} = \mathrm{d}\mathbf{Q}_{\xi} + 2\boldsymbol{\epsilon}_{\boldsymbol{\mu}} \xi^{\nu} \mathcal{E}^{\mu}{}_{\nu}$$

Noether charge $(D-2)$**-form**

$$\mathbf{Q}_{\xi} = -\boldsymbol{\epsilon}_{\mu\nu}\left(\bar{P}^{\mu\nu\alpha\beta}\nabla_{\alpha}\xi_{\beta} + 2\nabla_{\beta}\bar{P}^{\mu\nu\alpha\beta}\xi_{\alpha}\right)\,, \quad \bar{P}_{\mu\nu\alpha\beta} = P_{\mu\nu\alpha\beta} - P_{\mu[\nu\alpha\beta]}$$

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On-shell $\mathrm{d}\boldsymbol{J}_{\xi} = 0$, so we have a conserved quantity

$$Q_{\xi} = \int_{V_{D-1}} \boldsymbol{J}_{\xi} = \int_{\mathbb{S}^{D-2}_{\infty}} \mathbf{Q}_{\xi}$$

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Observe: integration always at infinity, because $\mathrm{d}\boldsymbol{Q}_{\xi}=\boldsymbol{J}_{\xi}=-\frac{1}{2}\star\boldsymbol{\xi}\mathcal{L}\neq 0$

Charges from Komar integrals

Define the **Noether-Komar charge-$(D-2)$ form** Kastor '08; Ortín '21

$$\tilde{\mathbf{Q}}_{\xi} = \mathbf{Q}_{\xi} - \mathbf{\Omega}\,, \quad \mathrm{d}\mathbf{\Omega} = -\frac{1}{2}\star\xi\mathcal{L}$$

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$\tilde{Q}_\xi$ is the Noether charge if $\int_{\mathbb{S}^{(D-2)}_\infty} \mathbf{\Omega} = 0 \longrightarrow$ compute charge from integral on the horizon $\mathcal{H}$

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Black hole entropy: Iyer-Wald formula

$$S = -2\pi \int_{\mathcal{H}} d^{D-2}x\sqrt{h}P^{\mu\nu\alpha\beta}\epsilon_{\mu\nu}\epsilon_{\alpha\beta}$$

String-inspired gravity

Heterotic string effective action on $\mathbb{T}^6$ at first order in α'

Bergshoeff, de Roo '89; PAC, Ruipérez '21

$$I_{\text{heterotic}} = \frac{1}{16\pi G}\int d^4x\sqrt{|g|}\left[R - \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}e^{2\varphi}(\partial\psi)^2 + \frac{\alpha'}{8}\left(e^{-\varphi}\mathcal{X}_4 - \psi R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}\right)\right]$$

where φ is the dilaton, ψ is the dual of $B_{\mu\nu}$ and

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$\Longrightarrow$ Stabilize the dilaton

$$I_{\text{stabilized}} = \frac{1}{16\pi G}\int d^4x\sqrt{|g|}\left[R - 2\Lambda - \frac{1}{2}(\partial\psi)^2 - \frac{\alpha'}{8}\psi R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma} + \frac{\alpha'}{8}\mathcal{X}_4\right]$$

General EFT of a shift-symmetric pseudoscalar field coupled to gravity (dynamical Chern-Simons gravity)

String-inspired gravity

Equations of motion

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}=-\frac{\alpha'}{2}\nabla^{\rho}\nabla^{\sigma}\left(\tilde{R}_{\rho(\mu\nu)\sigma}\psi\right)+\frac{1}{2}\left(\partial_\mu\psi\partial_\nu\psi-\frac{1}{2}g_{\mu\nu}\left(\partial\psi\right)^2\right).$$

$$\nabla^2\psi=\frac{\alpha'}{8}R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}$$

Observe

- ψ corrected at $\mathcal{O}(\alpha')$, $g_{\mu\nu}$ corrected at $\mathcal{O}(\alpha'^2)$
- Entropy also corrected at $\mathcal{O}(\alpha'^2)$ $\longrightarrow$ cannot apply Reall and Santos method

Near-horizon extremal geometry with $\mathrm{SO}(2,1)\times\mathrm{U}(1)$ symmetry

$$ds^2 = g(y)\left(-r^2dt^2+\frac{dr^2}{r^2}\right)+\frac{a^2}{f(y)}dy^2+f(y)(d\phi-2ardt)^2\,,$$

$$y = y_0\cos\theta\,,\quad \phi\sim\phi+\frac{2\pi}{\omega}$$

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Determined by $f(\pm y_0)=0$, $f'(\pm y_0)=\mp 2a\omega$

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Explicit solution $(\Lambda=0)$

$$\psi=0$$

$$g=a^2(1+y^2)$$

$$f=\frac{1-y^2}{1+y^2}$$

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Explicit solution $(\Lambda = 0)$

$$\psi = 0-\frac{\alpha'}{2a^2}\left(\arctan y+\frac{y\left(y^4+2y^2-7\right)}{2\left(y^2+1\right)^3}\right)$$

$$g = a^2(1+y^2)+\frac{\alpha'^2}{a^2}\left[\frac{5y\arctan y}{8}+\frac{4095y^{10}+19285y^8+35014y^6+35610y^4-5861y^2+241}{6720\left(y^2+1\right)^5}\right]$$

$$f = \frac{1-y^2}{1+y^2}-\frac{\alpha'^2}{a^4}\left[\frac{y\left(3y^2+43\right)\arctan y}{32\left(y^2+1\right)^2}\right.$$
$$\left.+\frac{\left(315y^{12}+5495y^{10}+21966y^8+31030y^6+51263y^4+963y^2+1192\right)}{3360\left(y^2+1\right)^7}\right]$$

Entropy and angular momentum

$$S = -2\pi \int_{\mathcal{H}} d^{D-2}x\sqrt{h}P^{\mu\nu\alpha\beta}\epsilon_{\mu\nu}\epsilon_{\alpha\beta} = \frac{2\pi a^2}{G} + \alpha' \frac{\pi}{2G} - \alpha'^2 \frac{\pi\,(805\pi + 2846)}{4480 a^2 G}$$

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$$J = -2\int_{\Sigma}\left[\epsilon_{\mu\nu}\left(\bar{P}^{\mu\nu\alpha\beta}\nabla_{\alpha}\xi_{\beta} + 2\nabla_{\beta}\bar{P}^{\mu\nu\alpha\beta}\xi_{\alpha}\right) + \mathbf{\Omega}\right] = \frac{a^2}{G} - \alpha'^2\frac{469 + 150\pi}{1920a^2G}$$

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The relation $S(J)$ reads

$$\boxed{S = 2\pi J + \alpha' \frac{\pi}{2G} - \alpha'^2 \frac{\pi}{JG^2}\left(\frac{493}{3360} + \frac{3\pi}{128}\right) + O(\alpha'^3)}$$

Observations:

- $\mathcal{O}(\alpha')$ correction is topological
- $\mathcal{O}(\alpha'^2)$ is negative: effect of truncating the dilaton?

EFT expansion of the Einstein-Hilbert action

$$\mathcal{L} = R + \mathcal{L}_{(4)} + \mathcal{L}_{(6)} + \mathcal{L}_{(8)} + \ldots$$

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- Four-derivative corrections $\mathcal{L}_{(4)}$ are topological in $D = 4$ (GB density $\mathcal{X}_4$)
- Six-derivative corrections $\mathcal{L}_{(6)}$: only two independent densities

$$R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\alpha\beta} R_{\alpha\beta}{}^{\mu\nu} \,, \quad R^{\mu\nu}{}_{\alpha\beta} R^{\alpha\beta}{}_{\rho\sigma} \tilde{R}^{\rho\sigma}{}_{\mu\nu}$$

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- Alternative choice

$$\mathcal{P} = 12R_{\mu}{}^{\rho}{}_{\nu}{}^{\sigma}R_{\rho}{}^{\alpha}{}_{\sigma}{}^{\beta}R_{\alpha}{}^{\mu}{}_{\beta}{}^{\nu} + R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\alpha\beta}R_{\alpha\beta}{}^{\mu\nu} - 12R_{\mu\nu\rho\sigma}R^{\mu\rho}R^{\nu\sigma} + 8R_{\mu}{}^{\nu}R_{\nu}{}^{\rho}R_{\rho}{}^{\mu}$$

$$\tilde{\mathcal{P}} = 7R^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta}{}_{\rho\sigma}\tilde{R}^{\rho\sigma}{}_{\mu\nu} - \frac{9}{2}RR_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma} - 20\tilde{R}^{\mu\nu\rho\sigma}R_{\mu\rho}R_{\nu\sigma}$$

$\mathcal{P} \longrightarrow$ Einsteinian cubic gravity Bueno, PAC '16
$\tilde{\mathcal{P}} \longrightarrow$ Parity-violating version of ECG

EFT expansion of the Einstein-Hilbert action

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$$\mathcal{P} = 12 R_{\mu\ \nu}^{\ \rho\ \sigma} R_{\rho\ \sigma}^{\ \alpha\ \beta} R_{\alpha\ \beta}^{\ \mu\ \nu} + R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\alpha\beta} R_{\alpha\beta}{}^{\mu\nu} - 12 R_{\mu\nu\rho\sigma} R^{\mu\rho} R^{\nu\sigma} + 8 R_{\mu}{}^{\nu} R_{\nu}{}^{\rho} R_{\rho}{}^{\mu}$$

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$\tilde{\mathcal{P}} \longrightarrow$ Parity-violating version of ECG

$$I_{\text{cubic}} = \frac{1}{16\pi G} \int d^4x \sqrt{|g|} \left[R + \lambda \mathcal{P} + \tilde{\lambda} \tilde{\mathcal{P}} \right]$$

Near-horizon extremal geometries

$$ds^2_{N,f} = (a^2 + x^2)\left(-r^2dt^2 + \frac{dr^2}{r^2}\right) + \frac{dx^2}{f(x)} + N(x)^2 f(x)\left(d\phi + 2ardt\right)^2$$

$$x_1 \leq x \leq x_2\,, \quad \phi \sim \phi + \frac{2\pi}{\omega}$$

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$$x_1 \leq x \leq x_2\,, \quad \phi \sim \phi + \frac{2\pi}{\omega}$$

Key property of these theories: the EOM are solved by $N = 1$.

Einstein equations $\mathcal{E}_{\mu\nu}$ reduce to

$$\mathcal{E}_{\phi\phi}\Big|_{N=1} = f(x)^2 \mathcal{E}_{xx}\Big|_{N=1} = \frac{x^2 f(x)}{(a^2 + x^2)} \frac{d}{dx} \mathcal{E}_f(f, f', f'', x)$$

Six-derivative EFT of gravity

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We only have to solve

$$\mathcal{E}_f(f,f',f'',x) = n$$

n related to breaking of equatorial symmetry

Six-derivative EFT of gravity

Boundary conditions

The solution contains 5 parameters $\{a, x_1, x_2, \omega, n\}$. These are fixed by the BC

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We find

$$x_i - \frac{a^2}{x_i} + \frac{8\lambda\omega^2(3 \mp 2x_i\omega)}{x_i} \mp \frac{48a\tilde{\lambda}\omega^3}{x_i} = n$$

$$\left(a^2 + x_i^2\right)^2 (1 \pm 2x_i\omega) + 24\lambda\omega^2 \left(x_i^2 \pm a^2(4x_i\omega \mp 1)\right) + 48a\tilde{\lambda}\omega^2 \left(x_i \pm \omega(a^2 - x_i^2)\right) = 0$$

Exact relations to be satisfied $\longrightarrow$ 1 free parameter

Six-derivative EFT of gravity

Entropy and angular momentum

S and J computed through integrals on x. Remarkably, these are total derivatives

$$S = \int_{x_1}^{x_2} dx \frac{d}{dx}\mathcal{S}(f, f', x) = \mathcal{S}(f, f', x)\Big|_{x_1}^{x_2}$$

$$J = \int_{x_1}^{x_2} dx \frac{d}{dx}\mathcal{J}(f, f', x) = \mathcal{J}(f, f', x)\Big|_{x_1}^{x_2}$$

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$$S = \frac{\pi}{2\omega G}\left[x_2 - x_1 + 24\lambda\omega^2\left(\frac{x_1}{a^2+x_1^2} - \frac{x_2}{a^2+x_2^2}\right) + 24\tilde{\lambda}\omega^2\left(\frac{a}{a^2+x_1^2} - \frac{a}{a^2+x_2^2}\right)\right]$$

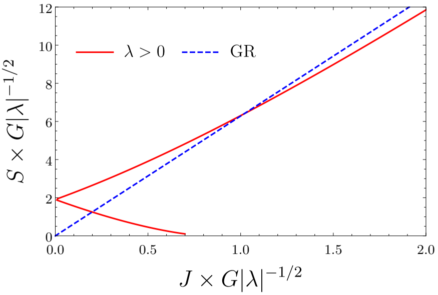
$$J = \frac{2a^2+x_1^2+x_2^2}{8aG\omega} - \frac{2\lambda\omega\left(5a^4+2a^2\left(x_1^2+x_2^2\right)-x_1^2x_2^2\right)}{aG\left(a^2+x_1^2\right)\left(a^2+x_2^2\right)} + \frac{6\tilde{\lambda}\omega(x_1+x_2)\left(a^2+x_1x_2\right)}{G\left(a^2+x_1^2\right)\left(a^2+x_2^2\right)}$$

The relation $S(J)$: perturbative expansion

$$S = 2\pi|J|\left[1 - \frac{\lambda}{(GJ)^2} + \frac{20\lambda^2 + 9\tilde{\lambda}^2}{2(GJ)^4} - \frac{184\lambda^3 + 117\lambda\tilde{\lambda}^2}{2(GJ)^6} + O\left(J^{-8}\right)\right]$$

- Leading correction coincides with Reall and Santos '19
- We get the subleading corrections. Parity-violating terms enter at order $\tilde{\lambda}^2$
- All the corrections are positive if $\lambda < 0$

The relation $S(J)$: Non-perturbative aspects



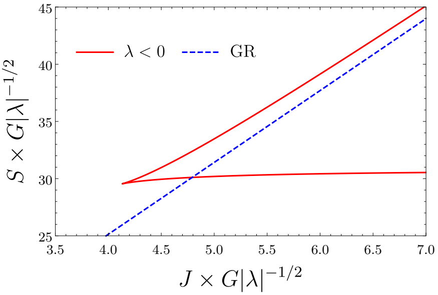


FIGURE: The relation $S(J)$ in the presence of the even-parity cubic correction. Left: $\lambda > 0$, right: $\lambda < 0$. The GR prediction $S = 2\pi|J|$ is shown for comparison.

For $\lambda < 0$ we have the bound

$$|J| > 4.1\frac{\sqrt{|\lambda|}}{G}$$

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- First-principles computation of angular momentum from Komar integrals
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- Cubic gravity: obtained exact result. Parity-violating terms affect the entropy
- Open problems: mass, extremality bound, global geometry
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Thank you for your attention

Angular momentum

$$J = -2\int_{\Sigma}\left[\epsilon_{\mu\nu}\left(\bar{P}^{\mu\nu\alpha\beta}\nabla_{\alpha}\xi_{\beta} + 2\nabla_{\beta}\bar{P}^{\mu\nu\alpha\beta}\xi_{\alpha}\right) + \mathbf{\Omega}\right]\,, \quad \xi^{\mu}\partial_{\mu} = \frac{1}{\omega}\partial_{\phi}$$

$$d\mathbf{\Omega} = -\frac{1}{2}\star\xi\mathcal{L}\,, \quad \int_{\mathbb{S}^2_{\infty}}\mathbf{\Omega} = 0$$

Bonus

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On the NH geometry, this leads to the choice

$$\mathbf{\Omega}=-\frac{ar}{2\omega}dt\wedge dy\,g(y)\mathcal{L}$$

Bonus

AdS case $\Lambda = -\frac{3}{L^2}$

The relation $S(J)$ can be studied parametrically $J(\zeta)$, $S(\zeta)$

$$J = \frac{L^2 \zeta \left(\zeta^2 - 1\right)}{G\left(3 - \zeta^2\right)^2}$$

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$$+\frac{8925+71155\zeta^2-104972\zeta^4+75970\zeta^6+7161\zeta^8+4235\zeta^{10}+630\zeta^{12}}{20160\left(-3+\zeta^2\right)\left(-1+\zeta^2\right)\left(1+\zeta^2\right)\left(-3+6\zeta^2+\zeta^4\right)}$$
$$\left.+\frac{\left(-166+675\zeta^2-1009\zeta^4+550\zeta^6+112\zeta^8+23\zeta^{10}+7\zeta^{12}\right)\arctan\zeta}{192\zeta\left(-3+\zeta^2\right)\left(-1+\zeta^2\right)\left(-3+6\zeta^2+\zeta^4\right)}\right\}$$

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For large black holes $J \gg L^2/G$

$$S(J) = \pi\left(\frac{2L^2 J}{\sqrt{3}G}\right)^{1/2}\left[1-\frac{\alpha'^2}{L^4}\left(\frac{2949}{2240}+\frac{107\pi}{72\sqrt{3}}\right)\right]$$

The relation $S(J)$: AdS black holes

Reminder: for GR we have

$$J_0=\frac{L^2\zeta\left(\zeta^2-1\right)}{G\left(3-\zeta^2\right)^2}\,,\quad S_0=\frac{\pi L^2\left(\zeta^2-1\right)}{G\left(3-\zeta^2\right)}$$

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With corrections we can set $J(\zeta) = J_0(\zeta)$ and we get

$$S = S_0\left[1 - \frac{16\lambda\zeta^2\left(\zeta^2-3\right)^2\left(\zeta^4+13\zeta^2-12\right)}{L^4\left(\zeta^2-1\right)^2\left(\zeta^6+7\zeta^4+3\zeta^2-3\right)} + \frac{64\zeta^2\left(\zeta^2-3\right)^4\lambda^2}{L^8\left(\zeta^4-1\right)^4\left(\zeta^4+6\zeta^2-3\right)^3}\left(\zeta^{18} \right.\right.$$
$$+70\zeta^{16} - 582\zeta^{14} - 746\zeta^{12} + 13444\zeta^{10} + 2946\zeta^8 - 34650\zeta^6 + 33282\zeta^4$$
$$\left.\left.-13797\zeta^2 + 2592\right) + \frac{1159\tilde{\lambda}^2\zeta^2\left(3-5\zeta^2\right)^2\left(\zeta^2-3\right)^4}{L^8\left(\zeta^4-1\right)^4\left(\zeta^4+6\zeta^2-3\right)} + \ldots\right]$$