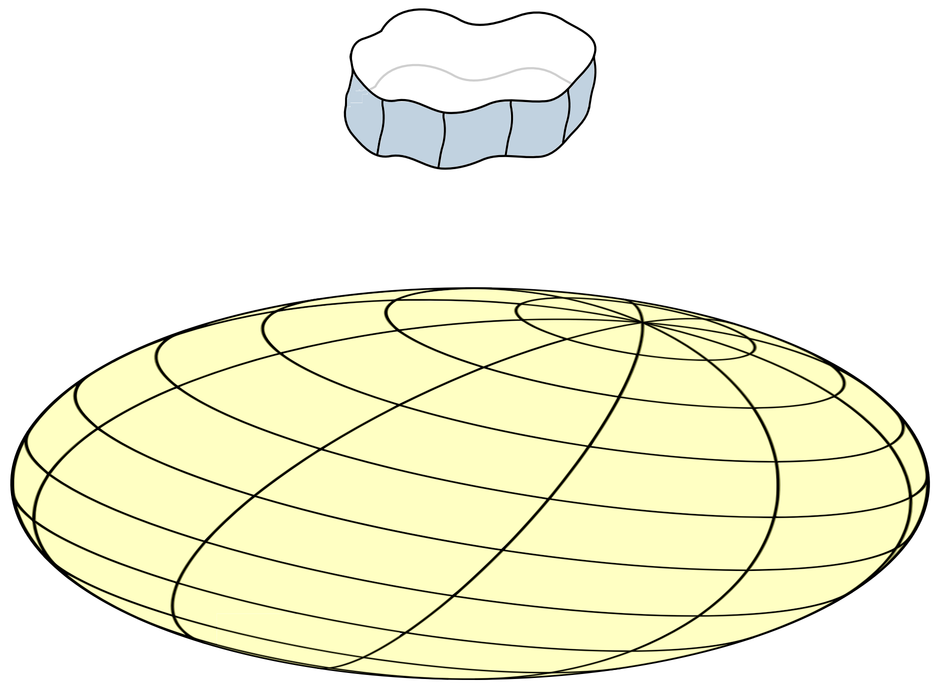
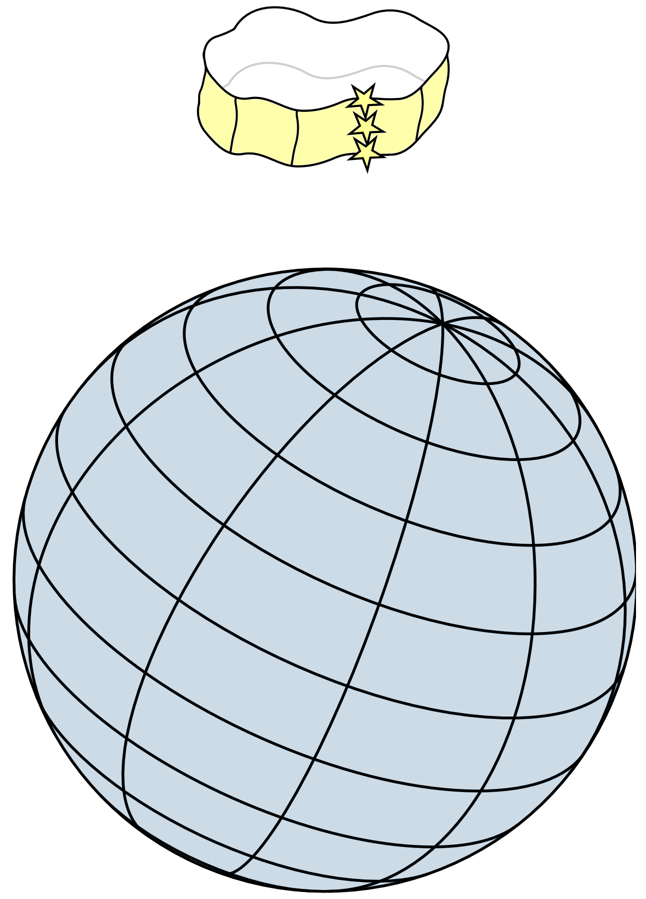
Eurostrings 2023, Gijón/Xixón

28th of April

Jordanian deformations of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstring





Sibylle Driezen

based on 2112.12025; 2207.14748; 2212.11269 with

Riccardo Borsato; J. Luis Miramontes; Juan Miguel Nieto García and Leander Wyss

ETH zürich

Motivation and setting

Why integrable deformations of worldsheet sigma-models

Integrability program proven to be very successful for **strongly coupled gauge-theories** and **strings in background fields**

→ tailored toolbox of analytic techniques (based on hidden symmetries) to derive exact results

→ rigid proofs for e.g. $\mathrm{AdS}_5 \times \mathrm{S}^5 \leftrightarrow \mathcal{N} = 4$ SYM

✦ **holography**: deform to AdS/CFT systems with **less manifest symmetries** (non-conformal, non-maximally susy) whilst preserving the ability to apply **integrability** program

✦ **more formally**: — generalise the integrable **toolbox** (underlying hidden structures at play)

— whilst being rare: **explore family** of integrable models and their mathematical formulation

Motivation and setting

Today: whole zoo of integrable deformations of worldsheet sigma-models

η-deformations

[Klimcik '02, '08; Delduc, Magro, Vicedo '13 …]

λ-deformations

[Sfetsos '13; Hollowood, Miramontes, Schmidtt '14 …]

$T\bar{T}$-deformations

[Smirnov, Zamolodchikov '16;
Cavaglia, Negro, Szécsényi, Tateo '16]

multi-parameter deformations

[Delduc, Georgiou, Hoare, Kameyama, Klimcik, Lacroix, Magro, Sfetsos, Siampos, Thompson, Vicedo, …]

…

a lot known about their SUGRA embedding [Borsato, Demulder, SD, Hassler, Hoare, Seibold, Sfetsos, Tseytlin, Thompson, Van Tongeren, Wulff,…]

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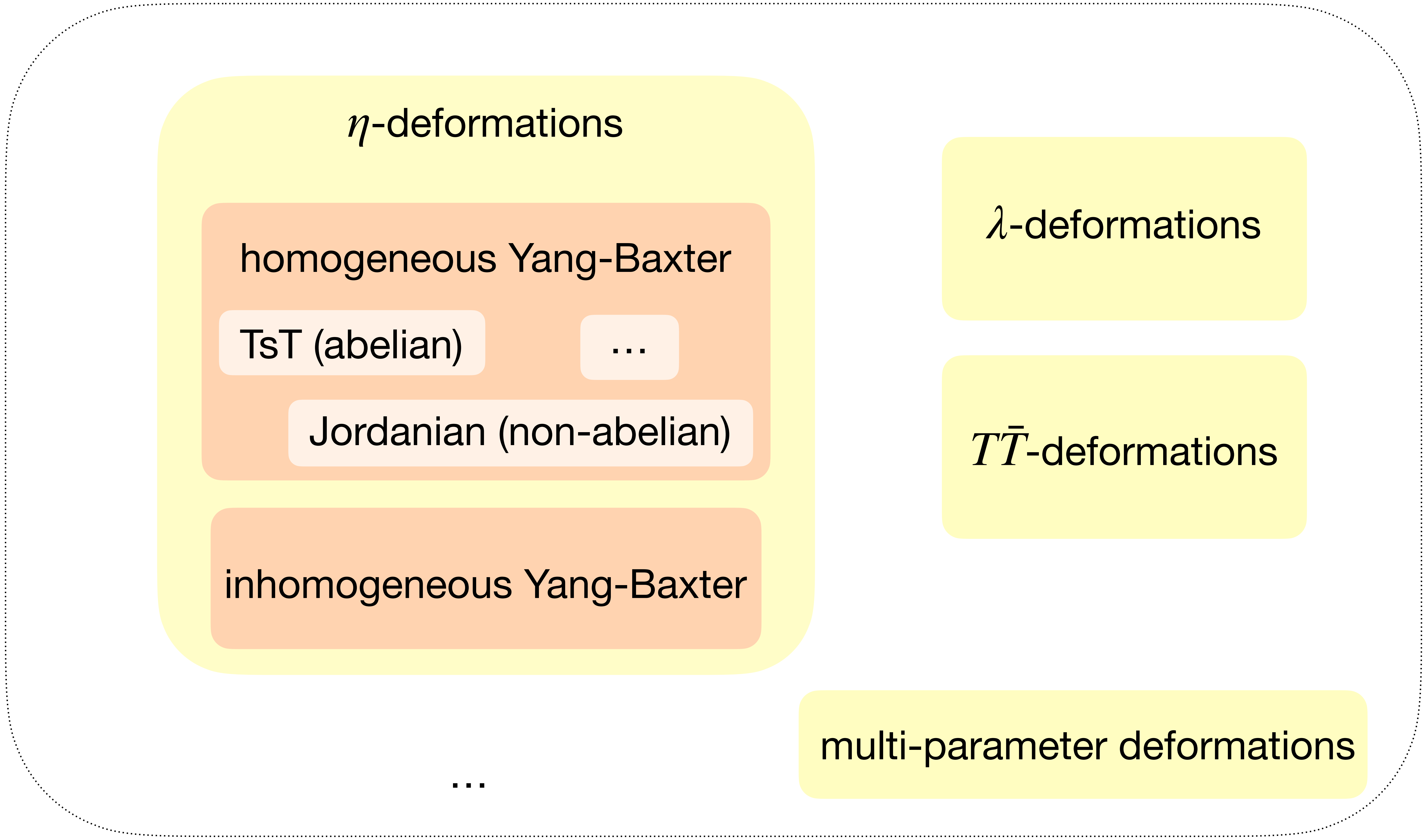


a lot known about their SUGRA embedding

Motivation and setting

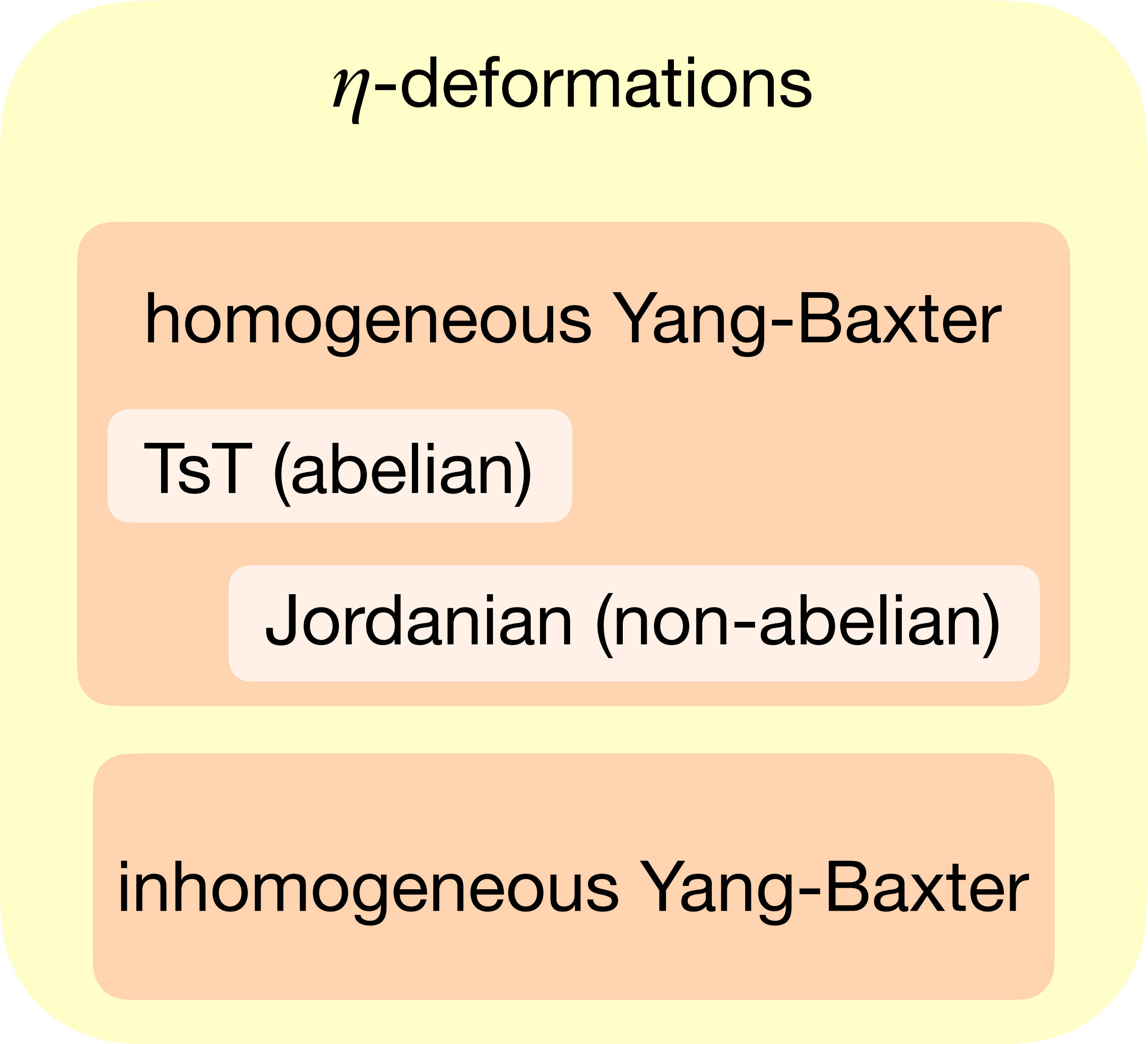
Extension of integrability methods developed for $\mathrm{AdS}_5 \times \mathrm{S}^5 \leftrightarrow \mathcal{N}=4$ **SYM to deformations?**

spectral problem known from first-principles for restricted class of **“diagonal” TsT**

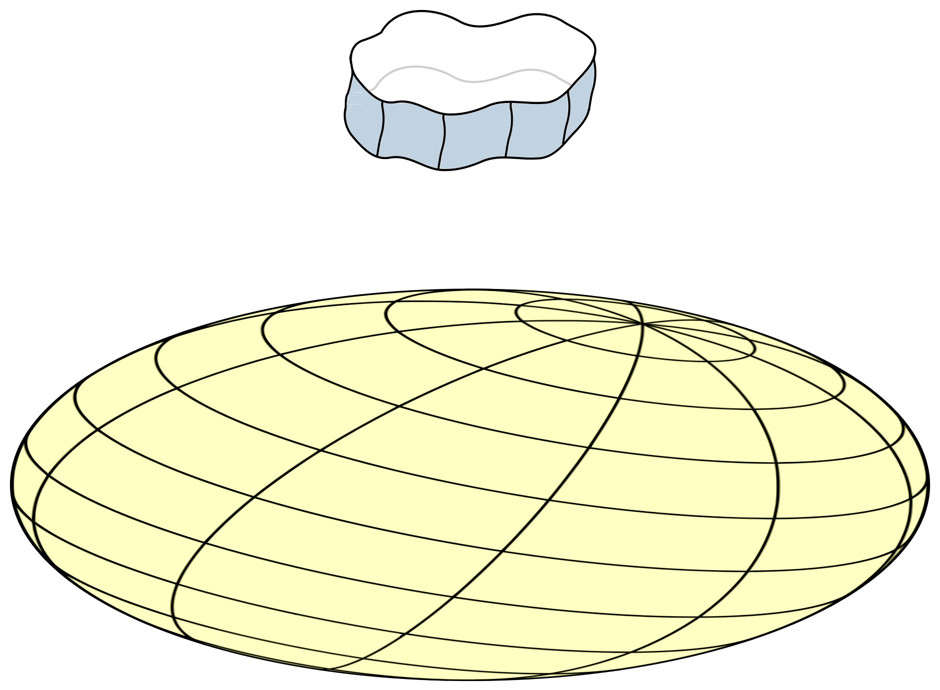
[Beisert, Roiban ’05; Gromov, Levkovich-Maslyuk ’10; de Leeuw, Van Tongeren ’12; Kazakov ’18…]

based on reformulation of deformed model as
undeformed model with **local twisted boundary conditions**

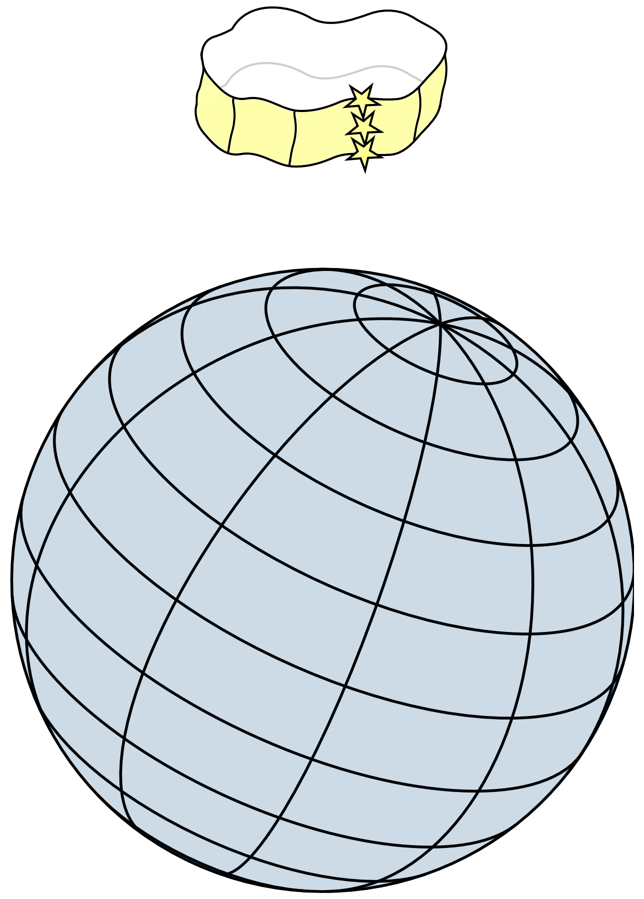
[Frolov, Roiban, Tseytlin ’05; Alday, Arutyunov, Frolov ’05]



Outline



- ✦ Brief intro to Homogeneous Yang-Baxter deformations
- ✦ Reformulation in terms of **undeformed twisted** models
- ✦ **Jordanian** deformations and why we care
- ✦ Classification of Jordanian deformations for $\mathrm{AdS}_5 \times \mathrm{S}^5$ preserving **preserving $D = 10$ IIB SUGRA**
- ✦ Spectral curve methods



Homogeneous Yang-Baxter (HYB) deformations

$g = g(X^\mu(\sigma^\alpha)) \in G = \mathrm{Lie}(\mathfrak{g})$

$\eta \in \mathbb{R}$

$R : \mathfrak{g} \to \mathfrak{g} : T_a \to R(T_a) = R_a{}^b T_b$

$$S = \int d^2\sigma \, \langle \partial_+ g g^{-1}, (1 - \eta R)^{-1} \partial_- g g^{-1} \rangle$$ **HYB of Principal Chiral Model**

$$[Rx, Ry] - R([Rx, y] + [x, Ry]) = 0$$ **classical Yang-Baxter equation**

$$R^T = -R$$ **antisymmetric**

$$R^{ab} f_{ab}{}^c = 0$$ **unimodularity**

— Preserves **worldsheet integrability** [Klimcik ’02, ’08; Delduc, Magro, Vicedo ’13] and **SUGRA** [Borsato, Wulff ’16]

— **TsT** = “abelian-R” ($\mathrm{Im}(R)$ is **abelian** subalgebra)

— $\mathrm{Im}(R)$ is **non-abelian** subalgebra of $\mathfrak{g}$

Twisted undeformed models

PCM		**HYB** (CYBE + $R^T = -R$)	
Currents	$\tilde{J}_\pm = \tilde{g}^{-1}\partial_\pm \tilde{g}$	Currents	$A_\pm = \mathrm{Ad}_g^{-1}(1 \pm \eta R)^{-1}\partial_\pm g g^{-1}$
Conserved (EOM) & Flat	$d \star \tilde{J} \approx 0 \quad \& \quad d\tilde{J} + \tilde{J} \wedge \tilde{J} = 0$	Conserved (EOM) & Flat	$d \star A \approx 0 \quad \& \quad dA + A \wedge A \approx 0$
Flat Lax connection	$\mathscr{L}_\pm(z) = \frac{\tilde{J}_\pm}{1 \pm z}, \quad z \in \mathbb{C}$	Flat Lax connection	$\mathscr{L}_\pm(z) = \frac{A_\pm}{1 \pm z}, \quad z \in \mathbb{C}$

(hallmark of classical integrability)

Twisted undeformed models

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Exploit on-shell equivalence:

$\tilde{J}_\pm[\tilde{g}] \approx A_\pm[g]$ $\Rightarrow$ relate $g = \mathscr{F}\tilde{g}$

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deformed periodic HYB

$g(2\pi) = g(0)$

Twisted undeformed models

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undeformed twisted PCM

$$\tilde{g}(2\pi) = W\tilde{g}(0) = \mathcal{F}^{-1}(2\pi)\mathcal{F}(0)\tilde{g}(0)$$

deformed periodic HYB

$$g(2\pi) = g(0)$$

$W[g]$: non-local expression [Matsumoto, Yoshida ’15; Vicedo ’15; Van Tongeren ‘18]

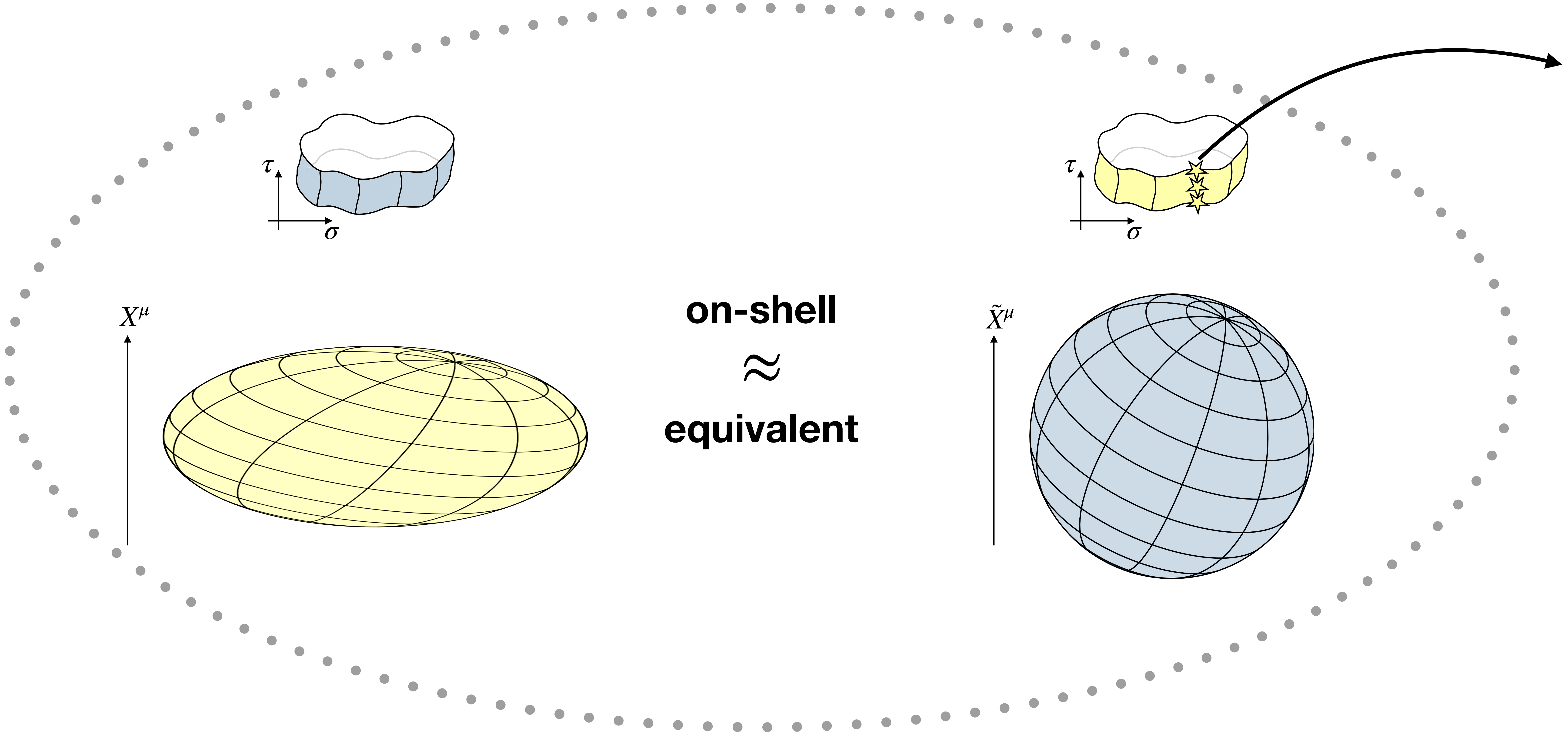
$W[\tilde{g}]$: local and closed expression [Borsato, SD, Miramontes ’21]

Twisted undeformed models

HYB-picture

PCM-picture

$$g(2\pi) = g(0) \quad \Rightarrow \quad \tilde{g}(2\pi) = W\tilde{g}(0),$$



$$W = e^{\eta R \mathbb{Q}[\tilde{g}]}$$

$$\partial_\tau W = 0$$

Jordanian deformations

When $\mathfrak{sl}(2,R)$ subalgebra of isometries $\quad [h, e_\pm] = \pm\, e_\pm \;,\; [e_+, e_-] = 2h \quad \Rightarrow \quad R(h) = \frac{e_+}{2},\; R(e_-) = -\,h,\; R(e_+) = 0$

Why

— truly **non-abelian**: $\mathrm{Im}(R) = \{h, e_+,$+ possibly supercharges$\}$ $\Rightarrow$ goes beyond TsT

— have a twist that is **always diagonal** [$\leftrightarrow$ abelian (TsT) & almost-abelian (sequence of TsT's)]

[Borsato, SD, Miramontes '21]

$\rightarrow$ important for **usability of twisted models** in semi-classical integrable methods

$$\Omega(z) = W^{-1}\,\mathrm{Pexp}\left(-\int \mathscr{L}(z)\right) = W^{-1}\left(1 + zQ\right) + \mathcal{O}(z^2)$$

has $\quad \partial_\tau \lambda(z) = 0$

✦ has **local** asymptotics $\Rightarrow$ **local charges** after going to diagonal or Jordan form

✦ has **diagonalisable** asymptotics $\Rightarrow$ allows to **reconstruct eigenvalues** of $\Omega(z)$ **on** $\mathbb{C}$ **for any sol.**

Long been "ignored" because **bosonic** R **is not unimodular**, i.e. $R^{ab} f_{ab}{}^c \neq 0 \;\Rightarrow$ **gSUGRA** ,

until [Van Tongeren '19] showing **inclusion of supercharges in** $\mathrm{Im}(R)$ can mitigate this, i.e. $\tilde{R}^{ab} f_{ab}{}^c = 0 \;\Rightarrow$ **SUGRA**

Classifying Jordanian models of $\mathrm{AdS}_5 \times S^5$

All (unimodular) Jordanian deformations of $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstring, $\mathfrak{g} = \mathfrak{psu}(2,2|4)$, classified in [Borsato, SD '22]

$$R = R^{ab} T_a \wedge T_b = h \wedge e_+ - \sum_{i=1}^{N} e_{-i} \wedge e_i$$

w.

$[h, e_+] = e_+,$ $\quad [h, e_{\pm i}] = (1/2 \pm \xi_i)\ e_{\pm i},$

$[e, e_{\pm i}] = 0,$ $\quad [e_i, e_j] = \delta_{i,-j} e$

CYBE (integrability) [Tolstoy '04]

$$\Rightarrow \quad R^{ab}[T_a, T_b] = 4(1 + N_0 - N_1)\ e_+ \quad \Rightarrow$$

unimodularity (SUGRA) if $N_1 = N_0 + 1$

N_0: # bosonic extensions; N_1: # fermionic extensions

Simplest case:

$N_0 = 0,$
$N_1 = 1$

Isometries	max. bosonic	supercharges
$g \to g_L g$	5 + 9	0
	3 + 4	4
	3 + 9	6
	5 + 4	8
$R = \mathrm{Ad}^{-1}_{g_L} R \mathrm{Ad}_{g_L}, \quad \mathrm{ad}_{T_{\tilde{a}}} R = R \mathrm{ad}_{T_{\tilde{a}}}$	5 + 9	12

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Spectral Curve methods

[Borsato, SD, Nieto, Wyss '22]

$$\Omega(z) = W^{-1}\,\mathrm{Pexp}\left(-\int \mathscr{L}(z)\right) = W^{-1}(1 + zQ) + \mathcal{O}(z^2)$$

has conserved eigenvalues $\lambda(z) = e^{ip(z)}$, quasimomenta $p(z) = \mathrm{diag}\{\hat{p}_1(z), \hat{p}_2(z), \hat{p}_3(z), \hat{p}_4(z) \,||\, \tilde{p}_1(z), \tilde{p}_2(z), \tilde{p}_3(z), \tilde{p}_4(z)\}$

$$\mathrm{AdS}_5 \sim \frac{SU(2{,}2)}{Sp(1{,}1)} \qquad S^5 \sim \frac{SU(4)}{Sp(2)}$$

Expansion around $z \sim 0$ to $\mathcal{O}(z)$ gives **local conserved charges**

$$\hat{p}_1(z) \sim -(E + Q_\Theta)z + \mathcal{O}(z^2) \qquad \hat{p}_2(z) \sim -\frac{i}{2}\mathbf{Q}_W + Q_\Theta z + \mathcal{O}(z^2)$$

$$\hat{p}_4(z) \sim (E - Q_\Theta)z + \mathcal{O}(z^2) \qquad \hat{p}_3(z) \sim +\frac{i}{2}\mathbf{Q}_W + Q_\Theta z + \mathcal{O}(z)$$

Evaluate $\Omega(z)$ and its eigenvalues $\lambda(z) = e^{ip(z)}$ on **simplest non-trivial classical solution** for field g

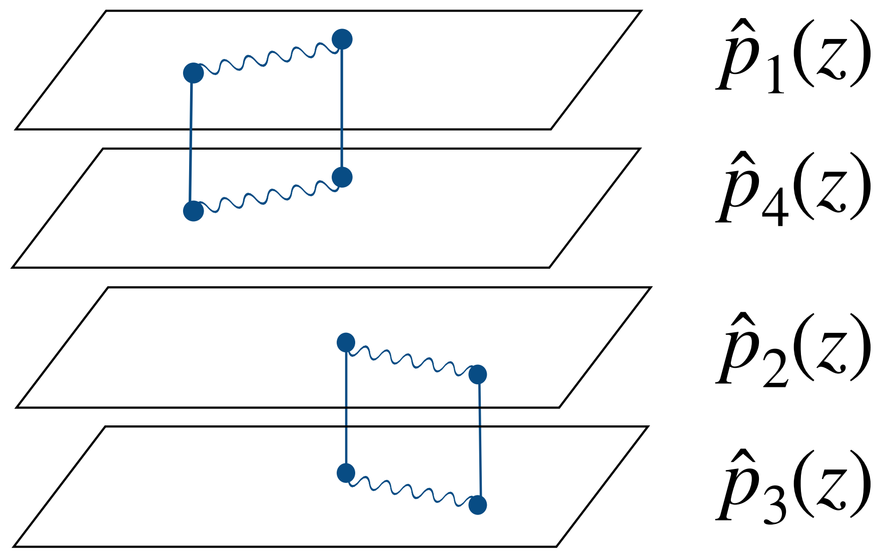
→ point-like in deformed picture "BMN-like"

$$T = \tau, \quad \dots$$

$$\hat{p}_1(z) = -\hat{p}_4(z) = \frac{2\pi\sqrt{z^2 - \eta^2}}{z^2 - 1},$$

$$\hat{p}_2(z) = -\hat{p}_3(z) = \frac{2\pi z\sqrt{1 - z^2\eta^2}}{z^2 - 1}$$

$E = 1, \quad \mathbf{Q}_W = 4\pi\eta, \quad Q_\Theta = 0$



Semi-classical corrections to the curve

Method of [Gromov, Vieira '07]: introduce quantum excitations in the form of microscopic cuts ~ quantum poles

$$p_i \to p_i + \delta p_i$$

Must fulfil number of analytical properties from the curve and from PSU(2,2|4)

- gluing conditions on cuts
- behaviour around poles of the Lax
- $\mathbb{Z}_4$ symmetry of PSU(2,2|4)
- asymptotic behaviour around $z \sim 0$

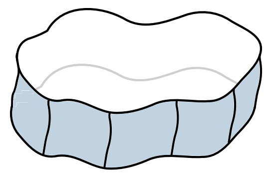
→ **restrictive enough to fix corrections completely**

$E_{1-\text{loop}} = E_{1-\text{loop}}(\eta)$ (6.39) and $\delta \mathbf{Q}_W = 0$

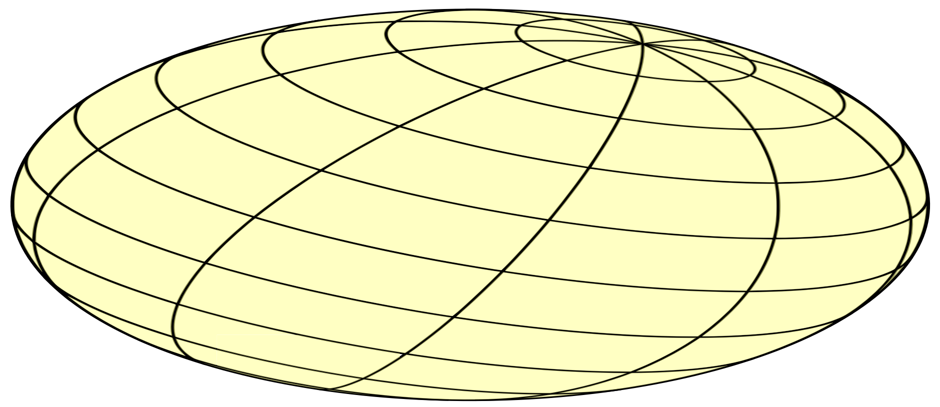
[Borsato, SD, Nieto, Wyss '22]

analogous to diagonal TsT / β-twisted SYM [Beisert, Roiban '05; de Leeuw, Van Tongeren '12…]

Conclusions and outlook



Homogeneous Yang-Baxter deformations as undeformed yet twisted models

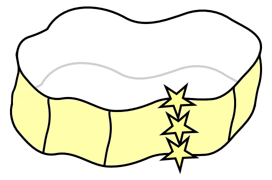


allows the use of the **classical spectral curve** method and its semi-classical quantisation unambiguously when the twist is **diagonal**

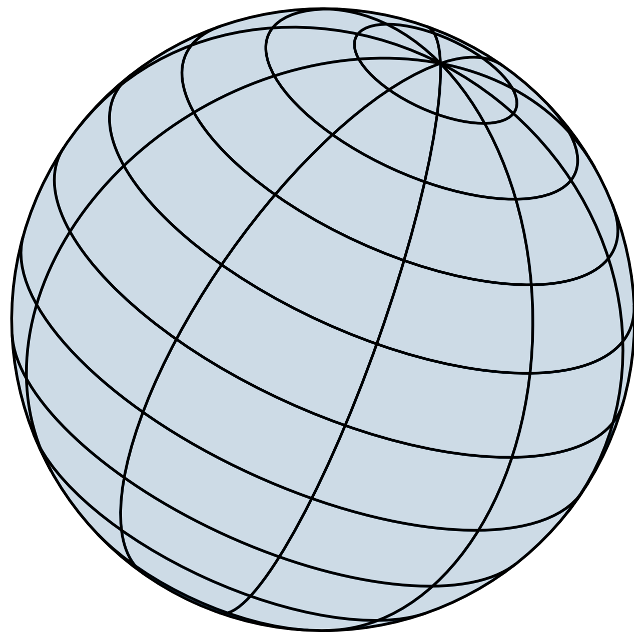
extend to the large web of dualities/deformations?

applied to a particular Jordanian deformation of $\mathrm{AdS}_5 \times \mathrm{S}^5$ to extract **1-loop correction to energy** of a BMN-like solution

relation and matching to (possible twisted spin-chain of) deformed $N = 4$ SYM side?
start up quantum integrable program and match with exact worldsheet results?



Classification of Jordanian deformations of $\mathrm{AdS}_5 \times \mathrm{S}^5$ which **preserve integrability and SUGRA**



gives many distinct backgrounds, possibly with wide range of applications to AdS/CFT

find well-behaved solutions
and use integrability to study non-symmetric backgrounds?