Entropy Functions for AdS Black Holes

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with

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Introduction

The microstate counting interpretation of black hole entropy S_{BH} is a major highlight of string theory

- Asymptotically flat black holes [Strominger,Vafa 96][…..]

- Asymptotically AdS black holes [Benini, Hristov, Zaffaroni 15][….]

Gravity: construct susy black holes/string solutions

Black hole horizon Σ : near horizon $AdS_2 \times \Sigma$ or $AdS_3 \times \Sigma$
Compute S_{BH} or c

Field theory: SCFT compactified on Σ - analyse IR behaviour using field theory techniques e.g. localistaion or anomaly polynomials

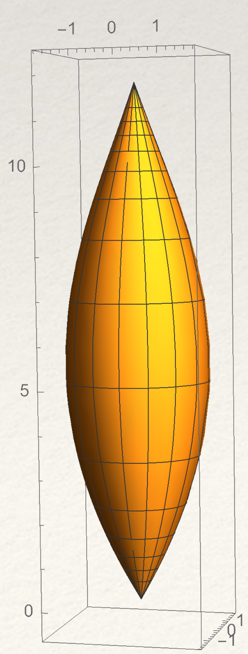
New approach on gravity side

- Focus on the near horizon susy AdS_2 and AdS_3 geometries. Develop extremisation framework to calculate S_{BH} and c without solving Einstein equations

- Much progress for $AdS_2 \times Y_9$ and $AdS_3 \times Y_7$ where Y_{2n+1} is a "GK geometry"

- Combined with field theory: state counting of entropy for infinite classes of black holes in $AdS_4 \times SE_7$ and an identification of d=2 SCFTs for infinite classes of black strings $AdS_5 \times SE_5$

- Black holes with spindle horizons $\Sigma(n_+, n_-)$:
  - -Arise as horizons of accelerating AdS black holes
  - -Susy preserved in a novel way
  - -Entropy expressed via gravitational blocks



GK Geometry $(Y_{2n+1}, g_{\mu\nu}, B, F_{\mu\nu})$ $n \geq 3$ [Gauntlett, Kim]

Type IIB

$$ds^2 = e^{B/2}[ds^2(AdS_3) + ds^2(Y_7)]$$
$$F_5 = vol(AdS_3) \wedge F + \ldots$$

[Kim]

- Sourced by D3-branes and dual to N=(0,2) SCFT in d=2 $U(1)_R$
- Near horizon limits of susy black strings in $AdS_5 \times SE_5$

D=11

$$ds^2 = e^{-2B/3}[ds^2(AdS_2) + ds^2(Y_9)]$$
$$G_4 = vol(AdS_2) \wedge F$$

[Kim,Park]

- Sourced by M2-branes and dual to N=2 SCQM with $U(1)_R$
- Near horizon limits of susy black holes in $AdS_4 \times SE_7$

Geometric extremal problem for GK [Couzens,Gauntlett,Martelli,Sparks 18]

Off shell GK geometry on Y_{2n+1}

- Impose susy: have Killing vector ξ and transverse Kahler J
- Impose flux quantisation
- Construct and extremise action: $S(\xi,[J])$

For $AdS_2 \times Y_9$ $\quad S(\xi_*) = S_{BH}$

i.e $S(\xi)$ is an off-shell entropy function!

For $AdS_3 \times Y_7$ $\quad S(\xi_*) = c_{SCFT}$

i.e $S(\xi)$ is an off-shell central charge!

Various ways to calculate $S(\xi,[J])$: especially in toric case

Sasaki-Einstein geometry

Type IIB $AdS_5 \times SE_5$ $F_5 = vol(AdS_5) + vol(SE_5)$

Dual to $\mathcal{N} = 1, d = 4$ SCFT: $a \propto \frac{1}{Vol(SE_5)}$

D=11 $AdS_4 \times SE_7$ $G_4 = vol(AdS_4)$

Dual to N=2 SCFT $\mathcal{N} = 2, d = 3:$ $F_{S^3} \propto \frac{1}{\sqrt{Vol(SE_7)}}$

- SE have canonical Killing vector ξ dual to R-symmetry
- Volume obtained by volume minimisation $\mathcal{V}_5(\xi)$ [Martelli,Sparks,Yau 05]
- SCFTs have global symmetries arising from KK reduction:

  Flavour symmetries: from isometries of SE

  Baryonic symmetries: from co-dimension 2 cycles of SE

Black strings in AdS5 and AdS3 examples

- Start with $AdS_5 \times SE_5$

Dual to d=4 N=1 SCFT: quiver gauge theory with flavour symmetries and baryonic symmetries

- Compactify d=4 SCFT on Σ and add magnetic fluxes n_i , M_a for global symmetry. Demand susy:

Riemann surface Σ_g : topological twist $\frac{1}{2\pi}\int_{\Sigma_g} F^R = 2(1-g)$

Spindle $\Sigma(n_+, n_-)$: twist $\frac{1}{2\pi}\int_{\Sigma} F^R = \frac{n_- + n_+}{n_- n_+}$

anti- twist $\frac{1}{2\pi}\int_{\Sigma} F^R = \frac{n_- - n_+}{n_- n_+}$

[Ferrero,Gauntlett,Martelli,Perez,Sparks] [Ferrero,Gauntlett,Sparks]

- If we flow to d=2 SCFT in IR then there should be a black string in $AdS_5 \times SE_5$ with $AdS_3 \times Y_7$ arising in the near horizon with a GK geometry Y_7 fibred as $SE_5 \hookrightarrow Y_7 \to \Sigma$

Can calculate c of d=2 SCFT using GK geometry. Can show that we can write $S(\xi, [J])$ in terms of a "master volume" $\mathcal{V}_5(\xi, [J])$ of SE fibre

- Riemann surface case [JPG,Martelli,Sparks 19]
- Spindle case - can be written in terms of gravitational blocks!

[Boido,JPG,Martelli,Sparks 22]

$$\xi = b_0 \partial_{\varphi_0} + b_i \partial_{\varphi_i}$$

$$S(\xi) = \frac{1}{b_0} \left(\mathcal{V}_5^+ - \mathcal{V}_5^- \right)$$

$$\mathcal{V}_5^\pm = \mathcal{V}_5^\pm (b_i^\pm, [J_\pm])$$

When the SE_5 is toric, with toric data $\vec{v}_a \in \mathbb{Z}^3$

$$S(\xi) = \frac{1}{b_0} \sum_{a<b<c} (\vec{v}_a, \vec{v}_b, \vec{v}_c) \left(R_a^+ R_b^+ R_c^+ - R_a^- R_b^- R_c^- \right) 3N^2 \,,$$

$R_a^\pm$ are dual to R-charges of baryonic operators in d=4 SCFT associated with D3-branes wrapping susy cycles in SE_5

Field theory: toric case can calculate c using anomaly polynomials and c-extremisation: find exact agreement (off shell)

Riemann surface: [JPG,Martelli,Sparks 19] [Hosseini,Zaffaroni 19]

Spindle: [Boido,JPG,Martelli,Sparks 22] [Hosseini,Hristov,Zaffaroni 19]

This provides an identification of an infinite classes of d=4 quiver field theories compactified on Σ with these $AdS_3 \times Y_7$ solutions

Caveat: provided that they both exist…

Black holes in AdS4 and AdS2 examples

- Analogous story for $AdS_2 \times Y_9$ solutions with with $SE_7 \hookrightarrow Y_9 \to \Sigma$ and SE_7 toric

- Using toric data use GK geometry to calculate an off shell entropy function as a function of geometric twists and fluxes

Can be identified with the entropy of magnetically charged black holes in $AdS_4 \times SE_7$

- Spindle horizons: [Boido,JPG,Martelli,Sparks 22]

-Entropy as sum of gravitational blocks $S(\xi) = \frac{1}{b_0}\left(\mathcal{V}_7^+ - \mathcal{V}_7^-\right)$

-Can be made further explicit for toric SE7

-The black holes are accelerating

-Regular D=11 solutions

- Field Theory

- Riemann surface case: off-shell calculation of twisted topological index $\mathscr{I}$ for certain d=3 quiver gauge theories on $S^1 \times \Sigma_g$

[Benini,Zaffaroni15] [Hosseini,Zaffaroni16]

Find exact agreement (off-shell)

[JPG,Martelli,Sparks 19]
[Hosseini,Zaffaroni 19] [Kim,Kim19]

Together, this gives a microscopic state count for the entropy of asymptotically AdS4 black holes

- Spindle case: Can we recover entropy from d=3 SCFT on a spindle? c.f. [Inglese,Martelli,Pittelli 23]

Summary

- New geometric techniques in GK geometry to calculate physical observables for susy AdS2 and AdS3 solutions and hence susy AdS4 black holes and AdS5 black strings

- Spindle horizons lead to entropy S_{BH} and central charge c in terms of gravitational blocks

- Microstate counting interpretation of black hole entropy for infinite classes of susy black holes.
  Identification of infinite classes of d=2 SCFTs

Many avenues for further work

- Spindles: better field theory understanding? Higher dimensional generalisations… [Cheung,Fry,JPG,Sparks 22]

- Can similar geometric techniques be developed for other classes of supersymmetric AdS solutions/AdS black holes?