APPROXIMATE HIGHER-FORM SYMMETRIES
AND TOPOLOGICAL PHASE TRANSITIONS

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[2301.09628] Armas, AJ

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MOTIVATION

- **Symmetries** are powerful guiding principle for developing effective theories for physical systems without a detailed understanding of their microscopic constituents.
- Equilibrium phases of matter can be organised according to their symmetries and whether these are **spontaneously broken** or **unbroken** in the ground state, commonly known as the **Landau paradigm**.
- Symmetries can even be useful when they are only **approximately** respected by the system.

MOTIVATION

- In recent years, the notion of symmetries has been **generalised** to include higher-form symmetries, higher-group symmetries, subsystem symmetries, non-invertible symmetries, etc. [Shao's review talk]
- These allow for a **generalised Landau paradigm**, that also includes exotic phases of matter, such as *topologically ordered states, spin liquids, fractons, topological insulators,* etc. [reviews by McGreevy 2022, Cordova et. al 2022]
- The focus of this talk is **continuous higher-form symmetries**, which concerns higher-dimensional charged objects, such as strings and surfaces. [Gaiotto et al. 2014]
- These describe **topological order** in many-body systems, such as *equipotential planes* in a superfluid, *lattice planes* in a crystal, *magnetic fields* in a plasma, or *electric fields* in a dielectric gas.

MOTIVATION

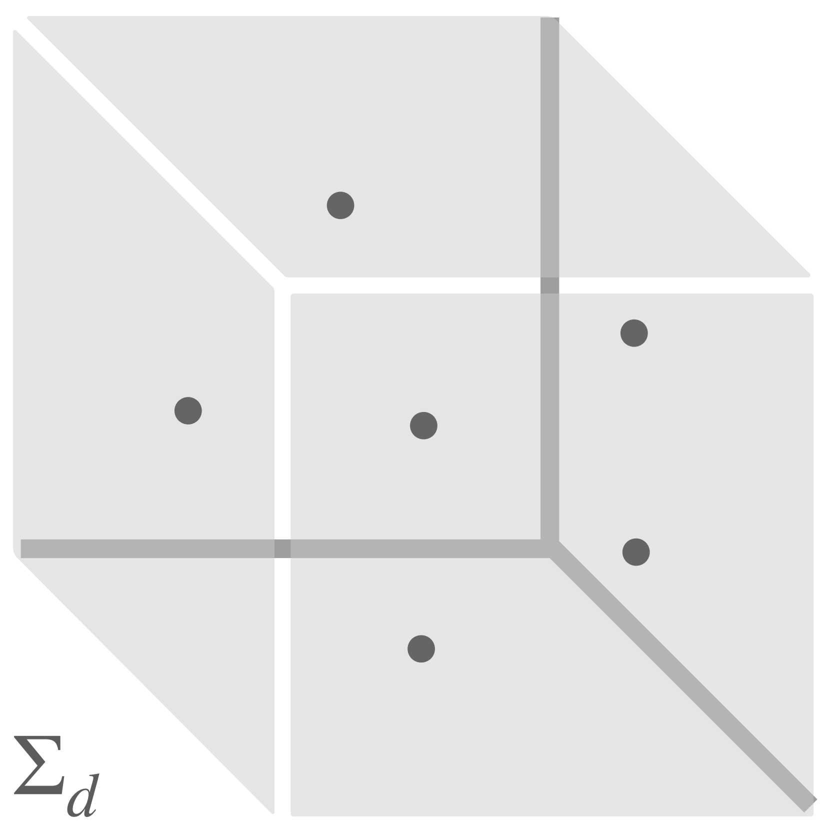
- Explicit breaking of higher-form symmetries describes **topological defects**, such as *superfluid vortices*, *crystal dislocations*, *magnetic monopoles*, or *free charges*.
- Topological defects mediate **topological phase transitions**,[1] wherein a spontaneously broken symmetry gets restored. Examples include *superfluid phase transition*, *melting*, and *plasma phase transition*.

[1]Not to be confused with phase transitions between topologically ordered phases.

HIGHER-FORM SYMMETRIES

- Continuous 0-form symmetry:

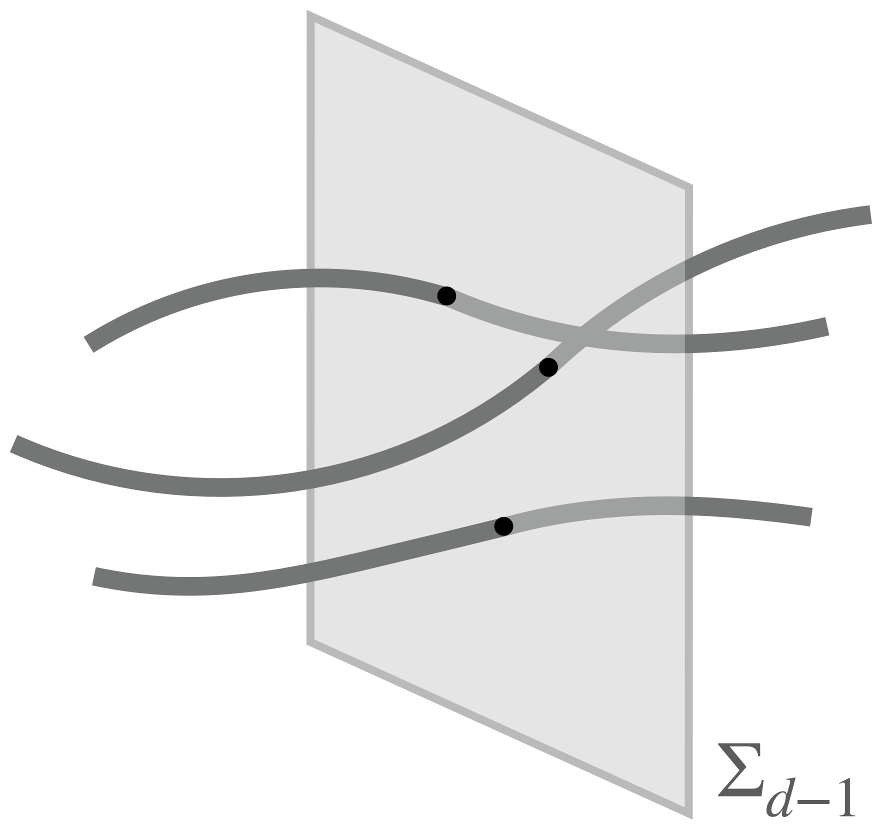
$$\partial_\mu J^\mu = 0$$



- The number of charged particles in a volume Σ_d is conserved in **time**.

- Continuous 1-form symmetry: [Gaiotto et al. 2014]

$$\partial_\mu J^{\mu\nu} = 0$$



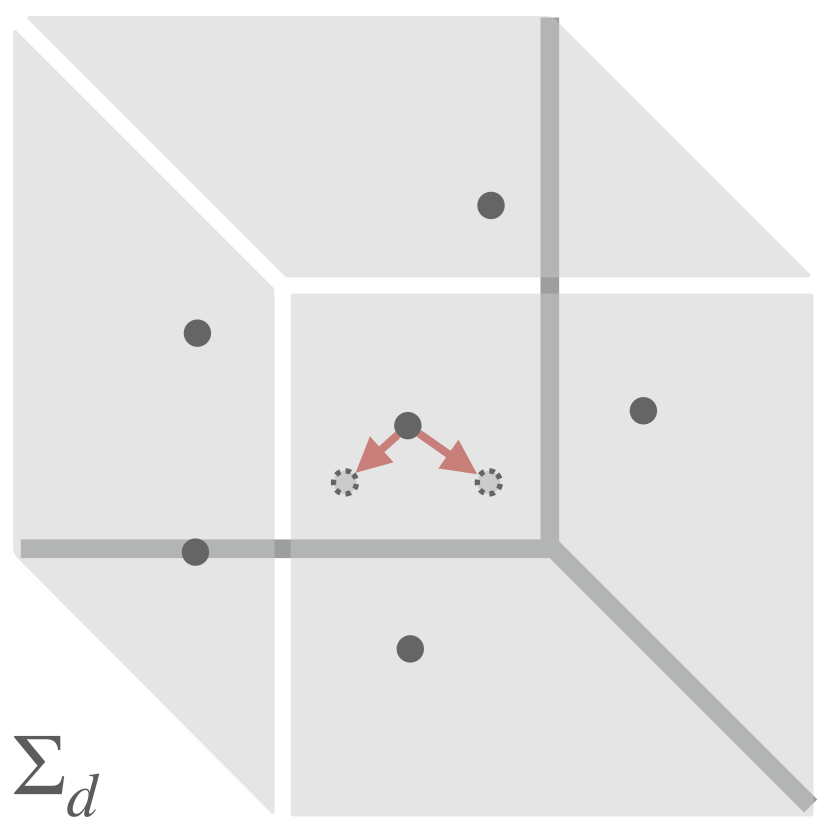
- The number of charged "strings" passing a cross-section Σ_{d-1} are conserved in **time** and under **spatial deformations** of Σ_{d-1}.

$J^{\mu\nu} = -J^{\nu\mu}$

APPROXIMATE HIGHER-FORM SYMMETRIES

- Continuous approximate 0-form symmetry:

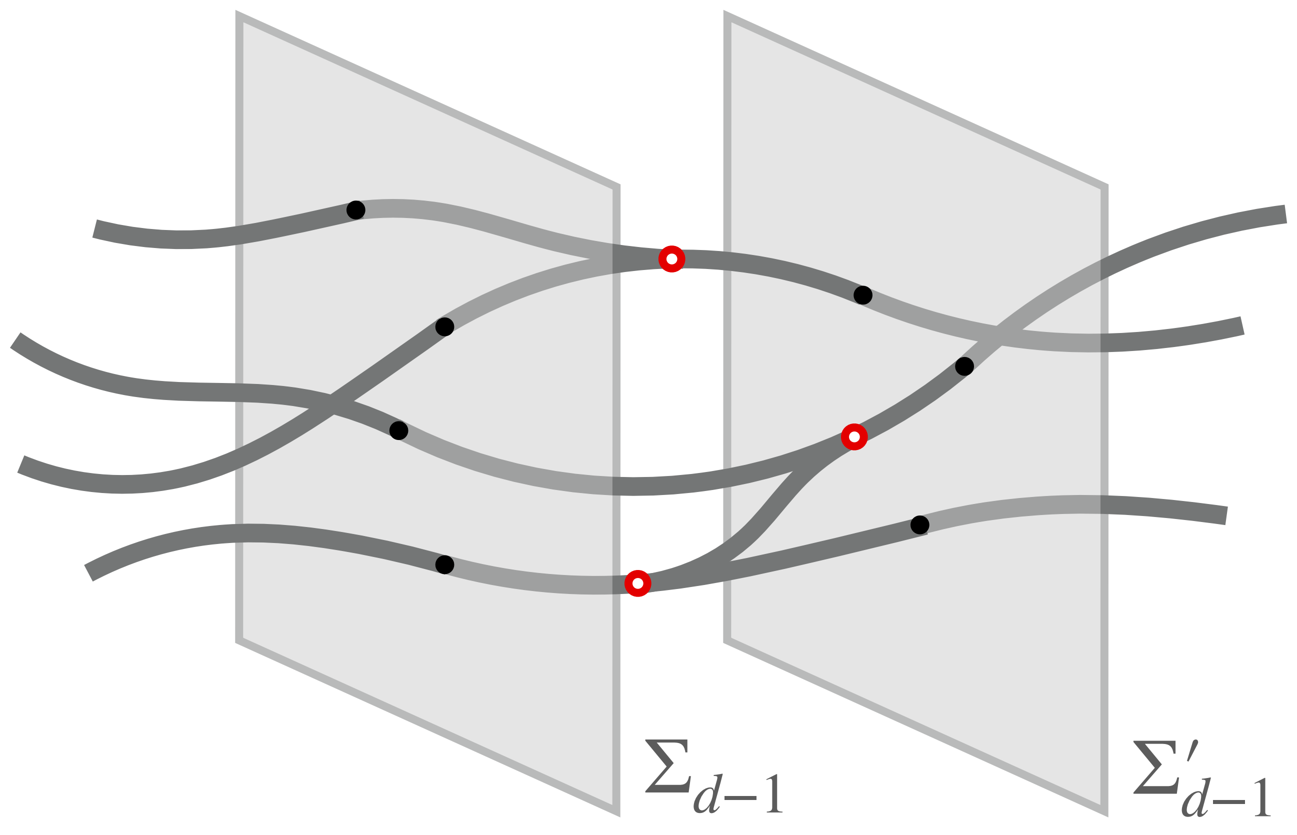
$$\partial_\mu J^\mu = -\ell L$$



- Charged particles can be created/annihilated in **time**.

- Continuous approximate 1-form symmetry:

$$\partial_\mu J^{\mu\nu} = -\ell L^\nu$$

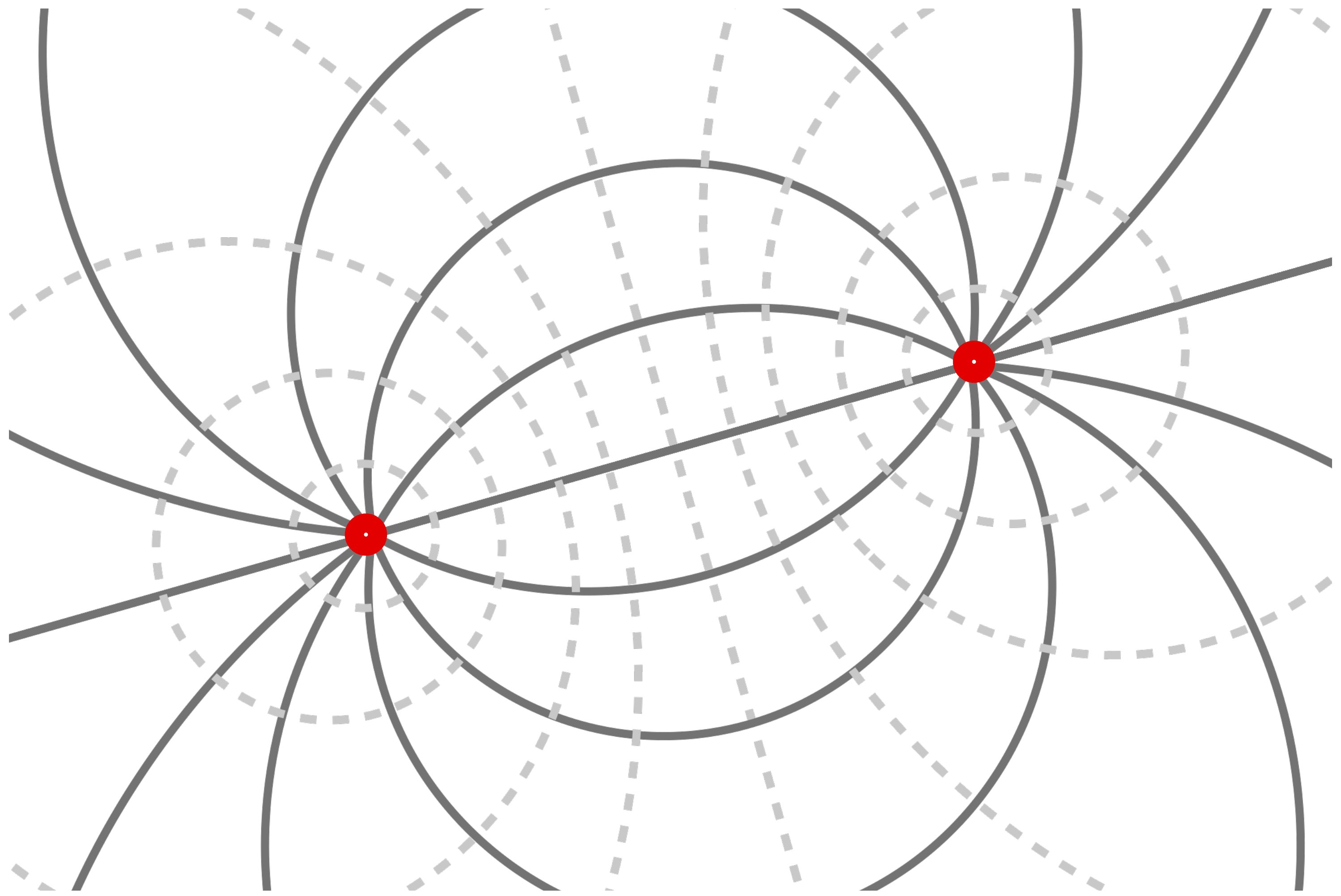


- Charged "strings" passing a cross-section Σ_{d-1} can be created/annihilated in **time** and under **spatial deformations** of Σ_{d-1}.
- Defects furnish a 0-form symmetry: $\partial_\mu L^\mu = 0$

EXAMPLE: ELECTROMAGNETISM

- Electromagnetism has an approximate **electric** 1-form symmetry, broken by **free charges**. (3+1)-dim version also has an exact **magnetic** 1-form symmetry.

[Gaiotto et al. 2014]
[Hofman, Iqbal 2018]
[Armas, AJ 2018]



$$J^{\mu\nu} = -\mathscr{F}^{\mu\nu} + \mathscr{M}^{\mu\nu}_{polarised} \qquad \ell L^\mu = g_{EM}\mathcal{J}^\mu_{free}$$

$$\tilde{J}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\mathscr{F}_{\rho\sigma}$$

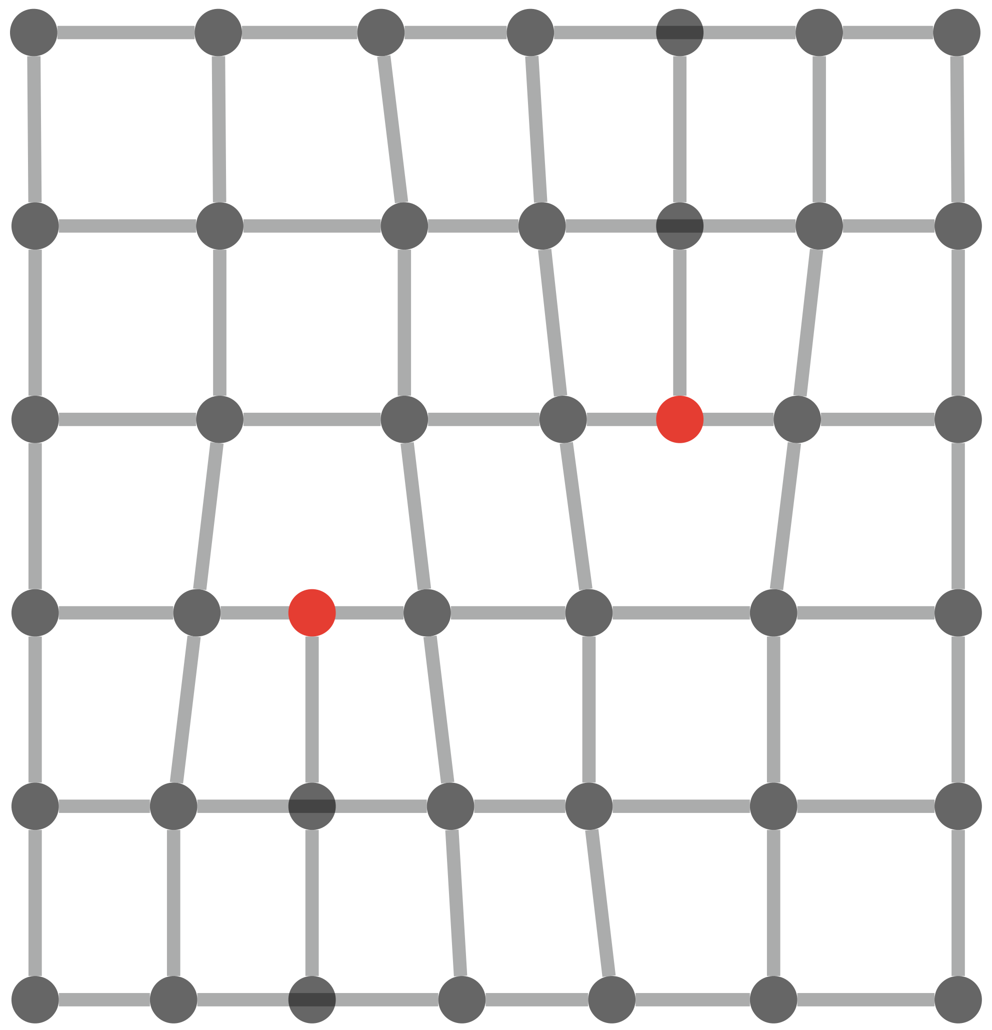
$$\Longrightarrow \qquad \partial_\mu J^{\mu\nu} = -\ell L^\nu, \qquad \partial_\mu \tilde{J}^{\mu\nu} = 0$$

- **Free charges** mediate the phase transition from **dielectric gas** to **polarised plasma**.

EXAMPLE: DEFECTED CRYSTALS

- (2+1)-dim crystals have approximate 1-form symmetries associated with the **lattice planes**, which are broken by **dislocations**.

[Grozdanov, Poovuttikul 2018]
[Armas, AJ 2019]



$$J^{I\mu\nu} = \epsilon^{\mu\nu\rho}\partial_\rho\phi^I \qquad \ell L^{I\mu} = \epsilon^{\mu\nu\rho}\partial_\nu\partial_\rho\phi^I \qquad \Longrightarrow \qquad \partial_\mu J^{I\mu\nu} = -\ell L^{I\nu}$$

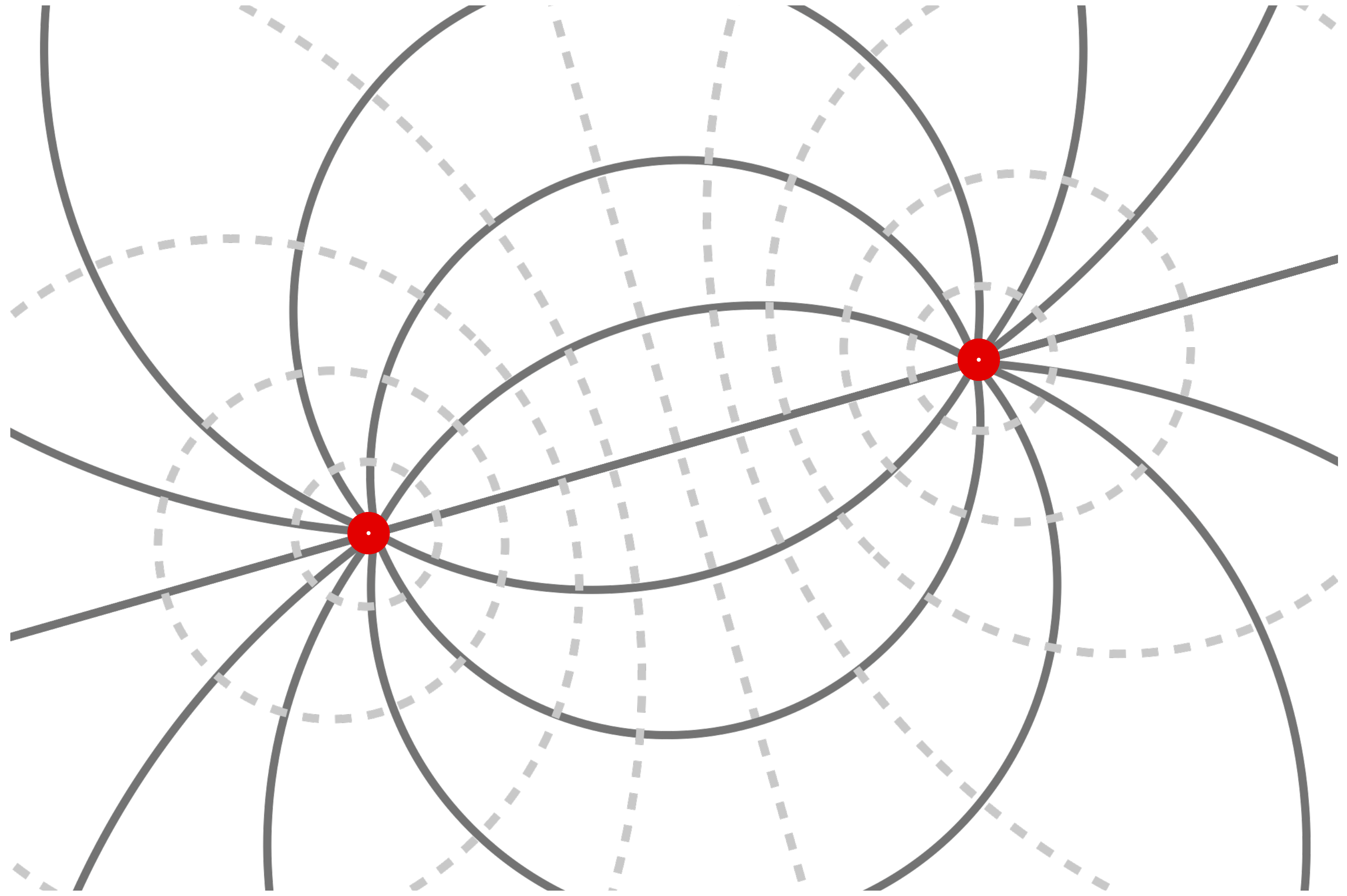
- **Dislocations** mediate the melting phase transition from **crystals/solids** to **fluids/liquids.**

[Berezinskii, Kosterlitz, Thouless 1972]
[Nelson Halperin 1979]

EXAMPLE: DEFECTED CRYSTALS

- (2+1)-dim superfluids have an approximate 1-form symmetry associated with **equipotential planes**, which are broken by **vortices**.

[Delacrétaz, Hofman, Mathys 2019]



$$J^{\mu\nu} = \epsilon^{\mu\nu\rho}\partial_\rho\phi \qquad \ell L^\mu = \epsilon^{\mu\nu\rho}\partial_\nu\partial_\rho\phi \qquad \Longrightarrow \qquad \partial_\mu J^{\mu\nu} = -\ell L^\nu$$

- **Vortices** mediate the phase transition from **superfluids** to **ordinary fluids.**

[Berezinskii, Kosterlitz, Thouless 1972]

BACKGROUND SOURCES

- Approximate 0-form symmetry:

$$\delta S[A,\Phi] = \int \mathrm{d}^{d+1}x \left(J^\mu \, \delta A_\mu + \ell L \, \delta\Phi \right)$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda \qquad \Phi \to \Phi - \Lambda \qquad \Longrightarrow \qquad \partial_\mu J^\mu = -\ell L$$

- Lorentz force:

$$\nabla_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho + \ell \Xi^\nu L$$

$F = \mathrm{d}A$

$\Xi = \mathrm{d}\Phi + A$

- Approximate 1-form symmetry:

$$\delta S[A,\Phi] = \int \mathrm{d}^{d+1}x \left(\frac{1}{2} J^{\mu\nu} \, \delta A_{\mu\nu} + \ell L^\mu \, \delta\Phi_\mu \right)$$

$$A_{\mu\nu} \to A_{\mu\nu} + 2\partial_{[\mu}\Lambda_{\nu]} \qquad \Phi_\mu \to \Phi_\mu - \Lambda_\mu + \partial_\mu \Lambda_\ell \qquad \Longrightarrow \qquad \partial_\mu J^{\mu\nu} = -\ell L^\nu, \quad \partial_\mu L^\mu = 0$$

- Lorentz force:

$$\nabla_\mu T^{\mu\nu} = \frac{1}{2} F^{\nu\rho\sigma} J_{\rho\sigma} + \ell \Xi^{\nu\rho} L_\rho$$

FINITE TEMPERATURE

- We are interested in systems at finite temperature with **approximate higher-form symmetries**.
- Carefully thinking about thermal equilibrium, we can classify phases of matter using the **spontaneous and explicit breaking pattern** of higher-form symmetries. [review by McGreevy 2022]
- We can leave thermal equilibrium perturbatively by formalising a **hydrodynamic framework** with approximate higher-form symmetries. This can be used to study dynamical transitions between different phases of higher-form symmetry.

THERMAL EQUILIBRIUM

- Many-body systems at thermal equilibrium can be characterised by their **thermal partition function**, defined on a Cauchy slice Σ.

$$\mathcal{Z}[A,\ldots] = \operatorname{tr}\exp\int \mathrm{d}\Sigma_\mu\Big[T^{\mu\nu}K_\nu + J^\mu\left(\Lambda_K + K^\lambda A_\lambda\right) + J^{\mu\nu}\left(\Lambda^K_\nu + K^\lambda A_{\lambda\nu}\right)\Big]$$

- Thermal frame: $K^\mu, \quad \Lambda_K, \quad \Lambda^K_\mu$

$$\pounds_K g_{\mu\nu} \;=\; \pounds_K A_\mu + \partial_\mu\Lambda_K \;=\; \pounds_K A_{\mu\nu} + 2\partial_{[\mu}\Lambda^K_{\nu]} \;=\; 0$$

Can take: $K^\mu = \delta^\mu_t/T_0, \quad \Lambda_K = \mu_0/T_0, \quad \Lambda^K_\mu = \delta^z_\mu\,\mu_0^{(1)}/T_0$

THERMAL EQUILIBRIUM

- Many-body systems at thermal equilibrium can be characterised by their **thermal partition function**, defined on a Cauchy slice Σ.

$$\mathcal{Z}[A,\ldots] = \operatorname{tr}\exp\int \mathrm{d}\Sigma_\mu \Big[T^{\mu\nu}K_\nu + J^\mu\left(\Lambda_K + K^\lambda A_\lambda\right) + J^{\mu\nu}\left(\Lambda^K_\nu + K^\lambda A_{\lambda\nu}\right) + \ell L^\mu\left(\Lambda^K_\ell + K^\mu \Phi_\mu\right)\Big]$$

- Thermal frame: $K^\mu, \quad \Lambda_K, \quad \Lambda^K_\mu, \quad \Lambda^K_\ell$

$$\pounds_K g_{\mu\nu} = \pounds_K A_\mu + \partial_\mu \Lambda_K = \pounds_K A_{\mu\nu} + 2\partial_{[\mu}\Lambda^K_{\nu]} = 0$$

$$\pounds_K \Phi - \Lambda_K = \pounds_K \Phi_\mu - \Lambda^K_\mu + \partial_\mu \Lambda^K_\ell = 0$$

Can take: $K^\mu = \delta^\mu_t / T_0, \quad \Lambda_K = \mu_0/T_0, \quad \Lambda^K_\mu = \delta^z_\mu \mu^{(1)}_0/T_0, \quad \Lambda^K_\ell = \mu^\ell_0/T_0$

THERMAL EQUILIBRIUM

- For systems with **spontaneously unbroken** symmetries, the **low-energy** thermal partition function is a "local" functional of the background fields [Banerjee et al. 2012] [Jensen et al. 2012]

$$\mathcal{Z}[A, \ldots] = \exp \int \mathrm{d}^d x \, \mathcal{F}(A, \ldots)$$

- For systems with **spontaneously broken** symmetries, the **low-energy** thermal partition function is a "non-local" functional of the background fields, given by a functional integral over time-independent configurations of the **Goldstone fields** [Bhattacharyya et al. 2012]

$$\mathcal{Z}[A, \ldots] = \int \mathcal{D}\phi \exp \int \mathrm{d}^d x \, \mathcal{F}(\phi, \ldots; A, \ldots)$$

HYDROSTATICS

- Spontaneously-unbroken 0-form symmetry:

$$\mathcal{F} \sim -\frac{\chi}{2}(\mu_0 + A_t)^2$$

$J^t \sim \chi \mu_0$

$$A_t \to A_t + \partial_t \Lambda$$

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- Spontaneously-unbroken 1-form symmetry:

$$\mathscr{F} \sim -\frac{1}{2}\chi \left(\mu_0 \delta_i^z + A_{ti}\right)^2 + \ldots$$

Not invariant under time-independent background 1-form gauge transformations.

$$A_{ti} \rightarrow A_{ti} + \partial_t \Lambda_i - \partial_i \Lambda_t$$

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$J^t \sim \chi\,\mu_0$

- Spontaneously-broken 0-form symmetry:

$$\phi \to \phi - \Lambda$$

$$\mathscr{F} \sim \frac{1}{2\tilde{\chi}}\left(A_i + \partial_i\phi\right)^2$$

$J^i \sim -1/\tilde{\chi}\,\partial^i\phi$ $\quad\quad$ $\partial_i\partial^i\phi = 0$

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- **Partially** spontaneously-broken 1-form symmetry:

$$\varphi \to \varphi - \Lambda_t$$

$$\mathcal{F} \sim -\frac{\chi}{2}(A_{ti} - \partial_i \varphi)^2$$

$J^{ti} \sim -\chi\,\partial^i \varphi$ $\qquad$ $\partial_i \partial^i \varphi = 0$

HYDROSTATICS

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- **Entirely** spontaneously-broken 1-form symmetry:

$$\phi_i \to \phi_i - \Lambda_i$$

$$\mathcal{F} \sim \frac{1}{4\tilde{\chi}}\left(A_{ij} + 2\partial_{[i}\phi_{j]}\right)^2$$

$J^{ij} \sim -2/\tilde{\chi}\, \partial^{[i}\phi^{j]}$ $\qquad$ $2\partial_k \partial^{[k}\phi^{i]} = 0$

HYDROSTATICS WITH APPROXIMATE SYMMETRIES

- Spontaneously-unbroken approximate 0-form symmetry:

$$\mathscr{F} \sim -\frac{\chi}{2}(\mu_0 + A_t)^2$$

$$J^t \sim \chi \mu_0$$

- Nothing particularly interesting happens in *hydrostatics* when the symmetry is approximate.

- **Partially** spontaneously-broken approximate 1-form symmetry:

$$\varphi \to \varphi - \Lambda_t$$

$$\mathscr{F} \sim -\frac{\chi}{2}(A_{ti} - \partial_i \varphi)^2 - \frac{\ell^2 \chi_\ell}{2}\left(\varphi - \Phi_t\right)^2$$

$$J^{ti} \sim -\chi \partial^i \varphi$$

$$L^t \sim -\ell \chi_\ell \varphi$$

$$\partial_i \partial^i \varphi = \frac{\ell^2 \chi_\ell}{\chi} \varphi$$

- String charges have **finite correlation length**.

HYDROSTATICS WITH APPROXIMATE SYMMETRIES

- Spontaneously-broken approximate 0-form symmetry: **Relaxed phase**

$$\phi \to \phi - \Lambda$$

$$\mathcal{F} \sim -\frac{\chi}{2}(\mu_0 + A_t)^2 + \frac{1}{2\tilde{\chi}}(A_i + \partial_i \phi)^2$$

$$J^t \sim \chi\,\mu_0$$

$$J^i \sim -1/\tilde{\chi}\,\partial^i \phi$$

$$\partial_i \partial^i \phi = 0$$

- Preserves $\Phi \to \Phi + a$ constant background shifts.
- ϕ is unscreened.

- **Entirely** spontaneously-broken approximate 1-form symmetry: **Coulomb phase**

$$\varphi \to \varphi - \Lambda_t \qquad \phi_i \to \phi_i - \Lambda_i$$

$$\mathcal{F} \sim -\frac{\chi}{2}(A_{ti} - \partial_i \varphi)^2 + \frac{1}{4\tilde{\chi}}\left(A_{ij} + 2\partial_{[i}\phi_{j]}\right)^2 - \frac{\ell^2 \chi_\ell}{2}\left(\varphi - \Phi_t\right)^2$$

$$J^{ti} \sim -\chi\,\partial_i \varphi$$

$$J^{ij} \sim -2/\tilde{\chi}\,\partial^{[i}\phi^{j]}$$

$$L^t \sim -\ell \chi_\ell\,\varphi$$

$$\partial_i \partial^i \varphi = \frac{\ell^2 \chi_\ell}{\chi}\varphi$$

$$2\partial_k \partial^{[k}\phi^{i]} = 0$$

- Preserves $\Phi_\mu \to \Phi_\mu + \partial_\mu \Lambda_\ell$ defect symmetry.
- φ is screened but ϕ_i is unscreened.

HYDROSTATICS WITH APPROXIMATE SYMMETRIES

- Spontaneously-broken approximate 0-form symmetry: **Pinned phase**

$$\phi \to \phi - \Lambda$$

$$\mathscr{F} \sim -\frac{\chi}{2}(\mu_0 + A_t)^2 + \frac{1}{2\tilde{\chi}}(A_i + \partial_i\phi)^2 + \frac{\ell^2 m^2}{2}(\phi - \Phi)^2$$

$$J^t \sim \chi\,\mu_0 \qquad J^i \sim -1/\tilde{\chi}\,\partial^i\phi \qquad L \sim \ell^2 m^2 \phi$$

$$\partial_i\partial^i\phi = 0$$

$$\partial_i\partial^i\phi = \ell^2 m^2 \tilde{\chi}\,\phi$$

- $\Phi \to \Phi + a$ constant background shifts are broken.
- ϕ is screened.

- **Entirely** spontaneously-broken approximate 1-form symmetry: **Higgs phase**

$$\varphi \to \varphi - \Lambda_t \qquad \phi_i \to \phi_i - \Lambda_i \qquad \phi_\ell \to \phi_\ell - \Lambda_\ell$$

$$\mathscr{F} \sim -\frac{\chi}{2}(A_{ti} - \partial_i\varphi)^2 + \frac{1}{4\tilde{\chi}}\left(A_{ij} + 2\partial_{[i}\phi_{j]}\right)^2 - \frac{\ell^2\chi_\ell}{2}\left(\varphi - \Phi_t\right)^2 + \frac{\ell^2 m^2}{2}(\phi_i - \Phi_i - \partial_i\phi_\ell)^2$$

$$J^{ti} \sim -\chi\,\partial_i\varphi \qquad J^{ij} \sim -2/\tilde{\chi}\,\partial^{[i}\phi^{j]} \qquad L^t \sim -\ell\chi_\ell\,\varphi \qquad L^i \sim \ell^2 m^2 \phi^i$$

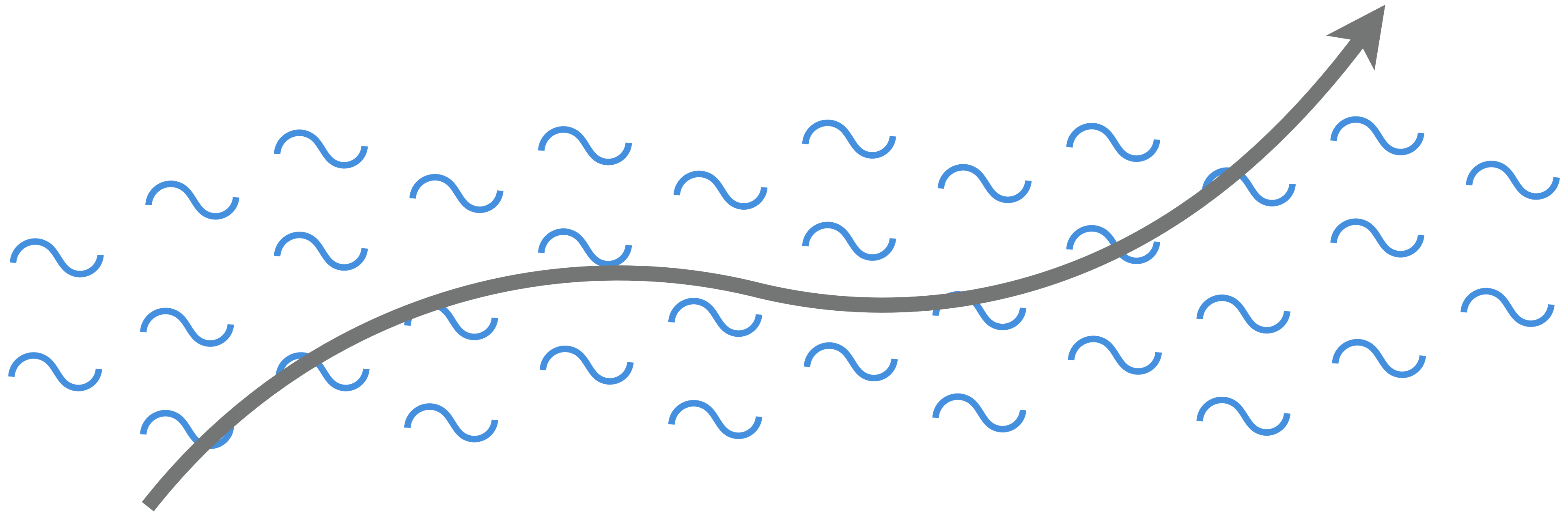
$$\partial_i\partial^i\varphi = \frac{\ell^2\chi_\ell}{\chi}\varphi$$

$$2\partial_k\partial^{[k}\phi^{i]} = \ell^2 m^2 \tilde{\chi}\,\phi^i$$

- $\Phi_\mu \to \Phi_\mu + \partial_\mu\Lambda_\ell$ defect symmetry is spon. broken.
- φ is screened but ϕ_i is unscreened.

HYDRODYNAMICS

- **Hydrodynamics** is a framework to capture perturbative departures of a many-body system from thermal equilibrium.
- The relevant hydrodynamic degrees of freedom are a set of **symmetry parameters** corresponding to each global symmetry (conserved charge) of the system.
- Additionally, we need to add massless **Goldstone fields** for each spontaneously broken global symmetry.



HYDRODYNAMICS WITH APPROXIMATE SYMMETRIES

- Approximate energy-momentum and charge conservation equations:

$$\nabla_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho + \ell \Xi^\nu L + \frac{1}{2} F^{\nu\rho\sigma} J_{\rho\sigma} + \ell \Xi^{\nu\rho} L_\rho$$

$$\nabla_\mu J^\mu = -\ell L$$

$$\partial_\mu J^{\mu\nu} = -\ell L^\nu$$

$$\partial_\mu L^\mu = 0$$

- Thermal frame fields are promoted to slowly varying dynamical fields

$$\delta\beta^\mu = \pounds_\chi \beta^\mu$$

$$\delta\Lambda_\beta = \pounds_\chi \Lambda_\beta - \pounds_\beta \Lambda$$

$$\delta\Lambda^\beta_\mu = \pounds_\chi \Lambda^\beta_\mu - \pounds_\beta \Lambda_\mu$$

$$\delta\Lambda^\beta_\ell = \pounds_\chi \Lambda^\beta_\ell - \pounds_\beta \Lambda_\ell$$

and we need the Goldstone for partially spontaneously-broken 1-form symmetry: $\delta\varphi = \pounds_\chi \phi - \beta^\mu \Lambda_\mu$

$$\frac{u^\mu}{T} = \beta^\mu$$

$$\frac{\mu}{T} = \Lambda_\beta + \beta^\mu A_\mu$$

$$\frac{\mu_\mu}{T} = \Lambda^\beta_\mu + \beta^\lambda A_{\lambda\mu} - \partial_\mu \varphi$$

$$\frac{\mu_\ell}{T} = -\ell\left(\varphi - \beta^\mu \Phi_\mu - \Lambda^\beta_\ell\right)$$

- Josephson equation for φ: $u^\mu \mu_\mu = 0$

HYDRODYNAMICS WITH APPROXIMATE SYMMETRIES

- Constitutive relations:

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + p\, g^{\mu\nu} - \chi\, \mu^\mu \mu^\nu - 2\eta\, P^{\mu\rho}P^{\nu\sigma}\nabla_{\langle\rho}u_{\sigma\rangle} - \zeta\, P^{\mu\nu}\nabla_\lambda u^\lambda$$

$$J^\mu = n\, u^\mu - \sigma\, P^{\mu\nu}\left(T\partial_\nu \frac{\mu}{T} + u^\lambda F_{\lambda\nu}\right)$$

$$J^{\mu\nu} = 2u^{[\mu}n^{\nu]} - \sigma\, P^{\mu\rho}P^{\nu\sigma}\left(2T\partial_{[\rho}\frac{\mu_{\sigma]}}{T} + u^\lambda F_{\lambda\rho\sigma}\right)$$

$$L = -\,\ell\,\sigma_\ell\left(u^\mu \Xi_\mu - \mu\right)$$

$$L^\mu = n_\ell u^\mu - \sigma_\ell\, P^{\mu\nu}\left(T\partial_\nu \frac{\mu_\ell}{T} + \ell\, u^\lambda \Xi_{\lambda\nu} - \ell\,\mu_\nu\right)$$

$$\delta p = s\delta T + n\delta\mu + n^\mu \delta\mu_\mu + n_\ell \delta\mu_\ell \qquad \epsilon = Ts + \mu n + \mu_\mu n^\mu + \mu_\ell n_\ell - p \qquad \sigma, \sigma_\ell, \eta, \zeta \geq 0$$

- Relaxation:

$$u^\mu \partial_\mu n + \ldots = -\frac{\ell^2 \sigma_\ell}{\chi} n + \ldots$$

$$u^\mu \partial_\mu n^\nu + \ldots = -\frac{\ell^2 \sigma_\ell}{\chi} n^\nu + \frac{\ell\,\sigma_\ell}{\chi_\ell} P^{\nu\rho}\partial_\rho n_\ell + \ldots$$

$$\partial_\mu n^\mu = \ell\, n_\ell$$

$$P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

LINEARISED FLUCTUATIONS

- Let us assume that we are fluctuating around $\mu = \mu_\mu = 0$ state.
  In this limit, energy and momentum fluctuations decouple from charge fluctuations, and propagate via the **fluid sound** and **shear modes**.
- The 0-form charge gives rise to a **damped diffusive mode**

$$\mu : \quad \omega = -iD_n k^2 - i\Gamma$$

- The 1-form charge gives rise to two **damped diffusive modes**

$$D_n = \frac{\sigma}{\chi}, \qquad \Gamma = \frac{\ell^2 \sigma_\ell}{\chi}, \qquad \chi = \frac{\partial n}{\partial \mu}$$

$$\mu_\perp : \quad \omega = -iD_n k^2 - i\Gamma$$

$$\mu_\parallel : \quad \omega = -iD_\ell k^2 - i\Gamma$$

$$D_\ell = \frac{\sigma_\ell}{\chi_\ell}$$

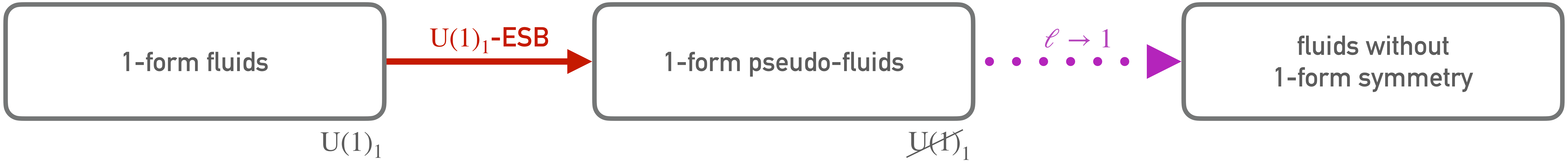
The $\mu_\parallel$ mode obeys a **damping-attenuation relation**: $\Gamma = D_\ell k_0^2$

$$\frac{1}{k_0} = \sqrt{\frac{\chi}{\ell^2 \chi_\ell}}$$

Such relations are generic features of fluids with **spontaneous+explicit** symmetry breaking. [Amoretti et al. 2018] [Ammon et al. 2019] [Donos et al. 2019] [Delacrétaz et al. 2021] [Armas, AJ, Lier 2021]

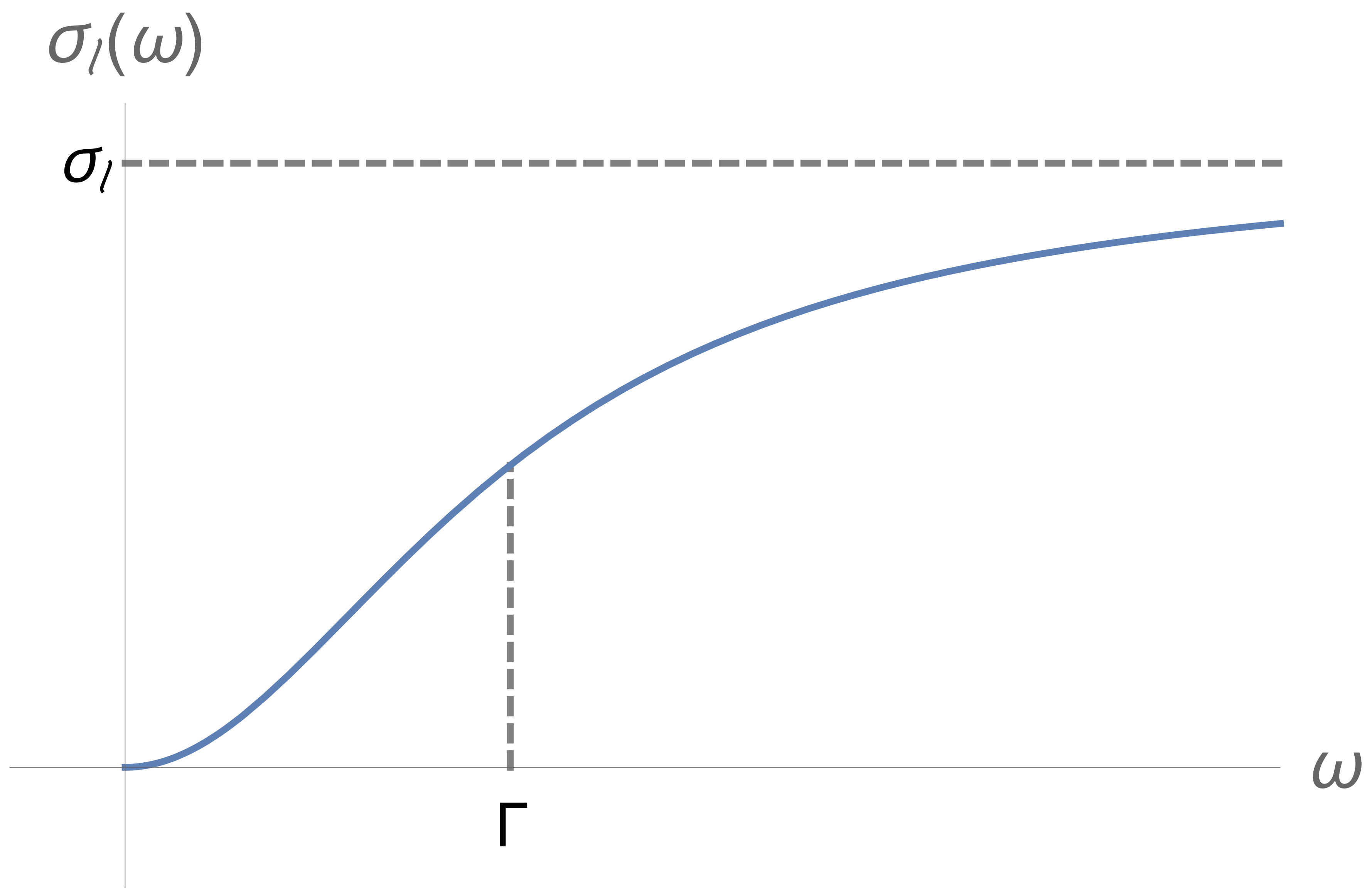
DEFECT PROLIFERATION

- If we increase the strength of defects, i.e. increase ℓ, the charge fluctuations gap out and we are left with a fluid without 1-form symmetry.



- Optical conductivity:

$$\sigma_\ell(\omega) = \mathrm{Re}\frac{i}{\omega}G^R_{L^xL^x}(\omega) \;\; = \sigma_\ell \frac{\omega^2/\Gamma^2}{1+\omega^2/\Gamma^2}$$



- This qualitative behaviour applies to crystal melting and superfluid-fluid phase transitions.

HIGHER-FORM SUPERFLUIDS

- The hydrodynamic construction can be generalised to **entirely spontaneously broken** approximate 1-form symmetries.
- In **Coulomb phase**, the defect 0-form symmetry remains spontaneously unbroken: string charge fluctuations are relaxed, while superfluid component fluctuations are long-lived.
  - After a proliferation of defects, we arrive at a $(d-2)$**-form fluid**. In the context of electromagnetism, this describes **magnetohydrodynamics** with conserved magnetic field lines.
- In **Higgs phase**, the defect 0-form symmetry becomes spontaneously broken: all charged fluctuations are relaxed.
  - After a proliferation of defects, we arrive at a **neutral fluid**. In the context of electromagnetism, this describes the **Meissner effect**, i.e. expulsion of all electromagnetic fields inside a superconductor.

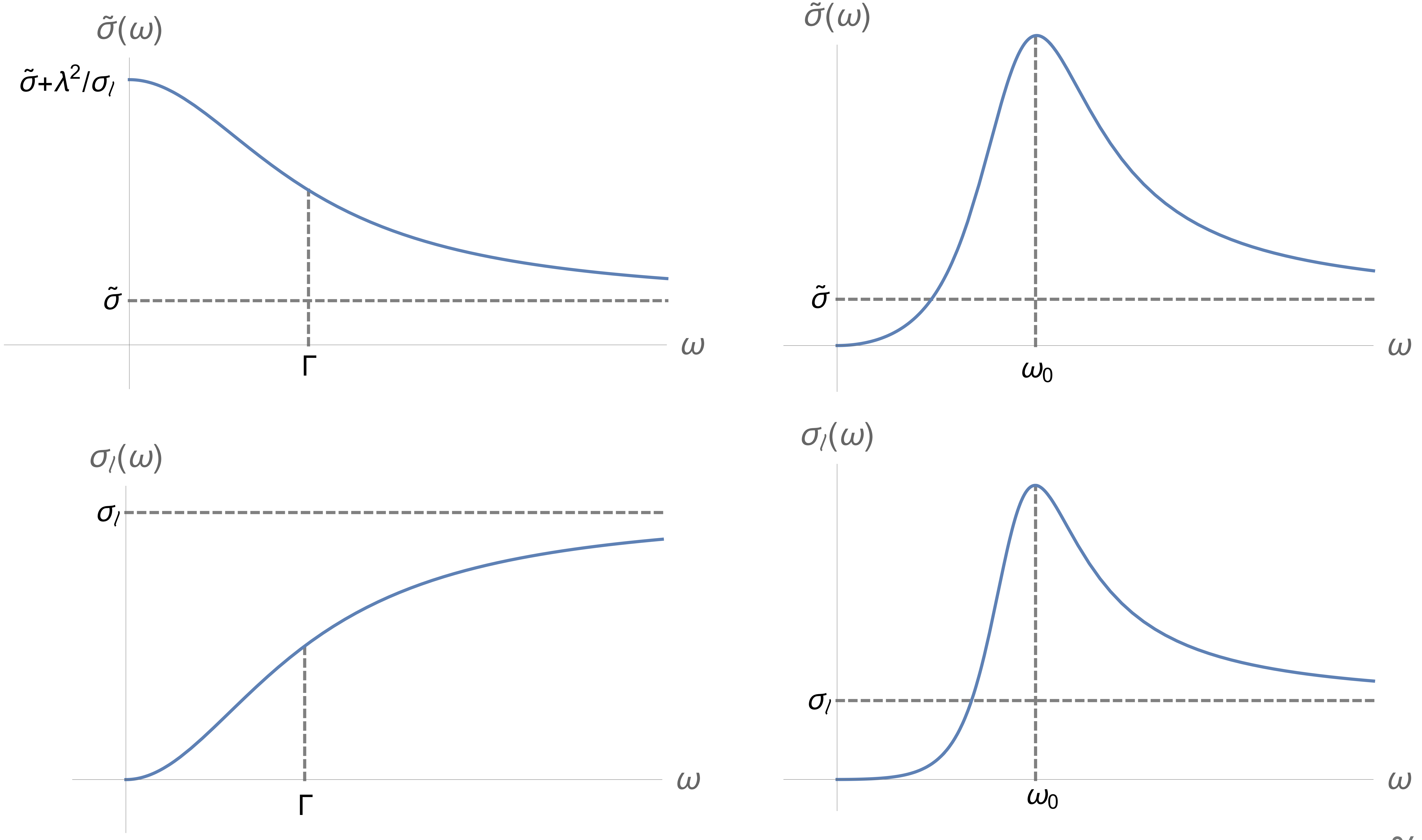
DEFECT PROLIFERATION

- Optical conductivities:

Coulomb phase **Higgs phase**

$$\tilde{\sigma}(\omega) = \mathrm{Re}\frac{i}{\omega}G^R_{\xi_{tx}\xi_{tx}}(\omega)$$

$$\sigma_\ell(\omega) = \mathrm{Re}\frac{i}{\omega}G^R_{L^xL^x}(\omega)$$



HIGHER-FORM SUPERFLUIDS

- Josephson equation for ϕ_μ:

$$u^\lambda \left(\partial_\lambda \phi_\mu - \partial_\mu \phi_\lambda\right) = \lambda\, \mu_\mu + \dots$$

In the absence of explicit symmetry breaking, the coefficient λ is 1.

- In the longitudinal sector, the theory admits a partially screened "photon" mode

$$\mu_\perp, \phi_\perp : \quad \left(i\omega - D_n k^2 - \Gamma\right)\left(i\omega - \tilde{D}_n k^2\right) + v_\perp^2 k^2 = 0$$

$$v_\perp^2 = \frac{\lambda^2}{\chi \tilde{\chi}}$$

$$\mathcal{F} \sim -\frac{\chi}{2}(\partial_i \varphi)^2 + \frac{1}{2\tilde{\chi}}\left(2\partial_{[i}\phi_{j]}\right)^2$$

- The speed of photon in a dissipative medium is different from the thermodynamic value. Similar results were found for pinned superfluids and crystals. [Armas, AJ, Lier 2021]

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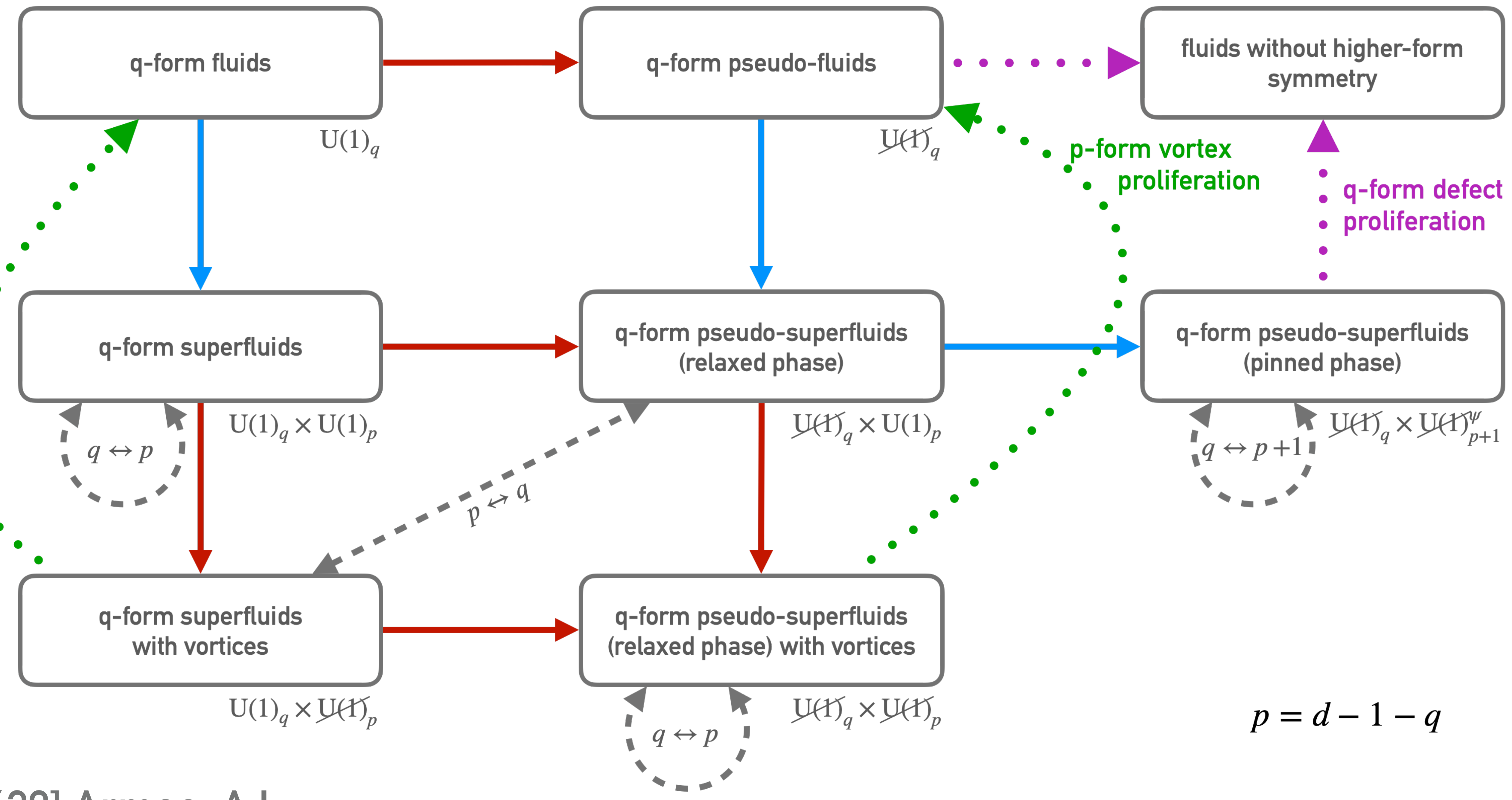
OVERVIEW

- **Higher-form symmetries** can be used to classify phases of matter with topological order.
- Explicit breaking of these symmetries is associated with **topological defects**, which mediate **topological phase transitions**.
- Hydrodynamics with approximate higher-form symmetries provides a model for dynamical phase transitions based on symmetries.



$p = d - 1 - q$

[2301.09628] Armas, AJ

OUTLOOK

- Further applications include emergent magnetic monopoles in spin ice, plasma phase transitions, melting phase transition in higher-dimensions, superfluid and superconductor phase transitions.
- Approximate higher-form symmetries in weakly-coupled QCD:

$$J^{\mu\nu} = -\mathscr{F}^{\mu\nu} + \mathscr{M}^{\mu\nu}_{hadron} \qquad \ell L^{\nu} = -ig_{YM}[\mathscr{A}_{\mu}, J^{\mu\nu}] + g_{YM}\mathscr{J}^{\nu}_{quark}$$

$$\tilde{J}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\mathscr{F}_{\rho\sigma} \qquad \tilde{\ell}\tilde{L}^{\nu} = -ig_{YM}[\mathscr{A}_{\mu}, \tilde{J}^{\mu\nu}]$$

  Connections to QCD phase transitions?
  Also have an analogous story in gravity. [Bueno's talk]

- There are also expected to be interesting interplays with **fractons** in the context of fracton/elasticity duality. [Pretko, Radzihovsky 2017]