String Duals of 2d YM & Symmetric Product Orbifolds

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Based on works in progress / to appear with **Pronobesh Maity** (ICTS, Bangalore)

Introduction & Motivation

- The idea of reformulating gauge theories in terms of strings has a long history.

Migdal, Makeenko, Polyakov,….

- String in non-critical dimensions → dynamical conformal mode

Das, Naik, Wadia, Polyakov, Distler, Kawai,…

$$S_{\text{Liouville}} \sim \int d^2z \left[\partial\phi\bar{\partial}\phi + e^{\phi}\right]$$

extra space direction

- Concrete realization: minimal strings, c=1 string

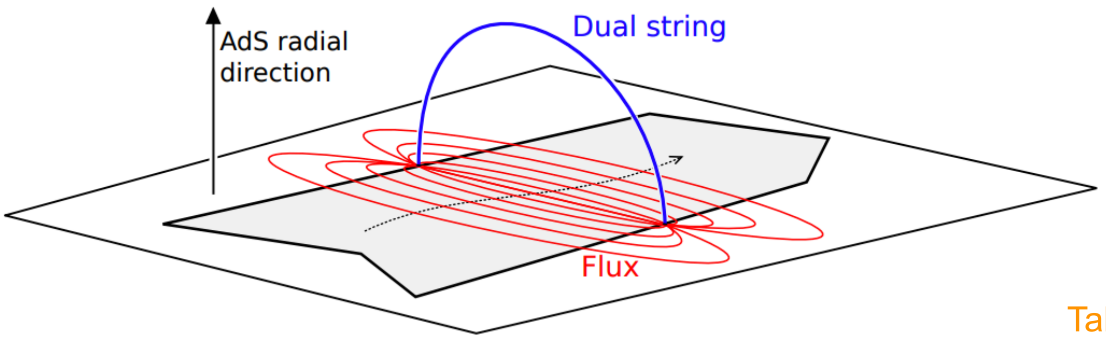
Large N matrix models ↔ Liouville + matter on the world sheet

- And of course, we have AdS/CFT

Large N SUSY gauge theories ↔ String in AdS

Common feature

Existence of radial/Liouville/holographic direction, which corresponds to extra scalar mode on the world sheet

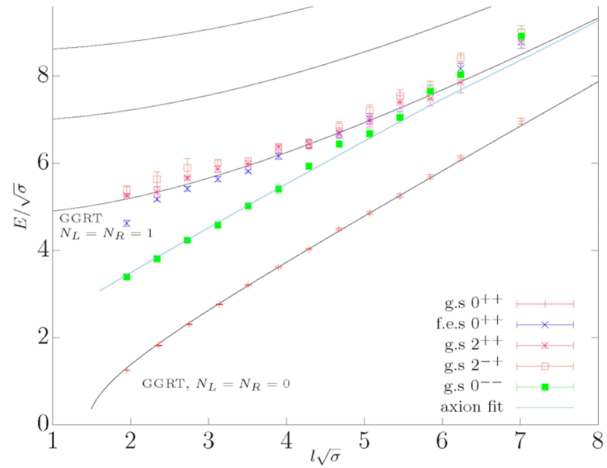


Taken from [Basso, Sever, Vieira]

Q: Is this always true?

An inconvenient truth

- Lattice studies of confining string in large N YM in 3d and 4d



Teper et al, Dubovsky et al.

3d

No extra mode on the worldsheet

4d

A pseudo-scalar resonance
on the worldsheet

Do not quite fit “radial/Liouville/holographic” paradigm.

Effective action for confining string

Polchinski, Strominger,…, Aharony, Komargodski,
…..Hellerman, Maeda, Maltz, Swanson,…..

- Polchinski-Strominger: Effective action around long string

$$S_{\text{Nambu-Goto}} + (26 - D)\underline{S_{\text{PS}}} + \cdots$$

Needed for restoring Weyl inv.

- Reformulation by Hellerman et al: "Composite linear dilaton"

$$S_{\text{Polyakov}} + \frac{26-D}{12\pi}\int d^2z\,\left(\partial\varphi\bar{\partial}\varphi + R\varphi/4\right) + \cdots \qquad \boxed{\varphi = \frac{1}{2}\log \partial X^{\mu}\bar{\partial}X_{\mu}}$$

- Reminiscent of Polyakov's Liouville action, but the extra mode is composite

- This is just an effective field theory, not UV complete theory.

Question

Can we write a string dual of confining gauge theory which

- does not involve an extra scalar
- is UV complete (i.e. path integral can be performed exactly)

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I will present one such realization for the simplest toy model, Chiral 2d YM

Punchline / Plan

- String dual for chiral 2d YM
- String dual for symmetric product orbifolds of arbitrary CFT
- Relation to $T\bar{T}$ deformation
- Potential relation to Eberhardt-Gaberdiel-Gopakumar
- Potential relation to matrix string

Speculative

Calculable, UV complete worldsheet theory for which the extra mode is composite.

Lightening review of 2d YM

- 2d YM = Yang-Mills theory in two dimensions

Lightening review of 2d YM

- 2d YM: Simplest possible confining theory.

  Linear potential, area law, meson spectrum…..

- Completely solvable even at finite N:

$$Z_{\mathscr{M}} = \sum_{\text{rep of U(N)}} \left(\dim R\right)^{2-2G} e^{-g_{\text{YM}}^2 A C_2(R)}$$

- At large N, the result factorizes into “chiral” and “anti-chiral” parts

$$Z_{\mathscr{M}} \overset{\text{large N}}{\rightarrow} Z_{\mathscr{M}}^{\text{chiral}} Z_{\mathscr{M}}^{\text{anti}}$$

- At finite N, each partition function receives corrections. In addition there will be interactions between chiral and anti-chiral parts.

$$Z_{\mathscr{M}}^{\text{chiral}} = Z_0^{\text{chiral}} + \frac{Z_1^{\text{chiral}}}{N^2} + \cdots \qquad Z_{\mathscr{M}} = Z_{\mathscr{M}}^{\text{chiral}} Z_{\mathscr{M}}^{\text{anti}} + \frac{Z_{\text{int}}}{N^2} + \cdots$$

2d YM as string theory

- Gross Taylor: 1/N expansion can be interpreted as string genus expansion.
- Chiral and anti-chiral parts correspond to worldsheets wrapping the target space with different orientations.
- But the explicit worldsheet action was not written down.

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- Horava: Proposed a “rigid topological string”. Much more complicated than standard string theory.
- Cordes, Moore, Rangoolam: “zero area limit” of 2d YM can be interpreted as topological string.
- Vafa: Aganagic, Ooguri, Saulina, Vafa: Partition functions on Riemann surfaces coincide with topological string partition function.

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- Vafa: Aganagic, Ooguri, Saulina, Vafa: Partition functions on Riemann surfaces coincide with topological string partition function.
- Our worldsheet theory: standard bosonic string. In principle we can use it to compute more general observables. We may be able to add boundaries to study mesons.

Worldsheet actions

- String dual for chiral 2d YM on a torus / cylinder

$$\int d^2z\,(\beta\bar\partial X + \bar\beta\partial\bar X + \lambda\partial X\bar\partial\bar X) - \frac{2}{\pi}\int d^2z\,\left(\partial\varphi\bar\partial\varphi + R\varphi/4\right)$$

$\beta\gamma$-system

Chiral analog of PS term

$$\varphi = \frac{1}{2}\log\partial X\bar\partial\bar X$$

$\lambda = g_{\rm YM}^2 N$

$X \sim X + 2\pi R$ etc

- String dual for symmetric product orbifolds for arbitrary CFT

$$\int d^2z\,(\beta\bar\partial X + \bar\beta\partial\bar X) - \frac{24-c}{12\pi}\int d^2z\,\left(\partial\varphi\bar\partial\varphi + R\varphi/4\right) + S_{\text{seed CFT}}$$

- The action looks complicated.
- Integrating out βs forces X to be holomorphic $\rightarrow$ path integral can be performed explicitly.
- Can be viewed as a non-critical version of non-relativistic string theory studied by Gomis and Ooguri.

Nonrelativistic string

Gomis, Ooguri,,…, Bergshoff, Gomis, Yan,….

$$\int d^2z \left(\beta\bar{\partial}X + \bar{\beta}\partial\bar{X}\right) + \text{(Polyakov action for transverse modes)}$$

- Worldsheet theory is relativistic, target space spectrum is non-relativistic.
- It can be obtained by a double-scaling limit of usual string theory in which one sends B-field large (critical) and $\alpha' \to 0$.
(Open string sector becomes noncommutative gauge theory)
- Roughly speaking, the non-relativistic limit kills half of the spectrum.
- Here taking the limit makes the theory chiral and kill "anti-strings" which do not exist in the symmetric product orbifolds.

Simple check: genus 1 partition function

$$\int d^2z\left(\beta\bar{\partial}X+\bar{\beta}\partial\bar{X}\right)+S_{\text{dilaton}}+\text{rest}$$

- Genus-1 partition function for the torus target space
- Integrating out $\beta \to X$: holomorphic

$$\int_{\text{fund}}\frac{d^2\tau}{\tau_2}\delta(\text{holo map})\,\det(\text{rest})\,e^{-S_{\text{dilaton}}}$$

- Holomorphic map from torus to torus: labelled by 4 integers.
- After doing all integration, we get for 2d YM

$$\sum_K\frac{1}{2K}e^{-\lambda K(\text{area})}\sum_{a\cdot d=K}(a+d)$$

- Reproduces the large N result by Gross and Taylor.
- We reproduced also scattering amplitudes, Wilson loop VEV.

Simple check: genus 1 partition function

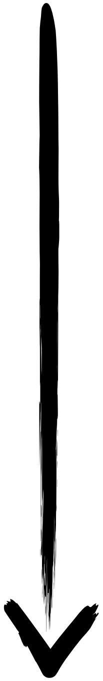
$$\int d^2z\left(\beta\bar{\partial}X+\bar{\beta}\partial\bar{X}\right)+S_{\text{dilaton}}+S_{\text{seed}}$$

- Doing the same, we can reproduce the (grand-canonical) partition function of symmetric product orbifolds.

- Result depends only on complex structure moduli (independent of Kahler moduli): special feature of nonrelativistic string.

Plan

- String dual for chiral 2d YM
- String dual for symmetric product orbifolds of arbitrary CFT
- Relation to $T\bar{T}$ deformation
- Potential relation to Eberhardt-Gaberdiel-Gopakumar
- Potential relation to matrix string



Speculative

Relation to $T\bar{T}$ deformation

$$\int d^2z \left(\beta\bar{\partial}X + \bar{\beta}\partial\bar{X}\right) + S_{\text{dilaton}} + S_{\text{seed}}$$

- Worldsheet matter stress tensor

$$T_{\text{w.s.}}(z) = -\beta\partial X - \frac{24-c}{12\pi}\{X, z\} + T_{\text{seed}} \overset{\text{Virasoro}}{=} 0$$

- Choose static gauge $\partial X = \bar{\partial}\bar{X} = 1$:

$$\beta \overset{\text{Virasoro}}{=} T_{\text{seed}}$$

- A naive guess for (symmetric orbifold of) $T\bar{T}$-deformation:

$$\int d^2z \left(\beta\bar{\partial}X + \bar{\beta}\partial\bar{X} + \beta\bar{\beta}\right) + S_{\text{dilaton}} + S_{\text{seed}}$$

- Integrating out β's, we arrive at an action almost identical to the description of $T\bar{T}$-deformed CFT by Callebaut, Kruthoff, Verlinde

Relation to Eberhardt-Gaberdiel-Gopakumar

- Free field representation of SL(2,R) WZW:

$$\int d^2z \left(\beta\bar{\partial}X + \bar{\beta}\partial\bar{X} + 4\underline{\partial\Phi\bar{\partial}\Phi} - \underline{e^{-2\Phi}\beta\bar{\beta}} - R\Phi\right)$$

  - Similar to our action, but Φ is not composite.
  - Additional term that looks like $T\bar{T}$-deformation

- They argued that, for tensionless string (k=1), the path integral localizes to

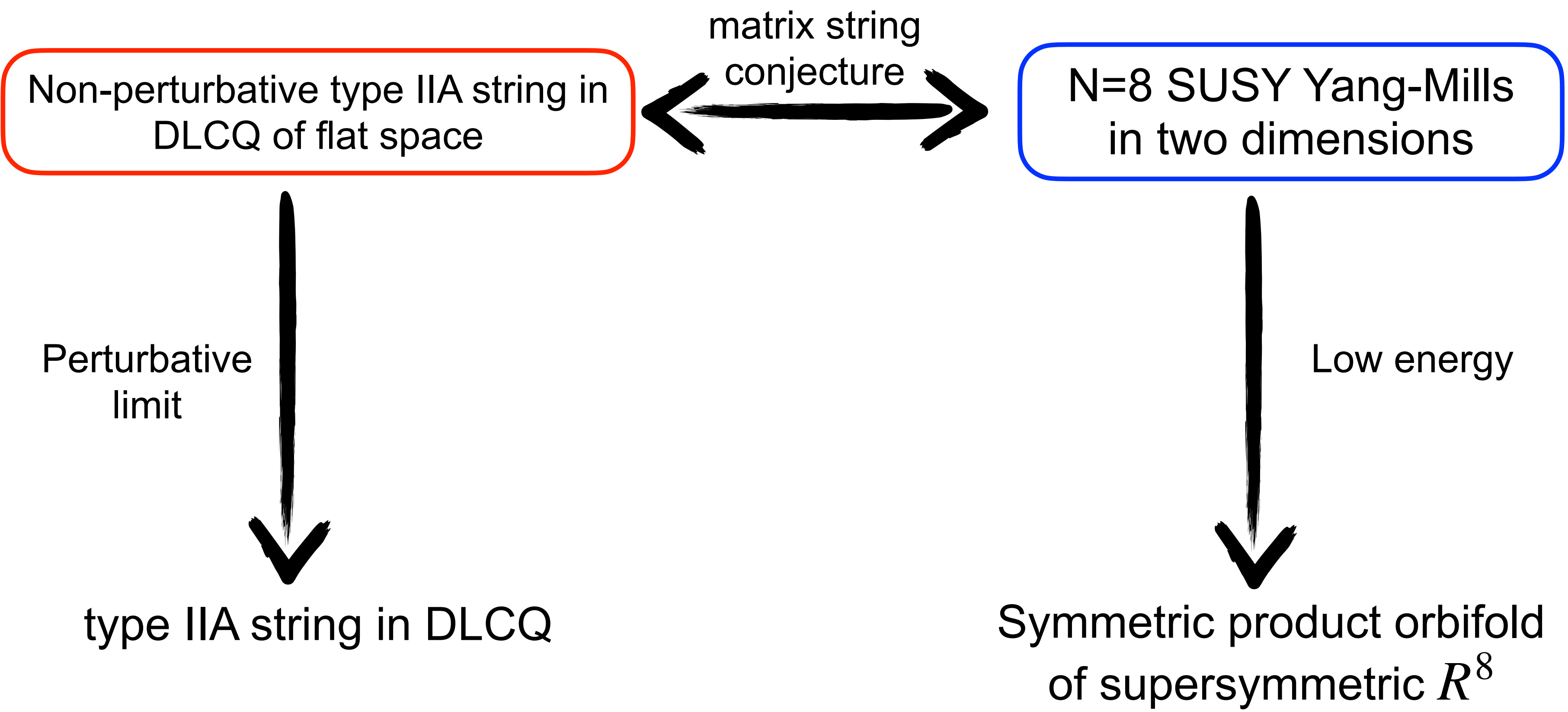
$$\Phi = -\log\epsilon + \log\partial X\bar{\partial}\bar{X}$$

- After the replacement, the action has the same structure as ours.

- The relation is not rigorous: their theory is supersymmetric, our theory is purely boson.

Relation to matrix string

Motl, Dijkgraaf, Verlinde, Verlinde



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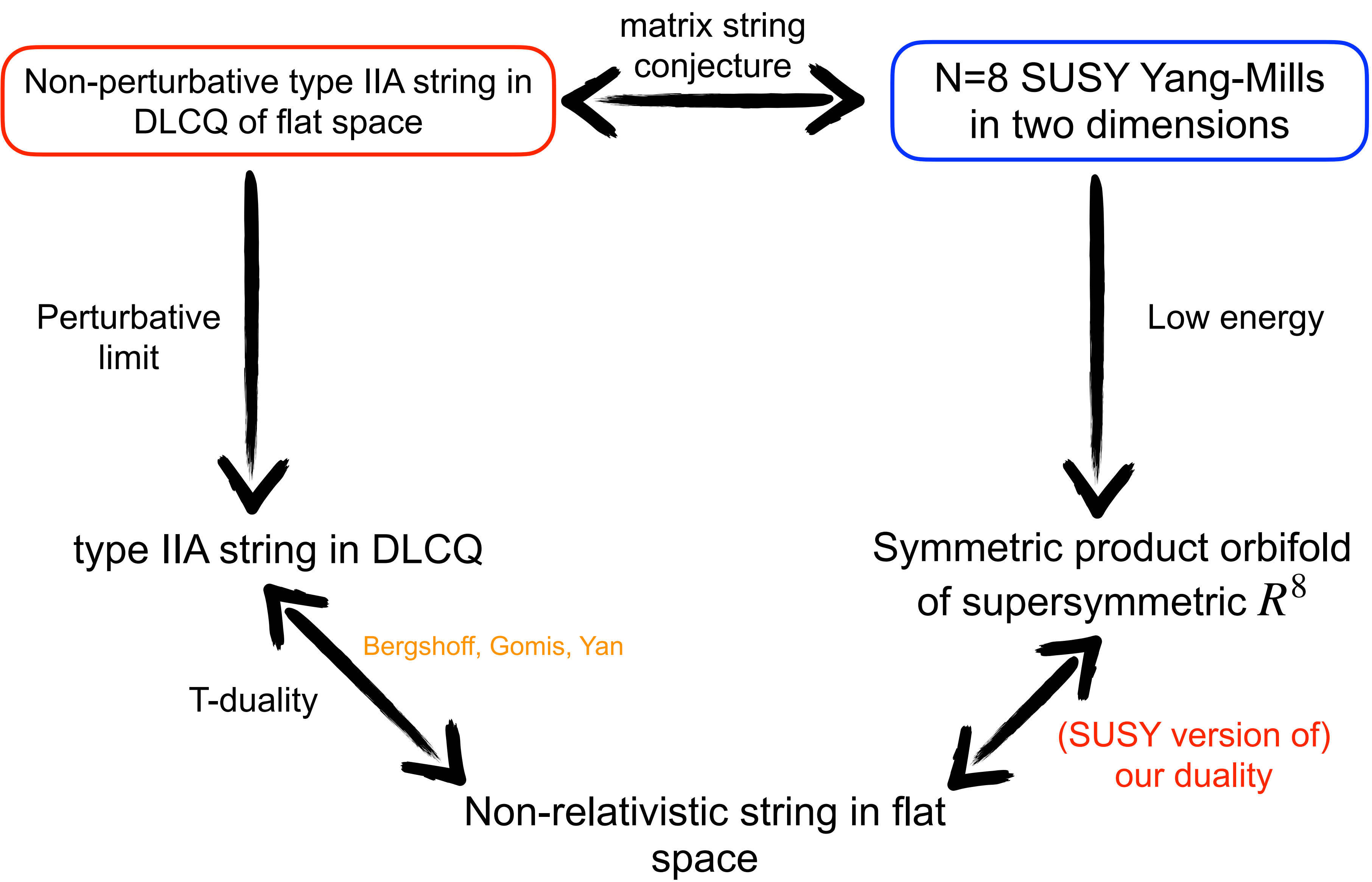

Conclusion

- Proposals for string duals to chiral 2d YM and symmetric product orbifolds.
- Duality without holographic directions.
- Interesting connections to recent developments in AdS_3/CFT_2, nonrelativistic string, matrix string,….

Future

- Meson spectrum.
- Non-perturbative corrections and D-instantons.
- Understand the relation with topological string by Vafa et al.
  - “Instantons beyond topological theory” Frenkel, Losev, Nekrasov
- Compute other observables for symmetric product orbifolds, supersymmetrize……
- Revisiting matrix string proposal?