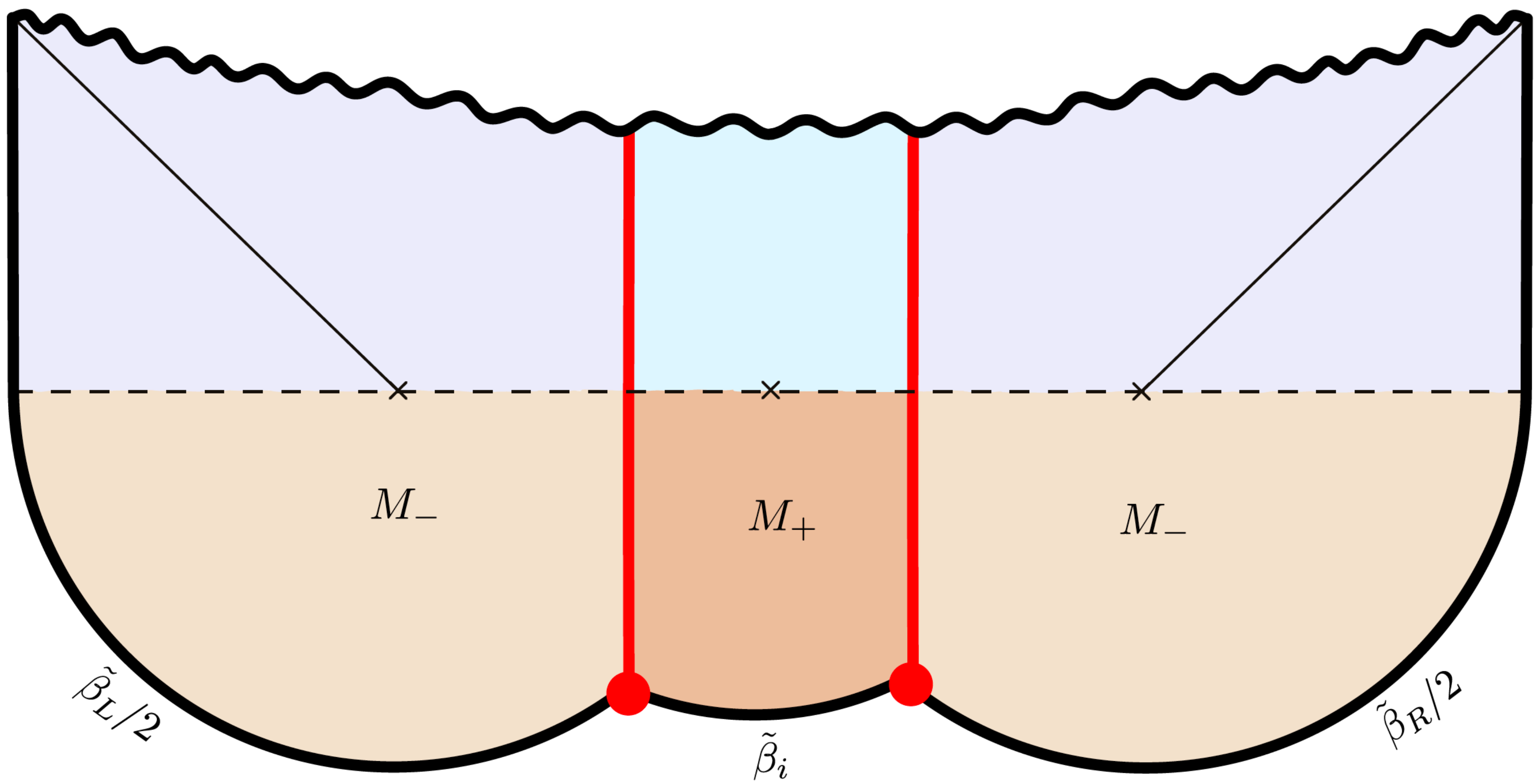
Microscopic origin of the entropy of black holes



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Context

- Black hole thermodynamics
  [Bardeen, Carter, Hawking, 1973]

- Hawking temperature and Bekenstein-Hawking entropy
  [Bekenstein, 1973] [Hawking, 1975]

- Gibbons-Hawking euclidean quantum gravity
  [Gibbons, Hawking, 1977] [Lewkowycz, Maldacena, 2013]

- Strominger-Vafa counting and the fuzzball approach
  [Strominger, Vafa, 1996] [Bena, Martinec, Mathur, Warner, 2022 Review]

- Wheeler's bags of gold geometries
  [Wheeler, 1964]

- Derivations of the Page curve
  [Pennington, Shenker, Stanford, Yang, 2019] [Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini, 2019]

Plan of the talk

- **Infinite microstates with geometric descriptions**
- **Wormholes, overlaps and universality**
- **The Hilbert space dimension**
- **Questions Raised**

Infinite families of microstate geometries

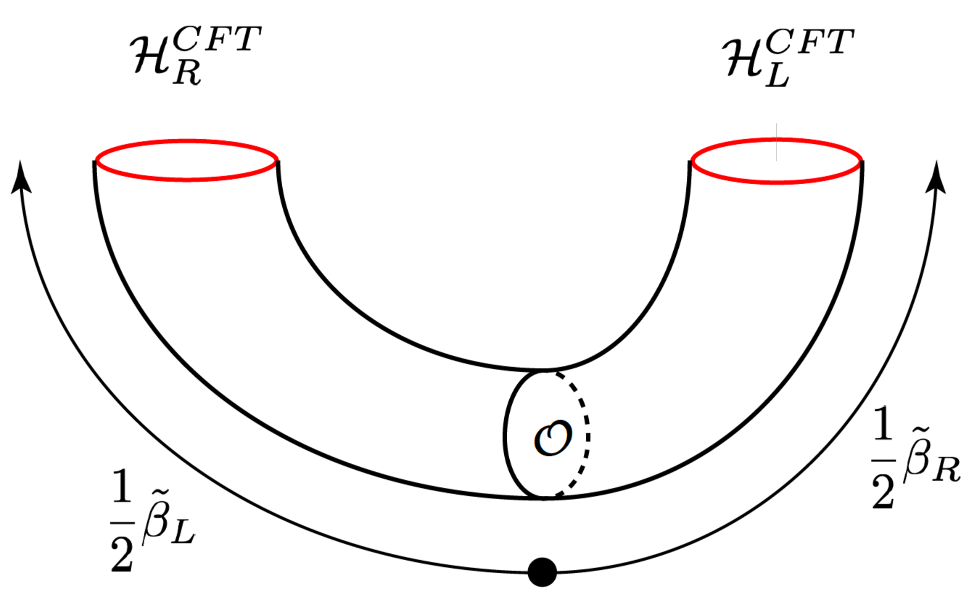
We seek study the entropy of eternal black holes in AdS.

Quantum Gravity with two asymptotic AdS boundaries is dual to a couple of CFT's

[AdS/CFT, Maldacena]

An insightful set of quantum states in these theories arise by inserting dust shell operators

The dust shell operator $\mathcal{O} = \prod_{i}^{N} \mathcal{O}(\theta_i)$ just creates a bunch of particles in particular positions



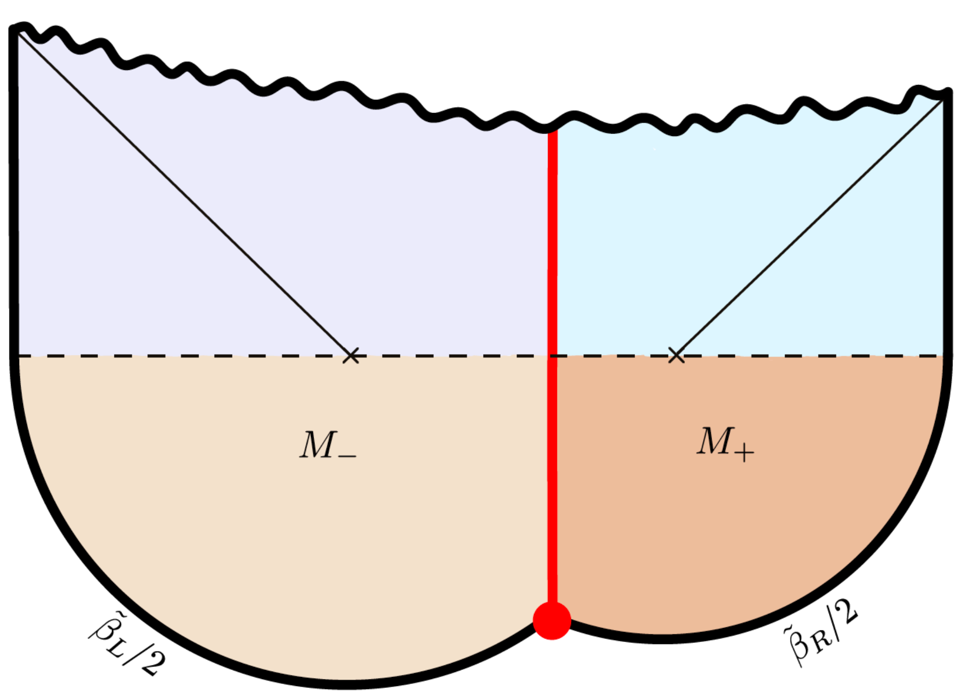
$$= \sum_{mn} e^{-\tilde{\beta}_L E_n/2 - \tilde{\beta}_R E_m/2} \, \mathcal{O}_{nm} |n, m\rangle$$

Infinite families of microstate geometries

Claim: For specific preparation temperatures, shell states are black hole microstates

This follows because these CFT microstates have effective geometric descriptions as a domain wall

$$T_{\mu\nu}\Big|_{\mathscr{W}} = \sigma\, u_\mu\, u_\nu$$



Birkhoff's theorem implies

$$ds^2_\pm = f_\pm(r)\, d\tau_\pm + \frac{dr^2}{f_\pm(r)} + r^2\, d\Omega^2_{d-1}$$

The shell trajectory can be parametrized by

$$r = R(T) \qquad \tau_\pm = \tau_\pm(T)$$

Such equations follow from Israel junction conditions

[Israel, 1966]

$$\Delta h_{ab} = 0 \qquad \Delta K_{ab} - h_{ab}\Delta K = -\,8\pi G T_{ab}$$

The proper mass of the shell, related to number of operator insertions, is unconstrained from above

Infinite families of microstate geometries

These infinite families naively overcount the Bekenstein-Hawking entropy

Wheeler's 'bags of gold'

In this AdS/CFT construction it is difficult to argue these states do not belong to the black hole Hilbert space

The question is: do they really overcount?

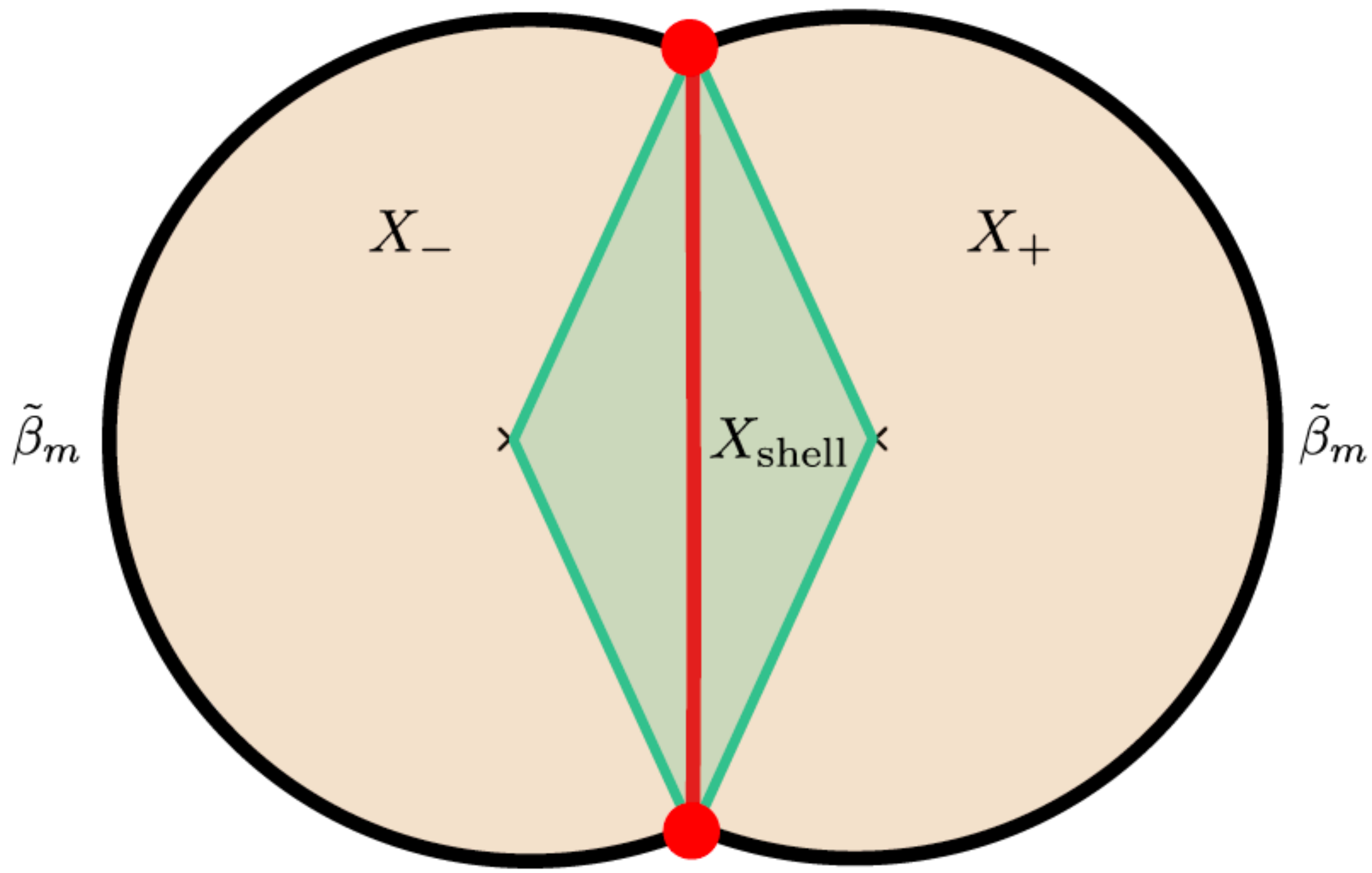
To analyze this question we need to compute the 'quantum overlaps'

Wormholes, overlaps and universality

To this end we first normalize the states. The norm is $Z_1 = e^{-I[X]}$, where $I[X]$ is the GR action

$$I[X] = -\frac{1}{16\pi G}\int_X (R - 2\Lambda) + \frac{1}{8\pi G}\int_{\partial X} K + \int_{\mathscr{W}} \sigma + I_{ct}$$

and X is the following euclidean manifold, solutions of the equations of motion



Wormholes, overlaps and universality

We then seek to compute the following quantities

$$\overline{\langle \Psi_{\mathbf{m}} | \Psi_{\mathbf{m}'} \rangle}\,, \qquad \overline{\langle \Psi_{\mathbf{m}} | \Psi_{\mathbf{m}'} \rangle \langle \Psi_{\mathbf{m}'} | \Psi_{\mathbf{m}''} \rangle}\,, \qquad \overline{\langle \Psi_{\mathbf{m}} | \Psi_{\mathbf{m}'} \rangle \langle \Psi_{\mathbf{m}'} \Psi_{\mathbf{m}''} \rangle \langle \Psi_{\mathbf{m}''} | \Psi_{\mathbf{m}'''} \rangle}\,, \quad \cdots$$

The overline notation means that we compute these quantities using the gravitational action

$$I[X] = -\frac{1}{16\pi G}\int_X (R - 2\Lambda) + \frac{1}{8\pi G}\int_{\partial X} K + \int_{\mathscr{W}} \sigma + I_{ct}$$

We choose a family with parametrically large mass differences so that

$$\overline{\langle \Psi_{\mathbf{m}} | \Psi_{\mathbf{m}'} \rangle} = \delta_{\mathbf{m},\mathbf{m}'}$$

The second then has the form

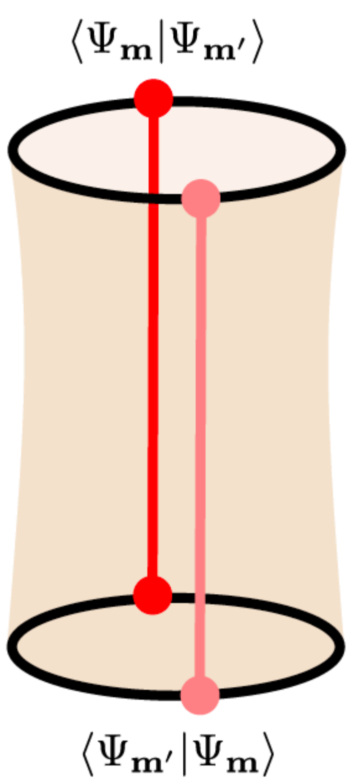
$$\overline{|\langle \Psi_{\mathbf{m}} | \Psi_{\mathbf{m}'} \rangle|^2} = \delta_{\mathbf{m},\mathbf{m}'} + \frac{Z_2}{Z_1 Z_1'} \qquad Z_2 = e^{-I[X_2]} \qquad Z_1 = e^{-I[X]} \qquad Z_1' = e^{-I[X']}$$

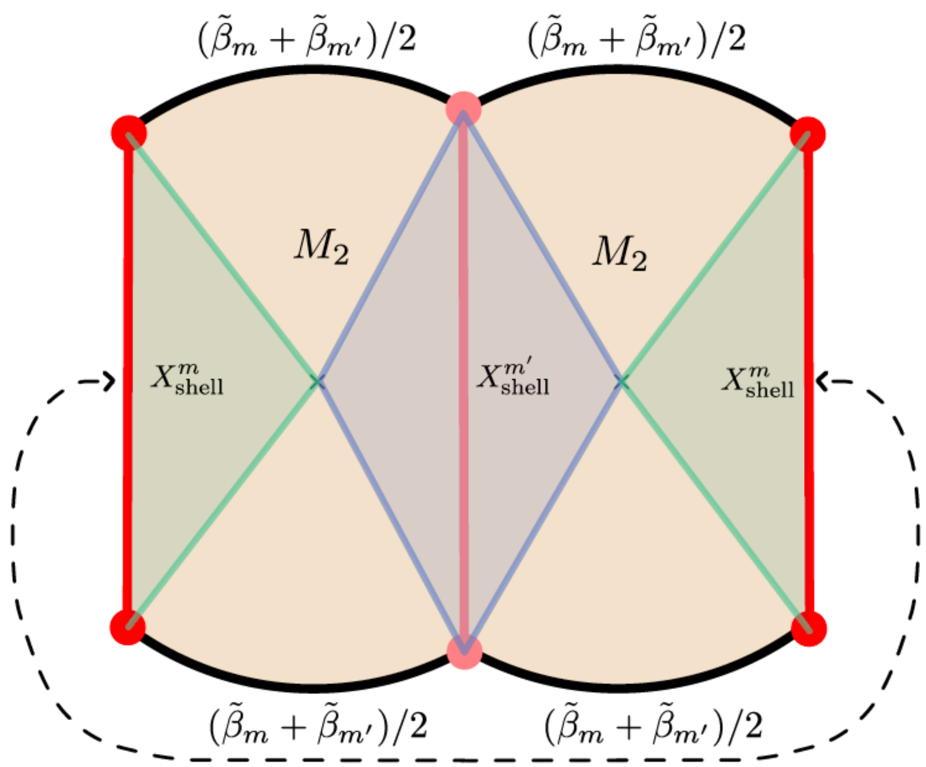
More generally

$$\overline{\langle \Psi_{\mathbf{m}} | \Psi_{\mathbf{m}'} \rangle \langle \Psi_{\mathbf{m}'} | \Psi_{\mathbf{m}''} \rangle \ldots \langle \Psi_{\mathbf{m}'\ldots'} | \Psi_{\mathbf{m}} \rangle}\,|_c = \frac{Z_n}{Z_1 Z_1' \cdots Z_1'^{\ldots'}}$$

Wormholes, overlaps and universality

The next step is to compute the action of the wormhole $Z_2 = e^{-I[X_2]}$, where X_2 is a wormhole connecting two different asymptotic boundaries.





[Chandra, Hartman, 2022]

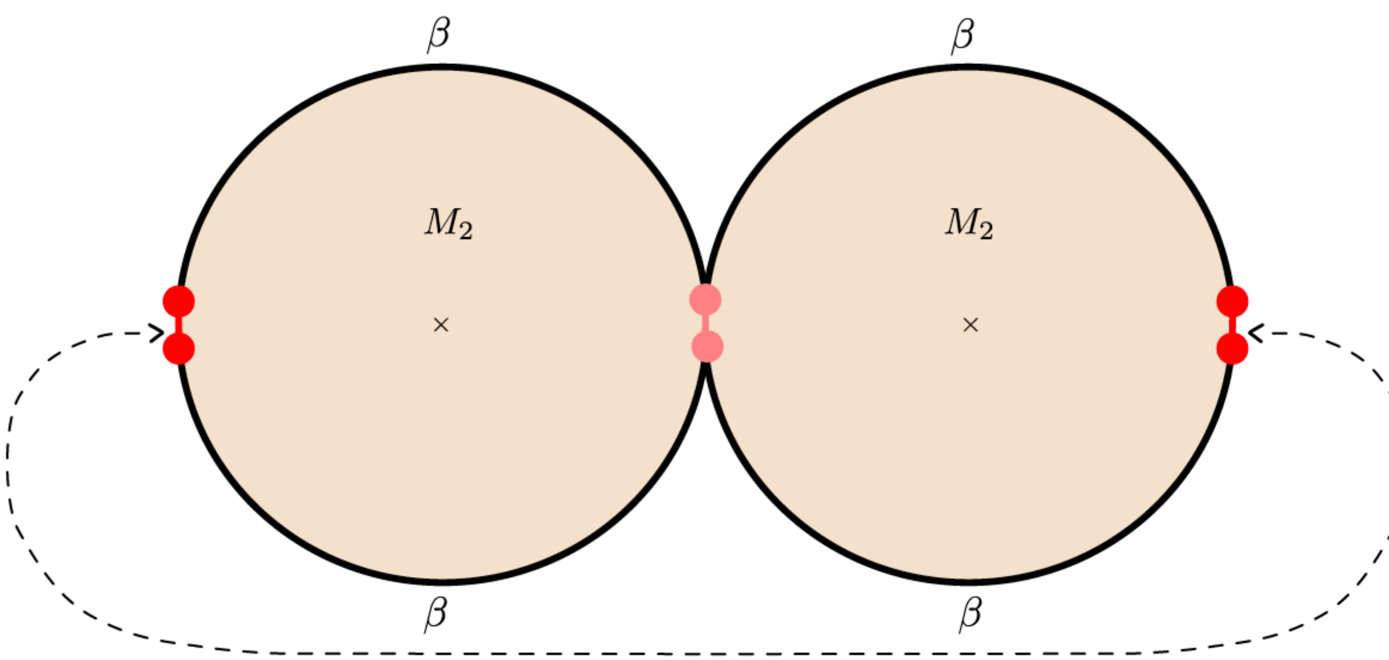
[Sasieta, 2022]

The wormhole arises by cutting two euclidean black holes with the two shells that prepare the states and then identifying each shell.

The action of the wormhole can be computed in a straightforward manner

Wormholes, overlaps and universality

The wormhole action can be computed for any mass. A simplification occurs in the limit of large mass.



The shell trajectory pinches the geometry and the inverse temperature of each wormhole black hole is twice the original black hole inverse temperature. The inner product squared simplifies to

$$\overline{|\langle \Psi_m | \Psi_{m'} \rangle|^2}\,|_c = \frac{Z_2}{Z_1 Z_1'} \approx \frac{Z(2\beta)^2}{Z(\beta)^4}$$

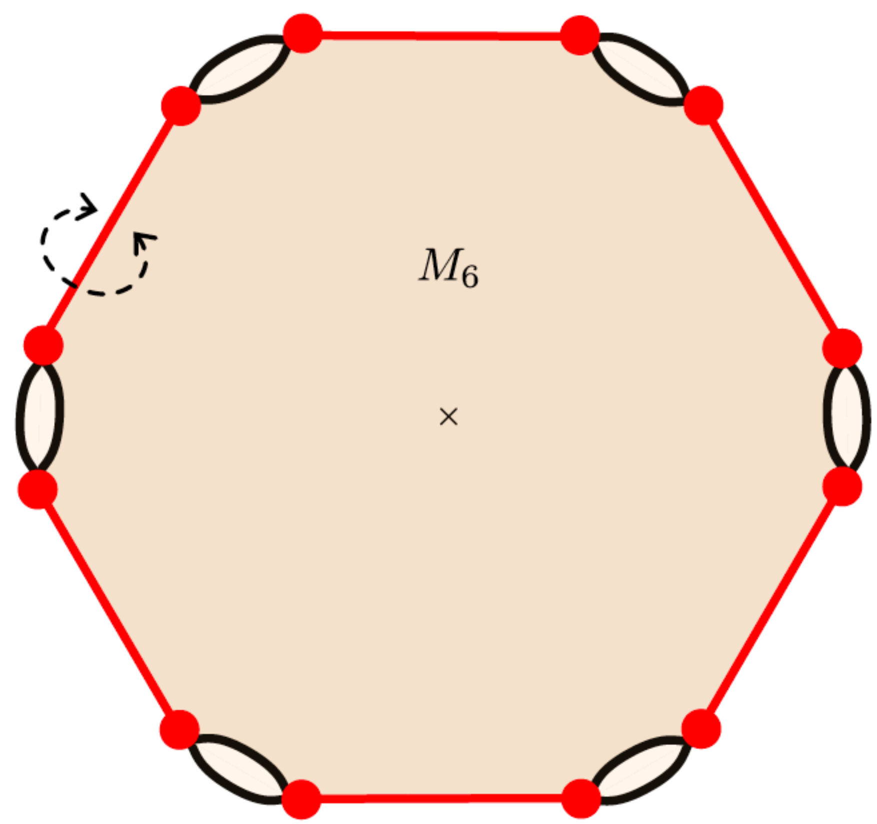
This result recontextualizes the Gibbons-Hawking euclidean gravity partition function. Here it is (still) NOT interpreted as a partition function counting states, but as the size of the quantum overlaps between interior states.

This wormhole provides the 'plateau' of the spectral form factor of the black hole.

[Cotler, Gur-Ari, Hanada, Polchinski, Saad, Shenker, Stanford, Streicher, Tezuka, 2017]

Wormholes, overlaps and universality

To compute higher moments we need the generalization of the previous wormhole to n-boundaries



In the limit of large shell masses all trajectories pinch, and the wormhole action becomes

$$\overline{\langle \Psi_{m_1} | \Psi_{m_2} \rangle \langle \Psi_{m_2} | \Psi_{m_3} \rangle \ldots \langle \Psi_{m_n} | \Psi_{m_1} \rangle} |_c = \frac{Z_n}{Z_1^{(m_1)} \ldots Z_1^{(m_n)}} \approx \frac{Z(n\beta)^2}{Z(\beta)^{2n}}$$

The same result appears for multi shell states.

The Hilbert space dimension

For a set of states $|\Psi_p\rangle$, the Hilbert space dimension is rank of the Gram matrix $G_{pq} = \langle \Psi_p | \Psi_q \rangle$.

Choose a set of Ω shell states with separated masses: $m_p = pm$, with $p = 1, \cdots, \Omega$ and sufficiently large m

The goal is to compute the rank of the Gram matrix for such set of states

From the gravity computation we know that

$$\overline{G^n_{pq}} = \frac{Z(n\beta)^2}{Z(\beta)^{2n}} \, \delta_{pq} \equiv \frac{Z_n}{Z_1^n} \, \delta_{pq} \qquad\qquad \frac{Z_n}{Z_1^n} \approx \frac{Z(n\beta)^2}{Z(\beta)^{2n}}$$

The rank is the number of non-zero eigenvalues. It follows from the density of states

This can be computed by using a trick in matrix theory. We first compute the resolvent of G_{pq}

$$R(\lambda) = \sum_i^{\Omega} R_{ii} \qquad\qquad R_{ij}(\lambda) \equiv \left(\frac{1}{\lambda 1 - G} \right)_{ij} = \frac{1}{\lambda} \delta_{ij} + \sum_{n=1}^{\infty} \frac{1}{\lambda^{n+1}} (G^n)_{ij}$$

And then compute its discontinuity along the real axis

$$D(\lambda) = \frac{1}{2\pi i} \left(R(\lambda - i\epsilon) - R(\lambda + i\epsilon) \right)$$

[Pennington, Shenker, Stanford, Yang, 2019]

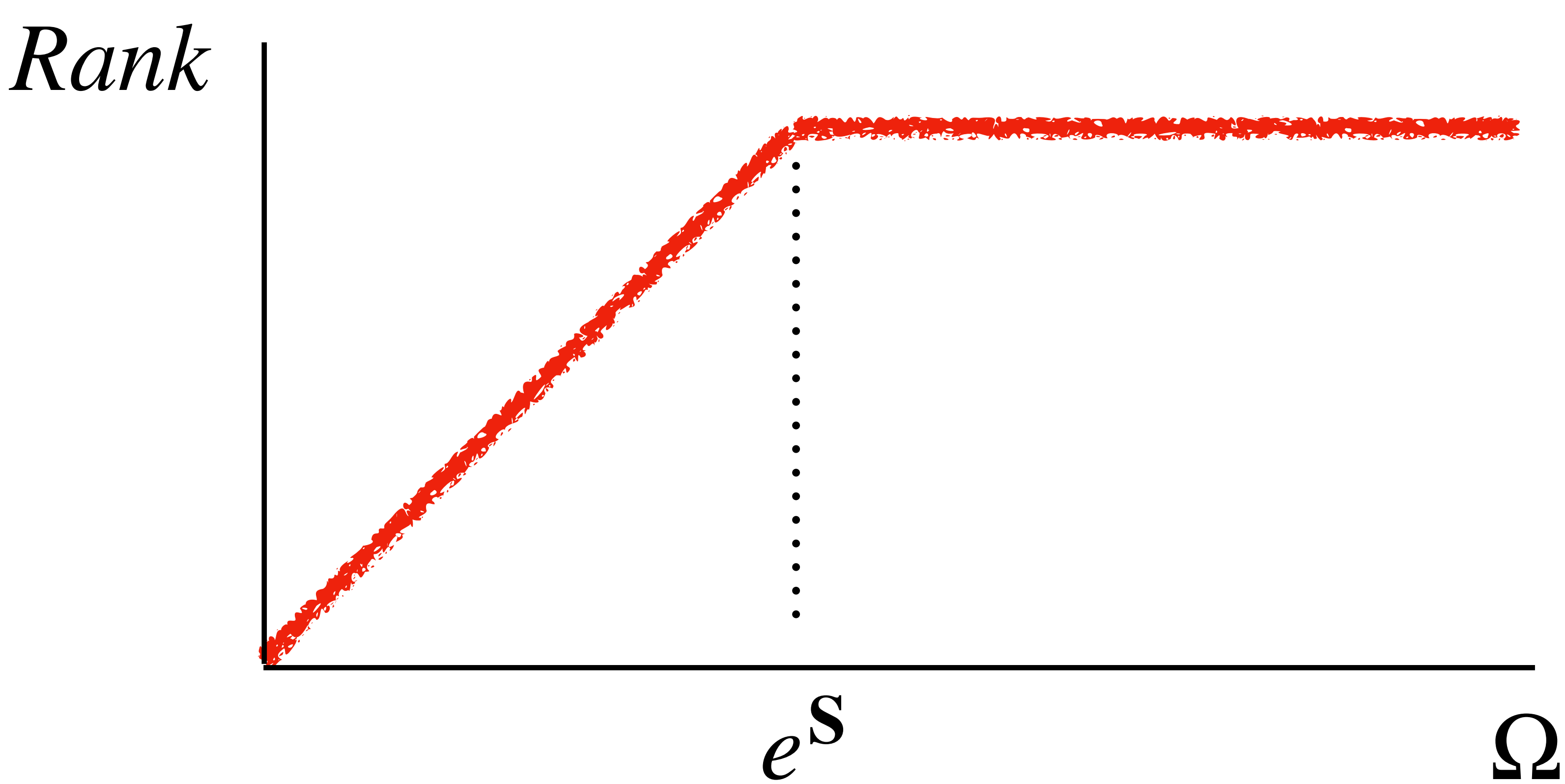
The Hilbert space dimension

The final form of the density of states is

$$D(\lambda) = \frac{e^{\mathbf{S}}}{2\pi\lambda}\sqrt{\left[\lambda - \left(1 - \Omega^{1/2}\, e^{-\mathbf{S}/2}\right)^2\right]\left[\left(1 + \Omega^{1/2}\, e^{-\mathbf{S}/2}\right)^2 - \lambda\right]} + \delta(\lambda)\left(\Omega - e^{\mathbf{S}}\right)\theta(\Omega - e^{\mathbf{S}})$$

where $\mathbf{S} \equiv 2\dfrac{A}{4G}$ is twice the Bekenstein-Hawking entropy

The rank of the Gram matrix as a function of the number of states Ω is then



Questions Raised

- Are these microstates or mesostates?

- Why is this a microscopic explanation?

- How microstates can have event horizons and singularities?

- Is this less microscopic than Strominger-Vafa counting?

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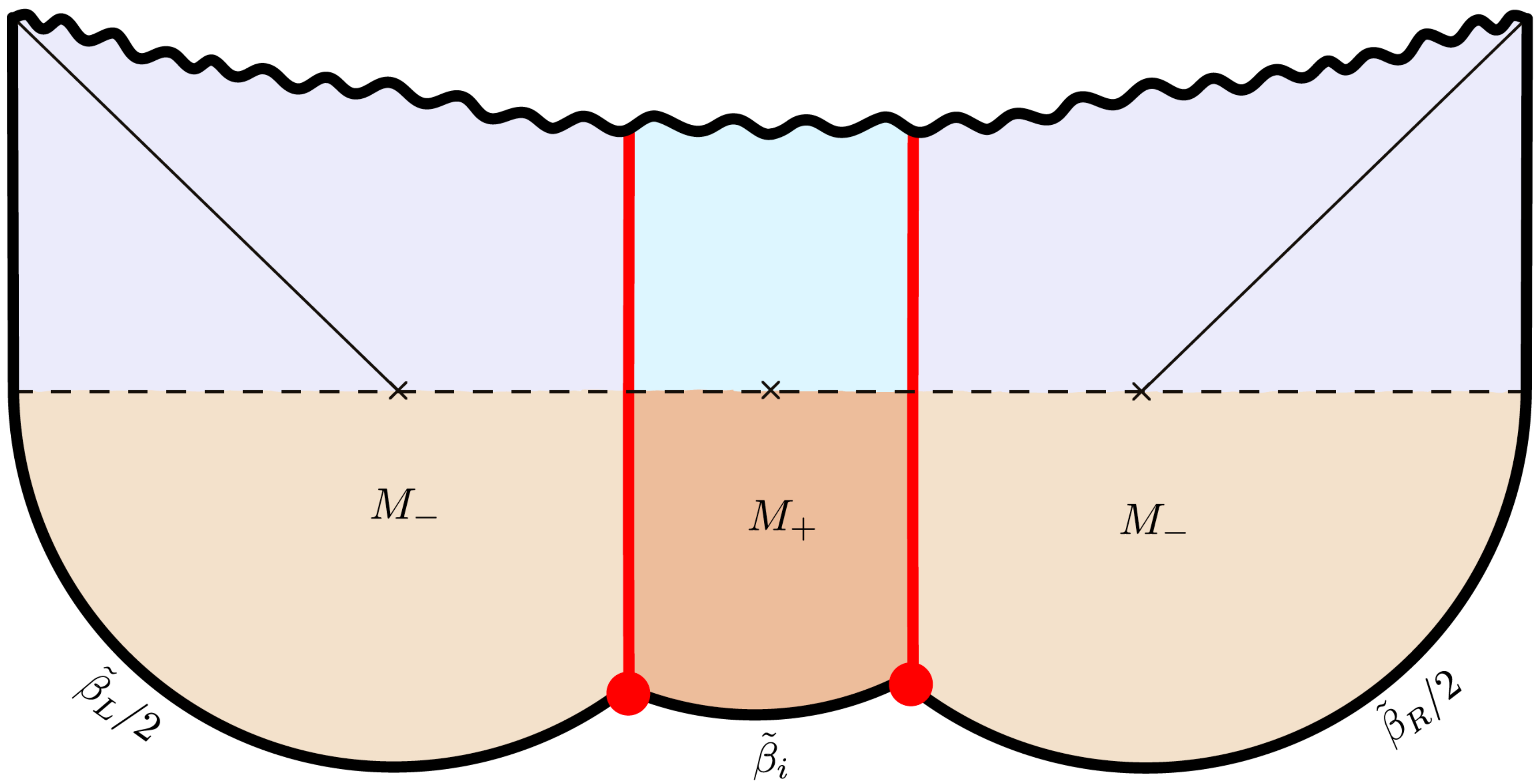

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