# Freely acting orbifolds of type IIB on $T^{5}$ 

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## Intro \& Motivation

## Spontaneous Supersymmetry

## Breaking in String Theory:

- Scherk-Schwarz (supergravity)
- Freely acting orbifolds (no fixed points) (Rohm '84; Ferrara, Kounnas, Porrati '88, ...)
- String Theory with duality twists (Dabholkar \& Hull '03)


## Scherk-Schwarz

- D+1 dimensional supergravity with symmetry group G

$$
\psi \rightarrow g \psi \quad g \in G
$$

- SS ansatz: $\psi\left(x^{\mu}, z\right)=\exp \left(\frac{M z}{2 \pi R}\right) \psi\left(x^{\mu}\right)$
- $M \in \mathfrak{g}$ is called the mass/twist matrix
- Monodromy: $\mathscr{M}=\exp (M) \in G$
- Results in gauged supergravity in D dimensions


## 6D to 5D Scherk-Schwarz

- IIB on $T^{4}: 6 \mathrm{D}(2,2)$ supergravity with U-duality group SO(5,5). T-duality group O(4,4). R-symmetry SO(5)xSO(5).
- Scalars SO(5,5)/SO(5)xSO(5): 25 scalars in $10 \times 10$ matrix

$$
\begin{gathered}
\mathscr{H} \rightarrow g \mathscr{H} g^{T} \quad g \in S O(5,5) \\
\mathcal{H}\left(x^{\mu}, z\right)=e^{M z} \mathcal{H}\left(x^{\mu}\right) e^{M^{T} z}
\end{gathered}
$$

- Twisting in the R-symmetry (partially) breaks supersymmetry in 5D, from $N=8$ to $N=6,4,2,0$.


## Twist Matrix

Twist in R-symmetry

$$
S O(5)_{L} \times S O(5)_{R} \simeq U S p(4)_{L} \times U S p(4)_{R}
$$

to break supersymmetry

$$
U S p(4) \rightarrow U S p(2) \times U S p(2) \sim S U(2) \times S U(2)
$$

Twist/mass matrix

$$
M_{\mathrm{L}}^{\mathfrak{u} \mathfrak{s p}(4)}=\left(\begin{array}{cccc}
0 & 0 & -m_{1} & 0 \\
0 & 0 & 0 & -m_{2} \\
m_{1} & 0 & 0 & 0 \\
0 & m_{2} & 0 & 0
\end{array}\right), \quad M_{\mathrm{R}}^{\mathfrak{u s p}(4)}=\left(\begin{array}{c}
0 \\
0 \\
m_{3} \\
0
\end{array}\right.
$$

| Fields | Representation | Masses |
| :---: | :---: | :---: |
| Scalars | $(\mathbf{5}, \mathbf{5})$ | $\left\| \pm m_{1} \pm m_{2} \pm m_{3} \pm m_{4}\right\|$ |
|  |  | $\left\| \pm m_{1} \pm m_{2}\right\|$ |
|  |  | $\left\| \pm m_{3} \pm m_{4}\right\|$ |
|  | $(\mathbf{4}, \mathbf{4})$ | $\left\| \pm m_{1,2} \pm m_{3,4}\right\|$ |
| Vectors | $(\mathbf{5}, \mathbf{1})$ | $\left\| \pm m_{1} \pm m_{2}\right\|, 0$ |
| Tensors | $(\mathbf{1}, \mathbf{5})$ | $\left\| \pm m_{3} \pm m_{4}\right\|, 0$ |
| Gravitini | $(\mathbf{4}, \mathbf{1})$ | $\left\| \pm m_{1,2}\right\|$ |
|  | $(\mathbf{1}, \mathbf{4})$ | $\left\| \pm m_{3,4}\right\|$ |
| Dilatini | $(\mathbf{5}, \mathbf{4})$ | $\left\| \pm m_{1} \pm m_{2} \pm m_{3,4}\right\|$ |
|  |  | $\left\| \pm m_{3,4}\right\|$ |
|  | $(\mathbf{4}, \mathbf{5})$ | $\left\| \pm m_{1,2} \pm m_{3} \pm m_{4}\right\|$ |
|  |  | $\left\| \pm m_{1,2}\right\|$ |

## Partial SUSY Breaking

Patterns of susy breaking:

$$
\begin{aligned}
& m_{1}=m_{2}=m_{3}=0 \rightarrow N=6 \\
& m_{1}=m_{2}=0 \rightarrow N=4(2,0) \\
& m_{1}=m_{3}=0 \rightarrow N=4(1,1) \\
& m_{3}=m_{4}=0 \rightarrow N=4(0,2) \\
& m_{4}=0 \rightarrow N=2(0,1) \\
& m_{1}=0 \rightarrow N=2(1,0)
\end{aligned}
$$

The $\mathrm{N}=4(2,0)$ and $(0,2)$ are related by parity, and are symmetric orbifolds $m_{1}=m_{2}, m_{3}=m_{4}$. $\mathrm{N}=6$ and $\mathrm{N}=2$ are asymmetric orbifolds.

## Orbifolds

- We consider orbifolds on $\mathbb{R}^{1,4} \times S^{1} \times T^{4}$ :
- A $T^{4} / \mathbb{Z}_{p}$ combined with a shift over $2 \pi R / p$ on the $S^{1}$
- Because of the shift, there are no fixed points (but still twisted sectors)
- Susy is spontaneously broken instead of explicitly: gravitini become massive instead of being projected out


## Orbifolds

- Symmetric orbifolds: $\mathbb{Z}_{p} \in G L(4 ; \mathbb{Z})$. This requires $p=2,3,4,5,6,8,10,12,24$
- $p=5,8,10,12,24$ break all susy, so susy requires $p=2,3,4,6$
- Asymmetric orbifolds: $\mathbb{Z}_{p} \in \operatorname{Spin}(4,4 ; \mathbb{Z})$; the twist matrix being element of the T-duality group. R-symmetry twists: $\mathbb{Z}_{p} \in \operatorname{Spin}(4) \times \operatorname{Spin}(4)$. Complete list for $p$ not known!


## Orbifolds

| Preserved supersymmetry | $(\mathrm{A})$ symmetric | Possible $\mathbb{Z}_{p}$ orbifold ranks |
| :---: | :---: | :---: |
| $\mathcal{N}=6$ | A | $p=2,3$ |
| $\mathcal{N}=4(0,2)$ | S | $p=2,3,4,6$ |
|  | A | $p=3,4,6,12$ |
| $\mathcal{N}=4(1,1)$ | A | $p=2,3,4,6,12$ |
| $\mathcal{N}=2$ | A | $p=2,3,4,6,12$ |
| $\mathcal{N}=0$ | S | $p=2,3,4,6,8,12,24$ |
|  | A | $p=3,4,6,8,12,24$ |

## Modular invariance

- For symmetric orbifolds, it is quite straightforward to construct modular invariant partition functions.
- For asymmetric orbifolds, it can lead to new constraints. E.g. for the $N=6 \mathbb{Z}_{p}$ orbifold, only $p=2,3$ lead to modular invariant partition function with integer coefficients in the $q \bar{q}$ expansion. For $p=4,6$ this integrality condition is not satisfied and these theories are (probably) in the swampland (Bianchi et al. '22). Similar issues arise in other asymmetric orbifolds.


## Example 1 : $\mathrm{N}=6$

- $m_{1}=m_{2}=m_{3}=0, m_{4}=\pi$. This is the $N=6 \mathbb{Z}_{2}$ asymmetric orbifold:
- $X_{L}^{a} \rightarrow-X_{L}^{a} ; a=1,2,3,4 Z \rightarrow Z+\pi R$
- Moduli space $\frac{S U^{*}(6)}{U S p(6)}$
- Only D1-D5 BPS bound state with equal $N_{1}=N_{5}$ (Bianchi '08)
- Generates an $R^{2} F^{2}$ term (cf. Bianchi, Bossard, Consoli '22)


## Example 2: $\mathrm{N}=4$

VM Moduli space: $\quad \mathscr{M}_{\mathcal{N}=4}=\mathrm{SO}(1,1) \times \frac{\mathrm{SO}(5, n)}{\mathrm{SO}(5) \times \operatorname{SO}(n)}$
[Awada\&Townsend, '85]
But we only find odd values for $n=1,3,5,7$
Pure $N=4$ supergravity in $D=5$ is/seems to be in the swampland.

For asymmetric ( $m_{2}=m_{4}=0$ ) orbifolds: no massless R-R states, all D-branes projected out. No S-duality. No BPS black holes from D-branes.

## Example 3: $\mathrm{N}=2$

Only asymmetric orbifolds. We find some of the magic $\mathrm{N}=2$ supergravities (Gunaydin, Sierra, Townsend), but not all. Dilaton sits in vector multiplet.
$m_{1}=m_{2}=m_{4}=\pi$ : same bosonic field content as $\mathrm{N}=6$. But D1-D5 bound state no longer BPS.
$m_{1}=m_{2}=m_{3}=2 \pi / 3 \mathbb{Z}_{3}$ orbifold. Single $\mathrm{N}=2$
hypermultiplet, quaternionic moduli space $S O(4,1) / S O(4)$
Was thought to not exist!

## Example 4: $\mathrm{N}=0$

- When all mass parameters are switched on, susy is completely broken. An example is

$$
m_{1}=m_{2}=m_{3}=m_{4}=\pi
$$

- At string theory level, the $\mathbb{Z}_{2}$ orbifold is $(-)^{F\left(T^{4}\right)}$ (similar to type OB), combined with a shift over the circle.
- This leaves all $\mathrm{D}=5$ scalars massless and lifts all the fermions. The resulting theory is tachyon free and the classical moduli space is $E_{6(6)} / U S p(8)$, the same as in $\mathrm{N}=8$.


## Outlook \& Future work

- Suprising new results for such an old and well-studied topic
- Study further D-branes in these orbifolds
- Make black holes in string theories with broken susy
- Fate of e.g. the D1-D5 system?
- $(4,4)$ CFT broken to $(2,4),(0,4),(2,2),(0,2)$ or maybe $(0,0)$ ?
- Similar analysis can be done in $\mathrm{D}=4$

