On the compactification of 5d SCFTs to 3d and discrete anomalies

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Motivation

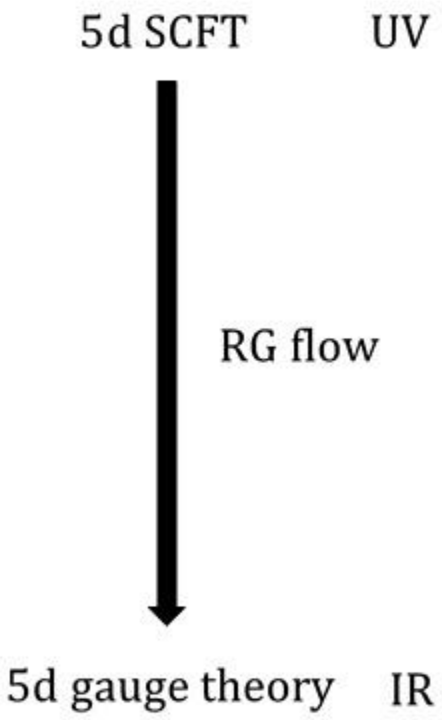
- Much can be uncovered about the dynamics of field theories from relationships between theories in different dimensions (compactification).
- Can be used to study the landscape of SCFTs and uncover strong coupling phenomena, like dualities.
- In such relations: the anomaly polynomial of the higher and lower dimensional theories are related.
- Are there similar relationships between anomalies in discrete symmetries?
- We study this question in a specific context: compactifications to 3d of 5d SCFTs.
- Can be used as a consistency check to understand compactifications. Imply relations between the higher and lower dimensional theories.

Outline

1. Introduction
   - Review on the surface compactification of 5d SCFTs
   - Across dimension anomaly matching
2. Across dimension anomaly matching for discrete symmetries
3. Summary and examples
4. Conclusions

5d SCFTs

- 5d theories with $\mathcal{N}$=1 superconformal symmetry. Based on the exceptional supergroup $F(4)$.
- They have $SO(5,2) \times SU(2)$ bosonic symmetries. They can also have global symmetries.
- Can be realized in string theory in various methods: brane systems, M-theory compactifications on CY3.
- Appear in field theory as UV completions of 5d gauge theories.
- 5d SCFT can be mass deformed → a 5d gauge theory whose inverse coupling constant squared is related to the mass deformation [Seiberg, 1996].



Examples

- One of the most well known 5d SCFTs are the Seiberg E_{N_f+1} theories.
- These are the UV completions of the 5d gauge theories $SU(2) + N_f F$, when N_f<8 [Seiberg, 1996].
- Have E_{N_f+1} global symmetry that is broken to $U(1) \times SO(2N_f)$ by the mass deformation.
- Many other 5d SCFTs exist. For instance, the UV completions of other gauge groups, like $SU(N)$ with fundamental matter.

Comment: $E_5 = SO(10)$, $E_4 = SU(5)$, $E_3 = SU(3) \times SU(2)$, $E_2 = SU(2) \times U(1)$, $E_1 = SU(2)$

General properties of compactification

- We wish to compactify 5d SCFTs on Riemann surfaces to 3d.
- To preserve SUSY: need to perform topological twisting. In general preserves only $\mathcal{N}$=2 SUSY in 3d (4 supercharges).
- May also include fluxes in global symmetries.
- Resulting theory depends on choice of 5d SCFT and compactification data (genus of Riemann surface, fluxes).
- Other properties of the surface (geometry, functional dependence of the flux) are generally irrelevant in IR.

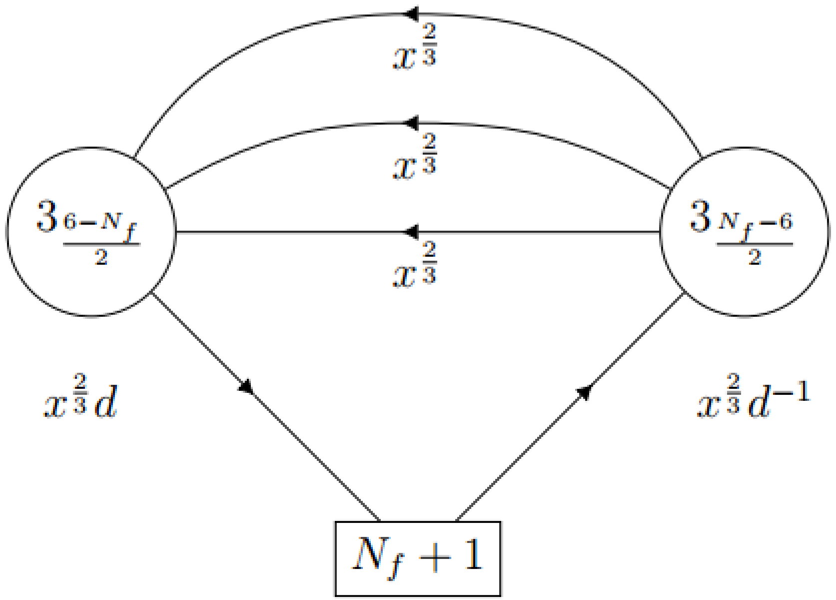
Resulting 3d theories

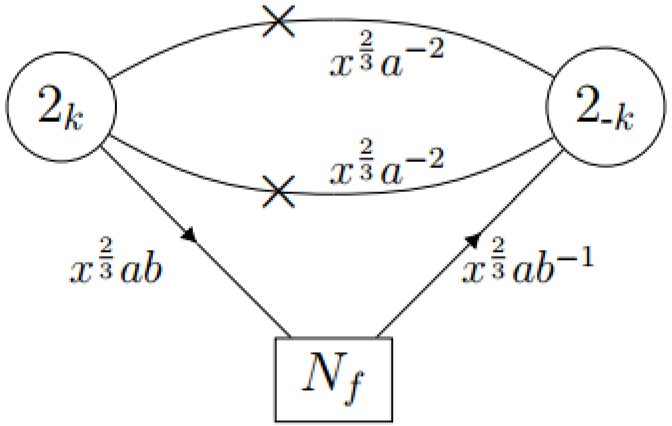
- Want to determine the resulting 3d theories.
- Progress can be made using a method that originated from the study of compactifications of 6d SCFTs to 4d [Kim, Razamat, Vafa, GZ, 2017,2018].
- The general idea is to start with compactifications on tori, were the analysis can be carried by reducing first to 4d, and then to 3d.
- Can incorporate global symmetry fluxes by choosing special flux configurations. Rely on the dependence of the resulting 3d theory only on the total flux, and not on its functional form.

Resulting 3d theories

- Can then use properties of 5d SCFTs compactified on a circle to carry the reduction to 4d.
- From these 4d theories: can formulate conjectures for the 3d theories, resulting from the reduction on the second circle. Given in terms of 3d $\mathcal{N}$=2 Lagrangian theories.
- The resulting 3d theories can then be put to various consistency checks.
- Once theories associated with compactifications on tori with fluxes are known, there are additional methods to get compactifications on surfaces with general genus [Razamat, Sabag, GZ, 2019; Razamat, Sabag, 2019].

Resulting 3d theories





Compactification of E_{N_f+1} 5d SCFTS on: torus with flux (above, $k=\frac{1}{2}(6-N_f)$), and genus 2 surface with no flux (left).

- This analysis was carried over in [Sacchi, Sela, GZ, 2021, 2023], resulting in conjectured Lagrangian 3d descriptions for the compactified theories.
- Some examples are given in the figures.

Tests

- The resulting 3d theories are checked via various consistency tests:
  - Global symmetry of 3d theory should match that of the 5d SCFT preserved by the flux. Non-trivial: symmetry many times enhances in the IR.
  - Identifying operators in 3d theory that descend from special operators in the 5d SCFT (notably conserved current and EM tensor).
  - Consistency of the construction: different pair-of-pants realizations of the same surface should yield dual theories.

Anomaly matching across dimensions

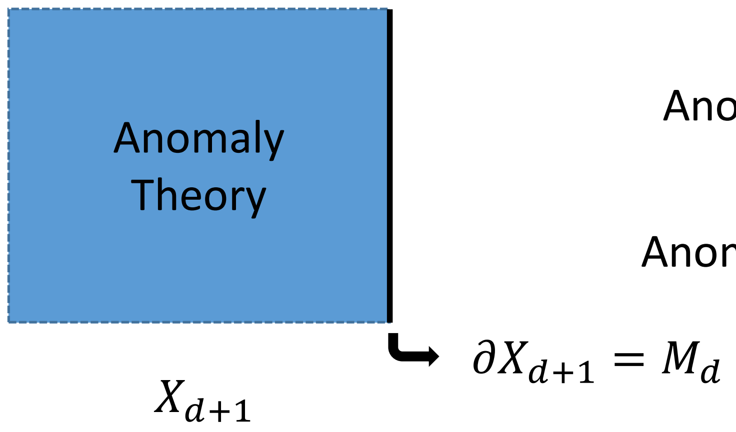
- All of the aforementioned tests were also used in the study of compactifications of 6d SCFTs to 4d.
- However in that case, there is an additional test that can be carried out: anomaly matching.
- The 6d SCFT can have anomalies in continuous symmetries. Can be packaged in a 8d anomaly polynomial.
- Similarly the 4d theory can have anomalies in continuous symmetries. Can be packaged in a 6d anomaly polynomial.
- The anomaly polynomials of the two theories should be related by [Benini, Tachikawa, Wecht, 2010]:

$$\int_{\Sigma} I_8(\mathcal{F}) = I_6^{(\Sigma,\mathcal{F})}$$

Anomaly matching across dimensions: discrete anomalies?

- For the case of 5d to 3d: no anomalies in continuous symmetries as both dimensions are odd.
- However, there can be anomalies in discrete symmetries.
- Can we perform across dimensions anomaly matching using discrete symmetries?
- In this case there is no anomaly polynomial, but we can still talk about an anomaly theory.

Anomaly theory



Anomaly theory action: $S_{AT} = 2\pi \int_{X_{d+1}} \omega(A)$

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Anomaly theory Lagrangian: $\mathcal{L}_{AT} = 2\pi\, \omega(A)$

- The anomaly theory is a d+1 dimensional topological theory living in a space whose boundary is the d dimensional spacetime where the field theory lives.
- Its action is a functional of background gauge fields. Under background gauge transformations: changes by boundary term, that cancels the anomalous variation of the d dimensional theory.

Reducing the anomaly theory

- Consider the case where $M_d = M_{d-2} \times \Sigma$, for some compact boundless 2d surface Σ. Can then take $X_{d+1} = X_{d-1} \times \Sigma$.
- Given a theory on M_d, can associate to it a theory on M_{d-2}, through compactification on Σ.
- It is then natural to expect that the anomaly theory describing the anomalies of the reduced theory to be given by compactification on Σ of the anomaly theory on X_{d+1}.
- Since the anomaly theory has no dynamical fields, we expect the Lagrangians to be related as:

$$\mathcal{L}_{d-1} = \int_{\Sigma} \mathcal{L}_{d+1}$$

- Shall test this relation in the context of 3d compactifications of 5d SCFTs.

Implications

- Can be used as a further consistency check on the resulting 3d theories.
- Illuminates symmetry structure between theories related via compactification.
- Can be used to predict the presence of certain anomalies in 3d.
- Can be used to detect the presence of anomalies in the 5d SCFTs.

Advantages of the 5d to 3d case

- Compactifications from 5d to 3d provide a great test case to study relations between discrete anomalies via compactification.
- Both dimensions are odd: no anomalies in continuous symmetries to test.
- By now: have decent understanding of anomalies in 5d SCFTs [Genolini, Tizzano, 2021, 2022; Apruzzi, Bhardwaj, Oh, Schafer-Nameki, 2021; Apruzzi, Bonetti, Garcia Etxebarria, Hosseini, Schafer-Nameki, 2021], and compactification to 3d [Sacchi, Sela, GZ, 2021,2023].
- Anomalies in 3d can in many cases be conveniently detected via the superconformal index [Bhardwaj, Bullimore, Ferrari, Schafer-Nameki, 2022, 2023; Sacchi, Mekareeya, 2022].

Anomalies of 5d SCFTs

- There are various known discrete anomalies in 5d SCFTs.
- Anomalies involving 1-form symmetries: the E_1 5d SCFT. Has $SO(3)$ 0-form symmetry and a $\mathbb{Z}_2$ 1-form symmetry with a mixed anomaly [Genolini, Tizzano, 2021]:

$$\frac{\pi i}{2}\int_{X_6} \mathcal{P}(B_2)\omega_2(SO(3))$$

- Anomalies involving discrete aspects of continuous symmetries: an $SU(2)$ global symmetry in 5d can carry a Witten anomaly [Witten, 1982; Intriligator, Morrison, Seiberg, 1997]. For SCFTs, there is always the $SU(2)$ R-symmetry. Has non-trivial Witten anomaly when rank is odd.
- Anomalies involving spacetime symmetries: 5d parity anomaly [Intriligator, Morrison, Seiberg, 1997].

Summary of main results

- Can analyze anomalies in the 3d theories conjectured to be the result of the compactification of 5d SCFTs, notably, the E_{N_f+1} SCFTs.
- Analysis done using the superconformal index.
- Can detect the presence of various anomalies in 3d.
- Can in some cases be matched with known 5d anomalies.
- In other cases: suggests the presence of new anomalies in 5d.
- 5d Witten anomaly for the $SU(2)$ R-symmetry → 3d parity anomaly for the $U(1)$ R-symmetry. Can indeed match the anomaly between 5d and 3d.

Examples: E_1 on a torus with flux

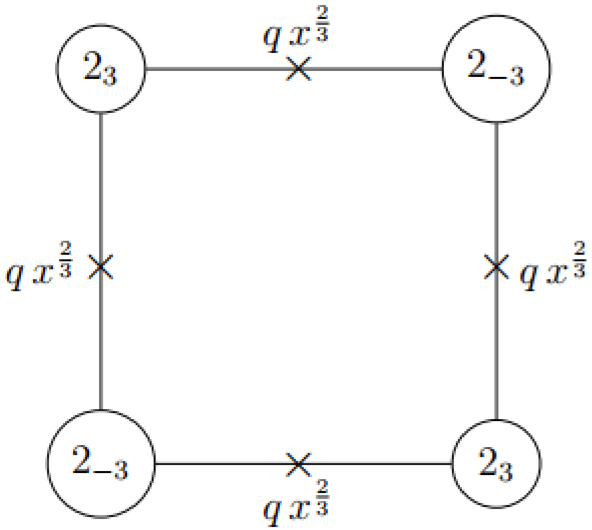
- Consider the E_1 5d SCFT. As just mentioned, has the mixed anomaly:

$$\frac{\pi i}{2}\int_{X_6}\mathcal{P}(B_2)\omega_2(SO(3))$$

- We can consider the compactification of E_1 SCFT on a torus with flux F in the Cartan of the $SO(3)$.
- Expect to get a 3d theory with $U(1)$ 0-form symmetry and a $\mathbb{Z}_2$ 1-form symmetry.
- By integrating the anomaly theory, we expect the mixed anomaly:

$$\pi i F\int_{X_4} B_2(C_1(U(1))\ mod\ 2)$$

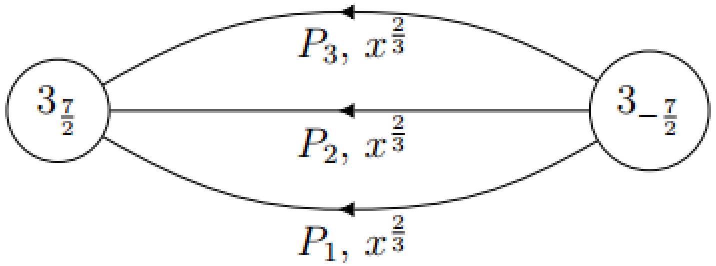
Examples: E_1 on a torus with flux



Compactification of E_1 5d SCFTS on torus with flux $F = 2$.

- Can compare against an explicit 3d calculation in the conjectured theory.
- Our results indeed suggests the presence of the expected anomaly.

Examples: E_0 on a genus 2-surface



- As another example, consider the E_0 5d SCFT. Has no continuous 0-form flavor symmetries, but has a $\mathbb{Z}_3$ 1-form symmetry [Morrison, Schafer-Nameki, Willett, 2020; Albertini, Del Zotto, Garcia Etxebarria, Hosseini, 2020].
- The 3d theory resulting from its compactification on a genus 2 surface was worked out in [Sacchi, Sela, GZ, 2023]. Indeed has no continues flavor symmetries, but has a $\mathbb{Z}_3$ 1-form symmetry.

Examples: E_0 on a genus 2-surface

- Can study anomalies involving the 1-form symmetry of the 3d theory using the superconformal index.
- Suggests the presence of the following mixed anomaly:

$$\frac{2\pi i}{3}\int_{X_4} B_2(C_1(U(1)_R)\ mod\ 3)$$

- Expect this anomaly to have a 5d origin. Indeed, it can be understood if the 5d SCFT has the mixed anomaly:

$$\frac{2\pi i}{3}\int_{X_6} B_2(C_2(SU(2)_R)\ mod\ 3)$$

- Results on compactification on higher genus also consistent with this anomaly. This suggests the presence of such anomaly in the 5d SCFT.

Conclusions

- Dimensional reduction of field theories leads to a relation between anomalies in continuous symmetries of the two theories.
- Natural to expect a similar relation to hold for discrete anomalies, where the associated anomaly theories are also related via dimensional reduction.
- We have preformed tests on this proposal by utilizing recent results on anomalies of 5d SCFTs and compactification of 5d SCFTs to 3d.
- Can also be used to test our understanding of anomalies and compactification of 5d SCFTs.
- Can be used to learn new information about the 5d SCFTs, like presence of new anomalies.

Open questions

- Generalizations: more anomalies, other 5d SCFTs, compactification from/to other dimensions.
- Can we detect the new anomalies in 5d SCFTs also by other methods?
- Extension to more general symmetry structures: 2-group, non-invertible symmetries …

Thank you