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A new stochastic frontier model with cross-sectional effects in both noise and inefficiency terms

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Abstract

This paper develops a new stochastic frontier model that allows for cross-sectional (spatial) correlation in both the noise and inefficiency terms. The model proposed is useful in efficiency analyses when there are omitted but spatially-correlated variables and firms benefit from best practices implemented by other (adjacent) firms. Unlike the previous literature, our model can be estimated by maximum likelihood using standard software. The model is illustrated with an application to the Norwegian electricity distribution sector.

Keywords: stochastic frontiers, correlated inefficiency, unobserved environmental conditions, incentive regulation, electricity distribution.

JEL: C21, C51, L25, L51, L94

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1. Introduction

This paper develops a new stochastic frontier (SF) model that allows for cross-sectional (spatial) correlation in both noise and inefficiency terms**.** A model specification with cross-sectional correlation in the noise term may be useful in efficiency analyses as it accounts for unobserved but spatially-correlated variables. Failure to account for spatial correlation effects in SF models might result in biased estimates of efficiency scores (e.g. Schmidt et al., 2009, and Orea et al (2017). On the other hand, a SF model with cross-sectional correlation in the inefficiency term could also be useful for examining whether firms benefit from best practices implemented in their adjacent firms. Spatial dependence in technical efficiency can also be found, for instance, in agricultural production (e.g. Areal et al., 2012, Vidoli et al., 2016, and Mate-Sánchez-Val et al., 2017), or when we are examining the efficiency of production units of a single firm they follow similar procedures of action designed by the holding they belong to.

In contrast to the previous literature (e.g. Areal et al., 2012, Tsionas and Michaelides, 2016; Schmidt et al., 2009, and Herwartz and Strumann, 2014), we develop a novel SF model with cross-sectional (spatial) correlated noise and inefficiency terms that can be estimated by ML using standard software. Our model can thus be viewed as a new application of the scaling property in SF analyses (for other uses of the scaling property, see Parmeter and Kumbhakar, 2014). Unlike the previous literature, our model permits separate examinations of the characteristics of the cross-sectional correlations of the noise and inefficiency terms, which are likely to be of a different nature. Furthermore, our model can easily be extended to incorporate global and local spatial spillovers. Our specification differs from Glass et al. (2013,2014) because rather than calculating efficiency from the cross-sectional specific effects, we compute efficiency by making an assumption about the distribution of the inefficiency term. In this sense, a spatial autoregressive (SAR) version of our spatial SF model can also be viewed as an extension of the Glass et al. (2016a) model.

The model is illustrated with an application to the Norwegian electricity distribution sector for the years 2004 to 2011. We have chosen this application for three reasons. First, this is a regulated sector where revenue caps are set by the Norwegian regulator (NVE) on the basis of total cost benchmarking. Thus, we expect the existence of some cross-sectional correlation in firms' inefficiency. Second, the NVE regulator aims to control for unobserved differences in environmental conditions among electricity distribution networks. As shown by Orea et al (2017), many weather conditions, geographical conditions and other unobserved cost drivers are likely to be spatially correlated. Third, we do not need a model with spatial or cross-sectional correlations in the frontier because the NVE staff in charge of network regulation do not see major systemic technical reasons for the cost of an electricity distribution firm to be affected by those of an adjacent firm to any significant degree.

The next section develops a SF model that allows for cross-sectional (spatial) correlation in both noise and inefficiency terms**.** Section 3 discusses the data used in the empirical analysis, and depicts the main results of our model with cross-sectional spillovers in either the noise term, the inefficiency term or both terms. Finally, Section 4 presents the conclusions and a future research agenda.

2. A cross-sectional stochastic frontier model

In this section we poresent our stochastic frontier model that allows for cross-sectional (spatial) correlation in both noise and inefficiency terms**.** We hereafter label this model as CSSF. Let us first assume that the firms' cost can be modelled using the following cost equation:

$$lnC_{it} = \beta X_{it} + \tilde{v}_{it} + \tilde{u}_{it} \tag{1}$$

where *i* stands for firms, *t* stands for periods, lnC_{it} is a measure of firms' cost in logs, X_{it} is a vector of *K* observable cost drivers, and $\tilde{v}_{it}$ and $\tilde{u}_{it}$ are two random terms where the former is an error term measuring random shocks and the latter a random term measuring firms' inefficiency. We allow both error terms to be cross-sectionally correlated. To this end, we adapt the popular SAR spatial stochastic process to a more general cross-sectional setting.

A cross-sectional or spatial autoregressive (SAR) specification for the two error terms in period t can be expressed as:

$$\tilde{v}_{it} = v_{it} + \rho W_i \tilde{v}_t \tag{2}$$

$$\tilde{u}_{it} = u_{it} + \tau W_i \tilde{u}_t \tag{3}$$

where v_{it} and u_{it} are two idiosyncratic noise and inefficiency terms, assumed to be distributed independently across the cross-sectional dimension, $\tilde{v}_t = (\tilde{v}_{1t}, \dots, \tilde{v}_{Nt})$ and $\tilde{u}_t = (\tilde{u}_{1t}, \dots, \tilde{u}_{Nt})$ are Nx1 vectors of the firms' random terms, ρ and τ are two cross-sectional autoregressive parameters, and $W_i = (W_{i1}, \dots, W_{iN})$ is a known 1xN cross-sectional weight vector with elements that are equal to zero if the noise/inefficiency term of a particular firm *j* is not assumed to be correlated with the noise/inefficiency of firm *i*, and equal to one otherwise. Quite often the weight vector is *row-standardized* with the number of adjacent units. In this case, $W_i v_t$ and $W_i u_t$ should be interpreted as the *average* values of the noise and inefficiency terms of adjacent firms. The coefficients ρ and τ measure the degrees of cross-sectional correlation between firms' noise and inefficiency terms respectively. The above transformations can be interpreted as a sort of *generalized* transformation of the original idiosyncratic noise and inefficiency terms. In what follows we will use the notation "~" to denote this transformation of the variables.

The noise and inefficiency terms can be rewritten using matrix notation as $\tilde{v}_t = M_\rho \cdot v_t$ and $\tilde{u}_t = M_\tau \cdot u_t$, where $M_\rho = (I_N - \rho W)^{-1}$, $M_\tau = (I_N - \tau W)^{-1}$ and $W = (W_1, W_2, \ldots, W_N)$ is an NxN spatial weight matrix. The constraints imposed by both the weighting structure together with the specific form of the cross-sectional process determine the covariance matrix for v_t and u_t. For the SAR structure in (2) and (3), the cross-sectional covariance matrices of both error terms will be full or not spare, which implies that shocks disturbing a particular firm might affect all other firms. In other words, the variance-covariance structure induced by the SAR model is *global*.

Note that the specification of firms' cost in equation (1) has the structure of a traditional SF model. However, the above model cannot be estimated using full maximum likelihood if we assume as customary that u_{it} is independently distributed across firms (see, for instance, Wang, 2003). Wang and Ho (2010) faced a similar problem using temporal transformations of their SF model. To get a closed form for the likelihood function once first-differences and within transformations of the model were carried out, they assumed that the inefficiency term u_{it} possesses the scaling property so that it can be multiplicatively decomposed into two components as follows:

$$u_{it} = h(m_{it}, \delta) \cdot u_i^* = h_{it} \cdot u_i^* \tag{4}$$

where $h_{it} \geq 0$ is a function of firm exogenous variables, and $u_i^* \geq 0$ is a firm-specific inefficiency term. Note that this implies that their within-transformed inefficiency term can be written as $\left(h_{it} - \frac{1}{T}\sum_{t=1}^{T} h_{it}\right) \cdot u_i^*$, where the distribution of u_i^* is not affected by the within-transformation. This key aspect of their model enabled them to get a tractable likelihood function for their transformed model.

Following Wang and Ho (2010), we next assume that the idiosyncratic inefficiency term (u_{it}) can be multiplicatively decomposed into two components, that is:

$$u_{it} = h(m_{it}, \delta) \cdot u_t^* = h_{it} \cdot u_t^* \tag{5}$$

where $h_{it} \geq 0$ is again function of firm and industry exogenous variables, and $u_t^* \geq 0$ is an industry-specific inefficiency term. For simplicity, we will assume that $u_t^* \sim N^+(0, \sigma_u)$. The above specification implies that firms' inefficiency in our model is obtained by stretching or shrinking the industry inefficiency level using a deterministic (scaling) function that scales up or down a random inefficiency value that is common to all firms in period *t*. As with WH, the random part of firms' inefficiency is used here as an anchor value to build the overall inefficiency level of each firm.

The above specification of u_{it} implies that the SAR-transformed inefficiency term in (3) can be written in matrix notation as:

$$\tilde{u}_t = (I_N - \tau W)^{-1} u_t = M_\tau u_t = M_\tau h_t u_t^* = \tilde{h}_t \cdot u_t^* \tag{6}$$

where $h_t = (h_{1t}, \dots, h_{Nt})$, and $\tilde{h}_t = (\tilde{h}_{1t}, \dots, \tilde{h}_{1t})$ are Nx1 vectors. The half-normal distribution of u_t^* is not affected by the cross-sectional transformation. As WH pointed out, this is the crucial aspect of the model that enables us to get a tractable likelihood function. In this sense, the industry nature of the anchor value used in (5) makes our specification of firms' inefficiency attractive in any SF model with cross-sectional spillovers in the one-sided error term. It should be noted as well that, neglecting the existence of individual effects, our model does not collapse to WH if the cross-sectional correlation tends to vanish. This model can be viewed as a transposed version of WH's model because we have changed the dimension of the basic random variable. We hereafter label this model as TWH.

We now adapt the WH likelihood function to a cross-section setting. In principle, the likelihood function of our CSSF model should be derived based on the joint distribution of $\tilde{v}_t$ and $\tilde{u}_t$ after some tedious derivations. However, as our CSSF model can be viewed as a transposed version of the WH model, the general specification of the likelihood function in WH is still valid in our setting. We only need to adapt the likelihood function in Wang and Ho (2010, p. 288) to a cross-sectional framework.

The cross-sectional adaptation requires carrying out three adjustments to their likelihood function. We first need to reverse the summation of partial log-likelihood functions used to get the overall likelihood function for the whole sample of observations. Second, we need to replace the temporal weight matrix implicitly used in WH with a cross-sectional weight matrix. Finally, we need to adjust the formula used by Wang and Ho (2010) to get firm-specific efficiency scores to our cross-sectional setting *and* our definition of firms' inefficiency.

As in WH, we should take into account that the Nx1 noise vector $\tilde{v}_t = (\tilde{v}_{1t}, \dots, \tilde{v}_{Nt})$ follows a multivariate normal distribution if we assume that the idiosyncratic noise term follows a normal distribution, i.e. $v_{it} \sim N(0, \sigma_v)$. Thus, the density function of the vector $\tilde{v}_t$ is:

$$g(\tilde{v}_t) = (2\pi)^{-\frac{N}{2}} |\Pi|^{-1/2} exp\left\{-\frac{1}{2}\tilde{v}_t{}'\Pi^{-1}\tilde{v}_t\right\} \quad (7)$$

where Π is the variance-covariance matrix of $\tilde{v}_t$, that can be written in general terms as $\Pi = \sigma_v^2 M_\rho{}' M_\rho$.

We now discuss the part of the likelihood function that has to do with the inefficiency term. Let us recall here that the truncated normal distribution of u_t^* is not affected by the cross-sectional transformation of the idiosyncratic inefficiency term, and thus $\tilde{u}_t = M_\tau h_t \cdot u_t^*$ is distributed as a heteroscedastic half-normal. The cross-sectional adaptation of equations (13)-(16) in Wang and Ho (2010) yields the following log-likelihood function for period t:

$$lnL_t = -\frac{N}{2} ln(2\pi) - \frac{1}{2} ln|\Pi| - \frac{1}{2}\tilde{\varepsilon}_t{}'\Pi^{-1}\tilde{\varepsilon}_t$$

$$+\frac{1}{2}\left(\frac{\mu_*^2}{\sigma_*^2} - \frac{\mu^2}{\sigma_u^2}\right) + ln\left[\sigma_* \Phi\left(\frac{\mu_*}{\sigma_*}\right)\right] - ln\left[\sigma_u \Phi\left(\frac{\mu}{\sigma_u}\right)\right] \quad (8)$$

where Φ is the standard normal cumulative distribution function, $\tilde{\varepsilon}_t = (\tilde{\varepsilon}_{1t}, \dots, \tilde{\varepsilon}_{Nt})$, $\tilde{\varepsilon}_{it} = lnC_{it} - \beta X_{it}$, and

$$\mu_* = \frac{\mu/\sigma_u^2 - \tilde{\varepsilon}_t{}'\Pi^{-1}\tilde{h}_t}{h_t{}'\Pi^{-1}\tilde{h}_t + 1/\sigma_u^2} \quad (9)$$

$$\sigma_*^2 = \frac{1}{\tilde{h}_t{}'\Pi^{-1}\tilde{h}_t + 1/\sigma_u^2} \quad (10)$$

The final log-likelihood function of the model is obtained by summing the above function from t=1 to t=T. Consistent parameters estimates can be obtained by numerically maximizing $lnL = \sum_{t=1}^{T} lnL_t$. Note that we first derive the partial log-likelihood function of all cross-sectional observations in period t, and then the overall likelihood function is obtained using the temporal dimension of our data. We have thus transposed the WH procedure, who first obtain the partial log-likelihood function using the temporal dimension of the panel (i.e. using all temporal observations of unit *i*) and then obtain the overall likelihood function as $lnL = \sum_{i=1}^{N} lnL_i$ using the cross-sectional dimension.

We finally discuss how we can obtain an efficiency score for each firm once the CSSF model has been estimated. Wang and Ho (2010) modified the Jondrow et al. (1982) formula and use the conditional distribution of u_{it} given the composed error term in differences (i.e. $\tilde{\varepsilon}_{it}$) to estimate u_{it}. Their estimator can be written using our notation as follows:

$$E(u_{it}|\tilde{\varepsilon}_{it}) = h_{it} \cdot \left[\mu_* + \frac{\sigma_* \phi\left(\frac{\mu_*}{\sigma_*}\right)}{\Phi\left(\frac{\mu_*}{\sigma_*}\right)}\right] = h_{it} \cdot E(u_i^*|\tilde{\varepsilon}_{it}) \tag{11}$$

We adapt here the temporal framework in WH to our cross-sectional setting. Given our model in (1) and our distributional assumptions, the analytical form for $E(\tilde{u}_{it}|\tilde{\varepsilon}_{it})$ can be written as follows:

$$E(\tilde{u}_{it}|\tilde{\varepsilon}_{it}) = \tilde{h}_{it} \cdot \left[\mu_* + \frac{\sigma_* \phi\left(\frac{\mu_*}{\sigma_*}\right)}{\Phi\left(\frac{\mu_*}{\sigma_*}\right)}\right] = \tilde{h}_{it} \cdot E(u_t^*|\tilde{\varepsilon}_{it}) \tag{12}$$

where $\tilde{h}_{it} = M_{i\tau} h_t$ and $M_{\mathrm{i\tau}}$ is the i^th row of M_τ. Two comments are in order regarding equation (21). In our model, firm's inefficiency is defined as $\tilde{u}_{it} = \tilde{h}_{it} u_t^*$ and not as $u_{it} = h_{it} u_t^*$. That is, the relevant and final scaling function is $\tilde{h}_{it}$ and not h_{it} as in WH. For this reason we are interested in $E(\tilde{u}_{it}|\tilde{\varepsilon}_{it})$ and not in $E(u_{it}|\tilde{\varepsilon}_{it})$. Second, the term in brackets in our model is an estimate of u_t^*, while in the WH model is an estimate of u_i^*.

3. Empirical illustration

We apply our empirical strategy to a balanced set of panel data for the Norwegian distribution utilities over the years 2004 to 2011. The data used in this study was obtained from the sector regulator, the Norwegian Water Resources and Power Directorate (NVE). We try to mimic the DEA analysis conducted by NVE and estimate (the negative of) a Translog input-oriented distance function with a single input - total firms' costs - that can be viewed as a cost function without input prices.

Our variable measuring firms' total cost (COST) includes operating expenses, capital depreciation and its opportunity cost, the cost of network energy losses, and the cost of energy not supplied to different user groups due to service interruptions. The cost variable has been deflated using the consumer price index and is expressed in 2004 real terms. Following the previous literature, we include the two main outputs: the number of customers (CUS), and the length of the network (NL). Finally, we include the percentage of overhead lines (OH) because firms' decisions on, for example, investment and maintenance of overhead and underground lines are different, and there could be a trade-off between both types of costs. Regarding firms' inefficiency, we use three inefficiency determinants: the percentage of overhead lines (OH), the number of transformer stations (ST), and a density variable (DEN) measured as the number of customers per kilometre of network. These three variables are measuring the urbanization (ruralisation) of the supplied areas and the complexity of networks as a whole.

Regarding the weight matrix W, we compute a spatial-based weight matrix aiming to capture cross-sectional spillovers in both firms' noise and inefficiency terms. To achieve this objective, we consider the distribution areas available on a map provided by NVE in October, 2015. In our study we follow the common approach in the literature for capturing and measuring the spatial interdependence using a physical contiguity matrix, whose elements are one for bordering areas, and zero otherwise. As is customary in the spatial econometric literature, we normalize this matrix with the number of adjacent spatial units, so that each of the non-zero elements of the matrix W_N equals the inverse of the number of adjacent service areas.

Table 1 provides a descriptive summary of the variables used in this study.

[Insert Table 1 here]

In Table 2 we show the parameter estimates of two simple SF models with no cross-sectional correlations. All of them use the same efficiency covariates and they differ only in the specification of u_{it}. The so-called WH model uses Wang and Ho (2010) and Orea and Kumbhakar (2004) specification of $u_{it} = h_{it} \cdot u_i^*$, where the u_i^* term follows a half-normal distribution. The TWH model replaces the time-invariant and firm-specific u_i^* term above with a u_t^* term that varies over time but which is common to all firms. All cross-sectional variation in firms' inefficiency is thus forced to be captured by the deterministic scaling function in the TWH model. The two models impose different assumptions on the random variation of u_{it}. The TWH model assumes that the cross-sectional and temporal variations of the basic inefficiency term are different (zero across firms, and positive over time). The WH model reverses the above two assumptions as it assumes a zero temporal variation of the random inefficiency component, but a positive cross-sectional variation. As the estimated coefficients of more comprehensive models are similar to those estimated using the aforementioned SF models with no cross-sectional spillovers, we also take advantage of Table 2 to briefly discuss the economic implications of the estimated frontier and inefficiency coefficients.

[Insert Table 2 here]

Regarding the frontier parameter estimates, the first conclusion that can be inferred from Table 2 is that both the WH and TWH models yield very similar results. Although the parameter estimates do not coincide, the first-order coefficients still have the expected sign and their magnitudes are also reasonable from a theoretical standpoint. The first-order coefficients of CUS and NL variables allows us to measure scale economies evaluated at the sample mean when the increase in the number of customers is complemented with an equivalent expansion of firms' network. The elasticity of scale in the TWH model is close to, but less than, unity. In line with Kumbhakar et al. (2015), this result suggests that a large number of firms have unexploited scale economies. On the other hand, the coefficient of OH is negative, indicating that a higher share of overhead lines reduces the total costs of utilities.

In addition to the frontier parameters, Table 2 displays the coefficients of the variables that are related to the inefficiency term. Notice that most of the estimated coefficients are statistically significant and have the same sign in both TWH and WH models. The same happens in the CSSF models examined in the next subsection. The coefficient of OH is negative and statistically significant, indicating that is costlier to manage firms with higher shares of overhead lines. We also obtain a negative coefficient for DEN and ST. It is difficult to conclude here whether larger utilities with larger number of stations tend to be more efficient than smaller utilities because the rural areas are generally larger than the urban areas but also have a smaller (larger) number of customers (overhead lines) per kilometre of network than more urbanized areas.

Table 2 also provides the descriptive statistics of the efficiency scores using the WH and TWH models. The standard deviation of the WH efficiency scores is slightly larger (11.4%), as expected given the large estimate of σ_u in this model, than the standard deviation of the TWH efficiency scores (10%). The average efficiency scores using these two models are quite high, on average about 80%. The high level of efficiency of this industry is attributable to the maturity of economic regulation, with the regulator having consistently been supervising and incentivizing the utilities to perform efficiently. Similar figures are obtained in Miguéis et al. (2012) using a DEA method for the period 2004 to 2007 (about 84-87%), and in Kumbhakar et al. (2015) with estimated efficiency in the range 82-87%. Growitsch et al. (2012) for the 2001-2004 period, and Orea et al. (2017) and Orea and Jamasb (2017) for the period 2004 to 2011 find larger efficiency scores of about 89-94% once they control for unobservable environmental conditions using different empirical strategies.

Figure 1 depicts Kernel densities of firms' efficiency. This figure shows that the efficiency distribution becomes less disperse when we use the WH model, as the minimum score in this model is 51.5% compared to 39.3% using the TWH model. However, the kernel density of the TWH model is more skewed to the left, meaning that relatively more of the observations are concentrated on the efficient part of the unit interval. In spite of these differences, overall we find that both distributions are close to each other. Thus, neglecting cross-sectional variations in firms' random inefficiency does not seem to be a crucial issue in our application.

[Insert Figure 1 here]

We next provide the main results of our CSSF model using full maximum likelihood estimation. For robustness analysis, the model has been estimated using SAR specifications for the spatial processes in either the noise term, the inefficiency term, or both terms. We also estimate a model where the overall error term follows a spatial process, as is customary in the spatial econometric literature that does not distinguish between different error terms. This is equivalent to imposing in our model that the ρ and τ coefficients are the same (i.e. $\tau = \rho$). As the estimated coefficients of all these models are similar to those estimated using the WH and TWH models, we will focus our discussion here on the estimated cross-sectional correlations, i.e. the coefficients ρ and τ.

In Table 3 we provide the parameter estimates, and other statistics, corresponding to four CSSF models that use SAR specifications for the spatial processes. Several comments are in order regarding the coefficients ρ and τ. First, the estimated ρ coefficient is always positive and statistically significant. This indicates that weather and geographical conditions as well as other unobserved cost drivers are spatially correlated.

The positive sign suggests that the unobserved conditions in neighbouring areas tend to be similar. This seems to be a robust result as the spatial noise correlation is still significant if we allow the inefficiency term to be correlated as well. Second, the so-called u-SAR model yields a positive and statistically significant τ coefficient. This coefficient even increases a little when we estimate a more comprehensive (uv-SAR) model that also controls for spatial spillovers in the noise term. However, the estimated spatial correlation in the inefficiency term is less than the spatial noise correlation. The positive, and significant, correlation in firms' inefficiency found in this paper seems to suggest that the Norwegian electricity distribution utilities emulate, at least partially, the performance of neighbouring utilities. This finding indicates that spatial proximity matters, and that the regulatory framework in Norway somehow encourages utilities to interact with each other. This is an expected result because the NVE regulator uses DEA techniques to benchmark the regulated firms, and in this setting the performance of each firm depends on the performance of its peers. It is worth mentioning that all data, benchmarking results and revenue cap calculations are published on the NVE web page every year.[1] However, a still unresolved empirical issue is whether the regulated utilities in Norway are using these data and models to identify (and mimic) their own peers. The peers are not necessarily neighbouring utilities, but the peers provided by the DEA exercise and the evaluated firms often have similar characteristics. Thus, if managers do not have precise information on the benchmarking results, each firm might find it profitable to keep an eye on neighbouring utilities because they have similar characteristics and at least some of them are likely to be used as peers by the regulator. Third, the uv-SARr model that imposes $\tau = \rho$ yields a similar coefficient of spatial correlation to that of the v-SAR model. Thus, it seems that the correlation of the inefficiency term is weaker than the correlation of the noise term. Finally, the model selection statistics provided in Table 3 corroborate the use of more comprehensive models as the preferred models are those that allow for both noise and inefficiency spillovers. In this sense, and as a last practical remark regarding the estimation of the uv-SAR model, it is worth mentioning that we have not found problems to simultaneously estimate ρ and τ coefficients in our application.

[Insert Table 3 here]

[1] See, e.g. http://www.nve.no/elmarkedstilsynet-marked-og-monopol/okonomisk-regulering-av-nettselskap/inntektsrammer/.

Figure 2 presents kernel densities of the efficiency score estimates from the SAR models above. We show this figure in order to examine the implications for efficiency analysis of neglecting the existence of spatial (cross-sectional) spillovers in either the noise or inefficiency terms. This discussion extends on previous papers (see e.g. Schmidt et al., 2009) that have examined the effects of overlooking spatial correlations in SF models without any allusion to noise or inefficiency terms. We observe that the kernel densities of the three models with spatial noise spillovers are very similar, regardless of whether or not we control for the existence of inefficiency spillovers. That is, ignoring the existence of spatial spillovers in the inefficiency term is not very important. However, Figure 2 shows that failure to account for spatial noise spillovers in a model that only aims to examine spatial correlations in firms' inefficiency (i.e. the *u*-SAR model) results in biased estimates of efficiency scores.

[Insert Figure 2 here]

We finally depict the annual evolution of the efficiency scores in Figure 3. All models yield similar temporal evolutions of the efficiency scores. Notice that similar trends are also found using a simple SF model that ignores any spatial correlation (i.e. the TWH model). The main difference among the models is the level of the estimated efficiency scores. Figure 3 also provides a very interesting, and somewhat unexpected, result regarding the dynamic performance of the regulated firms. To provide some context for this, our data set covers two regulatory periods, where the first three years (from 2004 to 2006) belong to the 2002-2006 price control review while the remaining years of our data set (from 2007 to 2011) cover the duration of the posterior price control review. The cost base for the revenue cap computations carried out by the NVE regulator is based on reported costs at t-2. Regarding the second regulatory period of our data set, the Norwegian regulator used the reported cost for 2005. Our results show that all SF and CSSF models yield much larger efficiency scores in 2005 than in other years. This outcome can be interpreted as an anecdotal evidence of the existence of gaming performance among regulated firms.

[Insert Figure 3 here]

4. Conclusions and future research agenda

This paper develops a new stochastic frontier model that allows for cross-sectional (spatial) correlation in both the noise and inefficiency terms**.** This model is useful in efficiency analyses when there are spatially correlated cost/production drivers that are not observed, and/or we expect the existence of cross-sectional (spatial) dependence in firms' efficiency. An attractive feature of the proposed model is that it can be estimated by full maximum likelihood using standard software. Unlike the previous literature, the model proposed permits separate examinations of the characteristics of the cross-sectional correlations of the noise and inefficiency terms, which are likely to be of a different nature. The model has been illustrated with an application to the Norwegian electricity distribution sector. Our CSSF models show that noise and firms' inefficiency are spatially correlated. We also find anecdotal evidence of the existence of gaming performance among regulated firms.

The paper can be extended in several ways. First, the regulated firms in Norway are incentivized to behave efficiently using DEA benchmarking techniques. Consequently, we should expect strong cross-sectional correlations in firms' inefficiencies when we compare each firm with their peers. Evidence on this correlation can be used as an empirical measure of the incentive power of the Norwegian regulation framework. We leave an examination of this issue for future research because it is unclear as yet how to construct the weight matrix capturing cross-sectional spillovers in the inefficiency term using DEA results. Second, it would also be interesting in future empirical research to use alternative spatial error processes such as the spatial moving average (SMA) process. The SMA structure yields more limited spillover effects across firms than the SAR structure. We expect a better performance of the SMA model compared to the SAR model if we were carrying out an application based on a DEA definition of the weight matrix because no other firms' inefficiency, except for the peers, should be correlated with the inefficiency of a particular firm. Finally, as noted by Simar et al. (1994), Wang and Schmidt (2002), and Álvarez et al (2006), some portions of a model possessing the scaling property can be estimated by non-linear least squares. This will allow us to know to what extend our results depend on the distributional assumptions on the random inefficiency variable.

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Table 1. Descriptive statistics of the data

		Mean	St.Dev.	Min	Max
COST	1000 NOK	94830.2	194260.2	1348	1797173
CUS	Number	21618.3	56893.1	348	552342
DEN	Number/Km	21.6	11.6	7.0	65.8
OH	%	67.4	18.1	14.2	97.1
ST	Number	969	1844.8	23	13525
NL	Km	769.7	1301.1	18	8648

Table 2. SF models. Parameter estimates and efficiency scores.

	TWH			**WH**		
	Coef.		s.e.	Coef.		s.e.
Frontier coefficients						
Intercept	3.568	***	0.075	3.527	***	0.017
lnCUS	0.763	***	0.094	0.530	***	0.041
lnNL	0.215	**	0.086	0.478	***	0.040
$0.5 \cdot \ln CUS^2$	-0.221	***	0.086	-0.208	***	0.070
$0.5 \cdot \ln NL^2$	-0.267	***	0.082	-0.266	***	0.088
lnCUS·lnNL	0.234	***	0.083	0.235	***	0.076
OH	-0.087		0.088	-0.319	***	0.072
Disturbance						
$\ln\sigma_v$	-1.948	***	0.023	-2.404	***	0.024
σ_v	0.143			0.090		
Inefficiency						
$\ln\sigma_u$	-1.699	***	0.471	-1.263	***	0.083
σ_u	0.183			0.283		
OH	-1.039	***	0.253	-3.213	***	0.418
lnST	-0.175	***	0.058	-0.302	***	0.080
lnDEN	-1.255	***	0.167	-0.577	***	0.186
Mean log-likelihood	1.2116			-1.7240		
#obs	1008			1008		
Log likelihood function	1221.3			-1737.8		
#parameters	12			12		
AIC	-2418.6			3499.6		
BIC	-2359.6			3558.6		
CAIC	-2418.3			3500.0		
HQIC	-2438.7			3479.5		
Efficiency scores						
Average	0.819			0.797		
St.Dev	0.100			0.114		
Min	0.393			0.515		
Max	0.981			0.989		

Note: *(**)(***) stands for statistically significance at 10%(5%)(1%).

Table 3. CSSF models. SAR specifications of u and v.

	u-SAR			**v-SAR**			**uv-SAR**			**uv-SARr**		
	Coef.		s.e.	Coef.		s.e.	Coef.		s.e.	Coef.		s.e.
Frontier coefficients												
Intercept	3.589	***	0.054	3.705	***	0.030	3.653	***	0.039	3.661	***	0.029
lnCUS	0.701	***	0.076	0.587	***	0.046	0.603	***	0.046	0.569	***	0.030
lnNL	0.272	***	0.068	0.391	***	0.041	0.374	***	0.041	0.401	***	0.029
$0.5\cdot\text{lnCUS}^2$	-0.174	**	0.072	-0.163	***	0.051	-0.170	***	0.052	-0.136	***	0.042
$0.5\cdot\text{lnNL}^2$	-0.238	***	0.069	-0.204	***	0.052	-0.214	***	0.053	-0.191	***	0.048
lnCUS·lnNL	0.194	***	0.069	0.174	***	0.050	0.181	***	0.051	0.153	***	0.044
OH	-0.129	*	0.076	-0.350	***	0.066	-0.342	***	0.063	-0.356	***	0.057
Disturbance												
$\ln\sigma_v$	-1.959	***	0.023	-1.992	***	0.022	-1.999	***	0.023	-1.997	***	0.022
σ_v	0.141			0.136			0.135			0.136		
Inefficiency												
$\ln\sigma_u$	-2.368	***	0.606	-3.214	***	0.732	-3.048	***	0.606	-3.572	***	0.515
σ_u	0.094			0.040			0.047			0.028		
OH	-1.256	***	0.323	-1.457	***	0.437	-1.314	***	0.379	-1.572	***	0.418
lnST	-0.321	***	0.094	-0.454	***	0.154	-0.446	***	0.112	-0.531	***	0.115
lnDEN	-1.428	***	0.215	-1.915	***	0.437	-1.857	***	0.338	-2.064	***	0.344
Spatial spillovers												
ρ				0.676	***	0.059	0.629	***	0.060	0.593	***	0.057
τ	0.392	***	0.144				0.423	***	0.121	0.593	***	0.057
Mean log-likelihood	1.2237			1.2655			1.2710			1.2698		
obs	1008			1008			1008			1008		
lnLF	1233.5			1275.7			1281.1			1279.9		
#parameters	13			13			14			13		
AIC	-2441.0			-2525.3			-2534.3			-2533.9		
BIC	-2377.1			-2461.4			-2465.5			-2470.0		
CAIC	-2440.6			-2524.9			-2533.9			-2533.5		
HQIC	-2416.7			-2501.0			-2508.1			-2509.6		
Efficiency scores												
Average	0.837			0.944			0.895			0.905		
St.Dev	0.088			0.060			0.076			0.071		
Min	0.445			0.572			0.509			0.544		
Max	0.982			0.999			0.993			0.994		

Note: $^{*}(^{**})(^{***})$ stands for statistically significance at 10%(5%)(1%).

Figure 1. Kernel densities of efficiency scores. SF models.

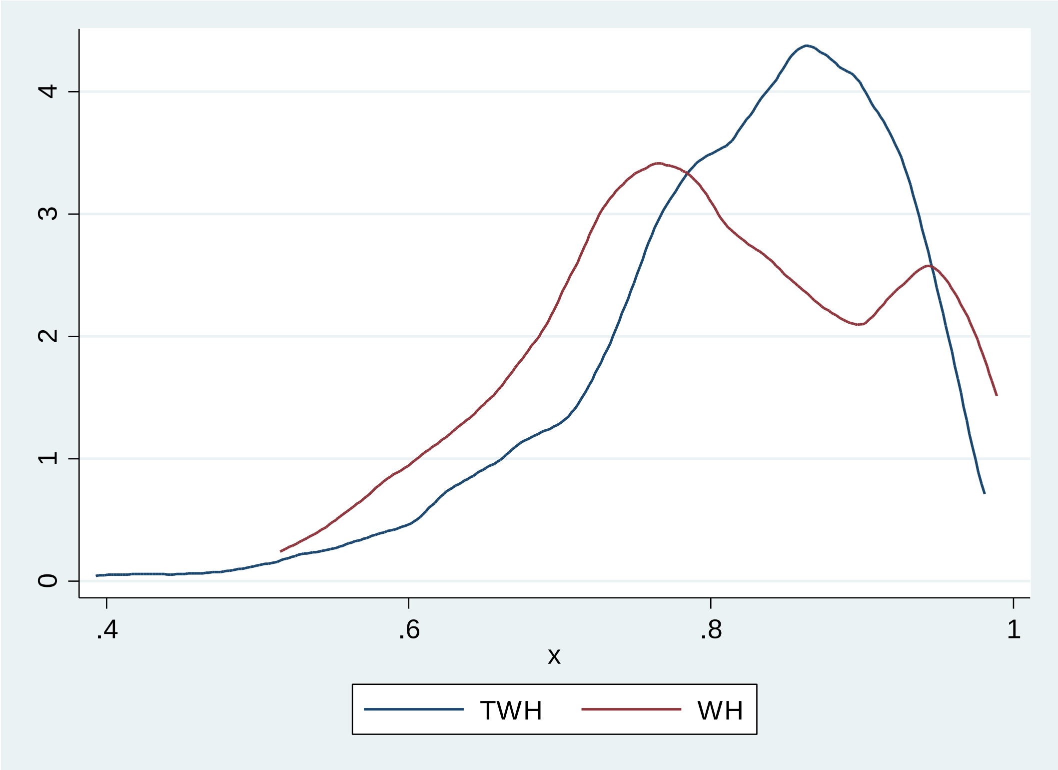

Figure 2. Kernel densities of efficiency scores.

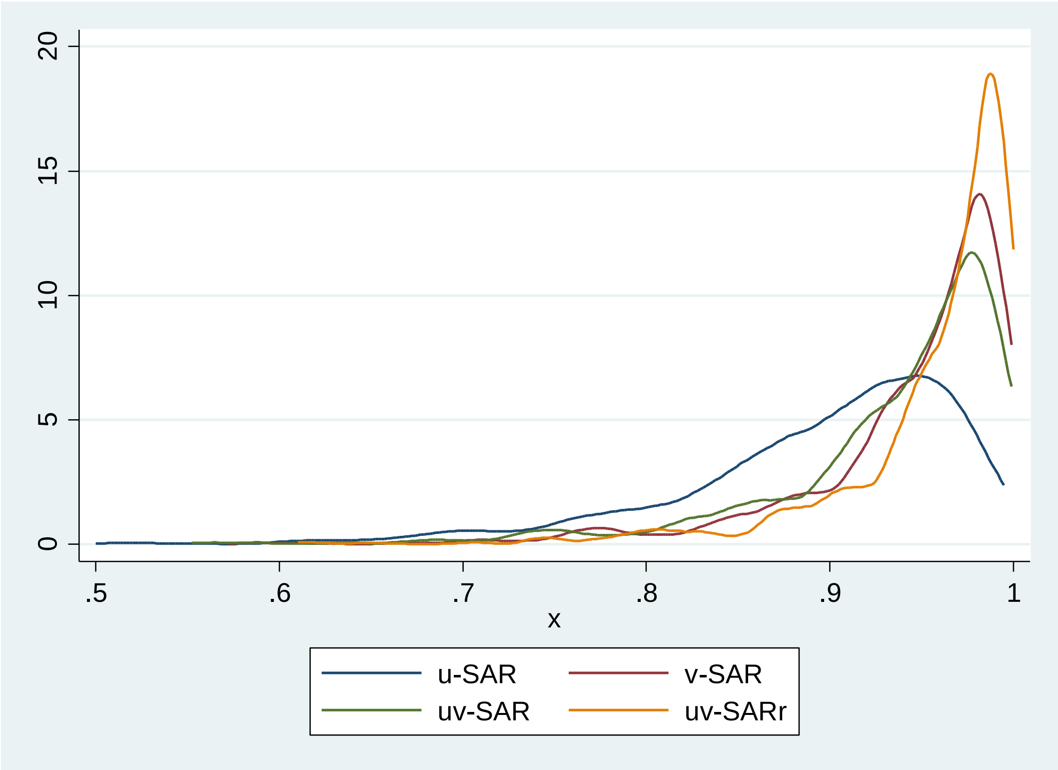

Figure 3. Annual average efficiency.