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IS IT MOLS OR COLS?

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IS IT MOLS OR COLS?

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Abstract. This paper assesses the terminology of modified and corrected ordinary least squares (MOLS/COLS) in efficiency analysis. These two approaches, while different, are often conflated.

"I am a proud member of the PWGQNKX: People Who Have No Idea How Acronyms Work."

modify: make partial or minor changes to (something), typically so as to improve it;

correct: put right (an error or fault).

1. Introduction

No doubt the estimation of the structural parameters of the stochastic frontier model through maximum likelihood is the most common approach in applied efficiency analysis. However, a variety of alternative proposals exist, prime amongst them is the two-step practice of first estimating the model via ordinary least squares (OLS) ignoring the existence of inefficiency and then shifting the estimated conditional mean up. How much to shift this curve up then leads to either the modified OLS estimator (MOLS) or the corrected OLS estimator (COLS). However, depending upon which paper (or textbook) you read, the use of the 'C' or the 'M' may be conflated with the other. I myself am not immune from this: in

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Parmeter & Kumbhakar (2014) the MOLS acronym appears while in Kumbhakar, Parmeter & Zelenyuk (2020) the COLS acronym is used for the same discussion. Both approaches offer the same essential ingredient, to adjust the intercept so that the estimated conditional mean from OLS acts as a frontier. However, the amount to shift the conditional mean differs based on the approach.

This paper reviews the historical use of the COLS and MOLS terminology in the field of efficiency analysis. As a point of writing, I will only use the term 'correct/corrected' in specific reference to COLS and 'modify/modified' in specific reference to MOLS. When the usage is to a generic change I will use 'adjust/adjusted' or 'shift/shifted' so as to avoid potential confusion on behalf of the reader. Also neither 'correct' or 'modify' is indicative of a specific statistical method so debate over which term to use is purely for historical purposes and the hope that one consistent notation can be used from this point forward.

2. The Methods

To avoid biasing the reader towards my own personal view on which method corresponds to which acronym I will introduce the two competing methods without distinction.

The two methods have the same end goal, to shift an estimated OLS conditional mean by some amount to produce a 'frontier'. What is important here is that not only do we care about shifting the estimated conditional mean up, but also by how much. And it is this subtle distinction that lies at the heart of differences in the terminology. Part of my belief as to why there is a potential for the confusion is that there are two distinct entities at play here that coincide to some degree with the development of the field. The first issue is the presence of noise. That is, are we operating under the assumption of a deterministic[1] or a

[1]The use of the word 'deterministic' is not meant pejoratively. Rather, the use is in keeping with the argot that arose at the time the field of frontier analysis developed, and is still to some degree used. Both deterministic and stochastic frontier models are fully specified statistical models (Sickles & Zelenyuk 2019) and each have their own set of assumptions under which estimators for these models perform admirably.

stochastic frontier. The second issue is then how much to shift the estimated conditional mean up. Assuredly, in the context of the stochastic frontier, the shift is such that not all of the data are bounded by the estimated frontier. However, when a deterministic frontier is of interest, the estimated conditional mean can be shifted so that either **all** or **some** of the data are below the estimated frontier. It is this subtle distinction that I believe to be the root source for differences in acronyms.

To begin we have the benchmark linear parametric frontier model:

$$y_i = \beta_0 + \boldsymbol{x}_i'\beta + \varepsilon_i, \tag{1}$$

where y_i is the dependent variable (output), β_0 is the intercept, $\boldsymbol{x}_i$ is a $k \times 1$ vector of inputs, β is the $k \times 1$ parameter vector, and ε_i dictates how y_i can deviate from the production frontier.

When $\varepsilon_i = v_i - u_i$, where v_i is stochastic noise and u_i accounts for inefficiency which serves to reduce observed output below the frontier then the model in Equation (1) is a stochastic frontier model. Alternatively, when $\varepsilon_i = -u_i$, we have the deterministic frontier model. The classic estimation of the stochastic frontier model is to assume a parametric distribution for v_i and u_i, derive the density of the composite error, $\varepsilon_i = v_i - u_i$, and estimate the model via maximum likelihood. The main approach to estimating the deterministic frontier is the use of programming methods (either linear or quadratic) to enforce the constraint that observed output can be no higher that the estimated production frontier.

The economic meaning of both of these methods is that the presence of inefficiency is a neutral shifter of technology. This is important as it pertains to how the two methods, after OLS has been deployed to estimate the conditional mean, then construct the frontier.

2.1. **Method A.** Method A is clearly entrenched in the stochastic frontier setting and seeks to use information on the assumed parametric structure of the density of u to recover information concerning $E(u)$. To see this note that the model in Equation (1) can be written as

$$
\begin{aligned}
y_i =& \beta_0 + \boldsymbol{x}_i'\beta + v_i - u_i \\
=& \beta_0 + \boldsymbol{x}_i'\beta + v_i - u_i - E(u) + E(u) \\
=& (\beta_0 - E(u)) + \boldsymbol{x}_i'\beta + v_i - (u_i - E(u)) \\
=& \beta_0^* + \boldsymbol{x}_i'\beta + \varepsilon_i^*, \qquad (2)
\end{aligned}
$$

where $\beta_0^* = \beta_0 - E(u)$ is the efficiency biased intercept and $\varepsilon_i^* = v_i - (u_i - E(u))$ is the mean zero error term.

Method A seeks to estimate $E(u)$ to then adjust the OLS intercept for the downward shift present due to ignorance of the composite error structure. Once this is done $\widehat{\beta} = \widehat{\beta}^* + \widehat{E(u)}$ can be constructed, which will then provide a consistent estimator of the intercept of the stochastic frontier model. Solutions for the estimator of $E(u)$ based on various distributional assumptions for u and v exist: the Half Normal solution is found in Aigner, Lovell & Schmidt (1977) (in passing) and Olson, Schmidt & Waldman (1980) specifically, the Exponential appears in Greene (1980*a*), the Gamma in Greene (1990), the Uniform in Li (1996), the Binomial in Carree (2002), the truncated Normal in both Harris (1992) and Goldstein (2003) (in entirely different forms) and the generalized Exponential in Papadopoulos (2021). All of these methods assume that v is Normally distributed and, save the truncated Normal, an analytical solution for $E(u)$ exists. There have also been proposals that have allowed different distributional assumptions on the noise component of ε: Nguyen (2010) provides a solution for the Laplace-Exponential model; Goldstein (2003) discusses the Student's t-Gamma composite error adjustment; and more recently, Wheat, Stead & Greene (2019) provide a closed form solution when the noise is assumed to stem from a Student's t distribution and the inefficiency is distributed Half Normal.

2.2. **Method B.** Method B seeks to construct a deterministic frontier by various manners which involve shifting the estimated OLS curve up so that some or **all** of the data are below the estimated curve. The simplest approach for this involves direct OLS estimation of the model in Equation (1) and then adding to $\widehat{\beta}_0$ the largest (positive) residual, $\widehat{\varepsilon}_{(n)} = \max_i \widehat{\varepsilon}_i$. This approach does not specifically require a distributional assumption on u_i.

Alternatively, similar to method A, if a distributional assumption is made on u, then $E(u)$ can be estimated and the intercept adjusted. Method A and B differ in this regard though because the lack of v in Method B then allows different moments to be used to recover the unknown $E(u)$.[2] Even though v is not present, correction of the OLS intercept in the context of Method B does not insure that the estimated frontier lies everywhere above all of the data.

2.3. **What These Methods Aim To Do.** Before diving into the specific semantic issues it is clear that both methods involve two distinct stages: first estimate the model ignoring the frontier structure or the explicit presence of u_i via OLS and second, shift/adjust the OLS estimate of β_0 to construct the frontier. The two distinct issues of these approaches are if distributional assumptions are imposed on v and u and if v exists at all, i.e. a deterministic vs. a stochastic frontier. As it turns out the debate over the acronyms depends heavily on which of the two issues one focuses on **first**. As we will hope to confer, the main distinction between the two approaches is linked to whether one is using a deterministic or a stochastic frontier, not if distributional assumptions are made on the components of the composite error.

[2] The work of Richmond (1974) is salient in this regard.

3. Prior to 1980

To be clear, both COLS and MOLS involve shifting the estimated OLS conditional mean up by some amount. Perhaps the earliest discussion of just such a shift is Winsten's (1957, pg. 283) comment to Farrell (1957) "It would also be interesting to know whether in practice this efficient production function turned out to be parallel to the average production function, and whether it might not be possible to fit a line to the averages, and then **shift** it parallel to itself to estimate the efficient production function." (bolding mine). Here the use of the words 'modified' or 'correct' are not present, but the idea of shifting the OLS regression line up was clear.

From then it was not until the late 1960s and early 1970s where there was a remarkable increase in interest in the estimation of frontier models. Aigner & Chu (1968) proposed a simple programming approach to estimate a deterministic frontier. Their work ensures that the estimated frontier lies above all of the data and no distributional assumptions are made on "disturbances".[3] It is clear from their baseline formulation that the proposed quadratic program they offer has the flavor of a restricted least squares approach.

The first paper to operate in a deterministic frontier setting, but to explicitly recognize that the estimated frontier may (should) not contain all of the data is Timmer (1971). Specifically, Timmer (1971, pg. 779) avers "The frontier is estimated in a probabilistic fashion by constraining X percent of the observations to fall outside the frontier surface." Another interesting aspect of Timmer (1971) is that his work was clearly not a shift/adjustment as the probabilistic component was enforced on the full estimation of the frontier, which then resulted in different estimates of the slope coefficients **and** the intercept (see Table 1, columns IIIa-IIIc in Timmer 1971). It is worth mentioning here that Timmer (1970, pg.

[3]See also Schmidt (1976) who attempts to discuss the Aigner & Chu (1968) setup in the context of OLS as well as the reply by Chu (1978) and Schmidt's (1978) rejoinder.

151) recognized that while average and frontier production could look dramatically different in his proposed approach, this was probably not to be expected: "The correlation ... is one further manifestation of the similar nature of frontier and average production functions and the relative neutrality of the shift from average to frontier." Timmer's (1971) approach is similar to Aigner & Chu (1968) except that it explicitly recognizes that without stochastic noise in the model (recall this is prior to the creation of the stochastic frontier model), the programming estimates are likely to be heavily influenced by any extremes in the data. In this case an arbitrary percentage of firms are allowed to be above the estimated frontier.

While Timmer (1970, 1971) was not concerned with shifting/adjusting an OLS estimated intercept, his work was perhaps the first to recognize that the estimated frontier may not lie above all of the data. Another important paper in this timeline is Afriat (1972) who studies a linear programming problem and introduced the idea of the Beta distribution as a means for introducing multiplicative (hence support on $[0, 1]$) inefficiency. Afriat (1972, pg. 575) discussed this as "This accounts for the "least-squares" principle of "estimating" α, β in the Cobb-Douglas production function ... But it is a principle belonging to a general statistical method which does not incorporate an economic meaning for error." A careful reading of Afriat (1972) also reveals that the modify/correct/shift verbiage does not appear and his approach is certainly not centered around least squares estimation of the conditional mean.

Richmond (1974), following the insights of Afriat (1972), proposed an explicit OLS procedure to estimate the parameters of the deterministic frontier model. He assumes that the error term in a multiplicative Cobb-Douglas model is the exponential of a Gamma distribution. With no stochastic error, this then leads to a simple (second order) moment condition of the OLS residuals that allows for estimation of the unknown parameter of the corresponding inefficiency term and the appropriate adjustment for the intercept.[4] The terms modify, correct, shift or adjust do not appear in Richmond (1974). An exemplar discussion of the literature just discussed here can be found in Sickles & Zelenyuk (2019, sections 11.2 & 11.3).

4. 1980

Why is 1980 an important demarcation? Prior to 1980 the explicit use of either the COLS or MOLS acronym did not appear in any published paper discussing estimation of the frontier model. However, in 1980 the first special issue dealing specifically with the stochastic frontier model appeared in the *Journal of Econometrics* (Aigner & Schmidt 1980). In this special issue the COLS terminology was explicitly introduced while the MOLS terminology was implicitly introduced.

From that special issue we have Olson et al. (1980, pg. 16) "The third estimator we consider is a corrected least squares estimator, which we will refer to as COLS, which was discussed briefly in ALS. This estimator is similar in spirit to the estimator suggested by Richmond (1974) in the context of a pure frontier." The corresponding statement in ALS is Aigner et al. (1977, pg. 28-29) "We note in passing that if estimation of $\boldsymbol{\beta}$ alone is desired, all but the coefficient in $\boldsymbol{\beta}$ corresponding to a column of ones in X is estimated unbiasedly and consistently by least squares. Moreover, the components of σ^2 can be extracted (i.e., consistent estimators for them can be found) based on the least squares results by utilizing eq. (9) for $V(\varepsilon)$ in terms of σ_u^2 and σ_v^2 and a similar relationship for a higher-order moment of ε, since $V(\varepsilon)$ and higher order mean-corrected moments of ε are themselves consistently estimable from the computed least-squares residuals." In a footnote Aigner et al. (1977, pg.

[4]Note that Richmond (1974) refers to the unknown parameter as n, which may lead some to confuse this with the sample size. Certainly it is not the best notation given the widespread use of n to represent the number of observations.

29) then give the exact third moment one would need for the correction to the OLS intercept. Greene (1980*a*, pg. 32) also references this same discussion.

The MOLS terminology can be inferred from Greene (1980*a*, pg. 35) "Of course, the (appropriately modified) OLS estimator is also consistent, and more easily computed." Here though Greene is referring to his earlier discussion on shifting the OLS curve up so that **all** of the observations lie on or below the frontier, i.e. a deterministic frontier. From Greene (1980*a*, pg. 34) "...then the OLS residuals can be used to derive a consistent estimate of α. We need only shift the intercept of the estimated function until all residuals (save for the one support point) have the correct sign."

The issue at this point is that Olson et al. (1980) explicitly use the COLS acronym while Greene (1980*a*) mentions a modified OLS estimator but never explicitly writes MOLS. Thus, it is clear from these paper what COLS is referring to and what MOLS can be attributed to.

5. After 1980

Based on the state of the field in 1980, if we were to zoom (no pun intended) through time to today it would be clear that COLS would refer to the adjustment of the OLS estimator by the expected value of the assumed distribution of inefficiency and MOLS would refer to the shifting of the OLS estimator by the largest positive residual. However, time is never so forgiving. Lovell (1993, pg. 21), crediting Gabrielsen (1975), baptizes MOLS as the method in which one adjusts the intercept based on a specific set of distributional assumptions (regardless of whether one is estimating a deterministic or stochastic frontier).[5] Prior to Lovell (1993) it is not clear where/when the explicit use of the acronym MOLS was used. While the word modified appeared in both Gabrielsen (1973) (in Norwegian) and Greene (1980*a*), neither specifically uses the acronym MOLS.

[5]Greene (2008) points the reader to Lovell (1993) as well for this nomenclature.

The exact wording in Lovell (1993, pg. 21) is "COLS was first proposed by Winsten (1957), although Gabrielsen (1975) is usually credited with its discovery. It makes no assumption concerning the functional form of the nonpositive efficiency component U_j. It estimates the technology parameters of (1.19) by OLS, and corrects the downward bias in the estimated OLS intercept by shifting it up until **all** corrected residuals are nonpositive and at least one is zero." (bolding mine). What is strange[6] about the Lovell (1993) chapter is that Olson et al. (1980) is not cited, which contains the first published use of the COLS terminology.

Moreover, the line "... although Gabrielsen (1975) is usually credited with its discovery" is odd given that the word "corrected" never appears in Gabrielsen (1975). It also is instructive to consider other substantive papers prior to Lovell (1993) that might have been able to shed light on the COLS/MOLS debate. Between the initial appearance of COLS in Olson et al. (1980) and Lovell's (1993) authoritative review, there is the review of Schmidt (1985), who also provides some glimpses into the evolution of this terminology. Schmidt (1985, pg. 302) "This was first noted by Richmond (1974). Furthermore, the estimated intercept can be "corrected" by shifting it upward until no residual is positive, and one is zero. This yields a consistent estimate of A, as shown by Gabrielsen (1975) and Greene (1980a). However, the asymptotic distribution of the "corrected" intercept is unknown ..." This is followed by Schmidt (1985, pg. 306) "Also, a simpler correct least squares estimator is possible, which estimates the model by ordinary least squares and then "corrects" the intercept by adding a consistent estimator of $E(u)$ based on higher (in the half-normal case, second and third) moments of the least squares residuals."

And the use of the word "correct" for both methods is not specific to Schmidt (1985); Førsund, Lovell & Schmidt (1980, pg. 12) also have a similar lack of distinction: "There is also an alternative method of estimation, apparently first noted by Richmond (1974), based

[6] As Lovell and Schmidt were coauthors on the original 1977 paper which indirectly referenced COLS.

on ordinary least squares results; we will call this correct OLS, or COLS. ... estimate by OLS to obtain best linear unbiased estimates of $(\alpha_0 - \mu)$ and of the α_i. If a specific distribution is assumed for u, and if the parameters of this distribution can be derived from its higher-order (second, third, etc.) central moments, then we can estimate these parameters consistently from the moments of the OLS residuals. Since μ is a function of these parameters, it too can be estimated consistently, and this estimate can be used to 'correct' the OLS constant term, which is a consistent estimate of $(\alpha - \mu)$. COLS thus provides consistent estimates of all of the parameters of the frontier." We note that the words "correct" and "modify" do not appear in Richmond (1974).

Continuing we have Førsund et al. (1980, pg. 12) "A difficulty with the COLS technique is that, even after correcting the constant term, some of the residuals may still have the 'wrong' sign so that these observations end up above the estimated production frontier. This makes the COLS frontier a somewhat awkward basis for computing the technical efficiency of individual observations. One response to this problem is provided by the stochastic frontier approach discussed below. Another way of resolving the problem is to estimate (4) by OLS, and then to correct the constant term not as above, but by shifting it up until no residual is positive and one is zero. Gabrielsen (1975) and Greene (1980*a*) have both shown that this correction provides a consistent estimate of α_0."

All of this discussion on page 12 was in the context of a 'deterministic' frontier, so no v was implicit in the model. But the usage of 'correct/corrected' was clearly linked to the mean of inefficiency being used to adjust the OLS intercept. Moreover, just a few pages later, when discussing stochastic frontiers, the COLS terminology again appears. Førsund et al. (1980, pg. 14) "Direct estimates of the stochastic production frontier model may be obtained by either maximum likelihood or COLS methods. ... The model may also be estimated by COLS by adjusting the constant term by $E(u)$, which is derived from the moments of the

OLS residuals. ... Whether the model is estimated by maximum likelihood or by COLS, the distribution of u must be specified."

The reviews of both Førsund et al. (1980) and Schmidt (1985) are interesting because they use COLS generically to refer to the adjustment of the OLS intercept estimate, whether in a deterministic or stochastic frontier. The MOLS terminology is not present in either review paper.

By 1990 the efficiency community was quite large and the field well developed. Another special issue of the *Journal of Econometrics* (Lewin & Lovell 1990) contained a survey of the field to date by Bauer (1990, pg. 42), who explicitly recognized COLS: "Estimates of this model can be obtained using corrected ordinary least squares (COLS) or by maximizing the likelihood function directly." No citation of Gabrielsen (1975) appears nor does any mention of the MOLS terminology.

Also in this special issue is Greene (1990, pg. 152) "... but the OLS constant term is biased ... Greene (1980*a*) obtained estimates for the parameters of the disturbance distribution, and the constant term, in the gamma frontier model by manipulating the OLS residuals." Gabrielsen (1975) is then cited in relation to the work of Greene (1980*a*). This is then followed by Greene (1990, pg. 153) "Only the first two [moments] are actually needed to correct the OLS intercept ... "

So it would seem that to resolve this issue Gabrielsen (1975) would need to be consulted. Perhaps this paper had explicitly used the MOLS terminology.

6. The Gabrielsen Dilemma

The Gabrielsen (1975) citation in Lovell (1993) (and several earlier papers) links the unpublished working paper to the Christian Michelsen Institute (CMI), Department of Humanities and Social Sciences, in Bergen, Norway. There is no record of this paper on the current website of the CMI so I contacted Mr. Reidunn Ljones at the Bergen Resource Centre for International Development on July 31st, 2017. Note that in 1992, the Department for Natural Science and Technology established the Christian Michelsen Research AS, and the CMR Group. The CMR Group is housed at the University of Bergen.

Upon first contact, Mr. Ljones wrote back to me on August 11, 2017: "I have now tried to find the publication you asked for. I can't find this title within the year 1975, or the number A-85 from another year. Neither his name Arne Gabrielsen. What I have found is that this title was published in Norwegian in 1973 with the number A-85. I think that they have translated the title in a reference/publication list. I can't find any trace suggesting that this publication was translated to English. Your reference must be wrong since 1975, A-85 is by another author with a different title."

After some further correspondence with Mr. Ljones about my desire to receive the paper (even in Norwegian) he wrote to me on August 14th, 2017: "I could not find any paper on any of the different references. I have been searching on the author again, and found this reference below also. You will find that this is also the year 1973, but it's DERAP paper; 53 and not A-85. Since all the publications for these different references is missing in our library archive, I have to visit our remote archive. Gabrielsen, Arne Estimering av "effisiente" produktfunksjoner : eksogene produksjonsfaktorer. - Bergen : CMI, 1973. - 33 p. (DERAP paper; 53)."

On September 15th, 2017 a scan of Gabrielsen (1973) was delivered to my inbox. I began using Google Translate to initially parse through the paper. From Gabrielsen (1973, pg. 2) we have "First, a modified version of the least squares method is presented." Actual wording is "Først utvikles en modifisert utgave av minstekvadraters metode." The Norwegian 'modified' (*modifisert*) does not appear again in the paper nor is the acronym MOLS ever used, but we now have the definitive link to "modified" and the early papers of the 1970s studying estimation of the deterministic stochastic frontier model.

Despite the lack of the exact MOLS, it is clear that Gabrielsen (1973, pg. 7) has in mind some form of adjustment to the OLS residual: "We will below develop the least squares estimators for the parameters of the model. We do this because the criterion function from the least-squares method in this model is different from what it would be if the residuals had expectation zero. However, it turns out that the least squares estimators for the limit elasticities in the model will be the usual least squares estimators we would get if the residuals had zero expectation. The difference occurs at the least squares estimates of the efficiency parameter, in our case, the multiplicative constant A and the least squares estimator to the expectation of the residuals."[7] This is the same intent that Richmond (1974) had.

At this point it might seem that Lovell (1993) had correctly attributed MOLS. However, Gabrielsen's (1973, eq. 5) statistical model was 'deterministic' in nature; there was only one-sided inefficiency and so the adjustment to the OLS intercept was with the intent of constructing a frontier that lied everywhere above the data, see Gabrielsen (1973, eqs. 11 and 12), or at least probabilistically lied above the data as in Timmer (1971). Thus it is not clear how the very specific COLS of Olson et al. (1980) for the stochastic frontier model came to be associated with the MOLS of Gabrielsen (1973) for the deterministic frontier model.

[7]Actual text: "Vi skal nedenfor utvikle minstekvadratersestimatorene for parametrene i modellen. Vi gjør dette fordi kriteriefunksjonen ved, minstekvadraters metode i denne modellen er forskjellig fra det den ville være om restleddene hadde forventning null. Det viser seg imidlertid at minstekvadratersestimatorene for grenseelastisitetene i modellen blir de vanlige minstekvadratersestimatorene vi ville fått om restleddet hadde forventning null. Forskjellen inntreffer ved minstekvadratersestimatorene for effisiensparameteren, i vårt tilfelle det multiplikative konstanten A og ved minstekvadratersestimatoren til forventningen av restleddet."

7. Textbook Treatment of the Acronyms

It is also instructive to observe how many of the leading textbooks on efficiency and productivity analysis approach this subject. For example, Kumbhakar & Lovell (2000, pg. 70) also has a section titled "Corrected Ordinary Least Squares (COLS)" which cites Winsten (1957) and then describes exactly the approach laid out in Greene (1980*a*) while another section titled "Modified Ordinary Least Squares (MOLS)" (Kumbhakar & Lovell 2000, pg. 71) describes explicitly the approach of Afriat (1972) and Richmond (1974), i.e. the deterministic frontier case. No citation of Gabrielsen (1973) exists. Further on, Kumbhakar & Lovell (2000, pg. 91), when describing Olson et al. (1980) refer to the method as MOLS: "This two-part estimation procedure amounts to the application of MOLS to a stochastic production frontier model." As we have seen Olson et al. (1980) never use the MOLS terminology.

Another prominent textbook in this area Coelli, Rao, O'Donnell & Battese (2005, pg. 242, Section 9.2) also wades into this terminology: "...while Richmond (1974) used a least squares technique, sometimes known as *modified ordinary least squares (MOLS)*." A few pages later, Coelli et al. (2005, pg. 245, Section 9.3) "One solution to this problem is to correct for the bias in the intercept term using a variant of a method suggest by Winsten (1957) – the resulting estimation is often known as the corrected ordinary least squares (COLS) estimator." There is a footnote in this passage that then states "Winston suggested the COLS estimator in the context of the deterministic frontier ..." Other contemporary textbook treatments are no more specific. Kumbhakar, Wang & Horncastle (2015, pg. 50) section 3.3.1 entitled "Correct OLS (COLS)" describes the approach of Winsten (1957) and Greene (1980*a*). See also section 4.3.1 which uses the same terminology. This textbook does not discuss the approach of Olson et al. (1980) whatsoever.

O'Donnell (2018) has separate chapters (7 and 8) dedicated to the estimation of deterministic and stochastic frontiers. Specifically, within each chapter he includes a section on least squares estimation (sections 7.3 and 8.2, respectively). In section 7.3 of O'Donnell (2018, pg. 268), adjustment of the OLS intercept from the deterministic frontier model is termed COLS: "…is the COLS estimate of the production frontier; by design, it runs parallel to the OLS line of best fit (the dotted line) and envelops **all** the points in the scatterplot." (emphasis mine).

When the least squares estimation of the stochastic frontier is discussed, O'Donnell (2018, pg. 302) introduces the MOLS acronym. There is also footnote 4 on the same page: "Elsewhere, these estimators are sometimes referred to as corrected ordinary least squares (COLS) estimators; see, for example, Horrace and Schmidt (1996, p.260). In this book, the term COLS is reserved for LS estimators for the parameters in deterministic frontier models." An interesting note about Horrace & Schmidt (1996) is that while they deploy the COLS terminology in reference to correction of a least squares estimate of the intercept of a stochastic frontier model, when they introduce (pg. 260) a panel data model estimated using generalized least squares (GLS) to account for the presence of random effects, they adjust the corresponding estimate here as well and term the estimator corrected GLS (CGLS), and so again we have use of the 'C' for adjustment in a stochastic frontier setting.

Finally, the most recent textbook in the field also discusses both terminologies. Section 11.2 in Sickles & Zelenyuk (2019) is named "Corrected OLS" and refers to what here has been described as MOLS, while Section 11.4.1 describes the Olson et al. (1980, pg. 372) approach and uses both COLS and MOLS terminology: "…in the usual deterministic COLS method and compared the ALS methodology with their version of COLS, which is often referred to as *modified* OLS or simply MOLS."

As is clear even the various textbooks in our area seem to use both terminologies in different manners.

8. Journal Publications Use of the Acronyms

To assess how differently the MOLS/COLS usage has been a Google Scholar search[8] for the term COLS in the *Journal of Productivity Analysis* turned up 29 articles. Each of these papers were read to determine if the intent of using COLS was to a stochastic or a deterministic frontier. This reduced the number of articles by five. Two additional relevant papers were found that used corrected OLS (without the COLS acronym). Further, a Google Scholar search for the term MOLS in the *Journal of Productivity Analysis* turned up six articles. Again, each of these papers were read to determine if the intent of using MOLS was to a stochastic or a deterministic frontier. This reduced the number of articles by one. One additional relevant paper was found that used modified OLS (without the MOLS acronym).

Of these 32 articles we have that all six papers using MOLS or modified OLS was always in the context of a stochastic frontier that shifts the OLS intercept based on a presumed distributional assumption for u. However, usage of the COLS acronym is mixed. 15 of the 26 articles used COLS (or corrected OLS) in the context of a deterministic frontier model, while eight articles used COLS (or corrected OLS) in the context of a stochastic frontier while 3 had no clear distinction between stochastic or deterministic frontier in their use.

Several of the papers invoking the terminology do so in ways that one might wonder how any of these terms came to be. For example, Amsler, Leonard & Schmidt (2013, pg. 294): "[COLS] was first suggested by Winsten (1957) – though it was literally a one-sentence suggestion– and then further developed by Greene (1980*a*), who proved the consistency of the COLS estimators of α and β." This passage is especially interesting as Peter Schmidt of Olson et al. (1980) was the first to use the COLS acronym (for a stochastic frontier) and here it is being used for a deterministic frontier and the references to both Winsten (1957) and Greene (1980*a*), as discussed prior, never use this terminology.

[8]Conducted on March 9, 2021

Similarly, in their influential work Simar, Van Keilegom & Zelenyuk (2017, pg. 190) proposed nonparametric estimation of the frontier itself coupled with an adjustment of the estimated conditional mean to construct a stochastic frontier. Simar et al. (2017, pg. 190): "Our approach can be viewed as a non- or semi-parametric version of the "modified OLS" (MOLS) method that was introduced as an alternative to MLE method for SFA in parametric setups." Later on page 192 they state "We will extend the idea of the Modified OLS (MOLS), originated in the full parametric, homoskedastic stochastic frontier models (see Olson et al. 1980) for our semi-parametric setup." But as we have discussed above Olson et al. (1980) called their procedure COLS.

The approach of estimating the stochastic frontier model via OLS and then adjusting the estimates to construct a stochastic frontier has also appeared in Wikström (2016) who deployed the MOLS acronym when detailing a panel data stochastic frontier model that involves shifting the estimates of the unobserved heterogeneity to take account of the two part nature. And there is Kumbhakar & Lien (2018, pg. 23) who developed the intercept adjustment for residuals estimated from a random effects panel data model for the generalized panel data stochastic frontier model, without any acronym connection and a simple descriptor of "method of moments estimation".[9] There is the important work of Amsler, Prokhorov & Schmidt (2016, p. 281) which discussed COLS estimation in the presence of endogeneity, what they term C2SLS: "This is a straightforward generalization of COLS, which perhaps surprisingly does not appear to have been discussed in the literature." Here is is clear that the intent of C2SLS is with respect to the COLS proposal of Olson et al. (1980).

Lastly, using the acronym OLSE+MME, Huynh, Pal & Nguyen (2021, pg. 8) have reinvented the method without any attribution to the stochastic frontier literature whatsoever:

[9]See also Kumbhakar & Parmeter (2019, sect. 3.1).

"To the best of our knowledge, the estimation of $\boldsymbol{\beta}$, σ, and λ that we present here is completely new and the estimators have fairly well structured closed forms."

9. The 'Final' Verdict

Given my age and lack of access to a time machine, it is impossible to know for sure the discussions relating to the argot that developed in the frontier literature after Aigner et al. (1977) at the various conferences that arose from the origins of this field. Far be it for me to lay down the gauntlet and suggest which term should refer to which model, especially in light of my own conflation of the COLS/MOLS acronyms. However, it is clear from the literature as reviewed above that the first use of COLS was with respect to a stochastic frontier model that was designed to correct the OLS estimator of the intercept up by the mean of inefficiency and thus not all of the data would be bounded by the corresponding estimated frontier. On the other hand, the use of the MOLS acronym does not directly appear in any of the early literature and the use of the word 'modify' **always** appeared in the context of a deterministic frontier.

Given that there exist two different distinctions as to which one might want to adjust an estimated conditional mean, the intent is important. Note that the explicit introduction of COLS (Olson et al. 1980) was for a stochastic frontier model, independent of a specific distributional assumption. The introduction of the word modify (Gabrielsen 1973), while also dependent upon a distributional assumption, was in the context of a deterministic frontier. It is this distinction that I believe to be important when adjudicating between the two acronyms. Should COLS/MOLS be used to refer to how much to shift the frontier up or by the type of frontier one is working with? Given that the type of frontier model being deployed is more important than the amount of adjustment, this should be the dominant force driving the information conveyed to a reader/listener when using either of the terms.

Thus, it is the hope of this article that COLS will be used to refer to those methods which correct the intercept in a 'stochastic frontier model' based on some type of distributional assumption on v and u and MOLS will refer to any method that constructs a true frontier where all of the data are bounded by the subsequently estimated frontier.

Or, recognizing the work of Førsund et al. (1980), Schmidt (1985) and Amsler et al. (2013), Peter Schmidt has used COLS in both the deterministic and the stochastic frontier setting. Perhaps it is best to just use COLS and retire MOLS?

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Appendix A. COLS in the JPA

A Google Scholar search for the term COLS in the *Journal of Productivity Analysis* turned up 29 articles. Each of these papers were read to determine if the intent of using COLS was to a stochastic or a deterministic frontier. This reduced the number of articles by five. Two additional relevant papers were found that used corrected OLS (without the COLS acronym).

Førsund (1992) – clear that intent of COLS is to a deterministic frontier model.

Seaver & Triantis (1992) – cites COLS in relation to Afriat (1972) and Greene (1980*b*) and it is clear that the intent is for a deterministic frontier model.

Neogi & Ghosh (1994) – cites COLS in relation to Richmond (1974) and clear that the intent is a deterministic frontier model.

Coelli (1995) – clear that intent of COLS is to Olson et al. (1980) and so a stochastic frontier model.

Wilson (1995) – cites COLS in relation to Greene (1980*b*) and it is clear that the intent is for a deterministic frontier model.

Banker (1996) – cites COLS in relation to Olson et al. (1980) but appears to be concerned with a deterministic frontier model.

Horrace & Schmidt (1996) – clear that intent of COLS is to Olson et al. (1980) and so a stochastic frontier model.

Bardhan, Cooper & Kumbhakar (1998) – uses COLS where it is clear that the intent is for a deterministic frontier model.

Gstach (1998) – not clear what the intended usage of COLS is.

Kerkvliet, Nebesky, Tremblay & Tremblay (1998) – cites COLS in relation to Greene (1980*b*) and it is clear that the intent is for a deterministic frontier model.

Zhang (1999) – cites Olson et al. (1980) and Coelli (1995) and it is clear that the intended usage is to a stochastic frontier model.

Cuesta (2000) – clear that intent of COLS is to a stochastic frontier model.

Førsund & Sarafoglou (2000) – cites COLS in relation to Richmond (1974) but not clear that the intended usage is to a deterministic/stochastic frontier model.

Fuentes, E. & Perelman (2001) – clear that the intent of COLS is to a deterministic frontier model.

Banker, Janakiraman & Natarajan (2002) – cites Richmond (1974), Greene (1980*a*) and Olson et al. (1980). Not clear what intended usage is for.

Jensen (2005) – cites COLS in relation to Winsten (1957) and it is clear that the intended usage is for a deterministic frontier model.

Smet (2007) – cites COLS in relation to Coelli (1995) and is clear that the intent of COLS is to a stochastic frontier model.

Simar & Wilson (2011) – use corrected OLS where it is clear that the intended usage is for a deterministic frontier model.

Amsler et al. (2013) – clear that the intended usage is for a deterministic frontier model.

Lai (2013) – uses corrected OLS where it is clear that the intended usage is for a stochastic frontier model.

Kuosmanen & Kortelainen (2012) – clear that the intent of COLS is to Greene (1980*a*) and so is referring to a deterministic frontier model.

Andor & Hesse (2014) – cites COLS in relation to Winsten (1957) and it is clear that the intended usage is for a deterministic frontier model.

Henningsen, Henningsen & Jensen (2015) – clear that the intent of COLS is to a stochastic frontier model.

Minegishi (2016) – clear that the intent of COLS is to Greene (1980*a*) and so is referring to a deterministic frontier model.

Wheat et al. (2019) – clear that the intent of COLS is to a deterministic frontier model.

Papadopoulos (2021) – clear that the intent of COLS is to a stochastic frontier model.

Appendix B. MOLS in the JPA

A Google Scholar search for the term MOLS in the *Journal of Productivity Analysis* turned up 6 articles. Each of these papers were read to determine if the intent of using MOLS was to a stochastic or a deterministic frontier. This reduced the number of articles by one. One additional relevant paper was found that used modified OLS (without the MOLS acronym).

Cummins & Zi (1998) – clear that the intended use of MOLS is to Greene (1990) and so is referring to a stochastic frontier model.

Serra & Goodwin (2009) – clear that the intended use of MOLS is to a stochastic frontier model.

Kuosmanen & Kortelainen (2012) – clear that the intent of MOLS is to Olson et al. (1980) and so is referring to a stochastic frontier model.

Minegishi (2016) – uses modified OLS with the intent of distinguishing it from COLS (for a deterministic frontier) and so is clear that this use is for a stochastic frontier model.

Wikström (2016) – cites MOLS in relation to Richmond (1974) and Greene (1980*b*) but clear that the intent of usage of MOLS is to a stochastic frontier model.

Simar et al. (2017) – clear that the intent of MOLS is to Olson et al. (1980) and so is referring to a stochastic frontier model.

All five of the articles identified used MOLS specifically to refer to the intercept correction in the context of a stochastic frontier model.