8TH ALAMA MEETING

ALAMA 2024

Linear Algebra, Matrix Analysis and Applications

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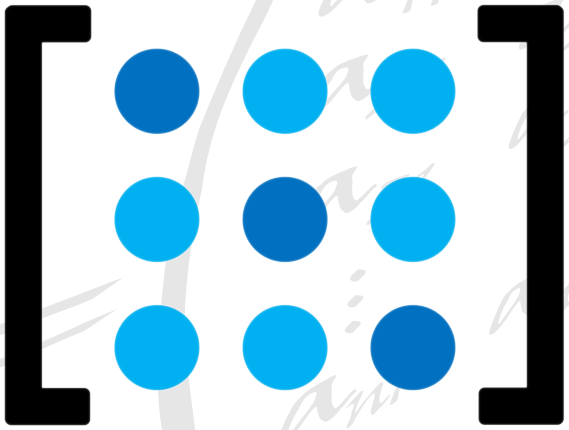
June 12-14, 2024

Gijón (Asturias)

Book of abstracts



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PRESENTATION

The Spanish Thematic Network of Linear Algebra, Matrix Analysis and Applications (ALAMA, https://www.red-alama.es) aims to bring together scientists whose research is related to Linear Algebra, Matrix Analysis, Matrix Theory and/or its applications in different contexts. It aims to cover a broad spectrum of interests, including both purely algebraic or analytical aspects, and numerical, combinatorial, geometric, probabilistic, didactic or historical aspects, without neglecting the applications of Linear Algebra in areas such as (but not restricted to) Control Theory, Cryptology and Code Theory, Graph Theory, Structure Mechanics, Signal and Image Processing, Artificial Intelligence or Data Mining.

The Network ALAMA organizes from June 12 to 14, 2024 its eighth biennial meeting, after the previous editions of Alcalá de Henares (2022), Alicante (2018), León (2016), Barcelona (2014), Leganés (2012), Vitoria-Gasteiz (2008), Valencia (2010), and Barcelona (2014). This time, it will take place at the Campus of Gijón of the Universidad de Oviedo (https://www.unioviedo.es/alama2024/). This meeting has been supported by the International Linear Algebra Society (https://ilasic.org/conferences/#ilas-endorsed-meetings)

The aim of this meeting is to renew contacts between members of the network and to share the progress achieved, as well as to review and plan the objectives of the network itself.

The meeting will consist of three plenary lectures. The plenary speakers are: Plamen Koev (San José State University), Yuji Nakatsukasa (University of Oxford) and S. Irene Díaz (Universidad de Oviedo).

In addition, there will also be 32 individual communications and 5 minisymposia grouping 36 communications more, which will be developed in different parallel sessions:

MS1: Structured matrices and high relative accuracy
MS2: Quantum computing and linear algebra
MS3: Algebraic methods for the recovery, correction and security of digital information
MS4: Orthogonal Polynomials and Matrix Analysis
MS5: On the inverse problem of recovery of the conductances

We would like to thank all the sponsors of the event: University of Oviedo, the Fundación Cajastur, the Mathematics Department of Universidad de Oviedo, the Spanish Society of Applied Mathematics (SEMA) and Ayuntamiento de Gijón (Asturias).

Finally, we hope the participants enjoy the scientific program as well as the stay in Gijón.

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Contents

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Communications

List of

List of Abstracts

Guest Speakers

Individual Contributions

List of Minisymposia

ORTHOGONAL POLYNOMIALS AND MATRIX ANALYSIS
Organizers: Luis Miguel Anguas, Dolores Barrios Rolanía, Juan C. García Ardila

SOBRE EL PROBLEMA INVERSO DE RECUPERACIÓN DE LAS CONDUCTANCIAS
Organizers: Ángeles Carmona, Andrés M. Encinas, María José Jiménez

STRUCTURED MATRICES AND HIGH RELATIVE ACCURACY
Organizers: Jorge Delgado, Pedro Alonso

Speakers

Guest

Irene Díaz
Universidad de Oviedo

Plamen Koev
San José State University

Yuji Nakatsukasa
University of Oxford

Challenges and strategies in ranking aggregation: from fairness to computational efficiency

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Abstract

Ranking aggregation plays a crucial role in various decision-making scenarios where multiple stakeholders' preferences need to be reconciled into a single ranking. Ensuring fairness in this process is indeed challenging, as it involves balancing conflicting objectives such as representing individual preferences accurately while also considering overall societal welfare or fairness criteria.

Over the years, numerous methods have been developed to address this challenge. These methods vary in complexity and the criteria they optimize for, ranging from simple averaging or scoring methods to more sophisticated approaches based on computational algorithms and optimization techniques. Some popular methods include Borda count, Copeland's method, Kemeny-Young method, and various variants of these.

Fairness in ranking aggregation can be approached from different perspectives. One common approach is to ensure that no individual or group is systematically disadvantaged or advantaged by the aggregation process. This can involve considering criteria such as proportional representation, monotonicity, consistency, and strategy-proofness.

However, achieving fairness in ranking aggregation often involves trade-offs between different fairness criteria, and there may not always be a universally optimal solution. Moreover, the choice of method may also depend on contextual factors such as the nature of the decision problem, the preferences of the stakeholders, and the available information [5].

Condorcet introduced an aggregation method centred on the majority criterion, laying a foundational framework for the field. His method entails determining the optimal ranking through pairwise comparisons among all alternatives. According to this principle, an alternative is prioritised over another if it wins in more than half of the individual comparisons (i.e., prevailing in the majority of pairwise contests). Consequently, the *Condorcet ranking* is established by applying this principle to every pair of alternatives, thus assigning the top positions in the consensus ranking to those alternatives that win most frequently against the others. However, the Condorcet method is vulnerable to what is known as the *voting paradox* problem [3], stemming from the non-transitive nature of this relationship.

In such case, a subset of alternatives could create a cycle in the overall ranking, even if all the individual rankings in the profile do not present any. The simplest cycle possible is the one generated by the situation in which a strict ranking cannot be established between three alternatives. Consider the set of three alternatives $\{a_1, a_2, a_3\}$ and a voting situation with three voters, where they give the following rankings respectively: voter 1) $a_1 \succ a_2 \succ a_3$, voter 2) $a_2 \succ a_3 \succ a_1$, and voter 3) $a_3 \succ a_1 \succ a_2$. In this case, attending the pairwise comparison of the alternatives, the voters prefer a_1 over a_2, a_2 over a_3, and a_3 over a_1. Given these pairwise preferences, no alternative attains an unequivocal overall majority preference (each pairwise comparison has exactly two votes), leading to a paradoxical situation where it is impossible not only to establish a consensus ranking, but to emerge a single alternative as the winning choice by agreement.

To seek a consensus solution from a set of rankings remains a challenging problem even today, yet one with considerable applicability, not only in voting processes but also in recommendation systems, for instance. In order to seek a consensus solution, integer matrices are commonly employed, exhibiting a specific structure derived from the rankings they represent. Many algorithms have been proposed to enhance the aggregation technique initially suggested by Condorcet, while also addressing challenges like the voting paradox. [4]. Also, there are other branches of fair ranking aggregation methods that exploit different aspects of fairness, some of these algorithms satisfy theoretical axioms but are still far

from being feasible in real situations due to their expensive computational costs, which prevent their use when quick response times are required [1] and the difficulty of exploring some relations over the matrices.

In this context, several algorithms have been proposed to implement efficient versions for computing the consensus ranking using classical aggregation methods [2,6–9]. Another problem is related to evaluating and comparing ranking aggregation algorithms as there is no standardised datasets containing profiles of rankings. And what is more important, obtaining these datasets is also a computational problem.

The purpose of this work is to demonstrate the challenges encountered when attempting to find consensus rankings, and to highlight the necessity of developing computationally efficient strategies, which involve studying the properties of ranking matrices and at the same time studying the behavior of these methods depending also on the properties of the profiles of rankings because even when profiles share similar dispersion characteristics, the distribution of profiles aligning with Condorcet criteria varies significantly if the number of voters and alternatives is modified.

Acknowledgements: The author was partially supported by the Government of Spain through project MCINN-23-PID2022-139886NB-100.

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DQDS algorithms and high relative accuracy computations

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Abstract

The 'dqds' in 'dqds algorithm' stands for 'differential quotient-difference with shift.' The name is for historical reasons [10] that have little to do with its main use today as one of the best ways to compute the singular values of bidiagonal matrices to high relative accuracy [6]. The second misnomer in the naming is the singular 'algorithm.' By our count, there are 5 dqds algorithms: regular [3], stationary [10], progressive [10], twisted [4], and one more that does not seem to have a name yet. These dqds algorithms and their use in high relative accuracy linear algebra calculations are the main topic of this talk.

The importance of the dqds algorithms goes way beyond singular values of bidiagonals. The ideas have been extended to perform virtually all linear algebra calculations to high relative accuracy, not only with bidiagonals, but with an arbitrary product of nonnegative bidiagonals [8,9]. The class of matrices that are products of nonnegative bidiagonals is known as *totally nonnegative* and is characterized by the fact that all minors of those matrices are nonnegative [5,7]. Examples include Vandemonde, Cauchy, Pascal, and many other related matrices [2].

Computing with high relative accuracy means getting the correct result, e.g., an eigenvalue or a singular value, no matter how tiny, with a correct sign and most leading digits. In stark contrast, the conventional linear algebra algorithms (e.g., those employed by LAPACK [1]), deliver only backward stability, meaning one gets the correct answers only for well behaved problems.

The dqds algorithms deliver high relative accuracy by avoiding the root cause of loss of such accuracy: subtractive cancellation. Subtractive cancellation is the loss of relative accuracy which occurs when subtracting quantities close in magnitude, which have already lost some digits to roundoff.

The key beneficial feature of the dqds algorithms within the scope of computing with totally nonnegative matrices is the ability to move factors around a product of nonnegative bidiagonals, changing those factors, but preserving the structure of the product.

For example, if L is a lower bidiagonal matrix, the *progressive dqds* allows one to compute an upper bidiagonal matrix U such that

$$LL^T = UU^T$$

without performing any subtractions and thus to high relative accuracy in each entry of U.

Remarkably, this can also be done in reverse: One can compute L from U also without performing any subtractions using the *regular* dqds [3].

In this talk we will review the common ideas in the dqds algorithms as well as their practical usefulness in various computations. In particular, we will address the challenges and the promises of dqds in designing an accurate eigenvector algorithm for symmetric tridiagonals: an algorithm which is optimal in running time ($O(n^2)$), delivers small residuals, a small relative error with respect to the relative gaps between the eigenvalues, orthogonal eigenvectors, and lastly, eigenvectors with correct oscillatory properties.

Acknowledgements: This research was partially supported by the NSF Award DMS-2309597 and by the Woodward Fund for Applied Mathematics at San José State University.

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Randomized methods for matrix computations

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Abstract

Among the most exciting recent developments in numerical linear algebra (NLA) is the advent of randomized algorithms that are fast, scalable, robust, and reliable. Originating in the work of Drineas, Kannan, and Mahoney [1], randomization has been used to a number of problems in NLA to design efficient algorithms that can enable computations of scale that are otherwise impossible. The reviews [2, 4, 5, 14] are now classics on the topic, and books are being written [8].

In this talk I will first try to explain why and how randomization can help reduce the complexity of a numerical algorithm. I will then describe some of the successful randomized algorithms in NLA, along with some open problems. These include low-rank approximation, solving linear systems, least-squares, and eigenvalue problems. The talk will draw on results from [3, 6, 7, 9–13].

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Abstracts

Clustering/distribution analysis and preconditioned Krylov solvers for the approximated Helmholtz equation and fractional laplacian

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Abstract

The current study investigates the asymptotic spectral properties of a finite difference approximation of nonlocal Helmholtz equations with a Caputo fractional Laplacian and a variable coefficient wave number μ (possibly unbounded with a power singularity), as it occurs when considering a wave propagation in complex media, characterized by nonlocal interactions and spatially varying wave speeds. More specifically, we are interested in the study of the following problem:

$$\begin{cases} (-\Delta)^{\alpha/2} u(x,y) + \mu(x,y)u(x,y) = v(x,y), & (x,y) \in \Omega \subset \mathbb{R}^2, \quad \alpha \in (1,2), \\ u(x,y) = 0, & (x,y) \in \Omega^c, \end{cases} \tag{1}$$

where $\mu = \mu(x,y)$ is a given variable-coefficient, complex-valued wave number with source term v and for simplicity $\Omega = [0,1]^2$. By employing fractional centered differences (FCD) to the problem in (1) we are lead to the following matrix-vector equation:

$$A_{\mathbf{n}} u := (B_{\mathbf{n}} + D_{\mathbf{n}}(\mu))u = f, \quad \mathbf{n} = (n,n)$$

where $B_{\mathbf{n}} = \frac{1}{h^\alpha}\hat{B}_{\mathbf{n}}$, with $\hat{B}_{\mathbf{n}}$ being the two-level symmetric Toeplitz matrix generated by

$$t_\alpha(\eta,\psi) = \left[4\sin^2\left(\frac{\eta}{2}\right) + 4\sin^2\left(\frac{\psi}{2}\right)\right]^{\frac{\alpha}{2}},$$

and

$$D_{\mathbf{n}}(\mu) = \underset{\mathbf{i}=\mathbf{1},\ldots,\mathbf{n}}{\operatorname{diag}} \mu\Big(\frac{\mathbf{i}}{\mathbf{n}}\Big).$$

For the sake of simplicity, the previous equation is written in the following scaled form

$$\hat{A}_{\mathbf{n}} u := (\hat{B}_{\mathbf{n}} + h^\alpha D_{\mathbf{n}}(\mu))u = v, \quad \mathbf{n} = (n,n).$$

Our main results concern the spectral distribution of the sequence $\{\hat{A}_{\mathbf{n}}\}_{\mathbf{n}}$ and related preconditioning.

Theorem 1. *Let $\mu(x,y)$ be a bounded complex-valued function and let $\bar{\mu}_h$ be a bounded sequence of complex numbers. Then*

- *a1 the matrix-sequence $\{h^\alpha D_{\mathbf{n}}(\mu)\}_n$ is zero-distributed;*
- *a2 $\{\hat{B}_{\mathbf{n}} + h^\alpha D_{\mathbf{n}}(\mu)\}_n$ is a GLT matrix-sequence with GLT symbol t_α, so that $\{\hat{B}_{\mathbf{n}} + h^\alpha D_{\mathbf{n}}(\mu)\}_n \sim_\sigma t_\alpha$;*
- *a3 $\{\tau(\hat{B}_{\mathbf{n}})\}_n$ and $\{\tau(\hat{B}_{\mathbf{n}}) + h^\alpha \bar{\mu}_h I_{n^2}\}_n$ are GLT matrix-sequences with GLT symbol t_α, so that $\{P_{\mathbf{n}}\}_n \sim_\sigma t_\alpha$, $P_{\mathbf{n}} \in \{\tau(\hat{B}_{\mathbf{n}}), \tau(\hat{B}_{\mathbf{n}}) + h^\alpha \bar{\mu}_h I_{n^2}\}$;*
- *a4 $\left\{P_{\mathbf{n}}^{-1}\left(\hat{B}_{\mathbf{n}} + h^\alpha D_{\mathbf{n}}(\mu)\right)\right\}_n$ is a GLT matrix-sequence with GLT symbol 1, so that*

$$\left\{P_{\mathbf{n}}^{-1}\left(\hat{B}_{\mathbf{n}} + h^\alpha D_{\mathbf{n}}(\mu)\right)\right\}_n \sim_\sigma 1,$$

with $P_{\mathbf{n}} \in \{\tau(\hat{B}_{\mathbf{n}}), \tau(\hat{B}_{\mathbf{n}}) + h^\alpha \bar{\mu}_h I_{n^2}\}$.

Theorem 2. *Let $\mu(x,y) = 1/(x+\hat{\imath}y)^\gamma$. Then, for every $\gamma \geqslant 0$ such that $\alpha > \gamma - 1$ ($\alpha \in (1,2)$), we have*

- *b1 $\{h^\alpha D_{\mathbf{n}}(\mu)\}_n \sim_\lambda 0$;*

b2 $\{\hat{B}_{\mathbf{n}} + h^{\alpha} D_{\mathbf{n}}(\mu)\}_n \sim_{\lambda} t_{\alpha}$.

Theorem 3. *Assume that there exist positive constants* $c \leqslant C$ *for which*

$$c/|x + \hat{\imath}y|^{\gamma} \leqslant |\mu(x, y)| \leqslant C/|x + \hat{\imath}y|^{\gamma}.$$

for every $x, y \in [0, 1]$. *Consider the preconditioner* $P_{\mathbf{n}} = \tau(T_{\mathbf{n}}(t_{\alpha}))$. *Then, for every* $\gamma \in [0, 1)$ *and for every* $\alpha \in (1, 2)$), *we have*

c1 $\{P_{\mathbf{n}}^{-1} h^{\alpha} D_{\mathbf{n}}(\mu)\}_n \sim_{\lambda} 0$;

c2 $\left\{P_{\mathbf{n}}^{-1} \left[\hat{B}_{\mathbf{n}} + h^{\alpha} D_{\mathbf{n}}(\mu)\right]\right\}_n \sim_{\lambda} 1$.

Finally, several numerical experiments and visualization results corroborating the theoretical analysis are provided and few open problems are presented.

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Stable computation of generalized matrix functions via polynomial interpolation

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Abstract

Generalized matrix functions (GMFs) extend the concept of a matrix function to rectangular matrices via the singular value decomposition. Several applications involving directed graphs, Hamiltonian dynamical systems, and optimization problems with low-rank constraints require the action of a GMF of a large, sparse matrix on a vector. We present a new method for applying GMFs to vectors based on Chebyshev interpolation. The method is matrix free and requires no orthogonalization and minimal additional storage. Comparisons against existing approaches based on Lanczos bidiagonalization demonstrate the competitiveness of our approach. We prove that our method is backward stable by generalizing the proof of the backward stability of Clenshaw's algorithm to the matrix case.

Dual simplex volume maximization for simplex-structured matrix factorization

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Abstract

Matrix factorization (MF) is a fundamental technique for extracting latent low-dimensional factors, with applications in numerous fields, such as data analysis, machine learning and signal processing. MF aims to decompose a given data matrix, $X \in \mathbb{R}^{m\times n}$, where the n columns represent m-dimensional samples, into the product of two smaller matrices, $W \in \mathbb{R}^{m\times r}$ and $H \in \mathbb{R}^{r\times n}$ called factors, such that $X \approx WH$. Often imposing additional constraints, such as sparsity or nonnegativity, on the factors is crucial, e.g., for interpretation purposes; see [6] and the references therein. A specific problem of this broad family assumes that each column of H belongs to the unit simplex, that is, for all j,

$$H(:,j) \in \Delta^r := \left\{ x \in \mathbb{R}^r \,\middle|\, x \geqslant 0, e^\top x = \sum_{i=1}^{r} x_i = 1 \right\},$$

where e is the vector of all ones of appropriate dimension. It has several applications in machine learning with two prominent examples including unmixing hyperspectral images where $H(i,j)$ is the proportion/abundance of the ith material within the jth pixel [3, 8], and topic modeling where where $H(i,j)$ is the contribution of the ith topic within the jth document [1, 2, 4].

This talk focus on the concept of duality and uses the correspondence between primal and dual spaces to provide a new perspective on fitting a simplex to the samples. The main contributions are as follows:

- We present a new formulation based on the concept of duality. This formulation provides a different perspective and bridges the gap between two existing families of approaches: volume minimization and facet-based identification.
- We prove the identifiability of the parameters with this new formulation.
- We develop an efficient optimization scheme that is not sensitive to initialization. We also provide an efficient extension for rank-deficient polytope factorization. A major advantage of this extension is that it does not rely on any additional parameters and is directly derived from the proposed full-rank formulation.
- We provide numerical experiments on both synthetic and real-world data sets, showing that the proposed algorithm competes favorably with the state of the art.

Our proposed approach is based on the polar operator. In order to recover the columns of W, which are the vertices of the simplex enclosing the columns of X, we focus on extracting the facets of its convex hull. The facets are implicitly obtained by calculating the vertices of the corresponding dual simplex. Let us define the polar of a set.

Definition 1 (Polar)**.** Given any set $\mathcal{S} \subseteq \mathbb{R}^d$, its *polar*, denoted $\mathcal{S}^*$, is defined as

$$\mathcal{S}^* := \left\{ \theta \in \mathbb{R}^d \,\middle|\, \theta^\top x \leqslant 1 \quad \text{for all } x \in \mathcal{S} \right\}.$$

Polars have many interesting properties. In particular, If $\mathcal{S}$ is a polytope containing the origin in its interior, any vertex of $\mathcal{S}$ corresponds to a facet of $\mathcal{S}^*$, and vice versa. Hence if $\mathcal{S}$ is a simplex, that is, an $(r-1)$-dimensional polytope with r vertices and r facets, then $\mathcal{S}^*$ is a simplex.

After a preprocessing, we can reduce to find a decomposition for a reduced $Y \in \mathbb{R}^{r-1\times n}$ satisfying $Y = PH$ where $P \in \mathbb{R}^{r-1\times r}$ and $H(:,j) \in \Delta^r$ for all j, or also said, we need to find P such that $\mathrm{conv}(Y) \subseteq \mathrm{conv}(P)$.

In the polar, we will have $\text{conv}(P)^* \subseteq \text{conv}(Y)^*$, where the vertices of $\text{conv}(P)^*$ are the facets of $\text{conv}(P)$, Hence any matrix $P \in \mathbb{R}^{r-1\times r}$ such that $\text{conv}(P)^* \subseteq \text{conv}(Y)^*$ corresponds to a feasible solution.

The constraint $\text{conv}(\Theta) = \text{conv}(P)^* \subseteq \text{conv}(Y)^*$ can be written as $Y^\top\Theta \leqslant 1_{n\times r}$ where $1_{n\times r}$ is the matrix of all-ones of size n by r. Given Θ, P can be recovered by computing the vertices of $\text{conv}(\Theta)^* = \{x \mid \Theta^\top x \leqslant e\} = \text{conv}(P)$, and vice versa.

We propose to solve the problem by maximizing the volume of $\text{conv}(\Theta)$ in the dual space. The rationale behind this choice is that the larger a set is, the smallest its dual is, since $\mathcal{S}_2^* \subseteq \mathcal{S}_1^*$ implies $\mathcal{S}_1 \subseteq \mathcal{S}_2$, and minimizing the volume in the primal has shown to be a powerful approach; see [5, 7, 9]. We therefore propose to solve the following model: Given $Y \in \mathbb{R}^{r-1\times n}$, solve

$$\max_{\Theta\in\mathbb{R}^{r-1\times r}} \text{vol}\big(\text{conv}(\Theta)\big) \quad \text{such that} \quad Y^\top\Theta \leqslant 1_{n\times r}. \tag{1}$$

We will discuss in details how we handle Problem 1 and how we can adapt it in the presence of noise, and also in rank-deficient cases. We also discuss the identifiability guarantees of solving (1) when the data is separable, or the SSC (Sufficiently Scattered Condition) holds or the data is η-expanded.

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Spectral analysis of block preconditioners for double saddle point linear systems with application to PDE-constrained optimization

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Abstract

Given positive integer dimensions $n \geqslant m \geqslant p$, consider the $(n+m+p) \times (n+m+p)$ double saddle point linear system of the form

$$\mathcal{A}w \equiv \begin{bmatrix} A & B^T & 0 \\ B & 0 & C^T \\ 0 & C & E \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} f \\ g \\ h \end{bmatrix} \equiv b, \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$ is a symmetric positive definite (SPD) matrix, $B \in \mathbb{R}^{m\times n}$ and $C \in \mathbb{R}^{p\times m}$ have full row rank, and $E \in \mathbb{R}^{p\times p}$ is a square positive semidefinite matrix. Such linear systems arise in a number of scientific applications including constrained least squares problems, constrained quadratic programming, magma-mantle dynamic. Similar block structures arise e.g. in the coupled Stokes-Darcy problem, and the preconditioning of such linear systems has been considered in [4]. Block diagonal preconditioners for (1) have been deeply studied in [1] and inexact block triangular preconditioners have been analyzed in [2, 3].

We consider the SPD preconditioner proposed in [5] in the framework of multiple saddle point linear systems, defined as $\mathcal{P} = \mathcal{P}_L \mathcal{P}_D^{-1} \mathcal{P}_L^{\top}$, where

$$\mathcal{P}_L = \begin{bmatrix} \widehat{A} & 0 & 0 \\ B & -\widehat{S} & 0 \\ 0 & C & \widehat{X} \end{bmatrix}, \qquad \mathcal{P}_D = \begin{bmatrix} \widehat{A} & 0 & 0 \\ 0 & \widehat{S} & 0 \\ 0 & 0 & \widehat{X} \end{bmatrix}, \tag{2}$$

and $\widehat{A}$, $\widehat{S}$ and $\widehat{X}$ are suitable SPD approximations of A and the Schur complement matrices, respectively. We show that the eigenvalues of the preconditioned matrix can be characterized in terms of the roots of a cubic polynomial for which we give tight bounds. We provide examples of large-scale PDE-constrained optimization problems, which give raise to double saddle-point linear systems, in which we check the validity of the bounds and, at the same time, we show the effectiveness of the proposed preconditioner.

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Doubly diagonally dominant matrices and their combined matrices

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Abstract

In this talk we consider different diagonally dominant matrices. When the matrix is nonsingular we study the corresponding combined matrix. We focus on properties of doubly diagonally dominant matrices. When the doubly diagonally dominant matrix is nonsingular we study the corresponding combined matrix. In particular, we study wether or not the combined matrix of a doubly diagonally dominant matrix preserves the doubly diagonally dominance property.

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On the consistency of the matrix equation $X^*AX = B$

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Abstract

As an introduction, in the previous Encuentro ALAMA in Alcalá 2022, we presented our findings regarding the consistency of the matrix equation $X^\top AX = B$ over the complex field for given square matrices A, B (not necessarily of the same size). This equation arises in several settings, in particular related to bilinear forms and matrix congruence. Note that two square matrices A and B of the same size represent the same bilinear form with respect to different bases if and only if A and B are congruent, that is, the matrix equation has a nonsingular solution X. Canonical forms for congruence were known since, at least, the 1930s, but we follow the one in [5]. When studying the consistency, two nontrivial reductions can be done: (i) we can assume that A and B are in CFC; (ii) we can assume that B, and X are full rank. With this starting point, we were able to solve (with a few exceptions) the case where B is symmetric or skew-symmetric in a series of recent papers (see [1–4]).

In this Encuentro ALAMA 2024, we will study the consistency of a related matrix equation,

$$X^*AX = B,$$

over the complex field for an arbitrary square matrix A, and an hermitian or skew-hermitian matrix B (not necessarily of the same size). In this case the right context is the *-congruence and there is a corresponding canonical form for *-congruence, or *-CFC (see [5]). The starting point to study this second matrix equation is almost identical than the one described before: (i) we can assume that A and B are in *-CFC; (ii) we can assume that B, and X are full rank.

But the differences start to mount quickly. First, the CFC and *-CFC are very different. One immediate and very relevant difference in our study of symmetric or hermitian matrices B is that:

- the CFC of a full rank $m \times m$ symmetric matrix is I_m.
- the *-CFC of a full rank $m \times m$ hermitian matrix is $I_{m_1} \oplus (-I_{m_2})$ for some $m_1, m_2 \geqslant 0$, such that $m_1 + m_2 = m$. Note that (m_1, m_2) is referred as the signature for B.

For our $X^\top AX = B$ equation this means that there is essentially one symmetric matrix of size $m \times m$, while for our $X^*AX = B$ equation this means that there are essentially $m + 1$ hermitian matrices of size $m \times m$. So, although the matrix equation is very similar, the symmetric and hermitian cases differ significantly. Not surprisingly, the same phenomenon occurs for skew-symmetric, and skew-hermitian matrices:

- the CFC of a full rank $2m \times 2m$ skew-symmetric matrix is $\left[\begin{smallmatrix} 0 & I_m \\ -I_m & 0 \end{smallmatrix}\right]$.
- the *-CFC of a full rank $2m \times 2m$ skew-hermitian matrix is $(\mathrm{i}I_{m_1}) \oplus (-\mathrm{i}I_{m_2})$ for $m_1 + m_2 = m$.

We will also explain that when we study the consistency of matrix equation $X^*AX = B$, the signature of $\frac{A+A^*}{2}$ will be relevant. While in the case of $X^\top AX = B$, the signature is not relevant at all.

In conclusion, one might think that the study of both matrix equations will be very similar, but it is not. The matrix equation $X^*AX = B$ offers new and intriguing challenges that we will explore, and we will also give some preliminary results.

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The Relative Gain Array in control theory

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Abstract

In the context of control theory, the Relative Gain Array (RGA) is a matrix that quantifies the interactions between the inputs and outputs of a multivariable system, called MIMO. The entries of the RGA matrix provide insight into how changes in one input affect each output, relative to changes in the other inputs. That is, the RGA provides a measure of interactions between the inputs and outputs and recommends pairing inputs and outputs such that the rearranged system has an RGA matrix close to identity, see [3]. The relationship between the RGA matrix and the transfer matrix lies in how the RGA matrix reflects the sensitivity of the system's outputs to changes in the inputs, while the transfer matrix represents the overall dynamics of the system.

The linear multivariable plant can be described by the transfer matrix $G(s) = [g_{ij}(s)]$, $i, j = 1, \dots, m$, where $g_{ij}(s)$ is the open-loop gain from the jth input to the ith output. If we consider $h_{ij}(s)$ as the gain of the ith output to the jth input, where all the loops except the ith output are under tight control, then RGA, $\lambda_{ij}(s)$, is defined as the open-loop gain divided by the gain between the same two variables when all other loops are under "perfect" control, that is $\lambda_{ij}(s) = g_{ij}(s)/h_{ij}(s)$. To minimize the modelling requirements, the interaction measure is evaluated at zero frequency ($s = 0$). Then, we denote $g_{ij}(0) = g_{ij}$, $G(0) = G$ and $\lambda_{ij}(0) = \lambda_{ij}$.

In mathematics, the RGA matrix is referred to as the combined matrix, $\Lambda(G) = [\lambda_{ij}] = G \circ G^{-T}$, where $\circ$ denotes the Hadamard (entrywise) product. This matrix has been studied by several authors, see [1, 2].

In this work, we study the relationship between the transfer matrix and its RGA matrix of the 2×2 multivariable plants. The entries of the RGA matrix are important: λ_{11} represents the relative gain between input 1 and output 1, while input 2 and output 2 remain constant. If the value of this entry is close to 1, it means that there is strong coupling between input 1 and output 1, and changes in input 1 will significantly affect output 1, even when input 2 is held constant. Conversely, if the value of this entry is close to 0, it means that input 1 has little effect on output 1, and input 2 is the primary driver of changes in output 1. Moreover, λ_{11} can be used to guide the design of effective control strategies.

Furthermore, depending on λ_{11}, the RGA and the transfer matrices can have mathematical structures that allow us to study some properties of them.

Sometimes, the behaviour of the system is incorrect and perturbations are used to rectify the problem. Perturbations in the RGA matrix of a 2×2 system can have an impact on the transfer matrix of the system. When a perturbation is applied to the RGA matrix, the system's model changes and, consequently, the transfer matrix changes too. Specifically, the diagonal entries of the transfer matrix, related to the magnitudes of the input and output signals, may be affected, as well as the off-diagonal entries of the transfer matrix, which are associated with the cross coupling effects between the input and output signals.

In this work we apply perturbations to the RGA matrix and the transfer matrix and give some results on the influence of these perturbations in the system.

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On doubly stochastic circulant combined matrices

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Abstract

An $n \times n$ circulant matrix is defined by n parameters and the entries in the first row, and each subsequent row is a cyclic shift forward of the one above. A circulant matrix is a doubly stochastic matrix if its entries are nonnegative numbers and the sum of its entries of any row or column is equal to 1. These matrices have some applications in different fields such as in numerical analysis because they are diagonalized by a discrete Fourier transform [4]. Also in cryptography, because a circulant matrix is used in the mixcolumns step of the advanced encryption standard. In signal processing [3], Markov chain Monte Carlo methods [5] or in machine learning [1].

Combined matrices are useful in control theory. They play a significant role in signal processing due to their ability to exploit the structure and properties of signals and systems. A combined matrix of a nonsingular matrix $A = (a_{ij})$ is the matrix $\mathcal{C}(A) = (c_{ij}) = A \circ A^T$ where $\circ$ means the Hadamard (entrywise) product, and A^T means the inverse transpose, $(A^{-1})^T$, of A, see [2].

In this work we consider a 3×3 doubly stochastic circulant matrices. These matrices have the following structure:

$$H = \begin{pmatrix} h_1 & h_2 & h_3 \\ h_3 & h_1 & h_2 \\ h_2 & h_3 & h_1 \end{pmatrix}, \quad \text{with} \quad h_j \geqslant 0.$$

Since the sum of the entries of its rows and its columns is the same, we construct a doubly stochastic matrix multiplying the matrix H by the inverse of that sum $d = h_1 + h_2 + h_3$, i.e.

$$U = \frac{1}{d} H.$$

The matrix U is the doubly stochastic matrix corresponding to the circulant matrix H. Note that U is also a circulant matrix.

In this work we consider the matrix U obtained from a positive circulant matrix H and we give some conditions on the set $\{h_1, h_2, h_3\}$ to find a real matrix A such that its combined matrix is U, that is, $\mathcal{C}(A) = U$. Moreover, we will extend, if it is possible, these results to matrices of larger size.

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Matrix representation for Clifford algebras

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Abstract

Let $z = a + ib$ a complex number then this complex number can be represented by the $2^1 \times 2^1$ matrices:

$$\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} + \begin{pmatrix} 0 & -b \\ b & 0 \end{pmatrix},$$

trying to extend this matrix representation into a high–order Clifford algebra type

$$\begin{aligned} e_0 &= 1, \\ e_j^{k_j} &= -\alpha_j, \quad j = 1, \dots, n, \\ e_i e_j + e_j e_i &= 2\gamma_{ij}, \quad i \neq j, \end{aligned}$$

and given the advantage to write elements of a Clifford algebra type $\mathbb{R}^*_{0,n}$ in a matrix form to manipulate a huge amount of information, we focus our attention not only in found a different way to write the elements of this algebra but also in have an operational computer tool to do the calculation [1]. This construction allows us to develop an algorithm that defines the direction of the research in a certain way.

An advantage of this approach is that all the information about the structure relations in the Clifford-type algebras is contained in the matrices. Thus, when we operate with such matrices, it is not necessary to consider these structure relations anymore.

To construct a matrix basis for $\mathbb{R}^*_{0,n}$ we define a specific order using a bijection with natural numbers and define the new fundamental matrices. The way that we define these fundamental matrices allows us to construct a computer code with the disadvantage that it was difficult to implement for Clifford–type algebras. But, the possibility to write by blocks the fundamental matrices that represent the vectorial part of the Clifford algebra, allows us to construct differently the fundamental matrices and the posterior computer implementation.

In this talk we construct a matrix representation of a Clifford type algebras $\mathbb{R}^*_{0,n}$ using two different ways and construct a computer code to obtain these matrices for each construction [2]. Additionally, we show some examples of how we get the products of two elements of a Clifford algebras in a matrix form.

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New insights in rank one perturbations of matrix pencils

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Abstract

In this talk we shall address the rank one perturbation problem for matrix pencils. This problem has been completely resolved by the authors and independently by I. Baragaña and A. Roca. It consists in determining the possible eigenstructure (i.e. Kronecker invariants) of a matrix pencil under rank one perturbations. Recently, we have noticed a direct relationship of this problem with a special one row-one column case completion of matrix pencils. We relate these problems directly, and show some open questions on this relationship.

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Riemannian optimization methods to compute the nearest singular pencil

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Abstract

Given a square pencil $A + \lambda B$, where A and B are complex (resp. real) square matrices, we consider the problem of finding the singular complex (resp. real) pencil nearest to it in the Frobenius distance. This problem is known to be very difficult, and the few algorithms available in the literature can only deal efficiently with pencils of very small size. We show that the problem is equivalent to minimizing a certain objective function over the Riemannian manifold $SU(n)\times SU(n)$ (resp. $SO(n)\times SO(n)$ if the nearest real singular pencil is sought), where $SU(n)$ denotes the special unitary group (resp. $SO(n)$ denotes the special orthogonal group). We propose three algorithms for computing the minimum of this objective function in the Riemannian framework:

1. a basic one which gives rise to a non-smooth optimization problem,
2. a smoothed regularized version of such basic approach, and
3. a third method, which is smooth and more expensive than the previous ones, but that solves also the additional problem of finding a nearest singular pencil with a specified minimal index.

This novel perspective is based on the generalized Schur form of pencils, and yields competitive numerical methods, by pairing it with algorithms capable of doing optimization on the Riemannian manifolds considered above. We provide numerical experiments that show that the resulting methods allow us to deal with pencils of much larger size than alternative techniques, yielding candidate minimizers of comparable or better quality. In the course of our analysis, we also obtain a number of new theoretical results related to the generalized Schur form of a (regular or singular) square pencil and to the minimal index of a singular square pencil whose nullity is 1.

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A square-root-free unitary QR algorithm

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Abstract

In 1968 Pal, Walker and Kahan presented a stable implementation of the QR algorithm for symmetric tridiagonal matrices that avoided the need to compute square roots [1]. Removing the square roots significantly lowered the computation time making it superior to the traditional QR algorithm when only the eigenvalues are needed. Here we present a square-root-free version of the QR algorithm for unitary upper-hessenberg matrices. This method is faster than the traditional version and just as stable making it a competitive method for computing the eigenvalues of unitary matrices.

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On Oppenheim's inequality for Hurwitz matrices

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Abstract

Oppenheim's inequality for positive definite matrices states that, if A and B are positive definite matrices of the same order, then

$$det(A \circ B) \geqslant det(A \cdot B),$$

where $\circ$ denotes the Hadamard, i.e., the coefficient-wise, product. In our talk, we discuss for different orders the validity of Oppenheim's inequality for Hurwitz matrices associated with (Hurwitz-) stable polynomials.

Hoffman-Wielandt type inequality for block companion matrices of certain matrix polynomials

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Abstract

One of the well known inequalities in matrix analysis is the Hoffman-Wielandt inequality which states as follows:

Let A and B be two $n \times n$ normal matrices with eigenvalues $\lambda_1, \dots, \lambda_n$ and $\mu_1, \dots, \mu_n$, respectively given in some order. Then there exists a permutation π of $\{1, \dots, n\}$ such that

$$\sum_{i=1}^{n} |\lambda_i - \mu_{\pi(i)}|^2 \leqslant \|A - B\|_F^2. \tag{1}$$

There have been many generalizations of the Hoffman-Wielandt inequality, the most general form is called the Hoffman-Wielandt type inequality goes as follows:
Let A be a diagonalizable matrix of order n and B be a normal matrix of order n, with eigenvalues $\alpha_1, \alpha_2, \dots, \alpha_n$ and $\beta_1, \beta_2, \dots, \beta_n$, respectively. Let X be a nonsingular matrix whose columns are eigenvectors of A. Then, there exists a permutation π of the indices $1, 2, \dots, n$ such that

$$\sum_{i=1}^{n} |\alpha_i - \beta_{\pi(i)}|^2 \leqslant \|X\|_2^2 \|X^{-1}\|_2^2 \|A - B\|_F^2. \tag{2}$$

In this presentation we examine the Hoffman-Wielandt and the Hoffman-Wielandt type inequality for block companion matrices of matrix polynomials whose coefficients are unitary/doubly stochastic matrices. In particular, if $P(\lambda)$ is a quadratic matrix polynomial whose coefficients are either unitary matrices or doubly stochastic matrices, then we prove that under certain conditions on these coefficients, the corresponding block companion matrix C is diagonalizable. Consequently, if $Q(\lambda)$ is another quadratic matrix polynomial of same size with corresponding block companion matrix D, then under certain conditions on the coefficients of $Q(\lambda)$, the Hoffman-Wielandt type inequality holds for the block companion matrices C and D. We also estimate the condition number that appears in Inequality (2) in this set-up. This talk is based on [1].

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Improving performance of contour integral-based nonlinear eigensolver with infinite GMRES

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Abstract

Nonlinear eigenvalue problems [4, 6] (NEP)

$$T(\lambda)v = 0, \qquad v \in \mathbb{C}^n \setminus \{0\}, \qquad \lambda \in \Omega, \qquad (\Omega \subseteq \mathbb{C}, \qquad T\colon \Omega \to \mathbb{C}^{n\times n})$$

are receiving increasing attention in chemistry, physics, and many other fields. To solve these problems, one of the most popular methods is contour integral-based algorithms [1,3,7]. However, a great challenge in applying this algorithm is that a series of linear systems need to be solved to approximate the moments, which could be a heavy workload, especially for large, sparse problems. In this work, we employ the recently developed infinite GMRES algorithm [2] to avoid the computation of costly factorizations at each quadrature node and solve the linear systems efficiently.

The original implementation of infinite GMRES is usually not satisfactory in practice. Therefore, several techniques are applied to make the infinite GMRES memory-friendly, computationally efficient, and numerically stable. We examine the connection between polynomial eigenvalue problems and their scaled linearizations. Through theoretical analysis and numerical experiments, we reveal that the scaling technique mentioned in [2, Remark 6.2] is, in fact, a weighting strategy in GMRES. With this insight, we propose a novel weighting strategy that significantly accelerates the convergence of infinite GMRES in this particular context. Additionally, a two-level orthogonalization [5] procedure is employed in infinite GMRES to reduce the memory footprint, bringing the memory usage from $\mathcal{O}(m^2n)$ to $\mathcal{O}(mn + m^3)$.

Several numerical examples are provided to illustrate the performance of our algorithm. It is shown that, without the proposed weighting strategy infinite GMRES can totally fail in some cases. When employed in Beyn's method, our algorithm can achieve a speedup of 30% to 50% on large non-polynomial test examples, and even over 500% on some specific examples. For large-scale problems with strong nonlinearity, our algorithm can offer better acceleration by performing fewer matrix factorizations.

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Diagonalizably realizable implies universally realizable

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Abstract

A spectrum $\Lambda = \{\lambda_1, \dots, \lambda_n\}$ of complex numbers is said to be *realizable* if it is the spectrum of an entrywise nonnegative matrix A. If the matrix A is diagonalizable/symmetric we say that Λ is *diagonalizably/symmetrically realizable* ($\mathcal{DR}/\mathcal{SR}$). The spectrum Λ is *universally realizable* ($\mathcal{UR}$) if it is realizable for each possible Jordan canonical form allowed by Λ. In 1981 Minc proved that if Λ is the spectrum of a diagonalizable positive matrix, then Λ is universally realizable. The question of whether Minc's result holds for nonnegative realizations has been open for almost 40 years. Recently several extensions have been obtained. One of the main open questions about the problem of universal realizability of spectra is under what conditions a $\mathcal{DR}$ list of complex numbers is $\mathcal{UR}$. We introduce the indices of *diagonalizable realizability* and of *universal realizability*, and we define the concept of *diagonalizably realizable extreme*. Then, we prove a surprisingly simple result, which shows how diagonalizably realizable implies universally realizable.

This work is based on the joint paper C. Marijuán and R. L. soto, Diagonalizably realizable implies universally realizable. *Electronic J. Linear Algebra*, 40, pp 382-395, April 2024.

It is well known that an irreducible nonnegative matrix A is similar, via a positive eigenvector, to an irreducible nonnegative matrix B with constant row sums. If A is a reducible nonnegative matrix only co-spectrality with a nonnegative matrix B with constant row sums can be assured. The matter is that the transformation of a diagonalizable matrix in a matrix with constant row sums, in general, does not preserve the diagonalizability. From the Perron Frobenius Theory we know that, to preserve diagonalizability, it is necessary and sufficient the existence of a positive eigenvector, as is said in the following result.

Theorem. *(Théorème 6, [1]) Let A be a reducible nonnegative matrix with spectral radius $\rho(A)$. Then, A has a positive eigenvector associated to $\rho(A)$ if and only if, in the Frobenius normal form of A, all the final components have a spectral radius $\rho(A)$ and the non final components have a smaller spectral radius.*

Guo proved in 1997 that, for a self-conjugate list of complex numbers $\Lambda = \{\lambda_2, \dots, \lambda_n\}$, there is a minimum nonnegative number $g_r(\Lambda)$ such that $\Lambda_\mu = \{\mu, \lambda_2, \dots, \lambda_n\}$ is realizable for all $\mu \geqslant g_r(\Lambda)$. We call $g_r(\Lambda)$ the *realizability index* of Λ.

In a similar way we obtain an index for diagonalizable realizability.

Theorem. *Let $\Lambda = \{\lambda_2, \dots, \lambda_n\}$ be a self-conjugate list of complex numbers. Then there is a nonnegative number $\lambda_0 \geqslant g_r(\Lambda)$ such that $\Lambda_\mu = \{\mu, \lambda_2, \dots, \lambda_n\}$ is diagonalizably realizable for every $\mu \geqslant \lambda_0$.*

Note that the set of μs such that Λ_μ is $\mathcal{DR}$ is infinite and bounded below by the realizability index $g_r(\Lambda)$. Then there exists the infimun $g_d(\Lambda)$ that we call the *diagonalizable realizability index* of Λ. We do not know if the index $g_d(\Lambda)$ is or not a minimum.

Now we obtain an index for universal realizability.

Theorem. *Let $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$ be a diagonalizably realizable list of complex numbers with a diagonalizable realizing matrix A having a positive eigenvector. Then there is a nonnegative number $\lambda_0 \geqslant g_d(\Lambda/\lambda_1)$ such that $\Lambda_\mu = \{\mu, \lambda_2, \dots, \lambda_n\}$ is $\mathcal{UR}$ for every $\mu \geqslant \lambda_0$.*

As above, there exists the infimum $g_u(\Lambda)$ that we call the *universal realizability index* of Λ.

Definition 1. Let $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$ be a diagonalizably realizable list of complex numbers. We say that Λ is diagonalizably realizable *extreme* if for all $\epsilon > 0$, $\Lambda_{-\epsilon} = \{\lambda_1 - \epsilon, \lambda_2, \dots, \lambda_n\}$ is not diagonalizably realizable.

The number λ_0 in the previous Theorem is λ_1 if Λ is $\mathcal{UR}$, or strictly greater than λ_1 if Λ is not $\mathcal{UR}$. The following results establish the biggest possible refinement of the result in this Theorem and clarify the connection between $\mathcal{DR}$ and $\mathcal{UR}$. In particular, they show how $\mathcal{DR}$ non-extreme implies $\mathcal{UR}$.

Theorem. *Let $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$ be a diagonalizably realizable list of complex numbers with a diagonalizable realizing matrix A having a positive eigenvector. Then $\Lambda_\epsilon = \{\lambda_1 + \epsilon, \lambda_2, \dots, \lambda_n\}$ is universally realizable for all $\epsilon > 0$.*

Theorem. *Let $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$ be a diagonalizably realizable list of complex numbers. Then*

1) If Λ is non-extreme with $\Lambda_{-\epsilon} = \{\lambda_1 - \epsilon, \lambda_2, \dots, \lambda_n\}$, $\epsilon > 0$, being diagonalizably realizable by a matrix A with a positive eigenvector, then Λ is universally realizable.

2) If Λ is extreme, then $g_u(\Lambda) = g_d(\Lambda/\lambda_1)$.

Remark. The lists $\Lambda = \left\{a, \frac{\sqrt{5}-1}{4}a, \frac{\sqrt{5}-1}{4}a, -\frac{\sqrt{5}+1}{4}a, -\frac{\sqrt{5}+1}{4}a\right\}$ are $\mathcal{DR}$ extreme, but not $\mathcal{UR}$. However, the corresponding lists Λ_ϵ are still $\mathcal{DR}$ but non-extreme and, since the realizations of Λ are irreducible, the lists Λ_ϵ are $\mathcal{UR}$ and $g_u(\Lambda) = g_d(\Lambda/\lambda_1)$ with $g_u(\Lambda)$ not minimum. Then, the statement "If Λ_ϵ is $\mathcal{UR}$ for all $\epsilon > 0$, then Λ is $\mathcal{UR}$" is false, and so, in general, the universal realizability index $g_u(\Lambda/\lambda_1)$ is not a minimum.

Theorem. *The $\mathcal{SR}$ and the $\mathcal{UR}$ are independent properties.*

We use techniques from Graph Theory and from Linear Algebra, [**?**, 2–4].

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Totally positive matrices and polynomial least squares fitting

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Abstract

As we read in [1], polynomial least squares fitting naturally leads to solve the *normal equations*. A numerical difficulty is that the matrix of the normal equations is usually very ill-conditioned, and consequently the book presents a better method to solve the problem: the use of the *QR factorization* of the matrix A. This is the method carefully analyzed in the recent paper [4], an interesting work presented in the journal as a *classroom note*. This issue of solving least squares problems has also been studied in detail in the classical book [3] (see Lectures 11, 16 and 19 of that book), where the advantages of the QR factorization approach over the normal equations approach are clearly shown, both theoretically and by using numerical experiments.

The problem for researchers in the field of numerical linear algebra should be that this approach (the use of QR factorization) dates back to the work of G. H. Golub, in 1965. So, it would be interesting to see how this good approach could be improved by using results related to the work of Plamen Koev [2], taking into account that in the problem we are considering the involved matrices are structured matrices which can also be *totally positive matrices*. An additional issue to be considered is the computation of the projection matrix (the *hat matrix*) associated to these least squares problems.

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Approximate Inverse LU preconditioning applied to least squares problems

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Abstract

In this work we consider the application of approximate inverse LU preconditioners to compute preconditioners for the iterative solution of sparse least squares problems of the form

$$\min_x \|b - Ax\|_2, \tag{1}$$

where $A \in \mathbb{R}^{m\times n}$ $(m \geqslant n)$ is a large and sparse matrix with full column rank. We consider the solution of (1) with the CGLS [3] method which implicitly applies the conjugate gradient method to the normal equations

$$A^T Ax = A^T b. \tag{2}$$

The application of preconditioning is a technique required to solve large linear systems of equations with iterative methods. The main goal of it is to transform the initial linear system into a new equivalent one which can be solved more efficiently. Left, right or two side preconditioning are different variants of this technique. For instance, left preconditioning consists in solving iteratively the linear system $M^{-1}Ay = M^{-1}b$ where the nonsingular matrix M is called the preconditioner. The preconditioner M must satisfy some a priori conditions: it should approximate the matrix A in some sense and its application must be as cheap as possible from a computational point of view. If M^{-1} is not explicitly available its application involves the solution a linear system with M at the preconditioning step. By contrast, approximate inverse preconditioners store explicitly the matrix M^{-1} and its application through matrix by vector has multiple advantages in numerical computation.

In this work we apply the V–AISM preconditioner introduced in [2] which is a variant of the AISM preconditioner [1]. The main difference is that the Sherman-Morrison formula is applied multiplicatively that allows for a compact representation of the partial factors. From this compact representation an incomplete LU decomposition can be extracted. The results of numerical experiments show that this new preconditioner is efficient compared with other approximate inverse preconditioners that appear in the bibliography.

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Parallel implementation of a structure-preserving iterative solver for computing the interior eigenvalues of the Bethe-Salpeter equation

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Abstract

The Bethe–Salpeter equation is a relevant problem in many state-of-the-art computational physics applications. For instance, in the Yambo software [3] it is used to evaluate optical properties. It is commonly formulated as an eigenvalue problem with a block-structured matrix, $H = \begin{bmatrix} R & C \\ -C^H & -R^T \end{bmatrix}$, where R is Hermitian and C is complex symmetric. The problem can be solved more efficiently from a computational and numerical point of view if this structure is preserved throughout the computations. Some authors have shown how to implement direct structure-preserving methods for this equation [1]. For a good enough approximation of the optical absorption spectrum, it is sufficient to compute a few eigenvalues, and iterative Krylov methods can be used as a cheaper alternative, as is done in [4]. The eigenvalues of interest are those with the smallest magnitude, which in this case lie in the middle of the spectrum. The application requires to compute both the right and left eigenvectors, but the left eigenvectors can be computed inexpensively once the corresponding right ones are known.

The signs can be extracted from matrix H, transforming it into $B^{-1}A$, with $A = \begin{bmatrix} R & C \\ C^H & R^T \end{bmatrix}$ and $B = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$. Now both A and B are Hermitian and $B^{-1}A$ is self-adjoint with respect to A and to B. We show how to use different structure-preserving Lanczos methods with a Krylov-Schur restart.

To extract the eigenvalues with the smallest magnitude, we use the shift-and-invert technique, to transform the spectrum so the interior eigenvalues converge first. The Krylov vectors are computed by applying $(A-\sigma B)^{-1}B$, with σ equal to zero in this case. This matrix-vector product is carried out using a block Cholesky factorization, using the structure of A and B to compute the Schur complement and solve the linear systems in an optimized way.

This work was made in the context of the SLEPc parallel numerical library for eigenvalue computation [2]. We have implemented a new solver, specific for the Bethe–Salpeter equation, with the structure-preserving Lanczos and shift-and-invert transformation. The new solver, which will be available in SLEPc, was validated using Yambo test cases.

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Inverse Horn problem

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Abstract

Given two Hermitian matrices $A, B \in \mathbb{C}^{n\times n}$ with eigenvalues $\alpha_1 \geqslant \cdots \geqslant \alpha_n \geqslant 0$ and $\beta_1 \geqslant \cdots \geqslant \beta_n \geqslant 0$ respectively, the Horn problem consists in finding the set of all possible $\gamma_1 \geqslant \cdots \geqslant \gamma_n \geqslant 0$ eigenvalues of $A + B$ (see [1]). We denote this set as $E(\alpha, \beta)$.

We are interested in the inverse problem, that is, given $\alpha = (\alpha_1, \dots, \alpha_n)$, $\beta = (\beta_1, \dots, \beta_n)$ and $\gamma = (\gamma_1, \dots, \gamma_n)$ such that $\gamma \in E(\alpha, \beta)$, how to find Hermitian matrices $A \in \mathbb{C}^{n\times n}$ and $B \in \mathbb{C}^{n\times n}$ such that A has eigenvalues $\alpha = (\alpha_1, \dots, \alpha_n)$, B has eigenvalues $\beta = (\beta_1, \dots, \beta_n)$, and $A+B$ has eigenvalues $\gamma = (\gamma_1, \dots, \gamma_n)$. When $\alpha_i \geqslant 0$, $\beta_1 \geqslant 0$, $\beta_2 = \dots = \beta_n = 0$ and $\gamma_i \geqslant 0$ a solution to this problem was given in [2]. We present some partial results about the Inverse Horn Problem.

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Generalized inverses of linear operators on infinite-dimensional vector spaces and applications

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Abstract

In recent years, the study of different linear operators on infinite-dimensional vector spaces has been applied to some fields of mathematics. The notion of finite potent endomorphism on an arbitrary vector space was introduced by J. Tate in [22] as a basic tool for his elegant definition of Abstract Residues. In this article, the trace of a finite potent endomorphism was defined, while its determinant was defined in [8].

During the last decade the theory of finite potent endomorphisms have been applied to studying different topics related to Algebra, Arithmetic and Algebraic Geometry. Thus, A. Yekutieli in [23] and O. Braunling in [1] and [2] have addressed problems of arithmetic symbols by using properties of finite potent endomorphism and C. Debry in [3] and L. Taelman in [21] have offered results about Drinfeld modules.

The theory of generalized inverses of finite potent endomorphisms has been fundamentally developed by the author of this talk. Thus, the characterization of their sets of 1-inverses and generalized reflexive inverses has been carried out jointly with D. Alba Alonso in [5] and [4], and the main properties of the Drazin inverse, the Drazin-Moore-Penrose inverses, the Core-Moore-Penrose inverse, the G-Drazin inverses and Group inverse of a finite potent endomorphism have been defined and studied in [9], [15], [17], [18], [19] and [20]. Moreover, the Moore-Penrose inverse of a linear map on infinite-dimensional inner product vector spaces was analyzed in [7].

In addition, once the properties of bounded finite potent linear operators on Hilbert spaces in [16] have been studied, the notions of Drazin-Star and Star-Drazin inverses in [11] have been introduced. Furthermore, the relationship between the Drazin, Moore-Penrose, and DMP inverses of a bounded finite potent endomorphism and its adjunct on a Hilbert space has been analyzed in [14].

As an application of this theory, new properties of generalized inverses of square matrices have been obtained in [5], [4] and [19]; and results on the consistency of infinite systems of linear equations and on infinite systems of non-homogeneous difference equations have been offered in [6], [7], [12], [13] and [11].

A generalization of the notion of finite potent endomorphism is the Core-Nilpontent endomorphism that have been introduced in [10]. The set of these linear operators is the largest in which it makes sense to define the index and the Drazin inverse of an endomorphism.

The talk will consist of presenting the basic definitions and main properties of these generalized inverses, as well as their applications.

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Characterization and construction of ASTP matrices

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Abstract

This work introduces a method for generating almost strictly totally positive (ASTP) matrices of any order. The method is constructive, wherein starting from an ASTP matrix A of size $n \times m$, an ASTP matrix B containing A is constructed. This new matrix B can be of size $(n+1) \times m$, $n \times (m+1)$ or $(n+1) \times (m+1)$. Moreover, a characterization of ASTP matrices is provided based on the necessary conditions for their construction. This characterization determines the ASTP property of a matrix A of size $n \times m$ by checking at most $n \times m$ minors.

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On the distance matrix of the boundary of a pseudotree

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Abstract

A square matrix D is called a *dissimilarity matrix* if it is symmetric, all off-diagonal entries are (strictly) positive and the diagonal entries are 0. If, in adittion, it satisfies, for any triplet $i, j, k \in [n]$, the *triangle inequality*: $d_{ik} \leqslant d_{ij} + d_{jk}$, then it is said to be a *metric dissimilarity matrix*.

An integer metric dissimilarity matrix D of order n is called a *distance matrix* if , for every $i, j \in [n]$, if $d_{ij} > 1$, then there exists an integer $k \in [n]$ such that $d_{ik} = 1$ and $d_{ij} = d_{ik} + d_{kj}$. The *distance matrix* D_G of a graph $G = ([n], E)$ is the square matrix of order n such that, for every $i, j \in [n]$, $d_{ij} = d(i, j)$.

Theorem 1. [2, 5] *Let D be an integer metric dissimilarity matrix of order n. Then, D is a distance matrix if and only if there is graph G such that $D_G = D$.*

An integer metric dissimilarity matrix D of order n is said to be *additive* if every subset of indices $\{i, j, h, k\} \subseteq [n]$ satisfies the so-called *four-point condition*, that is, if among the three sums $d_{ij}+d_{hk}$, $d_{ih}+d_{jk}$, $d_{ik} + d_{jh}$, the two largest ones are equal.

A block of a graph G is an induced subgraph of G that is 2-connected and maximal with respect to this property. A graph G is a *block graph* if every cycle of G induces a complete subgraph.

Theorem 2. *[2, 6] A graph G is a block graph if and only if its distance matrix D_G is additive.*

Theorem 3. *[4] If T is a tree on n vertices, then* $\det(D_T) = (-1)^{n-1}(n-1)2^{n-2}$.

Theorem 4. *[2] A graph of order n is a tree T if and only if D_T is additive and* $\det(D_T) = (-1)^{n-1}(n-1)2^{n-2}$.

Let S be a subset of vertices of order k of a graph $G = ([n], E)$. It is denoted by $D_{S,V}$ the submatrix of D_G of order $k \times n$ such that for every $i \in S$ and for every $j \in V$, $[D_{S,V}]_{ij} = d(i, j)$. Similarly, the so-called *S-distance matrix* of G, denoted by D_S, is the square submatrix of D_G of order k such that for every $i, j \in S$, $[D_S]_{ij} = d(i, j)$.

A set of vertices S of a graph $G = ([n], E)$ is called *resolving* if, for every pair of distinct vertices $i, j \in V$, $d(i,k) \neq d(j,k)$ for some vertex $k \in S$. A resolving set S of G is called *strong resolving* if, for every pair of distinct vertices $i, j \in V$, there is a vertex $k \in S$, such that either $d(i,k) = d(i,j) + d(j,k)$ or $d(j,k) = d(j,i) + d(i,k)$ [8].

Theorem 5. *[2, 7] A subset of vertices S of a graph $G = (V, E)$ is a strong resolving set if and only if G is uniquely determined by the distance matrix $D_{S,V}$.*

A vertex v of a graph G is said to be a *boundary vertex* if for some other vertex u of G, no neighbor of v is further away from u than v. The *boundary* $\partial(G)$ of G is the set of all of its boundary vertices [3]. If $S = \partial(G)$, then the *$\partial(G)$-distance matrix* $D_{\partial(G)}$ is also called the distance matrix of the boundary of G.

Proposition 6. [2, 7, 8] *The boundary $\partial(G)$ of every graph G is a strong resolving set. Hence, G is uniquely determined by the distance matrix $D_{\partial(G),V}$.*

Notice that the set of leaves $L(T)$ of a tree T is precisely its boundary, i.e., $\partial(T) = L(T)$.

Theorem 7. [2] *Let $\hat{B}$ be an integer metric dissimilarity matrix of order $\kappa \geqslant 3$. Then, $\hat{B}$ is the distance matrix $D_{L(T)}$ of the set of leaves of a tree T if and only if it is additive and*

1. *$\hat{b}_{ij} < \hat{b}_{ik} + \hat{b}_{jk}$, for every distinct $i, j, k \in [\kappa]$.*

2. $\hat{b}_{ij} + \hat{b}_{ik} + \hat{b}_{jk}$ *is even, for every distinct* $i, j, k \in [\kappa]$.

Theorem 8. *[2] Let T be a tree. Then, T is uniquely determined by the distance matrix $D_{L(T)}$ of the set of leaves $L(T)$ of T.*

Proposition 9. *[2] Let $G = (V, E)$ be a unicyclic graph of girth g such that C_g is its unique cycle. If $U(G)$ denotes the set of vertices of C_g of degree 2, then $\partial(G) = L(G) \cup U(G)$.*

Theorem 10. *[2] Let G be a unicyclic graph. Then, G is uniquely determined by the distance matrix $D_{\partial(G)}$ of the boundary $\partial(G)$ of G.*

We have also proved similar statements to those displayed in Theorems 8 and 10, for some other graph families, such as block graphs and cacti. On the other hand, it is relatively easy two find pairs of graphs having the same boundary (and thus also the same distance matrix of the boundary) but different order. See Figure 1, for three examples. Based on all of these results and also on the one stated in Proposition 6, we posed in [2] the following conjecture.

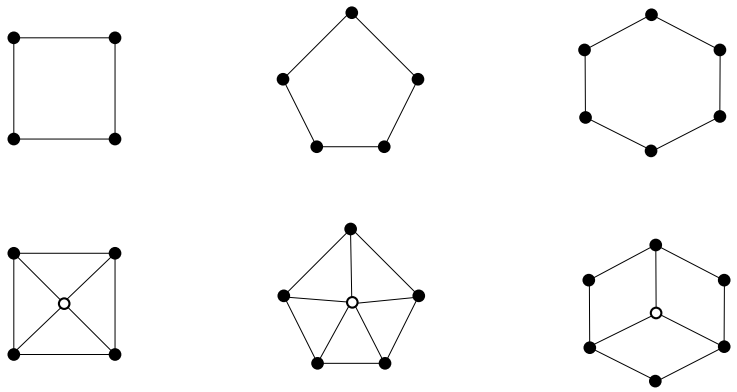

Fig. 1 Three pairs of graphs having the same $\partial(G)$-distance matrix but different order. In all cases, the boundary $\partial(G)$ is the set of black vertices.

Conjecture 11. [2] Let $\hat{B}$ an integer metric dissimilarity matrix of order κ. Let $G = ([n], E)$ be a graph on n vertices and κ boundary vertices such that $D_{\partial(G)} = \hat{B}$. If $G' = ([n], E')$ is a graph such that $D_{\partial(G')} = \hat{B}$, then G and G' are isomorphic.

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Universal realizability in low dimension

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Abstract

We say that a list $\sigma = \{\lambda_1, \dots, \lambda_n\}$ of complex numbers is realizable, if it is the spectrum of a nonnegative matrix. The Nonnegative Inverse Eigenvalue Problem (NIEP) is the problem of characterizing all possible realizable lists. If there exists a nonnegative matrix with spectrum σ for each possible Jordan canonical form allowed by σ, we say that σ is universally realizable ($\mathcal{UR}$). The problem of determining the universal realizability of spectra is called the Nonnegative Universal Realization Problem (URP). In terms of n, the NIEP is completely solved only for $n \leqslant 4$ and for $n = 5$ with trace zero. It is clear that for $n \leqslant 3$ the concepts of universally realizable and realizable are equivalent. We characterize the universal realizability of spectra of size 4:

Theorem. *Let $\sigma = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ be a list of complex numbers. If σ is nonreal, then σ is realizable if and only if σ is $\mathcal{UR}$. If σ is real with $\lambda_1 \geqslant \lambda_2 \geqslant \lambda_3 \geqslant \lambda_4$, then σ is $\mathcal{UR}$ if and only if σ is realizable and it does not satisfy $\lambda_1 = \lambda_2 > 0 > \lambda_3 = \lambda_4$ and $\lambda_1 + \lambda_3 + \lambda_4 < 0$.*

We also characterize the universal realizability of real spectra of size 5 with trace zero, and we describe a region for the universal realizability of nonreal spectra of size 5 with trace zero:

Theorem. *Let $\sigma = \{a, b, c, d, -(a+b+c+d)\}$ with $a \geqslant b \geqslant c \geqslant d$, $a > 0$ and $b + c + d \leqslant 0$. Then, the following statements are equivalent:*

i) *σ is $\mathcal{UR}$;*
ii) *σ is realizable and it is not one of the next spectra*

$$\{a, a, 0, -a, -a\},$$
$$\left\{a, b, -\frac{a+b}{3}, -\frac{a+b}{3}, -\frac{a+b}{3}\right\} \text{ with } a \neq b > \frac{a}{2}$$
$$\text{or } \left\{a, \frac{\sqrt{5}-1}{4}a, \frac{\sqrt{5}-1}{4}a, -\frac{\sqrt{5}+1}{4}a, -\frac{\sqrt{5}+1}{4}a\right\}.$$

Theorem. *Let $\sigma = \left\{a, b+ci, b-ci, -\frac{a}{2}-b+di, -\frac{a}{2}-b-di\right\}$, with $a, c, d > 0$.*

i) *If $b \neq -\frac{a}{4}$ or $d \neq c$, then σ is realizable if and only if σ is $\mathcal{UR}$.*
ii) *If $b = -\frac{a}{4}$ and $d = c$, then $\sigma = \left\{a, -\frac{a}{4}+ci, -\frac{a}{4}-ci, -\frac{a}{4}+ci, -\frac{a}{4}-ci\right\}$ is realizable if and only if $c \leqslant \frac{\sqrt{5}}{4}a$, and σ is $\mathcal{UR}$ if $c \leqslant \frac{a}{2}$.*
iii) *If $\sigma = \left\{a, -\frac{a}{4}+\frac{\sqrt{5}a}{4}i, -\frac{a}{4}-\frac{\sqrt{5}a}{4}i, -\frac{a}{4}+\frac{\sqrt{5}a}{4}i, -\frac{a}{4}-\frac{\sqrt{5}a}{4}i\right\}$, then σ is not diagonalizably realizable, and therefore is not $\mathcal{UR}$.*

Theorem. *Let $\sigma = \left\{a, b, c, -\frac{a+b+c}{2}+di, -\frac{a+b+c}{2}-di\right\}$, with $a \geqslant b \geqslant c$ and $d > 0$, and let $x^5+k_2x^3+k_3x^2+k_4x+k_5$ be its characteristic polynomial.*

i) *If $a > b > c$, then σ is realizable if and only if σ is $\mathcal{UR}$.*
ii) *If $a = b > c$ or $a = b = c$, then σ is not realizable.*
iii) *If $a > b = c$, then $\sigma = \left\{a, b, b, -\frac{a}{2}-b+di, -\frac{a}{2}-b-di\right\}$ (see Fig.1), and*
  1) *σ is realizable if and only if $k_2, k_3, k_4 - \frac{k_2^2}{4} \leqslant 0$;*
  2) *if σ is realizable and satisfies one of the following conditions*

$$b \leqslant 0 \quad \text{and} \quad b^2 + ab - d^2 + \frac{3a^2}{4} \geqslant 0, \text{ or}$$
$$\left(\frac{1}{4} - \frac{\sqrt{5}}{20}\right)a - \frac{\sqrt{5}\,b}{5} - \frac{\sqrt{10+2\sqrt{5}}\,d}{10}, \left(\frac{1}{4} + \frac{\sqrt{5}}{20}\right)a + \frac{\sqrt{5}\,b}{5} - \frac{\sqrt{10-2\sqrt{5}}\,d}{10} \geqslant 0,$$

then σ is $\mathcal{UR}$.

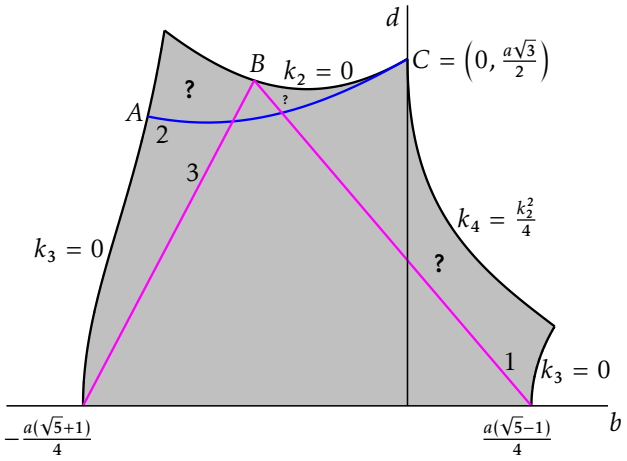

Fig. 1 ■ Region of realizability for $\{a, b, b, -\frac{a}{2} - b + d, -\frac{a}{2} - b - di\}$ in the bd-space for a fixed a, line 1 $\equiv \frac{\sqrt{5}}{5}b + \frac{\sqrt{10+2\sqrt{5}}}{10}d = \left(\frac{1}{4} - \frac{\sqrt{5}}{20}\right)a$, curve 2 $\equiv -\frac{\left(b+\frac{a}{2}\right)^2}{\left(\frac{a}{\sqrt{2}}\right)^2} + \frac{d^2}{\left(\frac{a}{\sqrt{2}}\right)^2} = 1$, line 3 $\equiv \frac{\sqrt{5}}{5}b - \frac{\sqrt{10-2\sqrt{5}}}{10}d = -\left(\frac{1}{4} + \frac{\sqrt{5}}{20}\right)a$, and regions with a question mark are the unknown region for universal realizability.

Theorem. *The spectra corresponding to the points of the border of the regions on Fig.1 with a question mark, i.e.* $k_3 = 0, k_2 = 0$ *and* $k_4 = \frac{k_2^2}{4}$*, are not* $\mathcal{UR}$*, except the spectra corresponding to the points* A, B *and* C *that are* $\mathcal{UR}$*. The identical real spectrum corresponding to the points* $\left(-\frac{a(\sqrt{5}+1)}{4}, 0\right)$ *and* $\left(\frac{a(\sqrt{5}-1)}{4}, 0\right)$ *is not* $\mathcal{UR}$ *.*

We use techniques from Graph Theory and from Linear Algebra, [1–3].

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Bounds for the Weyr characteristic of linear relations after one-dimensional perturbation

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Abstract

A linear relation S in $\mathbb{C}^n$ is a vector subspace of $\mathbb{C}^n \times \mathbb{C}^n$. Linear relations are related to matrix pencils. Indeed, given a matrix pencil $P(s) = sE - F \in \mathbb{C}[s]^{n\times m}$ we can associated to it two linear relations, the range and the kernel representations of it, which are $FE^{-1} = R\left(\begin{bmatrix} E \\ F \end{bmatrix}\right) \subseteq \mathbb{C}^n \times \mathbb{C}^n$, and $E^{-1}F := N(\begin{bmatrix} F & -E \end{bmatrix}) \subseteq \mathbb{C}^m \times \mathbb{C}^m$, respectively, i.e., they are the range and the kernel subspaces of the corresponding matrices. Conversely, given a linear relation, we can associate to it different pencils whose range or kernel representation is S. Among them, it is known which pencils are of minimal size.

Recently, the notion of the Weyr characteristics for a linear relation were introduced [4], and in [5] the Kronecker invariants of a matrix pencil were related with the Weyr characteristics of its kernel and range representations. On the other hand, the invariants of a matrix pencil obtained by a rank-one perturbation of another pencil are characterized in [2] (see also [3]). These results, thanks to the relationship between matrix pencils and linear relations, have allowed to characterize when a linear relation is a one-dimensional perturbation of another one [1].

From these results we explore bounds for the change of the Weyr characteristic of a linear relation which is a one-dimensional perturbation of another one. These bounds depend only on the corresponding Weyr characteristics of the unperturbed relation.

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Inner product free Shanks transformation for accelerating the convergence of vector sequences

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Abstract

Shanks transformation transforms a real sequence (s_n) into a real sequence (t_n) such that the convergence of (s_n) is accelerated via (t_n). Each term t_n appears as a solution of a linear system for which the size grows with n. Wynn's scalar Epsilon-algorithm is an appropriate way for computing t_n without explicitly solving the related linear system.

In the literature, generalisations of Shanks transformation to vector sequences (v_n) are derived with a strong coonection to the scalar case, by using inner product $y^T v_n$ for some choosen vector y. This approach gives rise to topological Epsilon-algorithms for computing the transformed sequence.

In this talk, we introduce an inner product free Shanks transformation for accelerating the convergnece of vector sequences. The construction of such transformation is achieved through the use of the powerful tools of Clifford algebra, which is a matrix algebra. Each term of the transformed sequence can be viewed as a solution of a generalized linear system with coefficients in Clifford algebra. We show also that the well-known vector Epsilon-algorithm introduced by P. Wynn allows us to compute the desired transformed sequence in an efficient way, without explicitly solving the generalized linear system.

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Combinatorics of integer partitions and bounded rank perturbations of matrix pencils

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Abstract

In this talk we present a new combinatorial result for partitions of integers. This result presents certain transitivity of different collections of partitions of integers, and generalizes previous related results on generalized majorization. We also show the application of this combinatorial result in the case of bounded rank perturbations of matrix pencils, where we give a solution in the minimal rank distance case.

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Some open questions on the solvability of Sylvester-like equations

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Abstract

Sylvester-like equations considered in this talk include:

$$AX + XD = E \qquad \text{(Sylvester equation)} \tag{1}$$
$$AXB + CXD = E \qquad \text{(Generalized Sylvester equation)} \tag{2}$$
$$AX + X^{\star}D = E, \quad \star = \top, * \qquad (\star\text{-Sylvester equation}) \tag{3}$$
$$AXB + CX^{\star}D = E, \quad \star = \top, * \qquad (\text{Generalized } \star\text{-Sylvester equation}), \tag{4}$$

where A, B, C, D, and E are given matrices of the appropriate sizes, and X is the unknown.

The solvability of these equations, as well as systems of these equations, has been extensively studied by many authors in many references since the seminal work by Sylvester from 1884 [1]. Under the notion of "solvability" we include:

- Characterizing the existence of solutions for either a fixed right-hand side E or for every right-hand side.
- Characterizing the existence of a unique solution, for either a fixed right-hand side E, or for every right-hand side.
- Obtaining explicit expressions for the solution.

The previous items include only theoretical questions, but also the computational aspect has been thoroughly studied, and efficient algorithms have been devised for the previous equations or systems of these equations.

In this talk, we will first review some of the main results regarding the solvability of these Sylvester-like equations, as well as systems of these equations, for general coefficients matrices. Then, I will discuss some relevant questions that are still open, which include:

- Characterize the consistency of $AX + X^{\star}D = E$, with E given, for general matrices $A, D^{\star} \in \mathbb{F}^{n\times m}$, with $\mathbb{F}$ being a field with characteristic 2.
- Characterize the uniqueness of solution of $AXB + CX^{\star}D = 0$ when A, B, C, and D are rectangular complex matrices.
- Obtain an explicit expression for the solution of the homogeneous Stein equation $X + CX^{\star}D = 0$, with $C, D \in \mathbb{C}^{m\times n}$.

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Minisymposia

Algebraic methods for the recovery, correction and security of digital information

Organizers: Noemí De Castro García, Luis Muñoz Casta, Miguel V. Carriegos

Computación cuántica y álgebra lineal

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Structured matrices and high relative accuracy

Organizers: Jorge Delgado, Pedro Alonso

ALGEBRAIC METHODS FOR THE RECOVERY, CORRECTION AND SECURITY OF DIGITAL INFORMATION

A convolutional variant with GRS codes for the McEliece cryptosystem

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Abstract

We present a variant of the McEliece cryptosystem that possesses several interesting properties, including a reduction of the public key for a given security level. In contrast to the classical McEliece cryptosystems, where block codes are used, we propose the use of a convolutional encoder to be part of the public key. The permutation matrix is substituted by a polynomial matrix whose coefficient matrices have columns with weight zero or at least weight two. This allows the use of Generalized Reed-Solomon (GRS) codes which translates into shorter keys for a given security level. Hence, the private key is constituted by a generator matrix of a GRS code and two polynomial matrices containing large parts generated completely at random. In this setting, instead of a single block message there is a sequence of messages and the errors are added throughout the sequence. Hence, the information vector encoded at each instant depends on the previous information vectors. In a usual convolutional setting, since the first vectors have none or few previous vectors, the first data received is more vulnerable. In order to protect the first instances, the encoded message is computed modulo a certain polynomial, turning all instances equally difficult to attack. After discussing possible structural and ISD attacks to this scheme, we conclude presenting the key sizes obtained for different parameters and estimating the computational cost of encryption and decryption process. The next table shows a comparison between the public key sizes between our new proposal and similar cryptosystems that haven't been broken.

	n	k	s	Wf	Public Key	Ciphertext size
New	90	66	30	$2^{129.14}$	207900	18900
Proposal	202	142	28	$2^{257.92}$	1147360	45248
	396	288	29	$2^{514.18}$	5132160	103356
Classic	2960	2288		2^{128}	1537536	672
McEliece	6960	5413		2^{256}	8373911	1547
	8192	6528		2^{256}	10862592	1664
GRS with	784	496		$2^{128.1}$	1428480	2880
Weight two mask	1820	1384		$2^{256.0}$	6637664	4360
Expanded RS	1258	1031		$2^{256.0}$	4624198	2724
Wild McEliece	852	618		$2^{128.0}$	≈ 712000	1170
with extension	858	672		$2^{128.0}$	624960	930
degree 2	892	712		$2^{128.0}$	634930	900

Tab. 1 Parameters, work factors, public key sizes and ciphertext sizes (in bits) of different PKCs

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ALGEBRAIC METHODS FOR THE RECOVERY, CORRECTION AND SECURITY OF DIGITAL INFORMATION

I/S/O representations and convolutional codes over modular integer rings: construction and good properties

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Abstract

A convolutional code is a type of error-correcting code commonly used in digital communication systems. It works by convolving the input data with a set of predefined code sequences to produce encoded output data. This helps in detecting and correcting errors that may occur during transmission.

Although the classical theory of convolutional codes deals with finite fields, Massey and Mittelholzer extended the concept to convolutional codes over rings in 1989, specifically to $\mathbb{Z}/M\mathbb{Z}$ where $M \in \mathbb{Z}^+$.

In this talk, we provide an overview of the existing approach to studying convolutional codes over certain commutative rings with identity from the perspective of linear dynamical systems, known as the *input/state/output* (I/S/O) representation. The initial results focused on convolutional codes over von Neumann noetherian rings, such as $\mathbb{Z}/n\mathbb{Z}$ where n is square-free ([1, 2]). Subsequently, these findings were generalized and extended to convolutional codes over artinian rings of principal ideals, such as $\mathbb{Z}/p^r\mathbb{Z}$ where p is prime and r is a positive integer ([3]). The available results provide a comprehensive study of the existence of minimal I/S/O representations for convolutional codes over rings of modular integers. Furthermore, obtaining the I/S/O representations is also applicable to the case of periodic convolutional codes over $R(z)$.

Moreover, we will discuss some applications of I/S/O representations and the challenges in generalizing this theory to rings, including the construction of non-catastrophic convolutional codes through reachable and observable systems, the construction of concatenated convolutional codes, and the development of new decoding algorithms. We will briefly outline a procedure to extend Rosenthal's decoding algorithm [4] for convolutional codes over a finite prime field to convolutional codes over the ring of modular integers $\mathbb{Z}/p^r\mathbb{Z}$.

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ALGEBRAIC METHODS FOR THE RECOVERY, CORRECTION AND SECURITY OF DIGITAL INFORMATION

Goppa codes. Geometric aspects and enumerative problems

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Abstract

Block q-ary codes of length n and dimension k are k-dimensional linear subspaces $\widehat{C} \subset \mathbb{F}_q^n$. On the other hand, non-catastrophic (or observable) q-ary convolutional codes of length n and dimension k are submodules, $C \subset \mathbb{F}_q[z]^n$, with locally free cokernel, so they are in one-to-one correspondence with k-dimensional linear subspaces $\widehat{C} \subset \mathbb{F}_q(z)^n$.

It is a classical result that the Grassmannian functor $Gr(k,n)^\bullet$ defined over the category of schemes is representable by a noetherian proper scheme defined over $\mathbb{Z}$, which we also denote by $Gr(k,n)$. Therefore, the same *algebraic variety* serves as a framework for block and observable convolutional codes:

$$\begin{aligned} Gr(k,n)^\bullet(\mathbb{F}_q) &:= \left\{ \begin{array}{l} q\text{-ary block codes of} \\ \text{dimension } k \text{ and length } n \end{array} \right\}, \\ Gr(k,n)^\bullet(\mathbb{F}_q(z)) &:= \left\{ \begin{array}{l} q\text{-ary observable convolutional} \\ \text{codes of dimension } k \text{ and length } n \end{array} \right\} \end{aligned} \tag{1}$$

Viewing the set of $[n,k]$ codes as a set of points in the Grassmannian has been exploited in the block case by using algebraic geometric techniques (such as geometric invariant theory etale cohomology, etc) to solve interesting problems (see for instance [3,6]).

One of the most relevant families of block and convolutional codes is the class of Goppa codes. These are defined from a geometrically irreducible smooth projective curve (of certain genus g), a set of n rational points and a divisor (of certain degree d) whose support does not contains none of the above points. If C is a Goppa code (block or convolutional), the geometric data defining it is called a Goppa structure. The most interesting numerical range for Goppa codes is $2g-2 < d < n$ (see [2,4]), so we assume this restriction.

The starting point of this research was to investigate the existence of a *common framwork/algebraic-geometric space* for the study of block and convolutional Goppa codes. Under the assumption $2g-2 < d < n$ and after slightly modifying the definition of Goppa code to admit arbitrary level structures [1,5], we will show that:

1. There exists an algebraic stack $P_{g,n,d}$ (the stack of Goppa structures), defined over $\mathbb{Z}$, whose set of $\mathbb{F}_q$-points (respectively, $\mathbb{F}_q(z)$-points) is identified with the set of block Goppa structures (respectively, convolutional Goppa structures) of genus g, length n and degree d.
2. The rule that attaches to a Goppa structure its associated code defines a morphism of stacks $\Theta : P_{g,n,d} \to Gr(k,n)$ (here $k = 1-g+d$). Consequently,

$$\begin{aligned} \mathrm{Im}(\Theta)(\mathbb{F}_q) &:= \left\{ \begin{array}{l} q\text{-ary block Goppa codes of} \\ \text{genus } g, \text{ length } n \text{ and degree } d \end{array} \right\}, \\ \mathrm{Im}(\Theta)(\mathbb{F}_q(z)) &:= \left\{ \begin{array}{l} q\text{-ary convolutional Goppa codes of} \\ \text{genus } g, \text{ length } n \text{ and degree } d \end{array} \right\} \end{aligned} \tag{2}$$

We will give some interesting geometric properties of the stack of Goppa structures (and their consequences in coding theory) as well as the geometric translation of certain problems in coding theory.

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ALGEBRAIC METHODS FOR THE RECOVERY, CORRECTION AND SECURITY OF DIGITAL INFORMATION

Flag codes and consistency with their projected codes

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Abstract

Let q be a prime power and consider a positive integer n. A constant dimension code is a nonempty collection of $\mathbb{F}_q$-subspaces of the vector space $\mathbb{F}_q^n$, all of them with the same dimension. A *flag code of type* $(t_1, \dots, t_r)$ is a nonempty set of *flags*, that is, a set of nested sequences of $\mathbb{F}_q$-subspaces of $\mathbb{F}_q^n$, all of them with the same increasing sequence of dimensions (the type).

Given a flag code $\mathcal{C}$ of type $(t_1, \dots, t_r)$, one can naturally obtain a family of constant dimension codes $\mathcal{C}_1, \dots, \mathcal{C}_r$: *its projected codes*, defined as the constant dimension codes $\mathcal{C}_i$ containing all the subspaces of dimension t_i appearing in flags in $\mathcal{C}$.

The parameters minimum distance and cardinality of a flag code $\mathcal{C}$ are related to the ones of $\mathcal{C}_1, \dots, \mathcal{C}_r$. However, these last ones do not determine the parameters of $\mathcal{C}$.

In this talk we introduce the the class of *consistent flag codes* (with respect to their projected codes) and show that, for this specific family, the flag code parameters are completely determined by the ones of its projected codes. We use this fact to study some structural properties that can be transferred from a flag code to its projected codes and vice versa, and also to describe a decoding algorithm for consistent flag codes.

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ALGEBRAIC METHODS FOR THE RECOVERY, CORRECTION AND SECURITY OF DIGITAL INFORMATION

A construction of spread codes based on Abelian non-cyclic groups

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Abstract

In the network coding setting, constant dimension codes have proved to be of utmost importance for the error correction in the transmission of information. Among them, the spreads stand out, as they are codes of maximum distance and maximum size for that distance.

The algebraic approach given by Kötter and Kschischang in [4], provided a rigorous mathematical setup for error correction when coding in non-coherent networks and, as a result, this theory was able to advance vastly. In this setting, the transmitted messages (*codewords*) are vector subspaces of a given vector space $\mathbb{F}_q^n$, and a *subspace code* is just a collection $\mathcal{C}$ of vector subspaces of $\mathbb{F}_q^n$. When all subspaces of $\mathcal{C}$ have the same dimension, we say that $\mathcal{C}$ is a *constant dimension code*.

One of the most important and studied families of subspace codes are *spread codes*, which are definied as a constant dimension code such that all its elements intersect pairwise trivially and their union covers the whole vector space. Spreads are clearly a relevant family of constant dimension codes since they reach the maximum distance and, at the same time, the maximum size for that distance.

A relevant technique to construct constant dimension codes is by considering the natural action of the general linear group, $\mathrm{GL}_n(\mathbb{F}_q)$, on the set of all k-dimensional vector subspaces of $\mathbb{F}_q^n$ (called Grassmannian, and denoted by $G_q(k,n)$). In this way, the codes arise as orbits under the action of some specific subgroup of $\mathrm{GL}_n(\mathbb{F}_q)$, and are called *orbit codes* [3,5,6,8]).

Most of these papers focus on the use of cyclic subgroups of $\mathrm{GL}_n(\mathbb{F}_q)$, called *cyclic orbit codes*. The studies in the use of other types of subgroups of the general linear group, is not enoughly explored yet. As a first step, in [2] we approach the study of orbit codes through the action of Abelian non-cyclic subgroups of $\mathrm{GL}_n(\mathbb{F}_q)$, giving a specific construction of maximum distance. Pursuing this line of research, the papers [1,7] are also concerned with Abelian non-cyclic orbit codes. Nevertheless, as far as we know, the only construction on spreads through the action of a non-cyclic Abelian group is given in [1]. Our main objective in this work is to pursue the research of orbit codes constructed by using non-cyclic Abelian groups.

Assume that $n = ks = k2t$, for some positive integers k, s, t. Firstly, we provide a k-partial spread of $\mathbb{F}_q^n$ (a subset of $G_q(k,n)$ such that $U \cap V = \{0\}$, for any $U \neq V$) with an orbital structure. For that, we construct an Abelian non-cyclic subgroup $\mathbf{H}$ of $\mathrm{GL}_s(\mathbb{F}_{q^k})$ with a suitable action on certain lines of $\mathbb{F}_{q^k}$.

Let $M_t \in \mathrm{GL}_t(\mathbb{F}_{q^k})$ be the companion matrix of a primitive polynomial of degree t over $\mathbb{F}_{q^k}$. We can consider that $\mathbb{F}_{q^{kt}} \cong \mathbb{F}_{q^k}[M_t]$ and, therefore, $o(M_t) = q^{kt} - 1$. On one hand, we consider $C = M_t^{q^k-1}$, whose multiplicative order is $r = \frac{q^{kt}-1}{q^k-1}$. And, we also consider a primitive element $\alpha \in \mathbb{F}_{q^k}$, whose multiplicative order $q^k - 1$.

We construct the matrices

$$h_1 = \begin{pmatrix} C & I_t \\ O_{t\times t} & \alpha I_t \end{pmatrix}, \quad h_2 = \begin{pmatrix} \alpha I_t & -I_t \\ O_{t\times t} & C \end{pmatrix} \in \mathrm{GL}_s(\mathbb{F}_{q^k}),$$

to define

$$\mathbf{H} = \langle h_1 \rangle \langle h_2 \rangle = \left\{ h_1^a h_2^b \mid 0 \leqslant a, b \leqslant q^{kt} - 2 \right\},$$

the group generated by the matrices h_1 and h_2, which is an Abelian non-cyclic subgroup of $\mathrm{GL}_s(\mathbb{F}_{q^k})$.

Consider the injective map given by:

$$\begin{array}{rccc} \varphi: & G_{q^k}(1,s) & \longrightarrow & G_q(k,n) \\ & \operatorname{rowsp}\begin{pmatrix} u_1 & \dots & u_s \end{pmatrix} & \mapsto & \operatorname{rowsp}\left(\begin{array}{c|c|c} \phi(u_1) & \dots & \phi(u_s) \end{array}\right), \end{array}$$

and the group monomorphism:

$$\begin{array}{rccc} \psi: & \mathrm{GL}_s(\mathbb{F}_{q^k}) & \longrightarrow & \mathrm{GL}_n(\mathbb{F}_q) \\ & \begin{pmatrix} a_{11} & \cdots & a_{1s} \\ \vdots & \ddots & \vdots \\ a_{s1} & \cdots & a_{ss} \end{pmatrix} & \mapsto & \left(\begin{array}{c|c|c} \phi(a_{11}) & \cdots & \phi(a_{1s}) \\ \hline \vdots & \ddots & \vdots \\ \hline \phi(a_{s1}) & \cdots & \phi(a_{ss}) \end{array}\right). \end{array}$$

We construct the following 1-dimensional orbit codes of $\mathbb{F}_{q^k}^s$, for $1 \leqslant i \leqslant t$:

$$C_i = \mathrm{Orb}_{\mathbf{H}}(\operatorname{rowsp}(\vec{e}_i)) = \{\operatorname{rowsp}((h_1^a h_2^b)_i) \mid 0 \leqslant a, b \leqslant q^{kt} - 2\} \subseteq \mathcal{G}_{q^k}(1,s).$$

Consider the vector subspace

$$U_{k,i} = \varphi(\operatorname{rowsp}(\vec{e}_i)) = \operatorname{rowsp}\left(\begin{array}{c|c|c|c|c} O_{k\times k} & \dots & I_k & \dots & O_{k\times k} \end{array}\right) \subseteq \mathcal{G}_q(k,n).$$

and the group $\bar{\mathbf{H}} = \psi(\mathbf{H}) \subseteq \mathrm{GL}_n(\mathbb{F}_q)$.

Therefore, for any $1 \leqslant i \leqslant t$ we can generate a k-partial spread as follows:

$$\bar{C}_i = \varphi(C_i) = \varphi(\mathrm{Orb}_{\mathbf{H}}(\operatorname{rowsp}(\vec{e}_i))) = \mathrm{Orb}_{\psi(\mathbf{H})}(\varphi(\operatorname{rowsp}(\vec{e}_i))) = \mathrm{Orb}_{\bar{\mathbf{H}}}(U_{k,i}).$$

Finally, we achieve to complet this orbit code with a nice family of k-subspaces of $\mathbb{F}_q^n$ in such a way the resulting code is a k-spread of $\mathbb{F}_q^n$ with an orbital structure which is not cyclic.

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ALGEBRAIC METHODS FOR THE RECOVERY, CORRECTION AND SECURITY OF DIGITAL INFORMATION

Performance of RNS-based processors for network coding

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Abstract

Network Coding [1, 2] (NC) is a new technique allowing intermediate nodes of networks communication to perform arithmetic operations on incoming packets, before forwarding them, in order to optimize the network resources. In fact, in addition to network throughput improvement, NC has been proven to achieve the maximum multicast capacity, improve network robustness and reduce energy consumption for several communication networks and technologies. However, node-wise arithmetic operation's complexity is a big problem interfering with its implementations.

A Residue Number System (RNS) [3, 4] is a finite integers ring $Z(M_k)$ formed by k co-prime moduli, $m_1, m_2, \dots, m_k$, where $M_k = \Pi_{i=1}^{k} m_i$. Without loss of generality, the moduli $m_1, m_2, \dots, m_k$, can be arranged in an ascending order.

An integer $X \in [0, M_k - 1]$ is represented via RNS by a vector $x = (x_1, x_2, \dots, x_k)$ of length k, where

$$x_i \equiv X \bmod m_i, \text{ for } i = 1, 2, \dots, k. \tag{1}$$

RNS transforms a computation within a large integer ring to parallel computations within smaller integers rings. That is, the multiplication and the addition of $x = (x_1, x_2, \dots, x_k)$ and $y = (y_1, y_2, \dots, y_k)$ in $Z(M_k)$ are computed respectively as k parallel multiplications

$$x_i.y_i \bmod m_i, \text{ for } i = 1, 2, \dots, k.$$

and k parallel additions

$$x_i + y_i \bmod m_i, \text{ for } i = 1, 2, \dots, k.$$

In this paper, we explore the benefits of using Residue Number System (RNS) based processors with network codes. Specifically, we study the complexity of the RNS-based processors over layered communication networks while computing NC arithmetic operations and we compare its results with the classic computing approach. The results show that for codewords of length n and RNS processors with k modulus, the performance of RNS-based processors linearly exceeds the classic approach by approximately $\frac{(9k+50)}{5k^2}\%$, $\frac{(3k+17)}{2k^2}\%$ and $\frac{(15k+84)}{9k^2}\%$ when comparing a single node, a network layer (3 nodes), and the whole network (3 layers) respectively. *i.e.*, for $k = 8$ RNS moduli of size $n = 256$ bits and using 1 Ghz processors, the time complexity of RNS-based processors is approximately 3 times smaller than classic computing.

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ALGEBRAIC METHODS FOR THE RECOVERY, CORRECTION AND SECURITY OF DIGITAL INFORMATION

Decoding of MDP convolutional codes over the erasure channel under linear systems point of view

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Abstract

This paper is trying to highlight the decoding capabilities of MDP convolutional codes over the erasure channel, by defining them as discrete linear dynamical systems, with wich the property on controllability and observability characteristics of linear system theory can be applied, in particular those of output observability, easily described using matrix language. Those are viewed against the decoding capabilities of MDS block codes over the same channel. Not only the time complexity is better, but also the decoding capabilities are increased with this approach, given that mostly on erasure channels, block codes are preferred.

COMPUTACIÓN CUÁNTICA Y ÁLGEBRA LINEAL

Optimización del número de puertas CNOT en circuitos aritméticos cuánticos

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Resumen

La puerta CNOT tiene una importancia crucial en el diseño de circuitos cuánticos debido a su papel en el procesamiento de información cuántica y la creación de entrelazamiento cuántico, lo que permite la correlación de estados entre dos qubits. Al combinar las puertas CNOT con rotaciones de un solo qubit, casi cualquier circuito cuántico puede ser simulado hasta cierto punto [4, 5]. Como resultado, muchas puertas de varios qubits están diseñadas utilizando solo puertas unitarias de un solo qubit y puertas CNOT. Sin embargo, las tasas de error significativas detectadas en las puertas CNOT utilizadas en ordenadores cuánticos experimentales son uno de los principales factores que contribuyen a su falta de fiabilidad [4]. Actualmente, nos encontramos en la era NISQ (Noisy Intermediate-Scale Quantum), donde los ordenadores cuánticos tienen un número moderado de qubits pero no lo suficientemente grande como para resolver problemas relevantes sin ser afectados por el ruido. En esta era, surgen dos problemas principales en el diseño de circuitos cuánticos: el número limitado de qubits disponibles y la complejidad de implementar circuitos tolerantes a fallos. Una técnica ampliamente utilizada para construir circuitos tolerantes a fallos es diseñarlos utilizando solo puertas Clifford+T, un conjunto universal generado por las puertas Hadamard, S, T, CNOT y las puertas Pauli, permitiendo la utilización de códigos de detección y corrección de errores [3]. Otro enfoque para la corrección de errores es reducir el número de puertas utilizadas, especialmente el número de puertas de dos qubits, en el diseño de circuitos, mejorando así la fiabilidad al reducir el número de operaciones del circuito [4]. Por lo tanto, el principal objetivo de este trabajo ha sido proponer circuitos aritméticos diseñados utilizando solo puertas del conjunto Clifford+T, al mismo tiempo que se optimizan estos circuitos minimizando el número de puertas CNOT requeridas. La plataforma utilizada para este propósito será IBM Quantum Platform (`https://learning.quantum.ibm.com/`).

Como se destacó anteriormente, reducir la dependencia de las puertas CNOT en el diseño de circuitos sigue siendo primordial para mejorar la fiabilidad del circuito [1, 3, 6]. En consecuencia, las métricas utilizadas para evaluar la optimización lograda incluyen el número de puertas CNOT y el número de qubits auxiliares requeridos, considerando la disponibilidad limitada de qubits. La Tabla 1 ilustra el número de puertas CNOT requeridas por las puertas comúnmente utilizadas y los qubits auxiliares que emplean.

Tab. 1 Métricas de las puertas más utilizadas en la literatura

Puerta	Número de puertas CNOT	Qubits Auxiliares
Pauli-X	0	0
Pauli-Z	0	0
CNOT	1	0
T	0	0
Hadamard	0	0
C-V y C-$V^{\dagger}$	2	0
S	0	0
TR [2]	7	0
Toffoli [2]	8	0
Peres	7	0
Temporary logical-AND	6	1

En este estudio, se examinarán varios circuitos aritméticos de la literatura con el objetivo de optimizarlos empleando diferentes técnicas para reducir el número de puertas CNOT requeridas, mientras se rediseñan exclusivamente utilizando puertas del conjunto Clifford+T. Entre estos circuitos, analizamos un sumador BCD reversible propuesto en [2], que incorpora varias puertas Toffoli y Peres. Tras aplicar diversas técnicas para reducir el número de puertas CNOT en este circuito, como reemplazar las puertas Toffoli y Peres con puertas Temporary logical-AND y su descomputación, logramos una reducción de aproximadamente el 20 % en el número necesario de puertas CNOT, mejorando así la eficiencia del circuito y permitiendo la utilización de códigos de detección y corrección de errores.

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COMPUTACIÓN CUÁNTICA Y ÁLGEBRA LINEAL

Avances en la clasificación de señales biomédicas con kernels cuánticos y otras técnicas de quantum machine learning

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Resumen

El estudio de las señales biomédicas es un campo de suma importancia, tanto para el ámbito de la investigación como para la sociedad. Descubrimientos en este campo pueden generar grandes avances tanto en la calidad de vida de las personas como en la sociedad, gracias a las mejoras de procesos médicos y reducción de costes. Por estos motivos, este trabajo está dedicado al análisis de los sonidos pulmonares, con el objetivo de clasificar las distintas enfermedades que pueden afectar al sistema respiratorio.

Normalmente, este proceso es realizado por un profesional médico, que ausculta al paciente con un estetoscopio en busca de distintas anomalías. Este proceso es subjetivo, depende en gran parte de la experiencia del técnico y puede dar lugar a diferentes interpretaciones. Debido a estas particularidades, la comunidad científica ha centrado sus esfuerzos en la mejora de la comprensión de estas señales pulmonares con técnicas de procesamiento de la señal y aprendizaje automático.

En este trabajo, se han recabado una serie de técnicas relevantes de procesamiento de señal, como los coeficientes cepstrales de Mel (MFCC); las transformadas de Fourier de tiempo corto (STFT) y los cocleogramas (CCGR) [6], en conjunción con técnicas de aprendizaje automático, como las máquinas de vectores de soporte (SVM) [4, 7], redes neuronales (NN) [12] y bosques aleatorios (RF) [11]. Con el fin de aportar a estas un enfoque apoyado en Quantum Machine Learning (QML) [3, 10], mediante el uso de redes neuronales cuánticas (QNN) [9] y máquinas de vectores de soporte cuánticas (QSVM) [5], para comprobar si la eficacia que estos modelos demuestran [1, 2], se puede traducir a esta tarea de clasificación.

Para llevar esto a cabo, se ha tomado el conjunto de datos públicamente disponible ICBHI 2017 Challenge [8], que consta de 6898 ciclos respiratorios etiquetados por expertos, que identifican la presencia o no de enfermedades. Se han seleccionado y preprocesado los datos, tomando 1959 muestras provenientes de un modelo específico de micrófono, se han muestreado a 4KHz, y se ha aplicado el análisis de componentes principales (PCA) como una técnica de reducción de la dimensionalidad para permitir que los modelos cuánticos puedan usar los datos debido a sus restricciones en tamaño. Finalmente, se han entrenado los modelos propuestos con la base de datos anterior, obteniendo los resultados que se muestran para los modelos clásicos en la Tabla 1 y para los modelos cuánticos en la Tabla 2.

Método	*Hiperparámetros*	*Exactitud Test*
SVM (con todas las características)	$C = 10$, $\gamma = 0{,}01$, núcleo: rbf	68,6 %
SVM (PCA)	$C = 100$, $\gamma = 1{,}0$, núcleo: rbf	67,9 %
NN (con todas las características)	lr = 0,001, unidades = 200, lotes = 20	65,6 %
NN (PCA)	lr = 0,001, lotes = 20	60,7 %

Fig. 1 Mejores resultados con los modelos ML clásicos.

Método	*Arquitectura*	*Exactitud Test*
QSVM	Codificación de amplitud	69,1 %
QNN	Representación angular, tree-tensor	66,1 %

Fig. 2 Mejores resultados con los métodos QML.

Los resultados evidencian un leve aumento en el rendimiento al emplear métodos de aprendizaje cuántico (QML) en comparación con sus contrapartes clásicas. Por ejemplo, el modelo SVM clásico más destacado logra una exactitud del 68,6 %, mientras que los QSVM alcanzan un 69,1 %. Este incremento en el desempeño de los modelos cuánticos se logra a pesar de la necesidad de reducir el número de características a un nivel manejable para el hardware actual, a diferencia de los modelos clásicos que conservan acceso al conjunto completo de características. De hecho, al utilizar el conjunto reducido de características con las SVM clásicas, la exactitud disminuye a un 67,9 %.

Esta disparidad es aún más evidente al comparar las redes neuronales clásicas con las cuánticas. La mejor NN clásica logra una exactitud del 65,6 % utilizando todas las características y solo del 60,7 % cuando se trabaja con el conjunto de datos reducido. Esta cifra es superada por las QNN, que alcanzan una exactitud del 66,1 % incluso cuando solo tienen acceso al conjunto reducido de características.

Esto demuestra que las técnicas de aprendizaje cuántico pueden llegar a superar a los métodos de aprendizaje clásicos en la clasificación de sonidos respiratorios, incluso cuando los modelos cuánticos tienen acceso a un número limitado de características. Específicamente, los QSVM lograron la mayor exactitud al utilizar la codificación de amplitud con 4 qubits y 16 características como mapa de características.

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COMPUTACIÓN CUÁNTICA Y ÁLGEBRA LINEAL

La jerarquía de Clifford desde una perspectiva algebraica

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Resumen

Existen varios mecanismos que permiten implementar puertas cuánticas resistentes a errores; uno de ellos es el esquema de teleportación cuántica introducido por Gottessman y Chuang [3].

Sea P_n el grupo de Pauli en n cúbits: el grupo generado por las puertas de n cúbits que son productos tensoriales de las puertas de Pauli (X, Y y Z) y de la puerta identidad en un cúbit. Sea C_n el grupo de Clifford en n cúbits: el normalizador de P_n en el grupo unitario de grado 2^n,

$$C_n := \{X \in U(2^n) \mid X(P_n)X^{-1} \subseteq P_n\}.$$

Partiendo de una implementación resistente a errores de las puertas de Pauli —que podría lograrse, por ejemplo, utilizando corrección de errores cuánticos [2]— el esquema de teleportación permite implementar con tolerancia a errores todas las puertas $U \in C_n$ del grupo de Clifford bajo la asunción de que, usando $2n$ cúbits ancilares, se pueda producir el *estado mágico*

$$|\psi_U\rangle := \frac{1}{\sqrt{2^n}}(I \otimes U) \sum_{k=0}^{2^n-1} (|k\rangle \otimes |k\rangle), \qquad (*)$$

donde I denota el operador identidad en n cúbits.

Por extensión, dado un conjunto cualquiera C de puertas que puedan implementarse de forma tolerante a errores, el esquema de teleportación nos permite implementar de forma tolerante a errores cualquier puerta U en el conjunto

$$C' := \{U \in U(2^n) \mid U(P_n)U^{-1} \subseteq C\}$$

siempre que se puedan preparar los estados de la forma $(*)$ para U. Esto nos permite definir inductivamente la *jerarquía de Clifford* para n cúbits como la sucesión de conjuntos $C_n^{(-)}$ dada por $C_n^{(1)} = P_n$ y

$$C_n^{(k+1)} := \{U \in U(2^n) \mid U(P_n)U^{-1} \subseteq C_n^{(k)}\}.$$

Se desprende fácilmente de la definición que esta jerarquía es una sucesión creciente de subconjuntos en el grupo unitario.

En la figura 1 se representa un circuito que utilizaría el esquema de teleportación para implementar una puerta $U \in C_n^{(k)}$ actuando sobre un estado arbitrario $|\alpha\rangle$. El esquema se limita a efectuar una medición

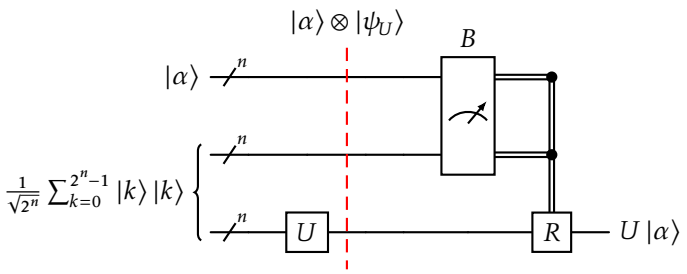

Fig. 1 Representación de cómo el esquema de teleportación se puede utilizar para implementar una puerta U en el k-ésimo nivel de la jerarquía en n cúbits actuando sobre un estado $|\alpha\rangle$ usando el estado mágico $|\psi_U\rangle$. La medición rotulada como B es una medición en la base de Bell. La puerta R es una puerta en el $(k-1)$-ésimo nivel de la jerarquía.

en la base de Bell en los cúbits de $|\alpha\rangle$ y en los primeros n cúbits del estado mágico. Tras la medición, el estado de los últimos n cúbits será $UP_t|\alpha\rangle$, donde P_t es una puerta del grupo de Pauli dependiente del resultado t obtenido en la medición. De este modo, fijando

$$R := U(P_t^{-1})U^{-1} \in C_n^{(k-1)}$$

y aplicando R en los últimos n cúbits, tenemos el estado

$$RUP_t|\alpha\rangle = U|\alpha\rangle,$$

como queríamos obtener.

Una puerta que se encuentre en el k-ésimo nivel de la jerarquía de Clifford —es decir, que pertenezca a $C_n^{(k)}$— y que no pertenezca a los niveles inferiores se podrá implementar de forma resistente a errores con $k-1$ aplicaciones del esquema de teleportación, y, en consecuencia, requerirá de la preparación de $k-1$ estados de la forma $(*)$.

Los dos primeros niveles de la jerarquía son, obviamente, grupos, pero esto ya no es cierto para los niveles sucesivos, lo cual dificulta su caracterización. Aún así, el estudio de la jerarquía de Clifford ha sido un área de investigación muy activa en los últimos años. Algunas obras se han centrado en el análisis de las propiedades que tienen las puertas que pertenecen en la jerarquía [4–6], mientras que otros trabajos han estudiado la generalización de la jerarquía de Clifford a sistemas de cúdits de dimensión (prima) arbitraria [1].

En este trabajo, estudiamos la jerarquía de Clifford desde un enfoque algebraico, lo que nos permite deducir propiedades generales sobre la cardinalidad de los distintos niveles de la jerarquía de Clifford y de todas las jerarquías análogas (como son las jerarquías en sistemas de cúdits). Además, extendemos las simulaciones numéricas que preparó De Silva [1] para calcular los operadores que pertenecen a los distintos niveles de las jerarquías en sistemas con cúdits.

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COMPUTACIÓN CUÁNTICA Y ÁLGEBRA LINEAL

QADS Rotacionales

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Resumen

En algunos procedimientos cuánticos de búsqueda, el operador iterativo actúa sólo en un subespacio invariante de pequeña dimensión, dejando las direcciones restantes invariantes. Este es el caso, por ejemplo, del operador de búsqueda de Grover, que actúa como una rotación en un subespacio bidimensional invariante y deja inalteradas las direcciones ortogonales [4]. Esta técnica de detección cuántica, y otras, como por ejemplo los caminos cuánticos de Szegedy [6], de Santos [5], de Wong [7], y el algoritmo de Deutsch-Jozsa [2], se encuentra bajo el marco teórico de Quantum Abstract Detecting Systems (QADS) [1].

De hecho, los QADS se introdujeron como un marco en común, para el estudio y diseño de algoritmos de detección en un entorno de computación cuántica. De hecho, dado un oráculo de caja negra para una función booleana f, los QADS construyen un estado inicial y un operador que puede usarse para detectar si la función es idénticamente cero o no.

Una de las ventajas que ofrece la introducción de esta metodología QADS es que permite construir nuevos QADS a partir de otros dados, lo que podría producir mejores probabilidades de detección. Estos QADS transformados son miembros del *cierre algorítmico* de QADS. La mayoría de estos procedimientos de cierre son bastante naturales, por ejemplo, ampliar el número de qubits utilizados, invertir el operador de detección, multiplicar los operadores de detección con el mismo estado inicial, la conjugación mediante un operador unitario o el control de un operador de detección con un qubit.

Ahora bien, si consideramos los QADS en los que el operador de detección se comporta como una rotación en un subespacio invariante bidimensional con un operador descrito por una matriz en $\mathrm{SO}(2) = \{A \in \mathrm{Mat}_{2\times 2}(\mathbb{C}) : A^t A = AA^t = I_2, \text{ y } \det(A) = 1\}$, donde $\mathrm{Mat}_{2\times 2}(\mathbb{C})$ denota el conjunto de todas las matrices con entradas en los números complejos de tamaño 2×2, el superíndice t denota la transpuesta de una matriz, y I_2 la matriz identidad de la dimensión 2, obtenemos una nueva familia de QADS llamados QADS rotacionales [3]. De hecho, si U_f es el operador detector de un QADS Q con estado inicial $|\varphi_0\rangle$, diremos que es un QADS rotacional, si existen $\alpha \in [0, 2\pi)$, estados ortonormales $|\varphi_1\rangle, |\varphi_2\rangle$, y $\beta_1, \beta_2 \in \mathbb{R}$, tales que

1. $|\varphi_0\rangle = \beta_1 |\varphi_1\rangle + \beta_2 |\varphi_2\rangle$
2. $U_f |\varphi_1\rangle = \cos\alpha \ |\varphi_1\rangle + \sin\alpha \ |\varphi_2\rangle$
3. $U_f |\varphi_2\rangle = -\sin\alpha \ |\varphi_1\rangle + \cos\alpha \ |\varphi_2\rangle$.

El operador de detección U_f de un QADS rotacional puede describirse directamente mediante una matriz

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \in SO(2), \tag{1}$$

ya que la matriz de coordenadas de U_f con respecto a una base ortonormal cuyos dos primeros elementos son $|\varphi_1\rangle, |\varphi_2\rangle$ es

$$\left(\begin{array}{cc|c} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ \hline 0 & 0 & I_{n-2} \end{array}\right). \tag{2}$$

Un primer ejemplo de un QADS rotacional es la búsqueda de Grover. Ahora bien, para esta nueva familia de QADS, encontramos una expresión explícita para la amplitud final, estudiamos sus propiedades, entre ellas su cierre algorítmico, derivando algunas equivalencias interesantes en términos de productos tensoriales y productos de raíces cuadradas de los QADS originales.

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COMPUTACIÓN CUÁNTICA Y ÁLGEBRA LINEAL

Estimación de fase con sistemas abstractos cuánticos de detección funcionales

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Resumen

Desde los inicios de la computación cuántica, la estimación de fase ha sido uno de los problemas más importantes a estudiar. ya que es posible una ventaja cuántica. Se han propuesto y mejorado numerosos algoritmos, y en este artículo proponemos un enfoque original basado en QADS funcionales.

Los QADS (Quantum Abstract Detecting Systems) fueron introducidos por Combarro, Ranilla y Rúa en [1]. Representan un marco teórico para aquellos circuitos cuánticos que dependen de una función booleana $f : \{0,1\}^k \longrightarrow \{0,1\}$ y fijan su estado inicial cuando $f \equiv 0$. Los QADS funcionales estudiados en este trabajo son una familia específica de QADS, que reciben cualquier otro QADS como input y aplican su operador U_f de acuerdo al siguiente circuito y a una función g.

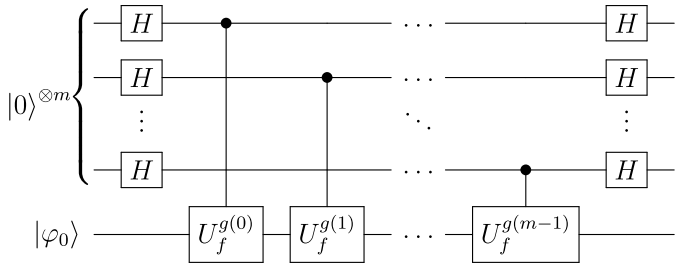

Fig. 1 Circuito de los QADS funcionales.

Los QADS funcionales tienen un buen comportamiento teórico: se puede obtener su clausura algorítmica y heredan algunas propiedades clave de los QADS en los que se basan. Además, si $F(m, U_f, g)$ es la matriz unitaria representando el circuito anterior, entonces la puerta QFT se puede reinterpretar como un producto de QADS funcionales:

$$QFT_n = \left(F(0, UROT_1, g_g) \otimes I^{\otimes n-1}\right)\left(F(1, UROT_2, g_g) \otimes I^{\otimes n-2}\right) \ldots \left(F(n-1, UROT_n, g_g)\right) H^{\otimes n}.$$

No obstante, los QADS funcionales y, en particular, los QADS geométricos (con $g(n) = 2^n$) han probado ser muy útiles para ciertos problemas prácticos, siendo el principal la estimación de fase. Los QADS combinatorios ($g(n) = 1$) ya habían sido estudiados para este problema [2], dando pie al m-test de Hadamard (el test de Hadamard para m qubits) y la búsqueda dicotómica. Ambos son superados por este nuevo algoritmo basado en QADS funcionales: el algoritmo de δ-aproximación.

Dado δ, necesita una primera aproximación de la fase β y, después, comprueba sucesivos intervalos de la forma $[\alpha - \delta, \alpha + \delta]$ hasta encontrar uno que contenga a β con alta probabilidad. Esto se logra aplicando circuitos cortos muchas veces, lo que reduce los errores debido al tamaño del circuito a cambio de hacer más mediciones. Este enfoque es ventajoso en ciertos contextos con ruido. Se prueba que los QADS geométricos son la elección óptima sobre la que basar este algoritmo. La comparación con los métodos mencionados se muestra en la tabla 1.

Una comparación con el método QPE también ha sido realizada. Debido a que aún no disponemos de una distribución teórica del algoritmo de δ-aproximación, esta comparación se ha hecho numéricamente. Para $\delta = 1/128$, y con el objetivo de lograr un nivel de confianza de 19/10.000, el QPE invierte al menos 15 qubits y 32.782 operaciones, en comparación con los 8 qubits y 20.316,98 operaciones de la δ-aproximación.

	Operaciones	Longitud media del intervalo	Porcentaje de intervalos fallidos
Búsqueda dicotómica	17784	0.024544	18.6
m-test de Hadamard	18000	0.029233	5.2
δ-aproximación	18176.97	0.02	0

Tab. 1 Comparación entre los tres algoritmos de estimación de fase basados en QADS funcionales para 1.000 intervalos de confianza y, aproximadamente, 18.000 operaciones.

Acerca de su comportamiento asintótico, puede ser comparado con el método de Kitaev [3] y con la *faster phase estimation* [5]. El resultado se muestra en la siguiente tabla. Se observa una ventaja en la anchura del circuito (número de qubits) y, a pesar de que realiza más operaciones, también garantiza que no se hagan consecutivamente (sin mediciones de por medio), lo que provocaría más errores al ser aplicado en un ordenador cuántico real. De nuevo, no disponer de una distribución teórica del método de δ-aproximación impide que este estudio asintótico sea totalmente fiable.

	Anchura	Profundidad	Tamaño
Kitaev	$O(m \log m)$	$O(m \log m)$	$O(m \log m)$
Faster Phase Estimation	$O(m \log^* m)$	$O(m \log^* m)$	$O(m \log^* m)$
δ-aproximación	$O(m)$	$O(m^3)$	$O(m^3)$

Tab. 2 Comportamiento asintótico del método de Kitaev, de la *faster phase estimation* y la δ-aproximación dependiendo de m, el número de potencias de U utilizadas.

Otra manera interesante de estimar una fase en con la estimación iterativa [4], donde el número de mediciones requeridas se reduce significativamente. Con input $\delta = 1/(2^l \times 3)$ en la δ-aproximación, resulta que, para que la estimación iterativa logre menor error, necesitaría, al menos, 210 mediciones, contra las 6155 mediciones de nuestro método. A pesar de ser una diferencia enorme, 6016 de ellas se invierten en el test de Hadamard para la aproximación inicial de β, y solo 139 en el resto. Aun así, esas 139 mediciones reducen el error de $1/10$ a $1/(2^7 \times 3)$. Esto recalca el hecho de que la segunda parte del algoritmo de δ-aproximación muestra un gran potencial para mejorar una estimación lograda por cualquier método de estimación de fase.

En conclusión, el método de δ-aproximación muestra un potencial interesante para la estimación de fase, y presenta la utilidad alternativa de mejorar una estimación dada por cualquier método aplicando circuitos cortos (en cuanto a operaciones y qubits), pero sin una cantidad significativa de mediciones. Aunque estos resultados parecen prometedores, es necesario un estudio más minucioso de este algoritmo para optimizarlo y capturar su comportamiento teórico, de manera que no debamos relegar las comparaciones a estudios numéricos.

Agradecimientos: Este trabajo ha sido parcialmente financiado por los proyectos PID2020-119082RB-C22 y PID2021-123461NB-C22 (MCIN/AEI/10.13039/501100011033) y el proyecto "QUANTUM ENIA - Quantum Spain" (Ministerio de Transformación Digital y Función Pública del Gobierno de España), y por la Unión Europea a través del Plan de Recuperación, Transformación y Resiliencia - NextGenerationEU en el marco de la Agenda España Digital 2026.

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COMPUTACIÓN CUÁNTICA Y ÁLGEBRA LINEAL

Esferas de Bloch para sistemas de varios qubits

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Resumen

La esfera de Bloch es un modelo muy usado para representar el estado de un qubit mediante un punto situado en la superficie de una esfera goniométrica cuyas coordenadas son valores esperados de magnitudes que caracterizan su estado cuántico. Sin embargo, hasta el momento no se ha consensuado una representación mediante este modelo para dos ó más qubits pese a poderse encontrar varios artículos en la literatura sobre el tema. Entre ellos, podemos citar como trabajo preliminar [2], donde se presenta una generalización de la representación de esferas estándar de Bloch para dos qubits en el dominio de fibraciones de Hopf. En [1] se muestra que los estados de un sistema de dos qubits se pueden representar en un álgebra geométrica de 6 dimensiones bastante similar a la Esfera de Bloch. En [4] se representa cualquier estado puro de dos qubits mediante seis ángulos reales. En [5] se representa un sistema de dos qubits mediante tres Esferas de Bloch, donde una de ellas puede tener radio distinto, también apoyado en fibraciones de Hopf. En [3] se aplica un Álgebra Geométrica para analizar el estado cuántico de varios qubits, donde un ángulo captura el entrelazamiento en terminos de la Entropia de Von Newman. Podemos resumir que existen modelos propuestos para dos qubits mediante 6 ó 7 dimensiones reales.

En este artículo proponemos una extensión del modelo de la Esfera de Bloch para un sistema conformado por más de un qubit, donde para el caso de dos qubits se necesitan exactamente tres esferas, es decir 6 dimensiones reales que se pueden capturar mediante tres esferas Bloch, cada una compuesta por dos ángulos, (θ_1, φ_1) y (θ_2, φ_2) encargadas de codificar cada uno de los dos qubits Q_1 y Q_2 respectivamente, y el último par (θ_3, φ_3) captura el entrelazamiento entre ellos. Para esto último vamos a apoyarnos en que cualquier puerta unitaria de un solo qubit se puede descomponer en las siguientes puertas de rotación a lo largo de los ejes Z e Y.

$$R_z(\varphi) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{bmatrix} \qquad R_y(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{1}$$

La Observación de las relaciones entre las fases de $|01\rangle$ y $|10\rangle$ con la fase de $|11\rangle$, junto a la caracteristica de su entrelazamiento, constituye otro elemento fundamental de la codificación propuesta.

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ORTHOGONAL POLYNOMIALS AND MATRIX ANALYSIS

Properties of eigenvalues and eigenpolynomials of finite order ordinary differential operators

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Abstract

Given a finite order differential operator

$$L \equiv \sum_{i=0}^{N} a_i(x)\partial_x^i, \tag{1}$$

we consider the sequence of eigenvalues $\{\lambda_n\}$ and the sequence of polynomials $\{P_n\}$ which are eigenfunctions of L, that is,

$$\sum_{i=1}^{N} a_i(x)\partial_x^i P_n(x) = \lambda_n P_n(x), \quad \forall n \in \mathbb{N}.$$

The main goal of this talk is to provide necessary and sufficient conditions for sequences $\{\lambda_n\}$ and $\{P_n\}$ to be the eigenvalues and eigenpolynomials of an operator L defined as in (1).

Regarding necessary conditions, we will highlight two results. The first one is that, given a differential operator L of order N, each eigenvalue λ_i with $i > N$ can be expressed as a linear combination of $\{\lambda_1, \dots, \lambda_N\}$.

The second result is related to the sequence $\{\delta_n^{(k)}\}$ introduced in [1] to obtain the explicit expression of the eigenvalues and eigenfunctions of a differential operator. Given L defined like in (1), the sequence $\{\delta_n^{(k)}\}$ is defined as

$$\delta_n^{(k)} = \sum_{i=k}^{n} \binom{n}{i} i! a_{i,i-k}, \quad k = 0, 1, \dots, n,$$

where $a_{i,i-k}, i = k, \dots, n$ are the coefficients of $a_i(x)$, this is,

$$a_i(x) = a_{i,i}x^i + \cdots + a_{i,1}x + a_{i,0}.$$

Then, we will prove that, for a fixed k, each $\delta_n^{(k)}, n > N$, depends linearly on $\delta_k^{(k)}, \dots, \delta_N^{(k)}$.

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ORTHOGONAL POLYNOMIALS AND MATRIX ANALYSIS

Matrix Sobolev inner products, generalized eigenvalues and zeros of polynomials

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Abstract

In a general framework of inner products induced by positive definite Hermitian infinite matrices in the space of polynomials, some results have been obtained applied to the problem of bounding the set of zeros of the orthogonal Sobolev polynomials. These results derive from the analysis of the asymptotic behavior of the generalized eigenvalues associated with a set of infinite Hermitian positive definite matrices that generalize a type of Sobolev inner products ([1] and [2]). In this context, we present new matrix aspects and several examples are explored.

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ORTHOGONAL POLYNOMIALS AND MATRIX ANALYSIS

On Sobolev bilinear forms and polynomial solutions of second-order differential equations

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Abstract

Let $\mathcal{L} \equiv \phi\, D^2 + \psi\, D$ be a linear second-order differential operator with non zero polynomial coefficients of degree at most 2, a sequence of real numbers $(\lambda_n)_{n\geqslant 0}$, and a Sobolev bilinear form

$$\mathcal{B}(p,q) = \sum_{k=0}^{N} \langle \mathbf{u}_k,\, p^{(k)}\, q^{(k)} \rangle, \quad N \geqslant 0,$$

where $\mathbf{u}_k$, $0 \leqslant k \leqslant N$, are linear functionals defined on space of polynomials. In this talk, we deal with the orthogonality of the polynomial solutions of the differential equation $\mathcal{L}[y] = \lambda_n y$ with respect to $\mathcal{B}$. We show that such polynomials are orthogonal with respect to $\mathcal{B}$ if the Pearson equations $D(\phi\, \mathbf{u}_k) = (\psi + k\,\phi')\, \mathbf{u}_k$, $0 \leqslant k \leqslant N$, are satisfied by the linear functionals in the bilinear form. Moreover, we use our results as a general method to deduce the Sobolev orthogonality for polynomial solutions of differential equations associated with classical orthogonal polynomials with negative integer parameters.

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ORTHOGONAL POLYNOMIALS AND MATRIX ANALYSIS

Solutions of linear systems of moment differential equations and generalized matrix exponentials

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Abstract

In the talk, we define and construct the so-called generalized exponential matrix by means of kernel operators for generalized summability. This matrix generalizes the classical exponential matrix and the fractional exponential matrix.

This object serves as a practical tool to express the solutions of linear systems of moment differential equations in a compact manner, in the spirit of the classical exponential matrix. More precisely, given a sequence of moments associated to some Laguerre-like measure, $m = (m(p))_{p\geqslant 0}$, and $A \in \mathbb{C}^{n\times n}$, the general solution of the system of moment differential equations

$$\partial_m y = Ay$$

is determined.

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ORTHOGONAL POLYNOMIALS AND MATRIX ANALYSIS

Structured matrices and Darboux Transformations of multiplication polynomial operators

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Abstract

We deal with Gram matrices associated with measures supported on the real line (Hankel matrices) and on the unit circle (Toeplitz matrices). By using perturbations of Sobolev type of these matrices get new structured matrices. We focus our attention on multiplication operators by a polynomial which are symmetric (in the first case) or unitary (in the second case). For them, we study Darboux transformations of the corresponding matrices which can be read in terms of matrix orthogonal polynomials on the real line (resp. the unit circle). A connection with bispectral problems is stated in the first case.

ORTHOGONAL POLYNOMIALS AND MATRIX ANALYSIS

On classical orthogonal polynomials and the Cholesky factorization of a class of Hankel matrices

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Abstract

Classical moment functionals (Hermite, Laguerre, Jacobi, Bessel) can be characterized as those linear functionals whose moments satisfy a second order linear recurrence relation. In this work, we use this characterization to link the theory of classical orthogonal polynomials and the study of Hankel matrices whose entries satisfy a second order linear recurrence relation. Using the recurrent character of the entries of such Hankel matrices, we give several characterizations of the triangular and diagonal matrices involved in their Cholesky factorization and connect them with a corresponding characterization of classical orthogonal polynomials.

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ORTHOGONAL POLYNOMIALS AND MATRIX ANALYSIS

Positive bidiagonal factorization of oscillatory tetradiagonal Hessenberg matrices

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Abstract

We identify conditions, articulated through continued fractions, that determine when an oscillatory tetradiagonal Hessenberg matrix is capable of possessing a positive bidiagonal factorization. To illustrate this, we investigate oscillatory tetradiagonal Toeplitz matrices as a specific case, exemplifying matrices that exhibit a positive bidiagonal factorization. Additionally, we establish a proof asserting that oscillatory banded Hessenberg matrices are structured in rays. Notably, the origin of the ray lacks a positive bidiagonal factorization, yet all interior points within the ray do exhibit such a positive bidiagonal factorization.

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ORTHOGONAL POLYNOMIALS AND MATRIX ANALYSIS

Sheffer-Dunkl sequences via umbral calculus in the Dunkl context

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Abstract

Umbral calculus refers to a series of techniques that can be used to prove some polynomial formulas. Nowadays, it mostly involves the study of Sheffer sequences. In this paper, we focus on a generalization of umbral calculus in a Dunkl context (that we call Dunkl-umbral calculus). Here, the operators of classical umbral calculus are naturally replaced by operators of the Dunkl theory on the real line. In this context we define for the first time Sheffer-Dunkl sequences, $\{s_{n,\alpha}(x)\}_{n=0}^{\infty}$, and provide some properties and examples. We also connect, via Dunkl-umbral calculus, properties of some polynomials in a Dunkl sense that have appeared in the literature in the recent years, like Bernoulli-Dunkl or Euler-Dunkl polynomials.

SOBRE EL PROBLEMA INVERSO DE RECUPERACIÓN DE LAS CONDUCTANCIAS

Experimental development of an EIT device and its proof of concept in the early detection of breast cancer

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Abstract

Electrical Impedance Tomography (EIT) is a non-invasive technology commonly used to visualize the variability of the electrical impedance of conductive objects, with emphasis on the early detection of breast cancer [1, 3]. Tomography is usually performed by solving the inverse problem of a linear transformation [5]. However, if new technologies are developed, experimental proof of concept is mandatory for their feasibility for real applications. The objective of this report is to show a new EIT prototype and its proof of concept based on the residual-signal methodology [4]. Our EIT prototype only uses 7 electrodes. In the experiments carried out, salty water, and small objects (comparable to a tumor less than 1 cm thick) are immersed in the EIT container. In all cases, our residual signal detects changes in impedance variability caused by the new invading object immersed in the EIT container, thus simulating, for example, the presence of a tumor in a breast. At the same time, a red LED turns on and off when the invading object is removed from the container. Detection takes a few seconds, but everything is done in real-time execution. See Figures 1 and 2. Experimentation is carried out in the multi-frequency excitation scheme. The cost of the electronic components of our technology does not exceed 150 euros, so it turns out to be a very low-cost prototype.

Definition 1. *Proof of Concept* (POC or PoC) is the realization of a certain idea, method or principle to demonstrate the viability that some concept or theory has practical potential [2].

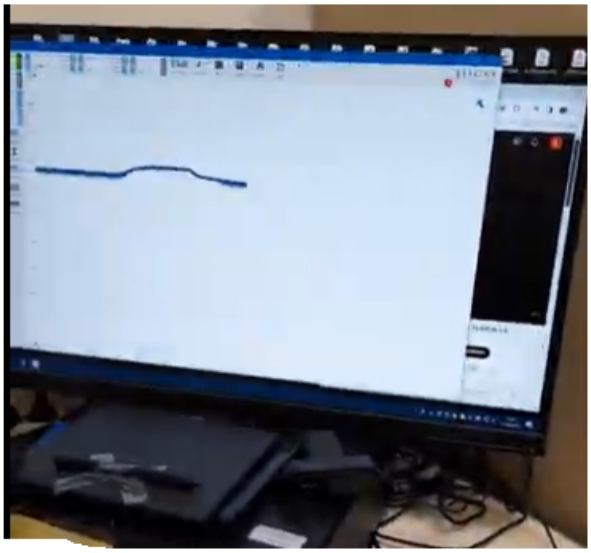

Fig. 1 Experimental results using a human finger. The figures show the red LED indicator (ponied by the pencil) and the residual signal generated.

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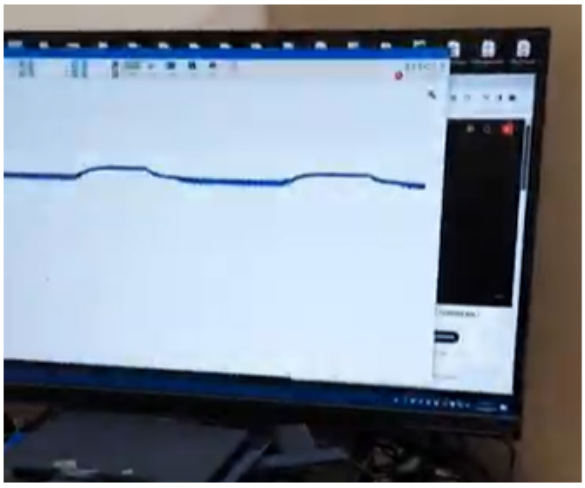

Fig. 2 Experimental results using a narrow metal rod of about 3 mm in diameter. The figures display the red LED indicator (ponied by the pencil) and the residual signal generated.

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SOBRE EL PROBLEMA INVERSO DE RECUPERACIÓN DE LAS CONDUCTANCIAS

Dirichlet-to-Neumann properties on spider networks

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Abstract

In this work, we define the Dirichlet–to–Neumann matrix associated with a Schrödinger type matrix on general networks, and we prove that it satisfies the *alternating property* which is essential to characterize those matrices that can be the response matrices of a network. We end with some examples of the sign pattern behavior of the alternating paths.

Let us consider the following *Dirichlet problem*: Given $f \in \mathbb{R}^m$ and $g \in \mathbb{R}^n$ find $u \in \mathbb{R}^{n+m}$ satisfying

$$\begin{bmatrix} I & 0 \\ -C(\delta(F);F)^\top & L(F;F) \end{bmatrix} \begin{bmatrix} u \end{bmatrix} = \begin{bmatrix} g \\ f \end{bmatrix}. \tag{1}$$

The existence and uniqueness of solution for System (1) were proved in [2]. In fact, the *Dirichlet Principle* tell us that for any data $f \in \mathbb{R}^m$ and $g \in \mathbb{R}^n$, Problem (1) has a unique solution.

The *Dirichlet–to–Neumann matrix*, denoted by Λ, is the Schur complement of $L(F;F)$ in L; that is,

$$\Lambda = L/L(F;F) = D - C(\delta(F);F) \cdot L(F;F)^{-1} \cdot C(\delta(F);F)^\top.$$

Observe that for any $g \in \mathbb{R}^n$, $\Lambda g = Dg - C(\delta(F);F)v_{g_{|F}} = L(\delta(F);\bar{F})v_g$.

Hence, Λ sends boundary Dirichlet date g to boundary Neumann currents Lv_g. The inverse problem is to recover the conductances C form Λ, see [1, 3, 4]. In this work we are not worried about this problem, but in studying some properties of Λ. In particular, we show that the Dirichlet–to–Neumann matrix has the alternating property, which may be considered as a generalization of the monotonicity property; see [5, Theorem 2.1] for the continuous version of this property.

Theorem 1 (Alternating paths)**.** *Suppose that $\delta(F) = A \cup B$, where A and B are disjoint subsets. Let $g \in \mathbb{R}^n$ such that $g_i \neq 0$ iff $i \in B$ and $p_1, \dots, p_k \in A$ such that*

$$(-1)^{i+1}\big(\Lambda g\big)_{p_i} > 0. \tag{2}$$

Then, there exist $q_1, \dots, q_k \in B$ such that

$$\big(\Lambda g\big)_{p_i} g_{q_i} < 0. \tag{3}$$

Moreover, for any $i = 1, \dots, k$, there exists a path from p_i to q_i such that $\gamma_i \setminus \{p_i, q_i\} \subset F$ and $g_{q_i} v_{g_{|(\gamma_i \setminus p_i)}} > 0$, where $v_g = Pg$.

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SOBRE EL PROBLEMA INVERSO DE RECUPERACIÓN DE LAS CONDUCTANCIAS

Bisymmetric nonnegative Jacobi matrix realizations

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Abstract

The spectral theory of Jacobi matrices; *i.e.*, real irreducible symmetric matrices $J(a,b)$ with main diagonal $a = (a_1,\dots,a_n)$ and second diagonal $b = (b_1,\dots,b_{n-1})$, $b > 0$, is nowadays a well-developed area of linear algebra and functional analysis. The inverse eigenvalue problems for Jacobi matrices have a long history and also have been studied in great detail in different ways, see for instance [1,3,7,8]. The case concerning to bisymmetric Jacobi matrices; that is, matrices that are symmetric about both of their main diagonals, has deserve special attention because its dynamic interpretation as mass-spring chains with symmetrically distributed beads, see [7,15]. In addition, is known that each ordered list of real numbers characterizes a unique bisymmetric Jacobi matrix realizing the list. Therefore, it is important to have examples of bisymmetric Jacobi matrices for which the spectrum is known. In fact, many of these of matrices are used as test problems to verify the efficiency of some algorithms to compute eigenvalues and also in regard to its stability, see the comments in [3,16,17]. The structure of both general bisymmetric matrices and their spectrum were described by A. Cantoni and P. Butler in [5].

In this communication, we present the recent results obtained by the authors in the spectral characterization of bisymmetric Jacobi matrices, see [6]. First, we present the characterization of this kind of matrices whose eigenvalues form a linear or quadratic monotone progression. This result unifies many different works over the last hundred years that appear recurrently in the literature, see for instance [1, 2, 4, 9–11, 18, 20] and also [12, 13, 19] for some applications of this kind of matrices in the analysis of random walks and some problems from statistical mechanics or quantum physics.

On the second hand we also present our result on the Nonnegative Inverse Eigenvalue Problem for bisymmetric Jacobi matrices. In our treatment is that we focus the reasonings on the gaps between eigenvalues instead on the eigenvalues themselves. The idea of considering the sequence of gaps was implicit in many of the mentioned references, see for instance [16], where a more than implicit mention to gaps is made. Once we translate into terms of gaps the necessary conditions on a given list to be realizable by a nonnegative Jacobi matrix obtained in [14], for bisymmetric realizations we show that, for a given gap sequence, there exists a threshold value for the lower eigenvalue, assuring the non-negativeness. Moreover, the adaptation of the results by Cantoni and Butler to Jacobi matrices allows us to obtain explicitly two entries of the realizing matrix and hence a new necessary condition on the gaps for nonnegative bisymmetric Jacobi realizability. For the case of a low order list, meaning $n \leqslant 6$, we obtain the unique bisymmetric (non-negative) Jacobi realization for an ordered list and present some specific examples.

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SOBRE EL PROBLEMA INVERSO DE RECUPERACIÓN DE LAS CONDUCTANCIAS

The Dirichlet-to-Neumann matrix as the Schur complement of the Laplacian

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Abstract

The Schur complement plays an important role in matrix analysis, statistics, numerical analysis, and many other areas of mathematics and its applications. Our goal is to introduce the Dirichlet–to–Neumann matrix on general networks as the Schur complement of the Laplacian matrix with respect to an invertible submatrix defined throughout the interior vertices. Schur complement is a rich and basic tool in mathematical research and applications, so we display an important property that illustrates its power in solving the discrete inverse problem. A complete version of this work in terms of operators can be found in [1].

Let us consider the following *Dirichlet problem*: Given $\mathsf{f} \in \mathbb{R}^m$ find $\mathsf{u}_f \in \mathbb{R}^m$ satisfying

$$\mathsf{L}(F;F)\mathsf{u}_f = \mathsf{f} \tag{1}$$

The existence and uniqueness of solution for System (1) implies that matrix $\mathsf{L}(F;F)$ is invertible and its inverse is called *Green matrix* for F and it is denoted by G. Observe that G is a symmetric matrix.

The *Dirichlet–to–Neumann matrix*, denoted by , is the Schur complement of $\mathsf{L}(F;F)$ in L; that is,

$$= \mathsf{L}/\mathsf{L}(F;F) = \mathsf{D} - \mathsf{C}(\delta(F);F)\cdot G\cdot \mathsf{C}(\delta(F);F)^{\top}.$$

This map is the response matrix to a Laplacian matrix, and therefore it can be assumed to be known, since it provides boundary reactions to boundary actions, the type of data that we can measure.

We take advantage of the Schur complement properties to study existence and uniqueness of the so-called overdetermined boundary value problem (see [2]) that are essential in the study of the inverse problem for the recovery of the conductances, [3, 4].

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SOBRE EL PROBLEMA INVERSO DE RECUPERACIÓN DE LAS CONDUCTANCIAS

Tridiagonal M-matrices whose group inverses are tridiagonal

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Abstract

Motivated by the works [1] and [4] we extend their results to singular tridiagonal M-matrices having group inverse. We analyze under which conditions the group inverse is also a tridiagonal matrix and determine explicitly the pattern of A for this property holds. To do this, we take into account that the $1n$ entry of the group inverse of any irreducible Jacobi M-matrix of order n is negative, see [2]. In addition we also use several known results about the structural properties of the group inverse, see [3, 5].

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SOBRE EL PROBLEMA INVERSO DE RECUPERACIÓN DE LAS CONDUCTANCIAS

Inverse and Moore-Penrose inverse of a structured totally positive matrix: accurate and efficient computation

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Abstract

The design of efficient and accurate algorithms to solve linear algebra problems involving nonsingular structured totally positive matrices, a type of matrices that appear in many fields, is a current topic of research in the area of numerical linear algebra. The interest in this topic is motivated fundamentally by the ill-conditioning of these matrices, which makes no accurate results are obtained when using standard linear algebra algorithms to solve linear algebra problems that involve them.

In this work, we consider the problems of computing the inverse and the Moore-Penrose inverse of structured totally positive matrices in an efficient and accurate way. The first step of the two algorithms we present is the fast and accurate computation of the bidiagonal decomposition for such matrices, since, having this decomposition for a nonsingular totally positive matrix computed with high relative accuracy, virtually all numerical linear algebra problems can be solved accurately for that matrix [1], in particular, the problems of inverse and Moore-Penrose computation [2].

The results of the numerical experiments, which illustrate the good behavior of the proposed algorithms, are shown.

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SOBRE EL PROBLEMA INVERSO DE RECUPERACIÓN DE LAS CONDUCTANCIAS

A 3D–realistic numerical and physical model of female breast

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Abstract

In electrical impedance tomography tests the usual way to proceed [1] is to solve a so called inverse problem (from data that is obtained at the contour of a body we compute electrical impedances at the interior). Our goal in attempting a more exact solution aims to discover specific points within a woman's breast that have increased their natural conductivity, eventually due to the presence of malignant tumour cells.

Mathematically speaking, the inverse problem is solved numerically in an iterative process: from an initial arbitrary distribution of conductances/impedances, it is checked whether this distribution agrees with the obtained data (electrical potentials read at the border); should the degree of concordance be not satisfactory enough, then conductances are recomputed (with the intention of achieving new values that are closer to real desired ones that explain the border readings at least well enough). This process of computing and checking impedance values is a so called inverse problem that is contrasted at each iteration by a forward problem, until some amount of convergence is reached and a sufficiently adequate approximation is obtained.

Standard verification techniques use finite element methods to solve the forward problem in the schema. The difficulty of the task [2, 3] lies on the morphology of the breast (a very heterogeneous mix of tissues that conform a very unstable structure of great variability in the sense of different behaviours and states, due to so many factors).

In the talk, and after [4], we will present a rather particular state of the art on numerical models (3D) of different female breast models so as to be able to carry out, with rigour and all guarantees the forward part of the outline. As an ultimate goal of our work, we would like to obtain our own numerically, anatomically and morphologically realistic 3D model of a female breast.

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SOBRE EL PROBLEMA INVERSO DE RECUPERACIÓN DE LAS CONDUCTANCIAS

Stability in the recovery of conductances in spider networks under a piecewise constant conductance hypothesis

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Abstract

The discrete inverse conductance problem for a network with boundary consists in recovering its conductance function from its Dirichlet-to-Neumann map. It is the discrete analogous to Calderon's Inverse Conductivity Problem [1], which involves the determination of the conductivity of a bounded connected set of $\mathbb{R}^n$.

Curtis and Morrow proved that the inverse conductance problem has a unique solution for critical planar networks (see [5–8]). A particular type of those networks are the well-connected spider networks, which have been widely used both as a discretization of a continuous set to obtain an approximate solution to the continuous inverse problem and as the workspace to solve the discrete problem (see [2–4, 8]).

In [2], an explicit formula for solving the discrete inverse conductance problem on well-connected spider networks was obtained. Nevertheless, the problem is severely ill-posed, so often the numerical application of this algorithm yields a conductance that is far away from the real one, even for relatively small-sized networks.

Calderon's Problem is also severely ill-posed. Nevertheless, Alessandrini and Vessella proved in [1] that if it is a priori known the hypothesis that the conductivity is piecewise constant with a bounded number of unknown values, the problem becomes Lipschitz stable. We introduce an analogous hypothesis for the discrete problem: the hypothesis that the conductance is constant in each subset of a partition of the set of edges such that the number of subsets is much smaller than the total number of edges.

We propose (see [4]) to formulate the discrete problem as a polynomial optimization problem, introducing a regularization term which penalizes the deviation from that piecewise constant conductance hypothesis. We present several numerical experiments in which we solve this problem for spider networks with piecewise constant conductance, obtaining good approximations to the real conductance; which supports the stability of our approach.

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STRUCTURED MATRICES AND HIGH RELATIVE ACCURACY

Totally positive matrices and Gaussian Markov random fields

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Abstract

A matrix such that all its minors are non-negative (resp. positive) is called totally positive (resp. strictly totally positive). Both these types of matrices have greatly called the attention of the scientific community (see, e.g., [2]) because, among other reasons, non-singular totally positive matrices allow for a bidiagonal factorization, something that turns out to come very handy from a computational point of view. More specifically, as described by Koev [4], if a bidiagonal factorization of a matrix is available it is then possible to perform numerous algebraic operations with High Relative Accuracy (HRA), meaning that the relative error of the computations is of the order of machine precision. Examples of algebraic operations that benefit from this approach include the computation of the inverse matrix, triangular factorization, determination of eigenvalues and singular values, and resolution of certain linear systems.

In the context of statistics, algebraic operations with covariance matrices are commonplace. It is therefore not difficult to imagine the potential of identifying conditions under which a covariance matrix is assured to be totally positive. For instance, it was proven in [3] that, given a Gaussian Markov Random Field over a graph of paths [6], if the covariances between adjacent variables on the graph are non-negative, then there always exists a reordering of the variables that makes the resulting covariance matrix to be totally positive. Additionally, if the covariances between adjacent variables on the graph are non-positive, then there always exists a reordering of the variables that makes the inverse of the resulting covariance matrix to be totally positive. These results imply that some algebraic operations with the covariance matrix of specific Gaussian Markov Random Fields may be performed to HRA.

In this work, we present some experiments proving the benefits of the approach described by Koev for performing algebraic computations with HRA with non-singular totally positive covariance matrices. For instance, it is shown that the TNEigenValues routine [5] clearly outperforms the conventional eigenvalue algorithm of LAPACK [1] (as implemented by eig in MATLAB) in terms of precision when computing the eigenvalues of the covariance matrix of a Gaussian Markov Random Field over a graph of paths such that all covariances between adjacent variables on the graph are either (1) non-positive or (2) non-negative.

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STRUCTURED MATRICES AND HIGH RELATIVE ACCURACY

Total positivity and high relative accuracy for Kac-Murdock-Szegö matrices

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Abstract

The *Kac-Murdock-Szegö* (KMS) matrix is a symmetric matrix with applications to many fields such as statistics, finances or digital signal processing. For more details in these matrices you can see for example [2], and Section 1.3 of [1] and references in there. These matrices have been extended to the nonsymmetric case (see Section 1.4 of [1]).

Bidiagonal decompositions of totally positive matrices are very useful in order to devise high relative algorithms for solving some algebraic problems with them. So, in this talk we will show explicit expressions for the bidiagonal decomposition of nonsymmetric KMS matrices. Taking into account this bidiagonal decomposition we will analyze when the nonsymetric KMS matrices are nonsingular and totally positive. Then we will study when the bidiagonal decomposition can be computed with high relative accuracy (HRA) providing a linear time complexity algorithm. In this case, the bidiagonal decomposition may be used to solve some algebraic problems for the matrices with HRA.

Then, the case of the Hadamard product of nonsymmetric KMS matrices will be considered. In particular, we will analyze when linear algebraic problems with the Hadamard product of nonsymmetric KMS matrices can be carried out to HRA.

Finally, we will present some numerical experiments illustrating the accuracy of the new algorithms proved in the theoretical results. In addition, the accuracy of the new methods for KMS matrices will be compared with the accuracy of the usual general methods.

Acknowledgements: This work was partially supported by MCIU/AEI through the Spanish research grants PID2022-138569NB-I00 (MCI/AEI) and RED2022-134176-T (MCI/AEI), and by Gobierno de Aragón (E41_23R).

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STRUCTURED MATRICES AND HIGH RELATIVE ACCURACY

Total positivity and symmetric functions

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Abstract

When a basis of functions is totally positive, the entries of the bidiagonal decomposition of its collocation matrices at increasing sequences of nodes on the domain can be expressed in terms of symmetric functions of the nodes. This idea relies on two important facts. On the one hand, the elements of the bidiagonal decomposition of a nonsingular and totally positive matrix can be expressed as quotients of some particular minors of it (cf. [1]). On the other hand, the structure of the collocation matrix is such that any of its minors is an antisymmetric function of its nodes. For polynomial bases, an explicit realization of this result can be found in [2], allowing to derive criteria for the total positivity of the class of polynomial bases with a lower triangular change of basis matrix from the monomial basis.

In this talk, we shall exploit the link between totally positive bases and symmetric functions to present novel formulae to compute the elements of the bidiagonal decomposition of collocation matrices. Besides, we will show that these results naturally extend to other interesting algebraic objects related to bases of functions, such as Gram and Wronskian matrices.

Aknowledgements: This work was partially supported by Spanish research Grants PGC2018-096321-B-I00 (MCIU/AEI) and RED2022-134176-T (MCI/AEI) and by Gobierno de Aragón (E41-23R).

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STRUCTURED MATRICES AND HIGH RELATIVE ACCURACY

Accurate computations with Newton bases

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Abstract

Fundamental problems in interpolation and approximation require linear algebra computations with collocation, wronskian and gramian matrices of a given basis. These matrices may become ill-conditioned as their dimensions increase and then, standard routines implementing best traditional numerical methods do not obtain accurate results when computing the eigenvalues, the singular values, the inverse matrices or the solution of linear systems of equations. For this reason, a relevant topic in Numerical Linear Algebra is the design and analysis of algorithms to obtain calculations to high relative accuracy, whose relative errors will be of the order of the machine precision. In this field of research, a major step forward was taken when considering nonsingular totally positive matrices, since they can be written as a product of bidiagonal matrices [5]. This factorization can be seen as a representation of the matrices exploiting their total positivity property to achieve accurate numerical linear algebra computations.

The Vandermonde matrix with respect to a given sequence of nodes can be considered as the change of basis matrix between the monomial and the corresponding Lagrange polynomial basis. Vandermonde matrices have relevant applications in interpolation and numerical quadrature (see [4, 6]). It is well known that these matrices are totally positive when considering positive nodes. Interested readers can find many papers providing procedures to achieve accurate computations when dealing with totally positive Vandermonde matrices (see [1–3] and references therein).

A classical approach to the Lagrange interpolation problem is the Newton's form, in which the polynomial interpolant is written in terms of the Newton basis for the considered sequence of interpolation nodes. We shall focus on the change of basis matrices between monomial and Newton bases of the same dimension. These matrices have a triangular structure and their total positivity will be fully characterized in terms of the sign of the considered nodes. In fact, whenever all nodes have the same sign, computations to high relative accuracy for the resolution of algebraic problems related to these matrices will be achieved, even though they do not possess the total positivity property. In this sense, we shall provide novel conditions that guarantee that a given matrix is the inverse of a totally positive matrix. As a result, computations to high relative accuracy are also achieved for non totally positive matrices.

Stirling numbers of the second kind can be considered as divided differences of monomial polynomials at sets of nodes formed by the first consecutive nonnegative integers. Taking into account these considerations, Stirling matrices can be seen as particular cases of the linear transformation between monomial and Newton bases and so, algorithms to high relative accuracy for the resolution of algebraic problemas with collocation, wronskian and gramian matrices of Touchard polynomial bases can be derived.

Numerical experiments will illustrate the accuracy and effectiveness of the proposed methods, in stark contrast with traditional approaches.

Acknowledgements: This work was partially supported by Spanish research grants PID2022-138569NB-I00 (MCIU/AEI) and RED2022-134176-T (MCI/AEI) and by Gobierno de Aragón (E41-23R)

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STRUCTURED MATRICES AND HIGH RELATIVE ACCURACY

High relative accuracy with collocation matrices of q-Jacobi polynomials

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Abstract

A matrix is called totally positive if all its minors are nonnegative [1]. A remarkable property of nonsingular totally positive matrices is that they can be factorized as a product of nonnegative bidiagonal matrices. This representation is known as bidiagonal decomposition. If the entries that define the bidiagonal decomposition are known accurately, they can be used as a parameterization to solve many common problems in numerical linear algebra with high relative accuracy [2, 3].

In this talk, we will consider the case of the little q-Jacobi polynomials, a family of orthogonal polynomials q-analogue of the Jacobi polynomials. We will show cases where the bidiagonal decomposition of collocation matrices of the little q-Jacobi polynomials can be obtained accurately, and hence, used to compute their eigenvalues, singular values, inverses with HRA as well as the solution to some systems of linear equations.

Acknowledgements: This research was partially supported through the Spanish research grants PID2022-138569NB-I00 and RED2022-134176-T (MCIU/AEI), and by Gobierno de Aragón (E41_23R).

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STRUCTURED MATRICES AND HIGH RELATIVE ACCURACY

An algorithm for constructing Jacobi sign regular matrices of order n

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Abstract

In this work, we present a constructive method to obtain large tridiagonal SR matrices with low computational cost, as well as the necessary conditions to carry out this construction and the characterization of tridiagonal SR matrices in terms of the last elements of the matrix. This method is based on extending a tridiagonal SR known matrix $A = (a_{ij})_{1\leqslant i,j\leqslant n}$ to another tridiagonal SR matrix $B = (b_{ij})_{1\leqslant i,j\leqslant n+1}$ such that $B[1,\dots,n] = (b_{ij})_{1\leqslant i,j\leqslant n} = A$. An algorithm has been designed that allows the expansion of a tridiagonal SR matrix in all cases where it is possible.

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STRUCTURED MATRICES AND HIGH RELATIVE ACCURACY

The Lagrange basis: total positivity and least squares problems

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Abstract

In this work the standard Lagrange basis of the space of polynomials of degree less than or equal to n is considered. The problem addressed here consists of obtaining the coordinates in such basis of the n-degree least squares fitting polynomial corresponding to a set of $l+1$ data, with $l \geqslant n$. Solving this problem is equivalent to solve, in the least squares sense, the overdetermined linear system $Lc = b$, where L is the collocation matrix corresponding to the Lagrange basis and to the $l+1$ data. Since matrix L is an ill-conditioned structured matrix [3], standard linear algebra algorithms to solve the problem provide no accurate results and specific algorithms taking into account the structure of matrix L must be considered. Although L is not a totally positive matrix, it can be factorized as the product of a strictly totally positive matrix and a diagonal matrix [4]. This allows the use of results from the field of totally positive matrices, mainly involving the bidiagonal decomposition results in [1,2], to solve both fast and accurately the posed problem. Some numerical experiments illustrating the good performance of the given algorithms are shown.

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Program

Miércoles, 12 de junio			
9:30 - 10:00	OPENING		
10:00 - 11:00	*DQDS Algorithms and High Relative Accuracy Computations* - **Plamen Koev** - SALÓN DE ACTOS		
11:00 - 11:30	COFFEE BREAK		
	A2 ROOM	A3 ROOM	A4 ROOM
	MS1: STRUCTURES MATRICES AND HIGH RELATIVE ACCURACY *Chair*: **Pedro Alonso**	PARALLEL SESSION 1 *Chair*: **Carlos Marijuán**	PARALLEL SESSION 2 *Chair*: **Jürgen Garloff**
11:30 - 11:50	*High relative accuracy with collolation matrices of a q-Jacobi polynomials* **Héctor Orera**	*Parallel implementation of a structure-preserving iterative solver for computing the interior eigenvalues of the Bethe-Salpeter equation* **Blanca Mellado Pinto**	*Hoffman-Wielandt type inequality for block companion matrices of certain matrix polynomials* **Shrinath Hadimani**
11:50 - 12:10	*Totally positive matrices and Gaussian Markov Random Fields* **Juan Baz**	*Universal realizability in low dimension* **Miriam Pisonero**	*Inner product free Shanks transformation for accelerating the convergence of vector sequences* **Ahmed Salam**
12:10 - 12:30	*The Lagrange basis: total positivity and least squares problems* **Raquel Viaña**	*Diagonalizably realizable implies universally realizable* **Carlos Marijuán**	*On Oppenheim's Inequality for Hurwitz Matrices* **Jürgen Garloff**
12:30 - 12:50	*Total positivity and symmetric functions* **Pablo Díaz**	*Inverse Horn problem* **M. Eulàlia Montoro**	*On the distance matrix of the boundary of a pseudotree* **Ignacio M. Pelayo**
12:50 - 13:10			*Dual simplex volume maximization for simplex-structured matrix factorization* **Giovanni Barbarino**
13:30	LUNCH		
	MS1: STRUCTURES MATRICES AND HIGH RELATIVE ACCURACY *Chair*: **Jorge Delgado**	MS2: ALGEBRAIC METHODS FOR THE RECOVERY, CORRECTION AND SECURITY OF DIGITAL INFORMATION *Chair*: **Noemí De Castro**	PARALLEL SESSION 3 *Chair*: **Luca Bergamaschi**
15:30 - 15:50	*Accurate computations with Newton bases* **Esmeralda Mainar**	*Flag codes and consistency with their projected codes* **Miguel Ángel Navarro Pérez**	*Spectral analysis of block preconditioners for double saddle point linear systems with application to PDE-constrained optimization* **Luca Bergamaschi**
15:50 - 16:10	*An algorithm for constructing Jacobi sign regular matrices of order n* **Antonio Palacio**	*Goppa codes. Geometric aspects and enumerative problems* **Ángel Luis Muñoz**	*Clustering/Distribution Analysis and Preconditioned Krylov Solvers for the Approximated Helmholtz Equation and Fractional Laplacian* **Andrea Adriani**
16:10 - 16:30	*Total positivity and high relative accuracy for Kac-Murdock-Szegö matrices* **Jorge Delgado**	*A convolutional variant with GRS codes for the McEliece cryptosystem* **Paulo Almeida**	*Improving performance of contour integral-based nonlinear eigensolver with infinite GMRES* **Yuqi Liu**
16:30 - 16:50			*Approximate Inverse LU preconditioning applied to least squares problems* **José Mas**
17:00 - 17:30	COFFEE BREAK		
		MS2: ALGEBRAIC METHODS FOR THE RECOVERY, CORRECTION AND SECURITY OF DIGITAL INFORMATION *Chair*: **Miguel V. Carriegos**	PARALLEL SESSION 4 *Chair*: **Froilán M. Dopico**
17:30 - 17:50		*Performance of RNS-based processors for network coding* **El Mamoun Souidi**	*Riemannian optimization methods to compute the nearest singular pencil* **Froilán M. Dopico**
17:50 - 18:10		*Decoding of MDP convolutional code over the erasure channel under linear systems point of view* **Laurence E. Um**	*Some open questions on the solvability of Sylvester-like equations* **Fernando de Terán**
18:10 - 18:30		*ISO representations and convolutional codes: construction and good decoding properties* **Noemí De Castro**	*New insights in rank one perturbations of matrix pencils* **Marija Dodig**
18:30 - 18:50		*A construction of spread codes based on Abelian non-cyclic groups* **Verónica Requena**	*Combinatorics of integer partitions, and bounded rank perturbations of matrix pencils* **Marko Stošić**
18:50 - 19:10			*On the consistency of the matrix equation* $X^*AX = B$ **Roberto Canogar**

Jueves, 13 de junio			
9:30 - 10:30	*Randomized methods for matrix computations* - **Yuji Nakatsukasa** - SALÓN DE ACTOS		
10:30 - 11:00	COFFEE BREAK		
	A2 ROOM	A3 ROOM	A4 ROOM
		MS3: Sobre el problema inverso de recuperación de las conductancias *Chair*: **Ángeles Carmona**	Parallel Session 5 *Chair*: **Rafael Bru**
11:00 - 11:20		*Dirichlet-to-Neumann properties on spider networks* **Ángeles Carmona**	*Generalized inverses of linear operators on infinite-dimensional vector spaces and applications* **Fernando Pablos**
11:20 - 11:40		*The Dirichlet-to-Neumann matrix as the Schur complement of the Laplacian* **María José Jiménez**	*The Relative Gain Array in control theory* **Begoña Cantó**
11:40 - 12:00		*Stability in the recovery of conductances in spider networks under a piecewise constant conductance hypothesis* **Álvaro Samperio**	*Matrix Representation for Clifford Algebras* **Johan Ceballos**
12:00 - 12:20		*Experimental development of an EIT device and its proof of concept in the early detection of breast cancer* **Leonardo Acho Zuppa**	*Doubly diagonally dominant matrices and their combined matrices* **Rafael Bru**
12:20 - 12:40			*On doubly stochastic circulant combined matrices* **Rafael Cantó**
13:30	LUNCH		
	MS4: Orthogonal Polynomials and Matrix Analysis *Chair*: **Luis Miguel Anguas**	MS3: Sobre el problema inverso de recuperación de las conductancias *Chair*: **Andrés M. Encinas**	
15:30 - 15:50	*Properties of eigenvalues and eigenpolynomials of finite order ordinary differential operators* **Luis Miguel Anguas**	*A 3D–realistic numerical and physical model of female breast* **Enric Monsó**	
15:50 - 16:10	*Matrix Sobolev inner products, generalized eigenvalues and zeros of polynomials* **Carmen Escribano**	*Bisymmetric nonnegative Jacobi matrix realizations* **Andrés M. Encinas**	
16:10 - 16:30	*On Sobolev bilinear forms and polynomial solutions of second-order differential equations* **Juan C. García Ardila**	*Inverses and Moore-Penrose inverses of structured totally positive matrices: accurate and efficient computation* **Ana Marco**	
16:30 - 16:50	*Positive bidiagonal factorization of oscillatory tetradiagonal Hessenberg matrices* **Manuel Mañas**	*Tridiagonal M-matrices whose group inverses are tridiagonal* **Kadali Kranthi Priya**	
17:00 - 18:30	REUNIÓN DE LA RED ALAMA		
18:30 - 20:30	VISITA A GIJÓN		
21:30	CONFERENCE DINNER - **Restaurante Bellavista**		

Viernes, 14 de junio			
9:30 - 10:30	*Challenges and strategies in ranking aggregation: from fairness to computational efficiency* - **S. Irene Díaz** - SALÓN DE ACTOS		
10:30 - 11:00	COFFEE BREAK		
	A2 ROOM	A3 ROOM	A4 ROOM
	MS4: Orthogonal Polynomials and Matrix Analysis *Chair*: **Dolores Barrios**	MS5: Computación cuántica y álgebra lineal *Chair*: **José Ranilla**	Parallel Session 6 *Chair*: **Alicia Roca**
11:00 - 11:20	*Solutions of linear systems of moment differential equations and generalized matrix exponentials* **Alberto Lastra**	*QADS rotacionales* **Miguel Hernández Cáceres**	*Stable computation of generalized matrix functions via polynomial* **Jared L. Aurentz**
11:20 - 11:40	*Structured matrices and Darboux Transformations of multiplication polynomial operators* **Francico Marcellán**	*Estimación de fase con sistemas abstractos cuánticos de detección funcionales* **Guillermo Lugilde**	*A square-root-free unitary QR algorithm* **Mónica Esquivel**
11:40 - 12:00	*On classical orthogonal polynomials and the Cholesky factorization of a class of Hankel matrices* **Misael Marriaga**	*Avances en la clasificación de señales biomédicas con kernels cuánticos y otras técnicas de quantum machine learning* **Diego García**	*Characterization and construction of ASTP matrices* **Antonio Palacio**
12:00 - 12:20	*Sheffer-Dunkl sequences via umbral calculus in the Dunkl context* **Judit Mínguez**	*La jerarquía de Clifford desde una perspectiva algebraica* **Samuel González**	*Bounds for the Weyr characteristic of linear relations after one-dimensional perturbation* **Alicia Roca**
12:20 - 12:40		*Optimización del número de puertas CNOT en circuitos aritméticos cuánticos* **Laura M. Donaire**	*Totally positive matrices and polynomial least squares fitting* **José Javier Martínez**
12:40 - 13:00		*Esferas de Bloch para sistemas de varios qubits* **Fernando L. Pelayo**	
13:00	COCKTAIL		

Index

Author

Author Index

N

O

P

R

S

T

U

V