EEEIC
2021
BARI

21ST EEEIC
INTERNATIONAL CONFERENCE
ON ENVIRONMENT AND ELECTRICAL ENGINEERING

05TH I&CPS
INDUSTRIAL AND COMMERCIAL
POWER SYSTEM EUROPE

BARI.PUGLIA.ITALY
07TH > 10TH SEPTEMBER 2021

Conference Proceedings

2021 IEEE International Conference on Environment and Electrical Engineering and 2021 IEEE Industrial and Commercial Power Systems Europe (EEEIC / I&CPS Europe)

7 - 10 September, 2021, Bari, Italy

Editor: Zbigniew Leonowicz

Part Number: CFP2151I-CDR   ISBN 978-1-6654-3612-0

For all technical inquiries regarding your CD, contact: EEEIC International, Lososiowice 38. Wolow, 56100, POLAND, pc@eeeic.eu, phone +48505153080

List of Papers

ID	Title	Authors (* corresponding author)
1	Design of an Automated Bubble Detection Device for Neuroangiography	Eleanor Leung (York College of Pennsylvania); Kala Meah (York College of Pennsylvania)*; James Moscola (York College of Pennsylvania); Donald Hake II (York College of Pennsylvania); Richard Bosse (York College of Pennsylvania); Cara Sperbeck (York College of Pennsylvania); Chad Wickard (York College of Pennsylvania); Jennifer Owrutsky (York College of Pennsylvania); Kaitlyn Graf (York College of Pennsylvania); Robert Acierno (York College of Pennsylvania); Abigail Wright (York College of Pennsylvania); Jason Mickenberg (York College of Pennsylvania); Hank Smeltzer (York Hospital)
2	High Voltage Insulators Condition Analysis using Convolutional Neural Network	Arailym Serikbay (Nazarbayev University); Mehdi Bagheri (Electrical and Computer Engineering Department, Nazarbayev University)*; Amin Zollanvari (Nazarbayev University)
3	Hybrid Testing of a Solar Tracking Equipment using In-Circuit Testing and JTAG Debugging Strategies	Sorin Liviu Jurj (Politehnica University of Timisoara)*; Raul Rotar ("Politehnica" University of Timisoara); Flavius Opritoiu (Politehnica University of Timisoara); Mircea Vladutiu (Politehnica University of Timisoara)
4	ICT Tools for Supporting Telecommunication Infrastructure Sharing: A Nigerian Case Study	Kennedy N Amadasun (Teesside University)*; Michael Short (Teesside University); Tracey Crosbie (Teesside University)
5	GUPFC Modeling for Power Flow Analysis in MVDC Grids	Francisco Jurado (University of Jaen)*; Ayman Awad (Aswan University); Salah Kamal (Aswan Universiyt, Egypt); Abdalla A. Ibrahim (Aswan University)
9	Sustainable Revamping of Wool Carbonization Systems	Massimo Delogu (Dep. of Industrial Engineering); Marco Cocci (Dept . of Industrial Engineering); Lorenzo Berzi (Dep. of Industrial Engineering); Luca Pugi (University of Florence)*
11	Experience in exploitation and diagnostics of high voltage transformers fleet	Michał Kunicki (Opole University of Technology)*
12	Sensitivity Analysis and Frequency Dynamics of Grid-Connected Converters with Virtual Inertia	Rossano Musca (University of Palermo)*; Gaetano Zizzo (DEIM University of Palermo); Salvatore Favuzza (University of Palermo); Eleonora Riva Sanseverino (University of Palermo)
13	Integration of second-life battery packs for self-consumption applications: analysis of a real experience	Adrian Soto (Public University of Navarre)*; Alberto Berrueta (Public University of Navarra); Pablo Zorrilla (Public University of Navarra); Alvaro Iribarren (Public University of Navarra); Diego H. Castillo (Autonomous University of Tamaulipas); Wenceslao E. Rodríguez (Autonomous University of Tamaulipas); Adolfo J. Rodríguez (Autonomous University of Tamaulipas); David T. Vargas (Autonomous University of Tamaulipas); Ignacio R. Matias (Public University of Navarra); Pablo Sanchis (Public University of Navarre); Alfredo Ursúa (Public University of Navarra)

14	Optimal distribution of PV modules on roofs with limited space	Covadonga Bayon-Cueli (Oviedo University); Arsenio Barbon (Oviedo University); Alejandro Fernandez-Conde (Oviedo University); Luis Bayon (Oviedo University)*
15	Review on Energy Efficient Wireless Sensor Network Protocols	M RAVINDER (NMIMS ,university); Vikram Kulkarni (NMIMS University)*
16	Oxidation Impact of CIGS Photovoltaic Modules Performance after 10 Years of Operation	Panagiotis Eleftheriadis (Politecnico di Milano)*; Alberto Dolara (Politecnico di Milano); Sonia Leva (Politecnico di Milano); Riccardo Simoneti (Politecnico di Milano)
17	Recycling of c-Si PV Modules: an Energy Analysis and Further Improvements	Panagiotis Eleftheriadis (Politecnico di Milano)*; Sonia Leva (Politecnico di Milano); Andrea Mulazzani (Politecnico di Milano)
18	Coordinated Steering and Torque Vectoring Lateral Stability Sliding Mode Control Applied to an Electric In-Wheel Motors Vehicle	Tommaso Favilli (Dept. of Industrial Engineering)*; Luca Pugi (University of Florence); Lorenzo Berzi (Dep. of Industrial Engineering); Marco Pierini (Dep. of Industrial Engineering)
19	Open-loop control of a grid-tied multilevel inverter interfacing a fuel cell stack	Magdi Muftah Attai (Universiti Sains Malaysia); Mohamed Salem (School of Electrical & Electronic Engg., Universiti Sains Malaysia)*; Khlid Ben Hamad (Cape Peninsula University of Technology); Prof. Mohamad Kamarol Mohd Jamil USM ("Respected member, IPC, ICIEV")
20	Design and Simulation Tools for Hybrid Fuel Cell Trains	Lorenzo Berzi (Dep. of Industrial Engineering); francesco cirillo (Hitachi Rail); viola pagliazzi (university of florence); Luca Pugi (University of Florence)*; alessandro vecchi (hitachi rail)
22	Determining the dynamic insulation resistance of DC cables used in photovoltaic systems under operational conditions	Christos Melios (University of Cyprus)*; Andreas Demetriou (University of Cyprus); Vasilios Androvitsaneas (National Technical University of Athens); Ioannis Gonos (National Technical University of Athens); Charalambos A. Charalambous (University of Cyprus)
23	Harmonic Mitigation In Three Phase SPPS Via PRESH Controller	Uma Yadav (J C Bose University of Science and Technology YMCA Faridabad)*; Dr Anju Gupta (J C Bose University of Science and Technology YMCA Faridabad); Dr Rajesh Kr Ahuja (J C Bose University of Science and Technology YMCA Faridabad)
26	Application of energy routers for frequency support in an AC/DC multi-microgrid system	PHAM Minh Cong (G2elab)*; Quoc Tuan Tran (French Alternative Energies and Atomic Energy Commission (CEA)); Ahmad Hably (Gipsa); Seddik Bacha (G2elab INP Grenoble France)
27	A Detection Method for Double Line-To-Ground Faults in DC-AC Systems	Jose Manuel Guerrero (Universidad Politécnica de Madrid); Kumar Mahtani (Universidad Politécnica de Madrid); Gustavo Navarro (Ciemat); Carlos A. Platero (Universidad Politecnica de Madrid)*
28	Evonorm algorithm for the overcurrent protection coordination problem	Meng Yen Shih (UAC)*; Arturo Conde (UANL); Luis Trujillo Guajardo (UANL); Daniel Guillen (ITESM); Zbigniew M Leonowicz (Wroclaw University of Science and Technology); Francisco Lezama (UAC); Jorge Chan (UAC)

Optimal distribution of PV modules on roofs with limited space

Covadonga Bayón-Cueli
Polytechnic School of Engineering
University of Oviedo
Gijón, Spain
cobayon18@gmail.com

Arsenio Barbón
Department of Electric Enginnering
University of Oviedo)
Gijón, Spain
barbon@uniovi.es

Alejandro Fernández-Conde
Polytechnic School of Engineering
University of Oviedo
Gijón, Spain
uo257265@uniovi.es

Luis Bayón
Department of Mathematics
University of Oviedo
Gijón, Spain
bayon@uniovi.es

***Abstract*—The present work proposes a new algorithm for finding the optimal distribution of PV modules on flat roofs of urban building so that the total absorbed energy is maximised. The paper is divided in 3 sections. Section 1 shows an analytic technique that allows for annual irradiation of a PV module to be calculated for every tilt angle, consequently, this procedure can also be used to determine the optimal tilt that maximises the total irradiation of such module. In Section 2, a new packing algorithm is developed (specifically for this problem) so that total area of the PV modules of the flat roof is maximised. Studying the shadowing between modules and the needed spacing is key for this part. Lastly, in Section 3, it's checked whether modifying the fixed tilt of the PV modules to be installed in a flat roof results in an increase of the total PV surface that compensates the loss of energy per module deriving from not using the optimal tilt angle, so that the maximum possible total absorbed energy for a given terrace is achieved.**

***Index Terms*—Photovoltaic systems, urban buildings, energy maximization, optimum tilt.**

I. Introduction

Photovoltaic (PV) systems are destined to play a fundamental role in energy generation in the coming years [1], and it is estimated that PV energy generation by 2050 might reach 14.5% from residential roof installation. This work presents a new algorithm for achieving the optimal distribution of PV modules in areas where space is limited, as it's usually the case for flat roofs of urban buildings.

When working with PV systems without solar tracker (i.e. with constant tilt), knowing the two installation angles is essential. First, the *tilt angle* (β), i.e. the angle between the plane of the surface and the horizontal plane, and secondly the *surface azimuth angle* (γ), i.e. the angle between the projection on a horizontal plane of the normal to the tilted surface and the geographical South. The optimum value of γ, is well known [2], which, for the northern hemisphere is $\gamma_{opt} = 0$ (^{o}). However, calculating the optimal tilt angle, β_{opt}, is still the aim of numerous studies. There are many models, like the ones proposed by [3], that only require the latitude of the localization, while works like those of [4] or [5] use irradiation models to compute β_{opt}, with a location-specific approach.

For large-scale plants, as available space is not a limiting factor, optimal values can be used, whereas in situations with a lack of extra area (which are the object of the present study) it's as important to know β_{opt}, as it is to know the energy loss associated with the use of different tilt angles. This paper presents an analytical procedure to calculate the solar irradiation reaching the tilted surface of the module, for a given period of time, whatever tilt angle is used. Annual irradiation is considered to, consequently, calculate the angle β_{opt} that results in the *maximum irradiation for each module.*

In a second section, the form and orientation of the available flat roof area are taken as fixed values [6]. We use as decision variables the dimensions (length and width), orientation, number and position of the PV modules. In addition, minimum distances between rows of PV modules are considered in order to allow maintenance and to avoid shadowing effects. Only PV modules of commercial dimensions are taken into consideration for this study. The specific objective of this part is to *maximise the area of all PV modules* in the whole system, taking into account the available area and the physical restrictions.

We present a new optimization method, related to the classical packing mathematical problem. Various type of packing problems have been widely studied [7], however, to the best of our knowledge, this particular two-dimensional rectangle packing problem has not been addressed in the literature so far.

Lastly, in a third section, the analysis of shadowing produced between panel rows will take into consideration the possibility of placing the PV modules at angles other than their β_{opt}. That way it would be analysed which options yields more interesting results: if using β_{opt}, to assure receiving the

978-1-6654-3613-7/21/$31.00 ©2021 IEEE

maximum irradiance possible per module, or, on the contrary, taking $\beta \neq \beta_{opt}$, in pursue of obtaining a greater area of PV modules, thus reducing the irradiation on each individual module. Keep in mind the objective is maximising the total incident energy for all PV modules on a flat roof.

II. Influence of tilt angle over the irradiation

The total solar irradiance (I_t) on tilted surfaces is usually calculated *[2]* as the sum of three components: the beam, the diffuse and the ground reflected solar irradiance:

$$I_t(n,T,\beta) = I_{bh} \cdot \frac{\cos\theta_i}{\cos\theta_z} + I_{dh} \cdot \left(\frac{1+\cos\beta}{2}\right) + (I_{bh}+I_{dh}) \cdot \rho_g \cdot \left(\frac{1-\cos\beta}{2}\right) \quad (1)$$

where I_{bh} (W/m^2) is the beam irradiance on a horizontal plane, θ_z (^{o}) is the zenith angle of the Sun, θ_i (^{o}) is the incident angle, I_{dh} (W/m^2) is the diffuse irradiance on a horizontal plane, β (^{o}) is the tilt angle, and ρ_g (dimensionless) is the ground reflectance. The incident angle of the Sun θ_i (^{o}) on a tilted surface can be determined [2] as a function of the declination δ, the latitude λ, the tilt angle β, the azimuth angle γ, and the hour angle ω. Hence, the total solar irradiance $I_t(n,T,\beta)$ on tilted surfaces depends on the day of the year n, the solar time T, and, obviously, β.

From the Eq. (1), we can compute, by direct integration from sunrise, T_R, to sunset, T_S, the total irradiation on a tilted surface $H_t(n,\beta)$ ($W.h/m^2$) for each day of the year n and tilt angle β:

$$H_t\left(n,\beta\right) = \int_{T_R(n)}^{T_S(n)} I_t(n,T,\beta)dT \quad (2)$$

Now, for each specific location, satellite estimations of monthly-averaged global and diffuse solar irradiations received on a horizontal surface from $PVGIS$ database [8] are used. From these monthly values, using Fourier analysis [9], hourly perturbed distributions for the beam and diffuse solar irradiances are computed. Thus, this methodology takes into account the main environmental conditions of the site.

In order to compute β_{opt}, the interval $\beta \in [0,90]$ is discretized and, following the Cavaleri's principle, the integral is computed for each one of the β values:

$$H_t^{\beta}(\beta) = \int_1^{365} H_t\left(n,\beta\right)dn \quad (3)$$

and find the value β_{opt} such that maximises $H_t^{\beta}(\beta)$. In order to clarify the exposition, an specific location has been chosen: Almeria (Spain), with latitude $36^o50'07''N$, longitude $02^o24'08''W$ and altitude 22 (m). The function $H_t(n,\beta)$ (Wh/m^2) is shown in Fig. 1. The Fig. 2 contains the plot of H_t^{β} against β. A maximum in $\beta_{opt} = 30.3$ (^{o}) can be seen. Apart from obtaining β_{opt}, this analytic technique allows for the quantification of energy losses when non-ideal tilts are used on PV panels. Our recent calculations [10] show that installing PV modules with $\gamma = 0^o$ and tilt angle deviations of up to 10 (^{o}) with respect to the optimum tilt angle makes for a loss on the incoming solar irradiation of about 1%. When tilt angle deviations are around 20 (^{o}), energy losses around 5% should be expected, while, with a deviation around 30 (^{o}), about a 10% of the energy is lost, with subtle variations depending on the location's latitude. These calculations will play a key role in later energy maximization, once row-spacing variations are considered.

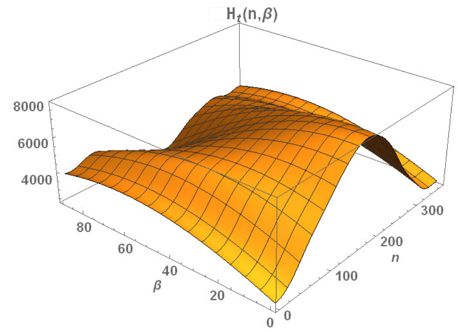

Fig. 1. Irradiation on a tilted surface $H_t\left(n,\beta\right)$.

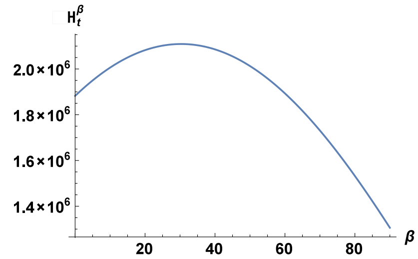

Fig. 2. Variation of H_t^{β} with tilt angle β.

III. Optimal distribution of PV modules on flat roofs

A. Statement

This section describes the mathematical problem where the objective function to be maximized is the total PV modules area (A_{PV}), given by:

$$A_{PV} = \sum_{i=1}^{N} W \cdot L \quad (4)$$

where N is the number of PV modules, W is the module width, and L is the module length. In this study it's assumed that all the modules are identical in size and only commercial modules are taken into consideration.

The roof-related parameters are: the roof area (A_r), the available roof area (A_{ar}), the roof form (F_r), and the roof orientation (O_r).

The available roof area (A_{ar}) is the area that can really be used for installation [11], and can be calculated using:

$$A_{ar} = A_r \cdot C_{BC} \cdot C_S \cdot C_{RT} \cdot C_{IA} \quad (5)$$

where C_{BC} is the building components coefficient (to take into account components such as chimneys, fans ...), C_S is the

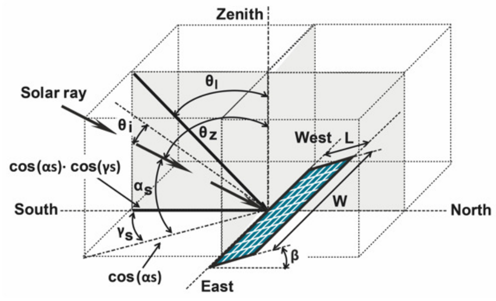

Fig. 3. Longitudinal study.

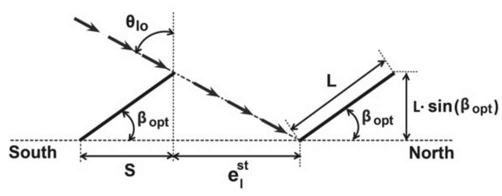

Fig. 4. Study of the shadows.

shadowing coefficient (to consider shadows caused by other buildings), C_{RT} is the roof-type coefficient and C_{IA} is the inclination angle coefficient. To simplify the exposition, flat roofs with neither shadowing nor components on their terrace are the only ones taken into consideration for this papers, so $A_{ar} = A_r$. The reader should notice how immediate the extension.

The form of the roof area (F_r) is defined as the ratio between length (a) and width (b) of the available roof area. The roof orientation (O_r) is defined by the angle that forms the North-South direction and the terrace edges. In this study, every PV module is aligned in the North-South direction and we assume that the terrace edges are parallel to our reference axes ($x - y$). Thus, the roof orientation is given by the angle (α) between the North-South direction and the positive half axis of y.

The optimization model also considers a *transversal installation distance* (e_t), needed for the adequate installation of the PV modules, and a *longitudinal maintenance distance* (e_l), which should be kept between rows of modules in order to allow a proper inspection, cleaning, and maintenance. Taking into account the Spanish Government Technical Report [12], a maintenance distance $e_l^m = 1$ (m) and a installation distance $e_t = 0.025$ (m) (due to clamps) are considered. In addition, there has to be a minimum distance between the terrace boundary and the modules, again, for maintenance purposes. This distance is designated e_b, and a value of 1 (m) is also assumed [12].

B. Study of the shadows

Apart from the longitudinal maintenance distance (e_l^m), which should be kept between rows of PV modules, another key aspect to consider are the shadows each preceding row casts over the next one.

Fig. 3 shows, for rows of modules facing South ($\gamma_{opt} = 0$ $(^o)$), the breakdown of solar radiation between its transversal and longitudinal component. From said image, it can be deduced that:

$$\tan\theta_l = \frac{\cos\alpha_s \cos\gamma_s}{\sin\alpha_s} = \frac{\cos\gamma_s}{\tan\alpha_s} = \tan\theta_z \cos\gamma_s \qquad (6)$$

where α_S is the height angle of the Sun (^{o}), θ_z is the zenith angle of the Sun (^{o}) and γ_S is the azimuth of the Sun (^{o}).

At the same time, the Spanish Government Technical Report [12] states that, in order to minimize shadowing effects, the distance between reflectors has to guarantee a minimum of 4 hours of sunshine around noon on the winter solstice. The angle θ_l, obtained from (6) when applying this standard on December 21 at $10:00$ will be referred as θ_{l0}.

It can immediately be obtained, from Fig. 4, that the required spacing to avoid shadowing between consecutive rows of modules that meets the previous criteria is:

$$e_l^{st} = S\frac{\tan\beta_{opt}}{\cot\theta_{l0}} = L\frac{\sin\beta_{opt}}{\cot\theta_{l0}} \qquad (7)$$

where $S = L\cos\beta_{opt}$ is the horizontal projection of L.

Therefore, the value to be imposed as *longitudinal spacing* should be:

$$e_l = \max[e_l^m, e_l^{st}] \qquad (8)$$

so that both ease of maintenance and compliance with the norm are guaranteed.

Note that the result depends on L (value to choose from commercial PV module models), θ_{l0} (value that only depends on the location) and β_{opt}, calculated in the previous section.

C. Packing algorithm E-W axis

Packing problems are a class of mathematical optimization problems that attempt to pack objects into containers [7]. In this paper we consider a very special type of two-dimensional rectangle packing problem. In this case, identical rectangles need/have to be packed in a fixed rectangle (as, for example, in [13]), taking into account two additional constraints: (i) A minimum space must be left between objects for installation and maintenance purposes and to avoid shadowing effects; (ii) The orientation of the objects is fixed with respect to the container, and generally, it's not orthogonal.

The optimization of the distribution of small-scale Linear Fresnel Reflectors ($SSLFRs$) on flat roofs of urban buildings has been analysed in previous studies like [14] where three different packing schemes are presented in order to solve the problem. Based on such work, one of the mentioned packing algorithms has been adapted to solve the present issue. To the best of our knowledge, this packing problem has not been addressed in the literature.

Given a terrace of fixed dimensions $a \times b$, we assume that the terrace edges are parallel to reference axes ($x - y$). Let α be the angle between the N-S direction and the positive axis y.

The new packing scheme consists of placing rows of South-facing PV modules (whose dimensions are $W \times S$) in an East-West direction. To achieve that (see Fig. 5), a base rectangle R_{11} is defined using two vertices A y B, which are located as close as possible to the upper-right corner of the terrace. The coordinates of the two *basic* vertices, $A(x_A, y_A)$ and $B(x_B, y_B)$, are given by:

$$R_{11}: \begin{cases} A(e_b, e_b + S\cos\alpha) \to D \\ B(e_b + S\sin\alpha, e_b) \to C \end{cases} \tag{9}$$

Once A and B have been settled, the other two vertices C and D can be calculated immediately (this process is repeated for every case study exposed in this paper):

$$R_{11}: \begin{cases} C(x_B + W\cos\alpha, y_B + W\sin\alpha) \\ D(x_A + W\cos\alpha, y_A + W\sin\alpha) \end{cases} \tag{10}$$

Firstly, the packing pattern is completed by placing vertically, from top to bottom, as many rectangles R_{1i} as possible. The increase of the y coordinates between one rectangle and the next is given by:

$$\Delta x = 0; \Delta y = \begin{cases} \dfrac{e_l}{\cos\alpha} + \dfrac{S}{\cos\alpha} & \text{if} \quad \alpha \neq \pi/2 \\ e_t + W & \text{if} \quad \alpha = \pi/2 \end{cases} \tag{11}$$

Thus, the coordinates of the four vertices A, B, C, and D of the rectangles in the first column are given by (with $i = 1, \ldots, n$):

$$R_{i1}: \begin{cases} A(x_A, y_A + (i-1)\Delta y) \to D \\ B(x_B, y_B + (i-1)\Delta y) \to C \end{cases} \tag{12}$$

From each rectangle in this first column, new rectangles R_{ij} (with $j = 1, \ldots, m$) are added in W-E direction, using:

$$\delta x = \begin{cases} e_t\cos\alpha + W\cos\alpha & \alpha \neq \pi/2 \\ e_t\sin\alpha + S\sin\alpha & \alpha = \pi/2 \end{cases} \tag{13}$$

$$\delta y = \begin{cases} e_t\sin\alpha + W\sin\alpha & \alpha \neq \pi/2 \\ 0 & \alpha = \pi/2 \end{cases} \tag{14}$$

$$R_{ij}: \begin{cases} A(x_A + (j-1)\delta x, y_A + (j-1)\delta y) \to D \\ B(x_B + (j-1)\delta x, y_B + (j-1)\delta y) \to C \end{cases} \tag{15}$$

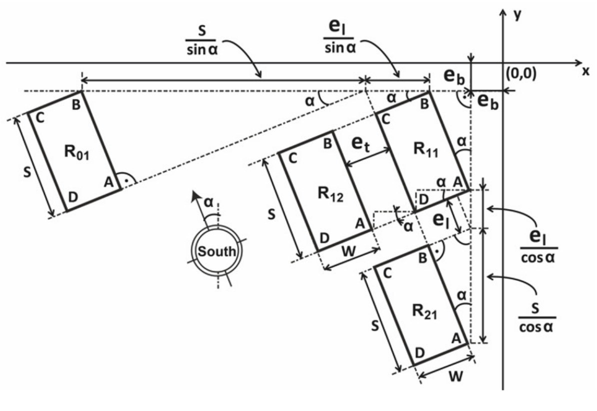

Fig. 5. Packing algorithm.

The packing pattern is completed placing new rectangles R_{k1} ($k = 0, -1, \ldots$) horizontally aligned with the base rectangle R_{11}. The vertices of rectangles R_{k1} are given by (with $k = 0, -1, \ldots$):

$$\Delta x = \begin{cases} \dfrac{e_l}{\sin\alpha} + \dfrac{S}{\sin\alpha} & \text{if} \quad \alpha \neq 0 \\ e_t + W & \text{if} \quad \alpha = 0 \end{cases} ; \Delta y = 0 \tag{16}$$

$$R_{k1}: \begin{cases} A(x_A + (1-k)\Delta x, y_A) \to D \\ B(x_B + (1-k)\Delta x, y_B) \to C \end{cases} \tag{17}$$

IV. Maximization of the Total Energy

Taking into consideration all the aforementioned formulae and criteria, the proposed method to maximize the total energy consists in 3 steps:

(1) Analytically calculate the $H_t^\beta(\beta)$ curve (see Fig. 2).

(2) Execute the packing algorithm for different values of β to obtain the $A_{PV}^\beta(\beta)$ curve.

(3) Calculate the maximum, β^*, from the Total Energy curve:

$$E_{PV}(\beta) = H_t^\beta(\beta) \cdot A_{PV}^\beta(\beta) \tag{18}$$

V. Numerical Examples

This section shows the results obtained with the results described in the previous sections. The optimization algorithm has been implemented using the commercial software Mathematica™. It's considered, without loss of generality, that $e_t = 0.025$ (m) and $e_l = 1$ (m), and $e_b = 1$ (m). For the specific location of Almeria (Spain) the value of θ_{l0} (6) of the standard on December 21 at $10:00$ results in 63.4^o. As previously noted, the optimal tilt is $\beta_{opt} = 30^o$ and the Total Annual Irradiation adds up to $H_t^{\beta_{opt}} = 2.10995$ $(MW.h/m^2)$.

The algorithm is able to deal with any configuration, no matter what form, F_r, or orientation, O_r, the roof has. As an example, a first case study is presented, where the dimensions of the terrace are: $a = 20$ (m) and $b = 10$ (m), so $F_r = 2$, and it's supposed that the angle the terrace forms with the North-South direction is $O_r = 30$ $(^o)$.

The 10 commercial models considered for this example are displayed in Table 1, they're from 5 different manufacturers: Era Solar (ES), Solar Power (SP), Talesun (TS), Jinko Solar (JS), and JA Solar (JA). Model name; technology used, either Polycrystalline (P) or Monocrystalline (M); and dimensions L (mm) and W (mm) are also shown for each module.

TABLE 1. 10 commercial PV modules.

n^o	Fab.	Mod.	Tec.	W	L
1	ES	BSP275P	P	991	1640
2	ES	ESPMC	P	992	1650
3	SP	REC TWIN PEAK	P	997	1675
4	ES	ESPMC	M	990	1650
5	ES	ESPMC	M	1002	1665
6	TS	TP672P	M	992	1960
7	ES	ESPMC	P	992	1956
8	ES	ESPMC	M	1002	1979
9	JS	HC 72 M	M	1002	2008
10	JA	MBB HALF-CELL	M	1052	2120

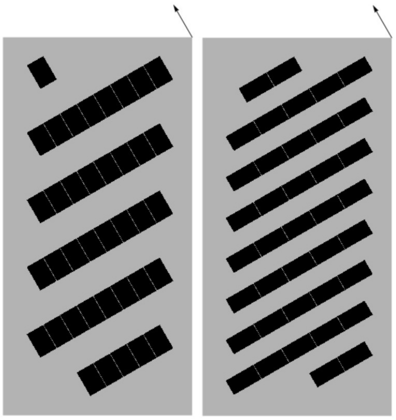

Fig. 6. Optimal solution for β_{opt}.

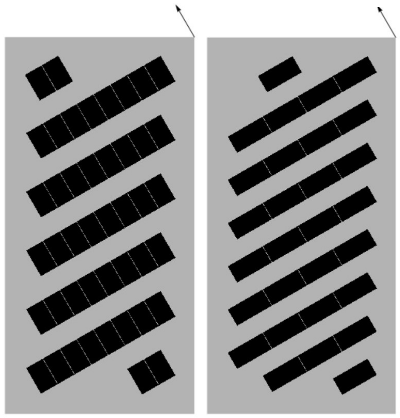

Fig. 7. Optimal solution for $\beta = 20^o < \beta_{opt}$.

It should be noted that the algorithm takes into consideration 4 possible rack configurations: $1V$, $1H$, $2V$ and $2H$, while rack configurations $3H$ and $3V$ are disregarded due to the excessive height of the resulting PV system. This classification goes as follows: (i) numbers 1,2,3,... represent the number of vertically consecutive modules in each row of the system (ii) for the letter, it's used V when referring to the rack configuration in which magnitude L is the reference for the tilt angle, and H for the one in which W is used as such reference.

So, the algorithm finds, using the packing scheme, the dimensions (i.e. the suitable model between those 10 mentioned), rack configuration ($1V$, $1H$, $2V$, $2H$), position and number of PV modules which maximize the total area of PV modules, A_{PV}. The optimization procedure is indeed a brute force algorithm that evaluates all the possible combinations of models and orientations. For this simple example, the algorithm running time is merely about 1-2 sec on a somewhat outdated personal computer (Intel Core™ i5-1035G1 CPU, 1.00GHz), which shows the outstanding potential this algorithm has for more complex, bigger problems.

The optimal solution of the packing scheme with $\beta_{opt} = 30$ (^{o}), for the particular case considered, is shown in Fig. 6, with the modules in $1V$ on the left, and in $1H$ on the right. The optimal $1V$ solution consist of 38 ESPMC (n^o 2) PV modules, with dimensions 992×1650 (mm) vertically oriented with an $A_{PV} = 62.20$ (m^2). The $1H$ optimal solution consist of 39 Rec Twin Peak (n^o 3) PV modules, with dimensions 997×1675 (mm) horizontally oriented with an $A_{PV} = 65.13$ (m^2). Therefore, for this case study the optimum is achieved for the $1H$ scheme: $A_{PV}^{\beta_{opt}} = 65.13$ (m^2). The spacing, e_l, obtained from (8), for $1V$ is 1.648, value imposed by the rule e_l^{st}, as the modules cast a considerable shadow when placed vertically. However, for $1H$ the shadow is noticeably smaller so the spacing e_l, obtained from (8), is just 1.0, corresponding with the maintenance requirement e_l^m.

The previously obtained solution employs the optimal tilt, β_{opt}, which guarantees the maximum annual irradiation $H_t^{\beta_{opt}}$, but now it'll be tested what changes when using a different tilt angle $\beta < \beta_{opt}$. Fig. 7 shows the two optimal solutions for $\beta = 20$ (^{o}), 10 (^{o}) less than optimal.

In this case, the new 1V solution (on the left of Fig. 7) employs 44 ESPMC (n^o 5) PV modules, with dimensions 1002×1665 (mm), for a total area of $A_{PV} = 73.41$ (m^2), needing a minimum spacing of $e_l = 1.137$. On the contrary, the optimal $1H$ solution (Fig. 7, on the right) now consists of 29 MBB Half-Cell (n^o 10) PV modules, with dimensions 1052×2120 (mm), a total area $A_{PV} = 64.68$ (m^2), and spacing $e_l = 1$. It should be noted the 1V total area, A_{PV}, has perceived a notable increase from its value for β_{opt}, while the $1H$ total area, A_{PV}, has been slightly reduced.

This occurrence can be explained due to the e_l equation (8). When placing the modules vertically with a reduced tilt β, the casted shadow is smaller so the value of e_l gets reduced from 1.648 to only 1.137 so total area A_{PV} gets increased. However, when placed horizontally, e_l is the maintenance value, e_l^m, which doesn't get altered with a reduced tilt β, so total area of modules A_{PV} ends up being reduced. Thus, for $\beta = 20$ (^{o}), the optimal solution has changed and the 1V option stands as the best one with an $A_{PV}^{20} = 73.41$ (m^2). Final question is: When reducing β, does the increase in area A_{PV} make up for the loss in annual irradiation per module? It's time to apply the total annual energy study from Section 1, comparing $H_t^{\beta} \cdot A_{PV}$ for both situations:

$$E_{PV}(\beta_{opt}) = 2.10995 \cdot 65.13 = 137.419 \ (MW{\cdot}h) \quad (19)$$

$$E_{PV}(20) = 2.08307 \cdot 73.41 = 152.911 \ (MW{\cdot}h) \quad (20)$$

So, in this particular case, reducing the tilt results in an 11% energy increase, though with a bigger number of modules needed.

If a tilt $\beta = 10$ (^{o}) is used, farther away from the optimal 20 (^{o}), the 1V solution consists of 38 n^o 6 PV modules for a total area $A_{PV} = 73.88$ (m^2), with $e_l = 1$ spacing; while, for the 1H option, 37 n^o 3 PV modules are needed for an area of $A_{PV} = 61.79$ (m^2), with minimal spacing $e_l = 1$. So, the maximum useful area results in $A_{PV}^{10} = 73.88$ (m^2). In

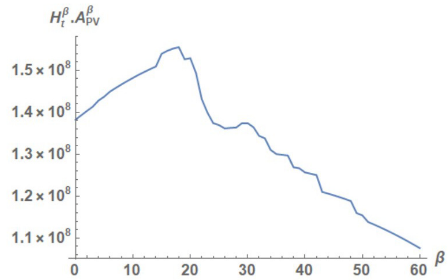

Fig. 8. Total Energy curve, $E_{PV}(\beta)$.

this case, once calculated the annual irradiation H_t^{10} for each module, the total energy adds up to:

$$E_{PV}(10) = 2.00602 \cdot 73.88 = 148.213\ (MW{\cdot}h) \qquad (21)$$

It can be concluded that employing $\beta = 10$ (^{o}) results in less energy absorbed than using $\beta = 20$ (^{o}). Again, the key is the influence of e_l^m: its fixed value makes it worthless to reduce β.

Numerous simulations have been made varying both the dimensions and form of the roof, while the angle the terrace forms with the North-South direction was modified between 0 and 90 (^{o}). It can be concluded that the optimal solution won't always consist in reducing (or increasing) the module tilt, so the packing algorithm and the irradiation calculations must be executed for each particular case study.

For the particular case of this paper, making a discretization of 1 (^{o}) for the value of β, the $E_{PV}(\beta)$ curve is shown in Fig. 8. The maximum value of the curve in Fig. 8, and therefore the optimal tilt for the PV modules that maximises the absorbed energy for the set of modules on the roof is $\beta^* = 18$ (^{o}), for a total energy of 155.527 ($MW{\cdot}h$).

VI. Conclusions

A mathematical method to optimize the installation of PV modules in urban residential buildings is presented and tested. A new packing algorithm specifically developed for this problem, combined with analytical calculation of annual irradiation for each possible tilt, allows for easy of obtention the optimal solution. The dimensions and arrangement of the PV modules are determined so that total absorbed energy is maximised, taking into account the shadowing effects. The results obtained show that the best solution depends on the characteristics of each particular problem, that is: the dimensions and orientation of the flat roof, latitude of the localization, dimensions of available PV modules, applicable laws, etc. In short, many variables need to be considered so the optimal strategy can't be known in advance.

References

[1] IEA, Trends in photovoltaic applications: 2019. International Renewable Energy Agency. Available from: https://iea-pvps.org/trends_reports/2019-edition/, accessed on 18 January 2021.

[2] J.A. Duffie, W.A. Beckman, Solar Engineering of Thermal Processes, 4 ed. New York: John Wiley & Sons; 2013.

[3] M.Z. Jacobson, V. Jadhav, World estimates of PV optimal tilt angles and ratios of sunlight incident upon tilted and tracked PV panels relative to horizontal panels, Solar Energy 169 (2018) 55-66.

[4] Y. Lv, P. Si, X. Rong, J. Yan, Y. Feng, X. Zhu, Determination of optimum tilt angle and orientation for solar collectors based on effective solar heat collection, Applied Energy 219 (2018) 11-19.

[5] A. Ullah, H. Imran, Z. Maqsood, N.Z. Butt, Investigation of optimal tilt angles and effects of soiling on PV energy production in Pakistan, Renewable Energy 139 (2019) 830-843.

[6] Y. Li, D. Ding, C. Liua, C. Wang, A pixel-based approach to estimation of solar energy potential on building roofs, Energy and Buildings 129 (2016) 563–573.

[7] H. Dyckhoff, A typology of cutting and packing problems, European Journal of Operations Research 44 (1990) 145-159.

[8] PVGIS, Joint Research Centre (JRC), Available from: https://ec.europa.eu/jrc/en/pvgis, accessed on 18 January 2021.

[9] A. Barbón, P. Fortuny Ayuso, L. Bayón, J.A. Fernández-Rubiera, Predicting beam and diffuse horizontal irradiance using Fourier expansions, Renewable Energy 154 (2020) 46-57.

[10] A. Barbón, C. Bayón-Cueli, L. Bayón, C. Rodríguez-Suanzes, Analysis of the tilt and azimuth angles of photovoltaic systems in non-ideal positions for urban applications, Applied Energy, Accepted/In Press (2021).

[11] S. Izquierdo, M. Rodrigues, N. Fueyo, A method for estimating the geographical distribution of the available roof surface area for large-scale photovoltaic energy-potential evaluations, Solar Energy 82 (2008) 929-939.

[12] IDAE, Technical Conditions for PV Installations Connected to the Grid [in Spanish]. Spanish Government Technical Report. Available from: http://www.idae.es. (Accessed 18 January 2021).

[13] E.G. Birgin, R.D. Lobato, R. Morabito, An effective recursive partitioning approach for the packing of identical rectangles in a rectangle, Journal of the Operational Research Society 61 (2010) 306–320.

[14] A. Barbón, N. Barbón, L. Bayón, J.A. Sánchez Rodríguez, Optimization of the distribution of small scale Linear Fresnel Reflectors on roofs of urban buildings. Applied Mathematical Modelling, Vol. 59, pp. 233-250, 2018.