International Journal of Computer Mathematics
Vol. 85, Nos. 3–4, March–April 2008, 345–358

An environmentally constrained economic dispatch: CFBC boilers in the day-ahead market

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(Received 30 October 2006; revised version received 22 December 2006; accepted 11 January 2007)

This paper presents an environmentally constrained economic dispatch algorithm for a hydrothermal system within the framework of a competitive and deregulated electricity market. The optimization problem of a firm in a competitive market is described, the objective function of which can be defined as profit maximization, and we consider that thermal plants are constrained to technical and environmental restrictions. An optimal control technique is applied and Pontryagin's theorem is employed. The proposed algorithm is implemented using the Mathematica© package and is applied to a sample system.

Keywords: Pollution emissions; Economic dispatch; Control problem; Hydrothermal systems; Pontryagin's maximum principle

AMS Subject Classification: 49J24; 92E20

1. Introduction

For decades, pulverized coal combustion (PCC) power plants have constituted the dominating technology among coal power generation technologies. Worldwide the majority of these PCC plants have no emission control equipment other than particulate removal systems. The technology for generating electricity from coal is undergoing change due to continued demand for cleaner power production. More efficient and cleaner power generation technologies [1] that will enable utilities to meet future environmental requirements while containing electricity costs will be the leading candidates in the decades to come. Fluidized bed combustion (FBC) has emerged as a viable alternative.

In a fluidized bed, solid inert material forms a bed on a perforated plate. Air is blown upward through the bed of particles at a sufficient velocity for the particles to overcome gravity; each particle will thus float on the gas stream like a boiling turbulent mass. Due to the good particle mixing, a uniform combustion temperature is obtained over the entire bed, resulting in a low combustion temperature. A temperature range of between 800 and 900 °C is commonly employed. If the gas is not pressurized, the system is termed atmospheric. As the gas velocity increases, hot particles are carried out of the combustion zone. After separation in a cyclone,

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International Journal of Computer Mathematics
ISSN 0020-7160 print/ISSN 1029-0265 online © 2008 Taylor & Francis
http://www.tandf.co.uk/journals
DOI: 10.1080/00207160701242334

the particles are recycled into the combustion chamber for further combustion. This scheme is termed *circulating* FBC or CFBC.

One of the main advantages of CFBC is the possibility of reducing the sulphur dioxide (SO_2) formed during combustion from the sulphur content of the fuel by adding a cheap absorbent material to the bed such as limestone ($CaCO_3$) or dolomite ($CaCO_3 \cdot MgCO_3$). If limestone is added to the bed, this undergoes a transformation called *calcination* to then form calcium oxide (CaO). The calcium oxide (a porous product) reacts with the SO_2 and oxygen to form calcium sulphate ($CaSO_4$), a transformation called *sulphation*. These reactions are optimum at temperatures of around 850 °C and this is one of the reasons why the operating temperatures in CFBC boilers are normally around 850 °C.

Another advantage of CFBC is the possibility of reducing the nitrogen oxides. Nitrogen oxides are formed during the combustion of coal; these oxides are normally abbreviated as NO_x. NO_x are partially formed by the nitrogen in the air (called thermal NO_x) and partially by the nitrogen bound in the coal (fuel NO_x). The reactions involving thermal NO_x are only significant at high temperatures ($>$1200 °C) and, since the combustion temperature in a CFBC boiler is below 900 °C, this extra NO_x is avoided. Fuel NO_x may be reduced with the aid of the CO present in the combustion gases that react with the nitrogen oxides. This is used in phase combustion, where the combustion in the first phase takes place under sub-stoichiometric conditions, resulting in CO being formed. Final combustion takes place in the following phase, once the remaining air has been added as secondary air.

On the other hand, the electricity supply industry is undergoing major restructuring. Traditional centralized regulation is being replaced by a competitive deregulated framework. This has been the case for Spanish utilities since 1 January 1998. In this paper the new short-term problems that are faced by a generation company in a deregulated electricity market are addressed and an optimization algorithm is proposed.

Several methods have been proposed for simulating competitive generation markets. The majority of these models [2] may be categorized into two main groups: models that represent all the generation companies and models that focus on a particular generation company. Models in the former group may be classified into two families: *equilibrium* and *simulation* models. Two approaches can be adopted to represent the spot market auctions when only one company is considered: price modelled as an exogenous variable and price modelled as a *function of the demand supplied* by the company under study.

In this paper we present the operation of one company in detail, including each of the company's generation units. Our model of the spot market explicitly represents the price of electricity as a *known exogenous* variable. We represent generation units at a high level of detail and our model distinguishes *individual generation units* and considers *inter-temporal constraints* such as hydro reserves.

In the Spanish market, two major electricity groups, Endesa and Iberdrola, control approximately 80% of all the electricity that is generated and distributed in Spain. The company that inspired our paper, HC, controls approximately only 7% of all the electricity that is generated. Accordingly, we consider our company as a price-taker, and this type of optimization model represents a market under perfect competition.

Traditionally, power generating plants have been dispatched following minimum fuel cost criteria (economic dispatch or optimal load flow) without considering the pollution produced. However, due to the ever increasing requirements of environmental regulations and social awareness, the opening up of these types of alternative strategies is becoming fundamental.

Numerous strategies exist [3] with the common goal of reducing the pollutant emissions of thermal power generation: minimization of total emissions (also known as emission dispatch) [4], minimization of the weighted sum of cost and emissions [5] and minimization of the cost with environmental constraints [6]. This is the typical economic dispatch, but maximum emissions are included among the operating constraints. This dispatch is called *environmentally constrained economic dispatch* (ECED). These are more realistic studies, as the majority of the regulations concerning environmental matters take the form of maximum pollution constraints.

This paper develops an ECED for a system that considers both thermal and hydro power plants, all within the framework of the new competitive deregulated electricity market, and will analyse the role of FBC plants in detail. The paper is organized as follows. In section 2 we consider a simple hydrothermal system with one hydro-plant. We set out our problem in terms of optimal control in continuous time, with the Lagrange-type functional, and use Pontryagin's minimum principle (PMP). In section 3 we present the optimal solution of the environmental and economic dispatch. In section 4 we present a solution algorithm. Section 5 illustrates the performance of our approach with a numerical example. Finally, section 6 summarizes the main conclusions of our research.

2. Statement of the hydrothermal problem

In this section the optimization problem of one company is described, the objective function of which can be defined as *profit maximization*. Let us assume that our hydrothermal system accounts for one hydro-plant and m thermal plants.

Let $\Psi_i : D_i \subseteq \mathbb{R} \longrightarrow \mathbb{R}$ $(i = 1, \ldots, m)$ be the cost functions (Euro/h) of the m thermal plants. The most usual cost function [7] of each generator can be represented as a quadratic function:

$$\Psi_i(P_i(t)) = \alpha_i + \beta_i P_i(t) + \gamma_i P_i^2(t), \quad i = 1, \ldots, m, \tag{1}$$

the solution of which can be obtained by conventional mathematical methods. In (1) P_i (MW) is the power generated, and we consider the thermal plants to be constrained by *technical restrictions* of the type

$$P_{i\,\min}^{\mathrm{Tch}} \leq P_i(t) \leq P_{i\,\max}^{\mathrm{Tch}}, \quad i = 1, \ldots, m, \ \ \forall t \in [0, T], \tag{2}$$

$[0, T]$ being the optimization interval. There are other models that we do not consider in this paper. For example, in [8], two cases of non-smooth cost functions are considered. One is the case with the valve-point loading problem where the objective function is generally described as the superposition of sinusoidal functions and quadratic functions. The other is the case with the multiple fuel problem where the objective function is expressed as piecewise quadratic cost functions.

On the other hand, several models have been used to represent the emissions function [3] of thermal plants. In [4] we construct a quadratic model for both emissions (SO_2 and NO_x) and calculate

$$E_i(P_i(t)) = \varepsilon_i P_i(t) + \sigma_i P_i^2(t), \quad i = 1, \ldots, m, \tag{3}$$

where E_i (mg/N m^3) is the pollutant emission (6% O_2) and P_i (MW) is the power generated, the parameters being computed via the least-squares criterion from several tests at thermal plants. Our problem considers the economic dispatch but also includes maximum emissions among the operating constraints: environmentally constrained economic dispatch (ECED).

Recently (November 2005), Spain formulated a *National Plan for Reducing Emissions from Existing Large Combustion Plants* (LCP). This plan contains the *emission limit values* (ELV) for SO_2 and NO_x (in mg/N m^3) for each plant, applicable for the period 2008–2015. Knowing the curve for each plant (3), and imposing the ELV of said plant, we immediately obtain *environmental restrictions* of the type:

$$P_{i\ \min}^{\text{Env}} \le P_i(t) \le P_{i\ \max}^{\text{Env}}, \quad i = 1, \dots, m, \;\; \forall t \in [0, T]. \tag{4}$$

In prior studies [9] it was proven that the problem of optimization of the fuel cost of a hydrothermal system with m thermal plants (with restrictions of type (2) or (4)) may be reduced to the study of a hydrothermal system made up of one single thermal plant, called the *thermal equivalent*. We shall denote as the *equivalent minimizer* of $\{\Psi_i\}_1^m$, the function $\Psi\colon D_1 + \cdots + D_m \to \mathbb{R}$ (where D_i are the domains of Ψ_i) defined by

$$\Psi(P(t)) = \min \sum_{i=1}^{m} \Psi_i(P_i(t)),$$

with $P(t)$ the power generated by said thermal equivalent.

Throughout the paper, no *transmission losses* will be considered, a crucial aspect when addressing the optimization problem from a centralized viewpoint. From the perspective of a generation company, and within the framework of the new electricity market, said losses are not relevant, since the generators currently do not participate in the sharing out of losses, thus receiving payment for all the energy they generate in power plant bars.

Let $H(t, z(t), \dot{z}(t))$ be the function of the *effective hydraulic contribution*, i.e. the power contributed to the system at instant t by the hydro-plant, $z(t)$ being the volume that is discharged up to instant t by the plant, and $\dot{z}(t)$ the rate of water discharge of the plant at instant t. If we assume that b is the volume of water that must be discharged during the entire optimization interval $[0, T]$, the following boundary conditions will have to be fulfilled:

$$z(0) = 0, \quad z(T) = b.$$

For the sake of convenience, we assume throughout the paper that these are sufficiently smooth and are subject to the following additional assumptions. Let us assume that the cost function $\Psi\colon \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ satisfies $\Psi'(x) > 0$, $\forall x \in \mathbb{R}^+$, i.e. it is strictly increasing. This constraint is absolutely natural: it reads more cost to more generated power. Let us also assume that $\Psi''(x) > 0$, $\forall x \in \mathbb{R}^+$, i.e. it is strictly convex. The models traditionally employed meet this constraint. Let us assume that the function of effective hydraulic generation $H(t, z, \dot{z})\colon \Omega_H = [0, T] \times \mathbb{R}^+ \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ is strictly increasing with respect to the rate of water discharge $\dot{z}$, i.e. $H_{\dot{z}} > 0$. Let us also assume that $H(t, z, \dot{z})$ is concave with respect to $\dot{z}$, i.e. $H_{\dot{z}\dot{z}} < 0$. Real models meet these two constraints; the former means more power to a higher rate of water discharge. It can be seen that we only admit non-negative thermal power $P(t)$ and we shall only admit non-negative volumes, $z(t)$, and rates of water discharge, $\dot{z}(t)$. Besides the previous statement, we consider $H(t, z(t), \dot{z}(t))$ to be bounded by technical restrictions

$$H_{\min} \le H(t, z(t), \dot{z}(t)) \le H_{\max}, \quad \forall t \in [0, T].$$

In our problem the *objective function* is given by revenue minus cost during the optimization interval $[0, T]$,

$$F(P, z) = \int_0^T [p(t)(P(t) + H(t, z(t), \dot{z}(t))) - \Psi(P(t))]\,\mathrm{d}t.$$

Revenue is obtained by multiplying the total production (thermal and hydraulic) of the company by the clearing price $p(t)$ in each hour t. The cost is given by Ψ, the cost function of the thermal equivalent, where $P(t)$ is the power generated by said plant. With the previous statement, our objective functional in *continuous time form* is

$$\max_{P,z} F(P,z) = \max_{P,z} \int_0^T L(t, P(t), z(t), \dot{z}(t))\, \mathrm{d}t,$$

with $L(t, P(t), z(t), \dot{z}(t)) = p(t)(P(t) + H(t, z(t), \dot{z}(t))) - \Psi(P(t))$, on the set

$$\Omega = \left\{ z \in \widehat{C}^1[0,T] \,\middle|\, \begin{matrix} & z(0) = 0, & z(T) = b, & \\ H_{\min} \leq & H(t, z(t), & \dot{z}(t)) \leq H_{\max}, & \forall t \in [0,T] \end{matrix} \right\},$$

where $\widehat{C}^1$ is the set of piecewise C^1 functions.

3. Optimal solution

We shall focus in the present paper on the development of the applications of optimal control theory (OCT) to this problem. If z satisfies Euler's equation for the functional F, we have that, $\forall t \in [0,T]$, Euler's equation is fulfilled:

$$L_z(t, P(t), z(t), \dot{z}(t)) - \frac{\mathrm{d}}{\mathrm{d}t} L_{\dot{z}}(t, P(t), z(t), \dot{z}(t)) = 0.$$

If we divide Euler's equation by $L_{\dot{z}}(t, P(t), z(t), \dot{z}(t)) > 0$, $\forall t$, and integrate, we have

$$L_{\dot{z}}(t, P(t), z(t), \dot{z}(t)) \cdot \exp\left[-\int_0^t \frac{H_z(s, z(s), \dot{z}(s))}{H_{\dot{z}}(s, z(s), \dot{z}(s))}\, \mathrm{d}s \right] = L_{\dot{z}}(0, P(0), z(0), \dot{z}(0))$$
$$= K \in \mathbb{R}^+, \quad \forall t \in [0,T].$$

We shall call this relation the *coordination equation* for $z(t)$, and the positive constant K will be termed the *coordination constant* of the extremal. Let us term the *coordination function* of $z \in \Omega$ the function in $[0,T]$, defined as follows:

$$\mathbb{Y}_z(t) = L_{\dot{z}}(t, P(t), z(t), \dot{z}(t)) \cdot \exp\left[-\int_0^t \frac{H_z(s, z(s), \dot{z}(s))}{H_{\dot{z}}(s, z(s), \dot{z}(s))}\, \mathrm{d}s \right],$$

with $L_{\dot{z}}(t, P(t), z(t), \dot{z}(t)) = p(t) H_{\dot{z}}(t, z(t), \dot{z}(t))$. Let us now obtain the fundamental result, which enables us to characterize the extremals of the problem and which is also the basis for elaborating the optimization algorithm that leads to the determination of the optimal solution of the hydrothermal system. We present the problem considering the *state variables* to be $z(t)$ and $P(t)$ and the *control variables* $u_1(t) = H(t, z(t), \dot{z}(t))$ and $u_2(t) = \dot{P}(t)$. Moreover, as $H_{\dot{z}} > 0$, the equation

$$u_1(t) - H(t, z(t), \dot{z}(t)) = 0$$

allows the *state equation* $\dot{z} = f(t, z, u_1)$ to be explicitly defined and we easily obtain

$$f_z = -\frac{H_z}{H_{\dot{z}}}, \quad f_{u_1} = \frac{1}{H_{\dot{z}}}.$$

The *optimal control problem* is thus

$$\max_{u_1(t), u_2(t)} \int_0^T L(t, P(t), u_1(t))\, \mathrm{d}t, \quad \text{with} \begin{cases} \dot{z} = f(t, z, u_1), \\ \dot{P} = u_2, \\ z(0) = 0, \qquad z(T) = b, \\ u_1(t) \in \Theta = \{x \mid H_{\min} \leq x \leq H_{\max}\}. \end{cases}$$

We shall use Pontryagin's minimum principle (PMP) [10] as the basis for proving this theorem.

THEOREM 3.1 (*theorem of coordination*) *If* $(z^*, P^*) \in (\widehat{C}^1, C^1)$ *is a solution of our problem, then* $\exists K \in \mathbb{R}^+$ *such that*

$$\mathbb{Y}_{z^*}(t) \begin{cases} \leq K, & if\ H(t, z^*(t), \dot{z}^*(t)) = H_{\min}, \\ =K, & if\ H_{\min} < H(t, z^*(t), \dot{z}^*(t)) < H_{\max}, \\ \geq K, & if\ H(t, z^*(t), \dot{z}^*(t)) = H_{\max}, \end{cases}$$

and

$$\dot{\Psi}(P^*(t)) = p(t).$$

Proof We shall term the *optimal controls* u_1^* and u_2^*, the *optimal states* will be $z^*(t)$ and $P^*(t)$, and the *co-state variables* will be λ_1 and λ_2. Let $\mathbb{H}$ be the Hamiltonian associated with the problem

$$\begin{aligned}\mathbb{H}(t, P, u_2, z, u_1, \lambda_2, \lambda_1) &= L(t, P(t), u_1(t)) + \lambda_1(t) \cdot f(t, z(t), u_1(t)) \\ &\quad + \lambda_2(t) \cdot u_2(t) \\ &= p(t)(P(t) + u_1(t)) - \Psi(P(t)) \\ &\quad + \lambda_1(t) \cdot f(t, z(t), u_1(t)) + \lambda_2(t) \cdot u_2(t).\end{aligned}$$

By virtue of PMP, there exist two $\widehat{C}^1$ functions, λ_1^* and λ_2^*, that satisfy the two following conditions:

$$\begin{aligned}\dot{\lambda}_1^*(t) &= -\frac{\partial \mathbb{H}(t, P^*(t), u_2^*(t), z^*(t), u_1^*(t), \lambda_2^*(t), \lambda_1^*(t))}{\partial z} \\ &= -\lambda_1^*(t) \cdot f_z(t, z^*(t), u_1^*(t)), \end{aligned} \tag{5}$$

$$\begin{aligned}\dot{\lambda}_2^*(t) &= -\frac{\partial \mathbb{H}(t, P^*(t), u_2^*(t), z^*(t), u_1^*(t), \lambda_2^*(t), \lambda_1^*(t))}{\partial P} \\ &= -L_P(t, P^*(t), u_1^*(t)), \end{aligned} \tag{6}$$

$$\begin{aligned}&\mathbb{H}(t, P^*(t), u_2^*(t), z^*(t), u_1^*(t), \lambda_2^*(t), \lambda_1^*(t)) \\ &\quad \geq \mathbb{H}(t, P^*(t), u_2^*(t), z^*(t), u_1(t), \lambda_2^*(t), \lambda_1^*(t)), \quad \forall u_1(t) \in \Theta, \end{aligned} \tag{7}$$

$$\begin{aligned}&\mathbb{H}(t, P^*(t), u_2^*(t), z^*(t), u_1^*(t), \lambda_2^*(t), \lambda_1^*(t)) \\ &\quad \geq \mathbb{H}(t, P^*(t), u_2(t), z^*(t), u_1^*(t), \lambda_2^*(t), \lambda_1^*(t)), \quad \forall u_2. \end{aligned} \tag{8}$$

From (5) it follows that

$$\lambda_1^*(t) = \lambda_1^*(0) \cdot \exp\left[-\int_0^t f_z(t, z^*(s), u_1^*(s))\, \mathrm{d}s\right]. \tag{9}$$

From (7) and (8) it follows that, for each t, $(u_1^*(t), u_2^*(t))$ maximizes on $\{u_1 \mid H_{\min} \leq u_1 \leq H_{\max}\}$ the function

$$\mathbb{F}(u_1, u_2) = \mathbb{H}(t, P^*(t), u_2(t), z^*(t), u_1(t), \lambda_2^*(t), \lambda_1^*(t)).$$

Hence, in accordance with the Kuhn–Tucker theorem, for each t there exist two real non-negative numbers, α and β, such that $(u_1^*(t), u_2^*(t))$ is a critical point of

$$\begin{aligned}\mathbb{G}(u_1, u_2) &= \mathbb{H}(t, P^*(t), u_2(t), z^*(t), u_1(t), \lambda_2^*(t), \lambda_1^*(t)) + \alpha(H_{\min} - u_1) \\ &\quad + \beta(u_1 - H_{\max}) \\ &= p(t)(P^*(t) + u_1(t)) - \Psi(P^*(t)) + \lambda_1^*(t) \cdot f(t, z^*(t), u_1(t)) \\ &\quad + \lambda_2^*(t) \cdot u_2(t) + \alpha(H_{\min} - u_1) + \beta(u_1 - H_{\max}),\end{aligned}$$

it being verified that if $H_{\min} < H(t, z^*(t), \dot{z}^*(t))$, then $\alpha = 0$, and $\beta = 0$ if $H(t, z^*(t), \dot{z}^*(t)) < H_{\max}$.

Hence we have

$$\frac{\partial \mathbb{G}(u_1^*, u_2^*)}{\partial u_1} = p(t) + \lambda_1^*(t) \cdot f_{u_1}(t, z^*(t), u_1^*(t)) - \alpha + \beta = 0, \tag{10}$$

$$\frac{\partial \mathbb{G}(u_1^*, u_2^*)}{\partial u_2} = \lambda_2^*(t) = 0, \tag{11}$$

and the three following cases.

Case 1. $H_{\min} < u_1^*(t) < H_{\max}$. In this case, from (10), $\alpha = \beta = 0$ and hence

$$p(t) = -\lambda_1^*(t) \cdot f_{u_1}(t, z^*(t), u_1^*(t)).$$

From (9) we have

$$p(t) = -f_{u_1}(t, z^*(t), u_1^*(t)) \cdot \lambda_1^*(0) \cdot \exp\left[-\int_0^t f_z(t, z^*(s), u_1^*(s))\,\mathrm{d}s\right],$$

$$\frac{p(t)}{f_{u_1}(t, z^*(t), u_1^*(t))} \cdot \exp\left[\int_0^t f_z(t, z^*(s), u_1^*(s))\,\mathrm{d}s\right] = -\lambda_1^*(0).$$

Bearing in mind that

$$f_{u_1} = \frac{1}{H_{\dot{z}}} \quad \text{and} \quad f_z = -\frac{H_z}{H_{\dot{z}}},$$

the following relation is fulfilled:

$$p(t) \cdot H_{\dot{z}}(t, z(t), \dot{z}(t)) \cdot \exp\left[-\int_0^t \frac{H_z(s, z^*(s), \dot{z}^*(s))}{H_{\dot{z}}(s, z^*(s), \dot{z}^*(s))}\,\mathrm{d}s\right] = -\lambda_1^*(0),$$

and the formula of Theorem 3.1 is verified:

$$\mathbb{Y}_{z^*}(t) = K = -\lambda_1^*(0).$$

Case 2. $u_1^*(t) = H(t, z^*(t), \dot{z}^*(t)) = H_{\max}$, then $\beta \geq 0$ and $\alpha = 0$. By analogous reasoning, we have

$$\mathbb{Y}_{z^*}(t) \geq K.$$

Case 3. $u_1^*(t) = H(t, z^*(t), \dot{z}^*(t)) = H_{\min}$, then $\alpha \geq 0$ and $\beta = 0$. By analogous reasoning, we have

$$\mathbb{Y}_{z^*}(t) \leq K.$$

Finally, from (11) and (6),

$$-L_P(t, P^*(t), u_1^*(t)) = 0 \Longrightarrow \dot{\Psi}(P^*(t)) = p(t). \qquad \blacksquare$$

Note. It is very important to stress that the problem is thus easily broken down into two sub-problems: thermal and hydro.

4. Optimization algorithm

To obtain the optimum operating conditions of the hydro-plant we use the coordination equation

$$\mathbb{Y}_z(t) = K, \quad \forall t \in [0, T]. \tag{12}$$

The peculiar form of the solution, expressed in Theorem 3.1, allows us to undertake its approximate calculation using numerical methods similar to those used to solve differential equations in combination with an appropriate adaptation of the classical shooting method. More precisely, we undertake two approximation processes.

- Approximate construction of z_K (*the adapted Euler method*).
- Construction of a sequence $\{K_j\}_{j\in\mathbb{N}}$ such that $z_{K_j}(T)$ converges to b (*the adapted shooting method*).

Step 1 Approximate construction of z_K (*the adaptad Euler method*). The problem will consist of finding, for each K, the function z_K that satisfies $z_K(0) = 0$ and the conditions of Theorem 3.1. From a computational point of view, the construction of z_K can be performed with the use of a discretized version of equation (12). In general, the construction of $\dot{z}_K$ cannot be carried out all at once over the entire interval $[0, T]$. The construction must necessarily be carried out by constructing and successively concatenating the extremal arcs and boundary arcs until completing the interval $[0, T]$. If the values obtained for z and $\dot{z}$ do not obey the constraints, we force the solution z_K to belong to the boundary until the moment when the conditions of leaving the domain (established in Theorem 3.1) are fulfilled.

The approximate construction of each z_K, which we shall call $\widetilde{z}_K$, is carried out by means of polygonals (Euler's method). We denote

$$\mathbb{Y}_{\widetilde{z}_K}(t_n) = p(t_n) \cdot H_{\dot{z}}(t_n, X_n, Y_n) \cdot \exp[-I_n],$$

and we consider the triple recurring sequence (X_n, Y_n, I_n) with $n = 0, \ldots, N-1$, $h = T/N$ and $t_n = n \cdot h$, which represents the following approximations:

$$z_K(t_n) \approx \widetilde{z}_K(t_n) := X_n,$$
$$\dot{z}_K(t_n) \approx \dot{\widetilde{z}}_K(t_n) := Y_n,$$
$$z_K(t) \approx \widetilde{z}_K(t) := X_{n-1} + (t - t_{n-1}) \cdot Y_{n-1}, \forall t \in [t_{n-1}, t_n],$$
$$\int_0^{t_n} \frac{H_z(s, z_K(s), \dot{z}_K(s))}{H_{\dot{z}}(s, z_K(s), \dot{z}_K(s))}\, \mathrm{d}s \approx I_n := \int_0^{t_n} \frac{H_z(s, \widetilde{z}_K(s), \dot{\widetilde{z}}_K(s))}{H_{\dot{z}}(s, \widetilde{z}_K(s), \dot{\widetilde{z}}_K(s))}\, \mathrm{d}s,$$

and which obeys the following recurrence relation:

$$X_0 = 0, \quad I_0 = 0,$$

– if $H(t_{n-1}, X_{n-1}, Y_{n-1}) = H_{\min} \to Y_n =$ the solution of $H(t_n, X_n, \chi) = H_{\min}$;

$$\text{if} \begin{cases} \mathbb{Y}_{\tilde{z}_K}(t_n) < K \longrightarrow Y_n = \text{the solution of } H(t_n, X_n, \chi) = H_{\min}, \\ \mathbb{Y}_{\tilde{z}_K}(t_n) \geq K \longrightarrow Y_n = \text{the solution of } p(t_n) \cdot H_{\dot{z}}(t_n, X_n, \chi) \cdot \exp[-I_n] \\ \qquad\qquad\qquad = K; \end{cases}$$

– if $\quad H_{\min} < H(t_{n-1}, X_{n-1}, Y_{n-1}) < H_{\max} \to Y_n =$ the solution of $p(t_n) \cdot H_{\dot{z}}(t_n, X_n, \chi) \cdot \exp[-I_n] = K$;

$$\text{if} \begin{cases} H(t_n, X_n, Y_n) < H_{\min} \longrightarrow Y_n = \text{the solution of } H(t_n, X_n, \chi) = H_{\min}, \\ H_{\min} \leq H(t_n, X_n, Y_n) \leq H_{\max} \longrightarrow Y_n = \text{the solution of } p(t_n) \\ \qquad\qquad\qquad \cdot H_{\dot{z}}(t_n, X_n, \chi) \cdot \exp[-I_n] = K, \\ H(t_n, X_n, Y_n) > H_{\max} \longrightarrow Y_n = \text{the solution of } H(t_n, X_n, \chi) = H_{\max}; \end{cases}$$

– if $H(t_{n-1}, X_{n-1}, Y_{n-1}) = H_{\max} \to Y_n =$ the solution of $H(t_n, X_n, \chi) = H_{\max}$;

$$\text{if} \begin{cases} \mathbb{Y}_{\tilde{z}_K}(t_n) > K \to Y_n = \text{the solution of } H(t_n, X_n, \chi) = H_{\max}, \\ \mathbb{Y}_{\tilde{z}_K}(t_n) \leq K \to Y_n = \text{the solution of } p(t_n) \cdot H_{\dot{z}}(t_n, X_n, \chi) \cdot \exp[-I_n] \\ \qquad\qquad\qquad = K, \end{cases}$$

$$X_{n+1} = X_n + h \cdot Y_n,$$
$$I_{n+1} = I_n + \int_{t_n}^{t_{n+1}} \frac{H_z(s, X_n + (s - t_n) \cdot Y_n, Y_n)}{H_{\dot{z}}(s, X_n + (s - t_n) \cdot Y_n, Y_n)} \, \mathrm{d}s.$$

Step 2 Construction of a sequence $\{K_j\}_{j \in \mathbb{N}}$ such that $z_{K_j}(T)$ converges to b (*the adapted shooting method*). Varying the coordination constant K, we search for the extremal that fulfils the second boundary condition $z_K(T) = b$. The procedure is similar to the shooting method used to resolve a two-point boundary value problem (TPBVP). A number of methods exist for solving these problems, including shooting, collocation and finite difference methods. Of the shooting methods [11, 12], the simple shooting method (SSM) and the multiple shooting method (MSM) appear to be the most widely known and used methods.

The simple shooting method transforms a TPBVP into an initial value problem where the initial values of selected parameters are varied to satisfy the desired end conditions. The boundary conditions are satisfied when the differential equation is integrated over $[0, T]$, using the initial condition obtained using the SSM. It should be noted that there can be serious problems with the convergence of the SSM if the starting initial condition is not close to the solution. This drawback of the SSM can be addressed by implementing what is known as the multiple shooting method (MSM).

The MSM is similar to the SSM, in that one selects unknown parameters at the initial time; however, one does not integrate the differential equation all the way to the final time. Instead, the ‘distance’ from a corresponding point on a pre-selected reference path is checked continuously as the integration proceeds, and the integration is aborted when the distance exceeds a tolerance value. Then, one starts the integration again from the corresponding point on the reference path and the previous step is repeated, until the system is integrated to the final time. An advantage of this approach over the SSM is that convergence can now be obtained for a larger class of TPBVPs. A serious disadvantage of this method is that if the differential equations are re-integrated to result in one continuous trajectory for the system, the actual final values may not be close to the desired final values. This is a common problem when solving TPBVPs that result from optimal control, like our problem.

Bearing in mind the above considerations, we implemented an SSM and obtained good results. Effectively, we may consider the function $\varphi(K) := z_K(T)$ and calculate the root of

$$\varphi(K) - b = 0, \tag{13}$$

which may be realized approximately using elemental procedures. In this work the secant method was used to calculate the approximate value of K for which (13) was verified. The algorithm shows rapid convergence to the optimal solution if we choose the next $K_{\min}$ and $K_{\max}$.

We set $H(t, z(t), \dot{z}(t)) = H_{\max}, \forall t \in [0, T]$. We calculate $Y_z(t), \forall t \in [0, T]$ and we choose $K_{\min} = \min_t Y_z(t)$.

We set $H(t, z(t), \dot{z}(t)) = H_{\min}, \forall t \in [0, T]$. We calculate $Y_z(t), \forall t \in [0, T]$ and we choose $K_{\max} = \max_t Y_z(t)$.

Note that $\forall K$ admissible (with the hypothesis $H_{\dot{z}\dot{z}}(t, z, \cdot) < 0$), we have

$$K_{\min} < K < K_{\max}.$$

The following proposition guarantees that, at the nodes $\{t_n\}_{n=0}^{N-1}$, the approximation $\widetilde{z}_K$ satisfies the condition established in Theorem 3.1.

PROPOSITION 4.1 *$\widetilde{z}_K$ satisfies in $\{t_n\}_{n=0}^{N-1}$ the following:*

$$\mathbb{Y}_{\widetilde{z}_K}(t_n) \begin{cases} \leq K, & if\ H(t_n, X_n, Y_n) = H_{min}, \\ = K, & if\ H_{min} < H(t_n, X_n, Y_n) < H_{max}, \\ \geq K, & if\ H(t_n, X_n, Y_n) = H_{max}. \end{cases}$$

Proof Let us bear in mind that

$$\mathbb{Y}_{z_K}(t_n) \approx \mathbb{Y}_{\widetilde{z}_K}(t_n) = p(t_n) \cdot H_{\dot{z}}(t_n, X_n, Y_n) \cdot \exp[-I_n],$$

if

$$H_{\min} < H(t_n, X_n, Y_n) < H_{\max} \Longrightarrow \mathbb{Y}_{\widetilde{z}_K}(t_n) = p(t_n) \cdot H_{\dot{z}}(t_n, X_n, Y_n) \cdot \exp[-I_n] = K.$$

Considering now that $H_{\dot{z}}(t_n, X_n, \cdot)$ is decreasing, we have

if $H(t_n, X_n, Y_n) = H_{\min} \Longrightarrow \exists \xi \mid H_{\min} \leq H(t_n, X_n, \xi)$ such that

$$p(t_n) \cdot H_{\dot{z}}(t_n, X_n, \xi) \cdot \exp[-I_n] = K \Longrightarrow \mathbb{Y}_{\widetilde{z}_K}(t_n) \leq K;$$

if $H(t_n, X_n, Y_n) = H_{\max} \Longrightarrow \exists \xi \mid H(t_n, X_n, \xi) \leq H_{\max}$ such that

$$p(t_n) \cdot H_{\dot{z}}(t_n, X_n, \xi) \cdot \exp[-I_n] = K \Longrightarrow \mathbb{Y}_{\widetilde{z}_K}(t_n) \geq K.$$ ■

To calculate the optimum power $P(t)$ of the thermal plant, we solve the equation

$$p(t) = \dot{\Psi}(P(t)), \quad \forall t \in [0, T].$$

The distribution among the thermal plants is immediate by means of the definition of the thermal equivalent, imposing the corresponding constraints (2) or (4) for each of the power plants.

A number of methods exist for solving the environmentally constrained economic dispatch (ECED) problem. Wong and Yuryevich [13] apply the evolutionary programming (EP) technique, El-Keib *et al.* [14] and Yalcinoz and Altun [15] propose a solution using modified genetic algorithms (GAs), and Santos and Vigo-Aguiar [16, 17] apply a dynamic programming algorithm to economic models. However, we could not find in the literature any work that considers hydro-plants (with a model containing a lot of detail) in the ECED problem, and, in addition, within the framework of the new competitive deregulated electricity market.

5. Example

A computer program was written using the Mathematica© package to apply the results obtained in this paper to a hydrothermal power system. In order to consider an example close to reality, we focused on a thermal system from Asturias (Spain). We consider a conventional 550 MW PCC plant belonging to the company HC, Aboño II, which has been studied by the present authors [4] and whose pollutant emissions were modelled, as well as another 50 MW CFBC plant belonging to the company Hunosa, La Pereda, which presents much more favourable environmental advantages than the former plant. The idea underlying this paper is to compare the two technologies. Therefore, given the small size of the La Pereda power plant, it was decided to take as an example the two CFBC plants that currently constitute a reference worldwide: Jacksonville (USA), generating 300 MW, and Gardanne (France), generating 250 MW. Using these two plants, we construct an equivalent CFBC plant [8], obtaining the parameters summarized in table 1.

The cost function Ψ_i used is a quadratic model (3) and the units for the coefficients are α_i (Euro/h), β_i (Euro/h.MW) and γ_i (Euro/h.MW^2). We consider $P_{i\,\min}^{\mathrm{Tch}} = P_{i\,\min}^{\mathrm{Env}} = 0$. To calculate $P_{1\,\max}^{\mathrm{Env}}$ for the PCC plant, we took the ELV published in the National Plan for Reducing Emissions from Existing LCP as reference. The Aboño II plant was assigned (from 2008 to 2015) 484 mg/N m^3 of SO_2 and 437 mg/N m^3 of NO_x. This means a reduction in SO_2 of 83% and a reduction in NO_x of 44%. With these data, we obtain $P_{1\,\max}^{\mathrm{Env}} = 100$ MW, a restriction that must be complied with from the year 2008 onwards.

For the CFBC plant we took the pollutant emissions published for Jacksonville, 90 mg/N m^3 of NO_x and 140 mg/N m^3 of SO_2, and for Gardanne, 240 mg/N m^3 of NO_x and 30 mg/N m^3 of SO_2. With these data, our equivalent CFBC does not exceed the ELV in any case, and we have $P_{2\,\max}^{\mathrm{Env}} = 550$ MW, which is hence equal to the technical restriction.

The hydrothermal system also considers one hydro-plant. We shall use the Salime plant in Asturias (Spain), which also belongs to a HC company. We use a *variable-head* model and the hydro-plant's effective hydraulic generation H (without transmission losses) is a function of $z(t)$ and $\dot{z}(t)$,

$$H(t, z(t), \dot{z}(t)) := A(t) \cdot \dot{z}(t) - B \cdot z(t) \cdot \dot{z}(t) - C \cdot \dot{z}^2(t),$$

with

$$A(t) := \frac{B_y}{G}(S_0 + t \cdot i), \qquad B = \frac{B_y}{G}, \qquad C = \frac{B_T}{G}.$$

Table 1. Coefficients of the thermal plants.

Plant	α_i	β_i	γ_i	$P_{i\max}^{\mathrm{Tch}}$	$P_{i\max}^{\mathrm{Env}}$
1 (PCC)	1615.35	36.676	0.03659	550	100
2 (CFBC)	1724.55	40.072	0.03511	550	550

Table 2. Hydro-plant coefficients.

G	b	i	S_0	B_y	B_T	H_{max}
519 840	6×10^6	133 200	239.5×10^6	4.34079×10^{-7}	2.94×10^{-5}	112

In variable-head models, the term $-B \cdot z(t) \cdot \dot{z}(t)$ represents the negative influence of the consumed volume and reflects the fact that consuming water lowers the effective height and hence the performance of the plant. The hydro-plant data are summarized in table 2. The units for the coefficients of the hydro-plant are: the efficiency G (m^4/h.MW), the constraint on the volume b (m^3), the natural inflow i (m^3/h), the initial volume S_0 (m^3), the coefficients B_y (m^{-2}) and B_T (m^{-2} h) (parameters that depends on the geometry of the tanks), and the maximum hydraulic generation H_{max} (MW).

We consider $H_{min} = 0$, a short-term hydrothermal scheduling (24 h) with an optimization interval $[0, 24]$ and we consider a discretization of 24 sub-intervals.

For this hydro-system we analyse two thermal systems: the one formed by the PCC plant and the hydro-plant and that formed by the equivalent CFBC plant and the hydro-plant. In both cases we shall carry out two studies: economic dispatch (ED) with technical restrictions, in which we shall maximize the profit for a given price, and the environmentally constrained economic dispatch (ECED), which includes among the operating constraints those referring to maximum emissions. The results obtained are shown below. In the figures, we use the terms $P(t)$ to denote the optimal power for the thermal plant and $H(t)$ for the hydro-plant. The clearing price $p(t)$ corresponding to 5 February 2006 (Sunday) for the Spanish electricity market and the profit obtained for all the cases are presented in figure 1.

Figures 2 and 3 show the economic dispatch with technical restrictions for the two systems. Comparing the two cases, we see that the PCC plant is more profitable, as expected given its lower electricity cost. With the new regulations, it is necessary to impose environmental restrictions; figures 3 and 4 clearly show that the equivalent CFBC plant is more profitable in

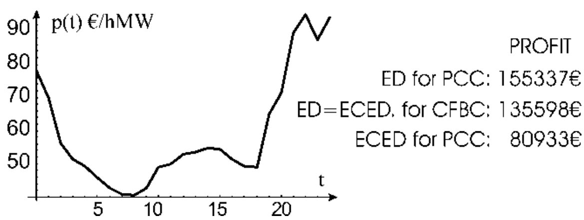

Figure 1. Clearing price $p(t)$ and profit.

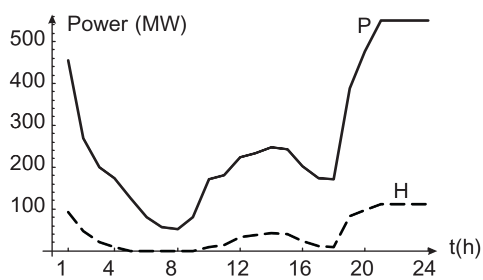

Figure 2. PCC plant with technical restrictions.

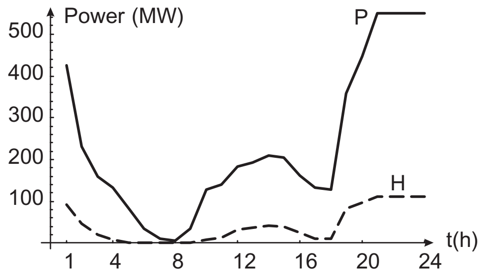

Figure 3. CFBC plant with technical or environmental restrictions.

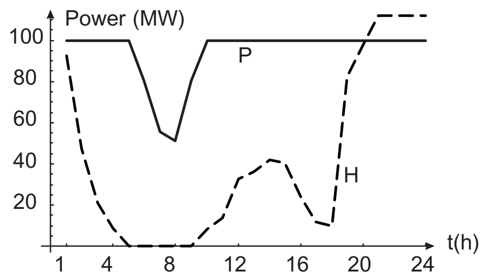

Figure 4. PCC plant with environmental restrictions.

this case. It is obvious that the CFBC plant solution is the same in the two dispatches, as it is likewise evident that the hydro-plant solution is the same in both cases, since, as we saw in Theorem 3.1, its functioning is independent of the behaviour of the thermal power plant.

In the PCC example, 14 iterations were needed and the time required by the program was 1.8 s on a personal computer (Pentium IV/2 GHz). In the CFBC example, 14 iterations were needed and the time required by the program was 2 s.

6. Conclusions

This paper develops an environmentally constrained economic dispatch for a hydro-thermal system within the framework of the new competitive deregulated electricity market, specifically in the day-ahead market under perfect competition. We compare the functioning of CFBC plants with conventional PCC plants.

It is generally known that CFBC plants have several advantages in comparison with conventional boilers: the possibility of employing a very large variety of solid fuels, reduced fuel preparation costs (if coal is used as fuel), high combustion efficiency, very reduced pollutant emissions in flue gases, with the possibility of complying with current ecological regulations, with no need for special supplementary installations, and the boiler construction being developed upwards, the seating surface being relatively restricted. With respect to economic/constructive characteristics, a CFBC that presents admissible pollution emission values according to international regulations is 55–65% cheaper than a conventional boiler with similar characteristics, which must be equipped with denox and desulphur systems and installations in order to comply with the requirements of these regulations.

In this paper we have examined the electricity cost of two different technologies, comparing the classical economic dispatch with technical restrictions (maximizing profit for a given price) with the ECED, which includes maximum emissions constraints. We have shown that a PCC plant is less advantageous under ECED than a CFBC plant. Thus, the replacement of conventional boilers with CFBC boilers may be a solution to satisfying the imposed requirements of current regulations regarding pollutant emissions into the atmosphere.

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