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An optimization problem in deregulated electricity markets solved with the nonsmooth maximum principle

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In this paper, the new short-term problems that are faced by a generation company in a deregulated electricity market are addressed and an optimization algorithm is proposed. Our model of the spot market explicitly represents the price of electricity as an uncertain exogenous variable. We consider a very complex problem of hydrothermal optimization with pumped-storage plants, so the problem deals with non-regular Lagrangian and non-holonomic inequality constraints. To obtain a necessary minimum condition, the problem was formulated within the framework of nonsmooth analysis using the generalized (or Clarke's) gradient and the Nonsmooth maximum principle. The optimal control problem is solved by means of an algorithm implemented in the commercial software package Mathematica. Results of the application of the method to a numerical example are presented.

Keywords: nonsmooth analysis; control problem; maximum principle; cyclic coordinate descent; electricity markets

2000 AMS Subject Classification: 49M20; 49J24; 91B24; 91B26

1. Introduction

Over the last decade, the electricity industry has experienced significant changes in terms of deregulation and competition. In this paper, we focus on the problem that a generation company faces when preparing its offers for the day-ahead market. Several methods have been proposed for simulating competitive generation markets. Most of these models [18] can be categorized into two major groups: models that represent all the generation companies and models that focus on a particular generation company. Two approaches can be adopted to represent spot market auctions when only one company is considered: price modelled as an exogenous variable and price modelled as a function of the demand supplied by the firm under study. In the former, the price of electricity does not depend on the company's decisions. This can be acceptable if the company is small enough. These models can again be classified into two sub-groups, depending on whether they use a deterministic [8] or probabilistic [17] price representation.

In this paper, we only represent the operation of one company in detail, including each of the company's generation units. Our model of the spot market explicitly represents the price

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of electricity as an uncertain exogenous variable. We represent generation units at a high level of detail and our model distinguishes individual generation units and considers inter-temporal constraints such as hydro reserves. In addition, we also consider pumped-storage hydro-plants.

The Spanish activity rules [6] have been used as a reference model for the market. The day-ahead market in the Spanish wholesale electricity market is organized as a set of 24 simultaneous hourly auctions. The simple bid format consists of a pair of (hourly) values: quantity q[MWh] and price p[€/MWh]. The utility company that inspires our paper, HC, controls approximately only 7% of all the electricity that is generated. So, we consider our company as a price-taker, and under this assumption, the volatility of the spot market price of electricity is represented by a stochastic model. Price forecasting techniques in power systems [7,10] are relatively recent procedures. In [12,13], an analysis based on differentiability is developed using stochastic models and a discretized version of a dynamic programming algorithm is applied to economic models. Although the problem of constructing the probability distribution exceeds the purpose of this paper, we suggest the following simplified approach based on ref. [1]. The idea is to search for past spot market sessions that can be considered similar to the session that the company is about to face. To identify the days, we classify the entire collection of sessions (using clustering techniques) according to the values of an explanatory variable. The most relevant information about the current session for our problem is the vector of 24 prices that has resulted from the day-ahead market clearing (in contrast with [1], in which the predicted chronological hourly demand curve is used). Once a group of S similar days has been identified, the company can assume that the probability distribution for the market session under study is completely defined by these past S market sessions (probability distributions with finite support). If we now focus on a particular auction, it is easy to understand that the S quantities and S prices decided by the company for that hour constitute the offer curve (nondecreasing) that the company must submit to that auction.

This paper addresses a very complex problem of hydrothermal optimization with pumped-storage plants. In this kind of problem (see the previous paper [4]), the Lagrangian is piecewise continuous and we consider constraints for the admissible generated power. Hence, this paper considers non-regular Lagrangian and non-holonomic inequality constraints (differential inclusions). The hydrothermal scheduling problem has been the subject of intensive investigation for several decades now. Dynamic programming [22] and mixed integer linear programming [16] methods have been widely used in different formulations, but these approaches require substantial simplifying assumptions to make the problem computationally tractable. Promising results have been obtained by using the Lagrangian relaxation technique to generate near optimal solutions [11]. The disadvantage of this approach lies in the primal solution, which is infeasible. As a result, some heuristic procedures are needed to get a feasible primal solution. In recent years, evolutionary computational optimization techniques is one tool that has shown certain ability in solving this problem. These evolutionary algorithms can be implemented in various forms, such as genetic algorithms [23], evolutionary programming [14], simulated annealing [21] and evolutionary strategy [20]. The main drawback with the majority of these finite-dimensional methods is the difficulty of treating large-scale systems.

In this paper, to obtain a necessary minimum condition, the problem is formulated within the framework of nonsmooth analysis [5,19] using the generalized (or Clarke's) gradient and the Nonsmooth Maximum Principle. This characteristic distinguishes our work from all the above and, to our knowledge, is the first paper in the literature in which this theory is applied to the proposed hydrothermal problem. The advantage of this infinite-dimensional technique compared to previous ones lies in the possibility of obtaining theoretical results (see the Main Theorem of Coordination in Section 3) whose implementation is feasible regardless of the size of the problem. Considering an elevated number of discretization intervals may make the use of other methods unviable, whereas our technique would in this case be more plausible while barely increasing the computational effort. Likewise, employing our technique (see the strategy inspired by the method of cyclic coordinate descent (CCD) in Section 4), the computational complexity of the problem does not become exorbitant when considering an elevated number of hydro-plants, whereas it may not be possible to address it using other techniques.

The paper is organized as follows. Section 2 presents the optimization models of the hydrothermal system, and the mathematical environment of our work: the nonsmooth Maximum Principle. In Section 3, we first consider a simple hydrothermal system with one hydro-plant. We shall set out our problem in terms of optimal control in continuous time, with the Lagrange-type functional, and we shall use nonsmooth analysis. In Section 4, we shall study the general case in which the system consists of n hydro-plants. We shall present the optimization algorithm that leads to determination of the optimal solution. Section 5 illustrates the performance of our approach with a real numerical example. Finally, Section 6 summarizes the main conclusions of our research.

2. Statement of the problem

In this section, the optimization problem of one company is described, the objective function of which can be defined as its profit maximization. Let us assume that our hydrothermal system accounts for n hydro-plants and m thermal plants: the $(H_{\mathbf{n}} - T_{\mathbf{m}})$ problem.

Let $\Psi_i : D_i \subseteq \mathbb{R}^+ \to \mathbb{R}^+$ $(i = 1, \ldots, m)$ be the cost functions (€$/h$) of the m thermal plants. The most usual cost function of each generator can be represented as a quadratic function:

$$\Psi_i(P_i(t)) = \alpha_i + \beta_i P_i(t) + \gamma_i P_i^2(t); \quad i = 1, \ldots, m,$$

where $P_i(MW)$ is the power generated, and we consider the thermal plants to be constrained by technical restrictions of the type

$$P_{i\,\min} \leq P_i(t) \leq P_{i\,\max}; \quad i = 1, \ldots, m, \quad \forall t \in [0, T],$$

$[0, T]$ being the optimization interval. In prior studies [2], it was proven that the problem with m thermal plants may be reduced to the study of a hydrothermal system made up of one single thermal plant, called the thermal equivalent: the $(H_{\mathbf{n}} - T_{\mathbf{1}})$ problem. We shall denote as the equivalent minimizer of $\{\Psi_i\}_1^m$, the function $\Psi : D_1 + \cdots + D_m \to \mathbb{R}$ defined by $\Psi(P(t)) = \min \sum_{i=1}^m \Psi_i(P_i(t))$; $P_{\min} \leq P(t) \leq P_{\max}$, with $P(t)$ the power generated by said thermal equivalent.

We assume that our system accounts for n hydro-plants that have a pumping capacity. The mapping $H : \Omega_H \to \mathbb{R}$, $H(t, z_1(t), \ldots, z_i(t), \ldots, z_n(t), \dot{z}_1(t), \ldots, \dot{z}_i(t), \ldots, \dot{z}_n(t)) = H(t, \mathbf{z}(t), \dot{\mathbf{z}}(t))$, is called the function of effective hydraulic contribution and is the power contributed to the system at the instant t by the set of hydro-plants, $z_i(t)$ being the volume that is discharged up to the instant t by the i-th hydro-plant, $\dot{z}_i(t)$ the rate of water discharge at the instant t by the i-th hydro-plant, and $\Omega_H \subset [0, T] \times \mathbb{R}^n \times \mathbb{R}^n$ the domain of definition of H.

We say that $\dot{\mathbf{z}} = (z_1, \ldots, z_n)$ is admissible for H if z_i belong to the class $\widehat{C}^1[0, T]$ (the set of piecewise C^1 functions), and $(t, \mathbf{z}(t), \dot{\mathbf{z}}(t)) \in \Omega_H$, $\forall t \in [0, T]$. The volume b_i that must be discharged up to the instant T is called the admissible volume of the i-th hydro-plant. Let $\mathbf{b} = (b_1, \ldots, b_n) \in \mathbb{R}^n$ be the vector of admissible volumes. In a general model, with hydraulic coupling between the n hydro-plants, we call $H_i(t, z_i(t), \dot{z}_i(t)) : \Omega_{H_i} = [0, T] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$

the function of effective hydraulic contribution by the i-th hydro-plant, being

$$H(t, \mathbf{z}(t), \dot{\mathbf{z}}(t)) = \sum_{i=1}^{n} H_i(t, z_i(t), \dot{z}_i(t)).$$

Besides, we consider $H_i(t, z_i(t), \dot{z}_i(t))$ to be bounded by technical constraints

$$H_{i\,\min} \le H_i(t, z_i(t), \dot{z}_i(t)) \le H_{i\,\max}; \quad i = 1, \dots, n, \ \ \forall t \in [0, T].$$

Throughout the paper, no transmission losses will be considered; a crucial aspect when addressing the optimization problem from a centralized viewpoint. From the perspective of a generation company, and within the framework of the new electricity market, said losses are not relevant, since the generators currently receive payment for all the energy they generate in power plant bars.

Let us assume that the cost function Ψ is strictly increasing, i.e. it reads more cost to more generated power. Let us assume as well that Ψ is strictly convex. The models traditionally employed meet this constraint.

Let us assume that the function H_i is strictly increasing with respect to the rate of water discharge $\dot{z}_i$, i.e. more power to a higher rate of water discharge and that $[\partial H_i/\partial z_i]_{\dot{z}_i=0} = 0$. Let us also assume that $\partial^2 H_i/\partial \dot{z}_i^2 < 0$. The real models meet these three constraints. In addition, pumped-storage plants are considered, and in this kind of problem, the derivative of H_i with respect to $\dot{z}_i$ presents discontinuity at $\dot{z}_i = 0$, which is the border between the power generation zone (positive values of $\dot{z}_i$) and the pumping zone (negative values of $\dot{z}_i$). In the real models, it is verified that $H_{\dot{z}}^+ \le H_{\dot{z}}^-$. In the $(H_\mathbf{n} - T_\mathbf{1})$ problem, the objective function is given by revenue minus cost during the optimization interval $[0, T]$. Revenue is obtained by multiplying the total production (thermal and hydraulic) of the company by the clearing price $p(t)$ in each hour t. Cost is given by Ψ, the cost function of the thermal equivalent, where $P(t)$ is the power generated by said plant. With this statement, our objective functional in continuous time form is

$$\max_{P,\mathbf{z}} F(P, \mathbf{z}) = \max_{P,\mathbf{z}} \int_0^T L(t, P(t), \mathbf{z}(t), \dot{\mathbf{z}}(t))\mathrm{d}t,$$

with:

$$L(t, P(t), \mathbf{z}(t), \dot{\mathbf{z}}(t)) = p(t)(P(t) + H(t, \mathbf{z}(t), \dot{\mathbf{z}}(t))) - \Psi(P(t))$$

on the set:

$$\Omega = \left\{ \mathbf{z} \in \left(\widehat{C}^1[0, T]\right)^n \middle| \begin{array}{c} z_i(0) = 0, z_i(T) = b_i \\ H_{i\,\min} \le H_i(t, z_i(t), \dot{z}_i(t)) \le H_{i\,\max}, \quad \forall t \in [0, T] \\ \forall i = 1, \dots, n \end{array} \right\}.$$

In the next section we shall present this problem as an optimal control problem. To solve it, we shall use the nonsmooth version of Pontryagin's Minimum Principle (PMP).

3. The $(H_1 - T_1)$ problem

We begin the development in this section by presenting the simple problem with one pumped-storage hydro-plant $(i = 1)$. In the $(H_1 - T_1)$ problem, we have $\mathbf{z} = z$ and our objective functional is

$$F(P, z) = \int_0^T L(t, P(t), z(t), \dot{z}(t))\mathrm{d}t,$$

with:

$$L(t, P(t), z(t), \dot{z}(t)) = p(t)(P(t) + H(t, z(t), \dot{z}(t))) - \Psi(P(t)),$$

on the set:

$$\Omega = \left\{ z \in \widehat{C}^1[0, T] \mid \begin{array}{c} z(0) = 0, z(T) = b \\ H_{\min} \le H(t, z(t), \dot{z}(t)) \le H_{\max}, \quad \forall t \in [0, T] \end{array} \right\}.$$

where $L(\cdot, \cdot, \cdot, \cdot)$ and $L_z(\cdot, \cdot, \cdot, \cdot)$ are the class C^0 and $L_{\dot{z}}(t, P, z, \cdot)$ is piecewise continuous ($L_{\dot{z}}(t, P, z, \cdot)$ is discontinuous in $\dot{z} = 0$). The problem involves non-holonomic inequality constraints (differential inclusions) and the previous assumptions guarantee that: $L_{\dot{z}\dot{z}}(t, P, z, \dot{z}) < 0$; $L_{\dot{z}}(t, P, z, \dot{z}) > 0$. We also assume that

$$H(t, b, \dot{z}(t)) \le H(t, z(t), \dot{z}(t)) \le H(t, 0, \dot{z}(t)), \quad \forall z \in \Omega.$$

These suppositions are fulfilled in all real hydrothermal problems, and bearing in mind the weak influence of $z(t)$, $(H(t, b, \dot{z}) \simeq H(t, z, \dot{z}) \simeq H(t, 0, \dot{z}))$, it is reasonable to substitute the restriction: $H_{\min} \le H(t, z(t), \dot{z}(t)) \le H_{\max}$ by others of the type: $H_{\min} \le H(t, b, \dot{z})$; $H(t, 0, \dot{z}) \le H_{\max}$. Thus, it is reasonable to substitute Ω by

$$\Omega^* = \left\{ z \in \widehat{C}^1[0, T] \mid \begin{array}{c} z(0) = 0, z(T) = b \\ H_{\min} \le H(t, b, \dot{z});\ H(t, 0, \dot{z}) \le H_{\max}, \quad \forall t \in [0, T] \end{array} \right\}.$$

The solution to the problem in Ω^* will be very close to that obtained with the set Ω, the advantage being that the mathematical treatment of sets of type Ω^* is much simpler than of those of type Ω. We shall focus in the present paper on the development of the applications of Optimal Control Theory (OCT) and nonsmooth analysis to this problem. Let us term as the coordination function of $z \in \Omega^*$ the function in $[0, T]$, defined by:

$$\mathbb{Y}_z(t) = L_{\dot{z}}(t, P(t), z(t), \dot{z}(t)) - \int_0^t L_z(s, P(s), z(s), \dot{z}(s))\mathrm{d}s,$$

denoting by $\mathbb{Y}_z^+(t)$ and $\mathbb{Y}_z^-(t)$ the expressions obtained when considering the lateral derivatives of L with respect to $\dot{z}$. Let us now see the fundamental result, which is the basis for elaborating the optimization algorithm. We present the problem considering the state variables to be $z(t)$ and $P(t)$ and the control variables $u_1(t) = \dot{z}(t)$ and $u_2(t) = \dot{P}(t)$. The optimal control problem is thus:

$$\max_{u_1(t), u_2(t)} \int_0^T L(t, P(t), z(t), u_1(t))\mathrm{d}t; \quad \text{with} \begin{cases} \dot{z} = u_1; & \dot{P} = u_2, \\ z(0) = 0, & z(T) = b, \end{cases}$$

$$u_1(t) \in \Theta = \{x \mid H_{\min} \le H(t, b, x);\ H(t, 0, x) \le H_{\max}\};\ u_2(t) \in (-\infty, \infty).$$

We shall use the nonsmooth version of PMP as the basis for proving this theorem.

THEOREM 1 (Main Theorem of Coordination) *If* $(z^*, P^*) \in (\widehat{C}^1, C^1)$ *is a solution of our problem, then* $\exists K \in \mathbb{R}^+$ *such that:*

(i) *If* $\dot{z}^*(t) = 0 \to \mathbb{Y}^+_{z^*}(t) \le K \le \mathbb{Y}^-_{z^*}(t)$.

(ii) *If* $\dot{z}^*(t) \neq 0 \to \mathbb{Y}_{z^*}(t)$ *is* $\begin{cases} \ge K & \textit{if } H(t, b, \dot{z}^*(t)) = H_{\min}, \\ = K & \textit{if } H_{\min} < H(t, z^*(t), \dot{z}^*(t)) < H_{\max}, \\ \le K & \textit{if } H(t, 0, \dot{z}^*(t)) = H_{\max}, \end{cases}$

and $\dot{\Psi}\,(P^*(t)) = p(t)$.

Proof We shall term the optimal controls u_1^* and u_2^*, the optimal states will be $z^*(t)$ and $P^*(t)$, and the co-state variables will be λ_1 and λ_2. For the sake of simplicity, in certain steps we shall omit the argument: (t) of the functions P, p, u_2, z, u_1, λ_2, and λ_1. Let $\mathbb{H}$ be the pseudo Hamiltonian associated with the problem.

$$\begin{aligned}\mathbb{H}(t, P, u_2, z, u_1, \lambda_2, \lambda_1) &= \lambda_1 \cdot u_1 + \lambda_2 \cdot u_2 - L(t, P, z, u_1) \\ &= \lambda_1 \cdot u_1 + \lambda_2 \cdot u_2 - p(P + H(t, z, u_1)) + \Psi(P).\end{aligned}$$

In virtue of PMP, there exists two functions λ_1^*, λ_2^* that satisfy:

$$-\dot{\lambda}_1^*(t) \in \partial_z \mathbb{H}(t, P^*, u_2^*, z^*, u_1^*, \lambda_2^*, \lambda_1^*), \tag{1}$$

$$-\dot{\lambda}_2^*(t) \in \partial_P \mathbb{H}(t, P^*, u_2^*, z^*, u_1^*, \lambda_2^*, \lambda_1^*), \tag{2}$$

$$\mathbb{H}(t, P^*, u_2^*, z^*, u_1^*, \lambda_2^*, \lambda_1^*) \ge \mathbb{H}(t, P^*, u_2^*, z^*, u_1, \lambda_2^*, \lambda_1^*); \quad \forall u_1(t) \in \Theta, \tag{3}$$

$$\mathbb{H}(t, P^*, u_2^*, z^*, u_1^*, \lambda_2^*, \lambda_1^*) \ge \mathbb{H}(t, P^*, u_2, z^*, u_1^*, \lambda_2^*, \lambda_1^*); \quad \forall u_2. \tag{4}$$

From the continuity of $\mathbb{H}$ with respect to z, and (1) we have that

$$-\dot{\lambda}_1^*(t) \in \frac{\partial \mathbb{H}(t, P^*, u_2^*, z^*, u_1^*, \lambda_2^*, \lambda_1^*)}{\partial z} = -L_z(t, P^*(t), z^*(t), u_1^*(t)). \tag{5}$$

From the continuity of $\mathbb{H}$ with respect to P, and (2) we have that

$$-\dot{\lambda}_2^*(t) = \frac{\partial \mathbb{H}(t, P^*, u_2^*, z^*, u_1^*, \lambda_2^*, \lambda_1^*)}{\partial P} = -L_P(t, P^*(t), z^*(t), u_1^*(t)). \tag{6}$$

If $\dot{z}^*(t) \neq 0$, from Equation (5), it follows that

$$-\dot{\lambda}_1^*(t) = -L_z(t, P^*(t), z^*(t), u_1^*(t)) = -p(t) \cdot H_z(t, z^*(t), u_1^*(t)). \tag{7}$$

If $\dot{z}^*(t) = 0$, from Equation (5), and $[\partial H_i / \partial z_i]_{\dot{z}_i = 0} = 0$, it follows that

$$\dot{\lambda}_1^*(t) = 0. \tag{8}$$

Hence, we can integrate (7), and bearing in mind (8), we obtain

$$\lambda_1^*(t) = \lambda_1^*(0) + \int_0^t L_z(s, P^*(s), z^*(s), u_1^*(s))\mathrm{d}s, \quad \forall t \in [0, T]. \tag{9}$$

From Equations (3) and (4), it follows that for each t, $(u_1^*(t), u_2^*(t))$ maximizes on $\Theta \times (-\infty, \infty)$ the function

$$\mathbb{F}(u_1, u_2) = \mathbb{H}(t, P^*, u_2, z^*, u_1, \lambda_2^*, \lambda_1^*).$$

Hence, in accordance with the Kuhn–Tucker Theorem, for each t there exists two real non-positive numbers, α and β, such that $(u_1^*(t), u_2^*(t))$ is a critical point of

$$\begin{aligned}\mathbb{G}(u_1, u_2) &= \mathbb{H}(t, P^*, u_2, z^*, u_1, \lambda_2^*, \lambda_1^*) + \alpha(H_{\min} - H(t, b, u_1)) + \beta(H(t, 0, u_1) - H_{\max})\\ &= \lambda_1^* \cdot u_1 + \lambda_2^* \cdot u_2 - p(t)(P^* + H(t, z^*, u_1)) + \Psi(P^*)\\ &\quad + \alpha(H_{\min} - H(t, b, u_1)) + \beta(H(t, 0, u_1) - H_{\max}),\end{aligned}$$

with $\alpha = 0$ if $H_{\min} < H(t, b, \dot{z}^*(t))$, and $\beta = 0$ if $H(t, 0, \dot{z}^*(t)) < H_{\max}$. If $\mathbb{G}$ attains a local maximum at (u_1, u_2), then

$$0 \in \partial_{u_1}\mathbb{G}(u_1^*, u_2^*), \tag{10}$$

$$0 \in \partial_{u_2}\mathbb{G}(u_1^*, u_2^*). \tag{11}$$

From the discontinuity of $H_{\dot{z}}(t, z, \cdot)$ in $\dot{z} = 0$, and (10) we have that

$$0 \in \lambda_1^*(t) - p(t)\partial_{u_1}H(t, z^*, u_1^*) - \alpha\partial_{u_1}H(t, b, u_1^*) + \beta\partial_{u_1}H(t, 0, u_1^*). \tag{12}$$

From the continuity of $\mathbb{G}$ with respect to $\dot{P}$, and (11) we have that

$$0 = \frac{\partial\mathbb{G}(u_1^*, u_2^*)}{\partial u_2} = \lambda_2^*(t). \tag{13}$$

We must analyse the three following cases:

Case 1 $\quad H_{\min} < H(t, z^*(t), \dot{z}^*(t)) < H_{\max}$. In this case $\alpha = \beta = 0$, and from Equation (12) we have that

$$0 \in \lambda_1^*(t) - p(t)\partial_{u_1}H(t, z^*, u_1^*).$$

If $\dot{z}^*(t) = 0$, then $H_{\dot{z}}(t, z, \cdot)$ is discontinuous, that is $\partial_{\dot{z}}H(t, z(t), \dot{z}(t)) = [H_{\dot{z}}^+, H_{\dot{z}}^-]$ *a.e.*, so we have

$$p(t) \cdot H_{\dot{z}}^+ \le \lambda_1^*(t) \le p(t) \cdot H_{\dot{z}}^-.$$

From Equation (9), we have

$$p(t) \cdot H_{\dot{z}}^+ \le \lambda_1^*(0) + \int_0^t L_z(s, P^*(s), z^*(s), u_1^*(s))\mathrm{d}s \le p(t) \cdot H_{\dot{z}}^-,$$

$$p(t)H_{\dot{z}}^+ - \int_0^t L_z(s, P^*, z^*, u_1^*)ds \le \lambda_1^*(0) \le p(t)H_{\dot{z}}^- - \int_0^t L_z(s, P^*, z^*, u_1^*)\mathrm{d}s,$$

and the formula of Theorem 1 is verified: $\mathbb{Y}_{z^*}^+(t) \le K \le \mathbb{Y}_{z^*}^-(t)$.

If $\dot{z}^*(t) \ne 0$, then $\mathbb{Y}_{z^*}^+(t) = \mathbb{Y}_{z^*}^-(t)$ and in such a case $\mathbb{Y}_{z^*}(t) = K$.

Case 2 $\quad H(t, 0, \dot{z}^*(t)) = H_{\max}$. In this case $\beta \le 0$, $\alpha = 0$, and from Equation (12) we have that

$$0 = \lambda_1^*(t) - p(t)H_{u_1}(t, z^*, u_1^*) + \beta H_{u_1}(t, 0, u_1^*).$$

Taking into account that $H_{u_1} > 0$, we have

$$\lambda_1^*(t) - p(t)H_{u_1}(t, z^*, u_1^*) \ge 0.$$

So, we have $\mathbb{Y}_{z^*}(t) \le K$.

Case 3 $H(t, b, \dot{z}^*(t)) = H_{\min}$. In this case $\alpha \leq 0$, $\beta = 0$, and by analogous reasoning, we have that $\mathbb{Y}_{z^*}(t) \geq K$.

Finally, from Equations (13) and (6)

$$-L_P(t, P^*(t), u_1^*(t)) = 0 \Longrightarrow \dot{\Psi}\left(P^*(t)\right) = p(t). \qquad \blacksquare$$

We shall call this relation

$$L_{\dot{z}}(t, P(t), z(t), \dot{z}(t)) - \int_0^t L_z(s, P(s), z(s), \dot{z}(s))\mathrm{d}s = K \in \mathbb{R}^+, \quad \forall t \in [0, T], \tag{14}$$

the coordination equation for $z(t)$, and the positive constant K will be termed the coordination constant of the extremal.

Note. It is very important to stress that the problem is thus easily broken down into the two sub-problems: Thermal and Hydro. In the thermal sub-problem, the power $P(t)$ of the equivalent thermal plant is distributed (as we see in ref. [2]) between the m thermal plants, and so is completely resolved.

4. Generalization to the ($H_\mathbf{n} - T_1$) problem the optimization algorithm

In this section, we present an algorithm of the numerical resolution of the problem of optimization of a hydrothermal system that involves n hydro-plants. The associated variational problem is related to solving a boundary-value problem for a system of differential equations. The algorithm uses a particular strategy related to the method of CCD [3]. The CCD method minimizes a function cyclically with respect to the coordinate variables. With our method, a problem of the type $H_\mathbf{n} - T_\mathbf{1}$ could be solved (under certain conditions) if we start out from the resolution of a sequence of problems of the type $H_\mathbf{1} - T_\mathbf{1}$. The algorithm for the $H_\mathbf{n} - T_\mathbf{1}$ problem carries out several iterations and at each j-th iteration calculates n stages, one for each hydro-plant. At each stage, it calculates the optimal functioning of a hydro-plant, while the behaviour of the rest of the plants is assumed fixed. For every $\mathbf{z} = (z_1, \ldots, z_n) \in \Omega$, we consider the functional $F_\mathbf{z}^i$ defined by

$$F_\mathbf{z}^i(P, v_i) = \int_0^T \left[p(t)(P(t) + H_\mathbf{z}^i(t, v_i(t), \dot{v}_i(t))) - \Psi(P(t))\right]\mathrm{d}t,$$

with

$$H_\mathbf{z}^i(t, v_i, \dot{v}_i) = H(t, z_1, \ldots, z_{i-1}, v_i, z_{i+1}, \ldots, z_n, \dot{z}_1, \ldots, \dot{z}_{i-1}, \dot{v}_i, \dot{z}_{i+1}, \ldots, \dot{z}_n),$$

where $H_\mathbf{z}^i$ represents the power generated by the hydraulic system as a function of the rate of water discharge and the volume turbinated by the i-th plant, under the assumption that the rest of the plants behave in a definite way. We call the i-th minimizing mapping the mapping $\phi_i : \Omega \to \Omega$, defined in the following way: for every $\mathbf{z} \in \Omega$

$$\phi_i(P, z_1, \ldots, z_i, \ldots, z_n) = (P^*, z_1, \ldots, z_i^*, \ldots, z_n),$$

where (P^*, z_i^*) minimizes $F_\mathbf{z}^i$. If we set $\Phi = (\phi_n \circ \phi_{n-1} \circ \cdots \circ \phi_2 \circ \phi_1)$ and

$$(P^j, \mathbf{z}^j) = \Phi(P^{j-1}, \mathbf{z}^{j-1}),$$

beginning with some admissible $(P^0, \mathbf{z}^0)$, we construct a sequence of $(P^j, \mathbf{z}^j)$ via successive applications of $\{\phi_i\}_{i=1}^n$ and the algorithm will search: $\lim_{j\to\infty}(P^j, \mathbf{z}^j)$. It is simple to justify the convergence of the algorithm in a finite number of steps, simply by considering the following solution set:

$$\{\mathbf{z} \mid F(P, \mathbf{z}) - F(\Phi(P, \mathbf{z})) < \varepsilon\}.$$

The application of every ϕ_i involves solving a problem of the type $(H_1 - T_1)$. To obtain the optimum operating conditions of the hydro-plant, we shall use the coordination Equation (14). To undertake the approximate calculation of the solution, expressed in Theorem 1, we use a similar numerical method to those used to solve differential equations in combination with an appropriate adaptation of the classical shooting method.

Step 1 Approximate construction of z_K (the adapted Euler method).

The problem will consist in finding for each K the function z_K that satisfies $z_K(0) = 0$, and the conditions of Theorem 1. From the computational point of view, the construction of z_K can be performed with the use of a discretized version of Equation (14). The approximate construction of each z_K, which we shall call $\widetilde{z}_K$, is carried out by means of polygonals (Euler's method). We denote

$$\mathbb{Y}_{\widetilde{z}_K}(t_n) = L_{\dot{z}}(t_n, P(t_n), X_n, Y_n) - I_n,$$

and we consider the triple recurring sequence (X_n, Y_n, I_n) with $n = 0, \ldots, N - 1$, with $h = T/N$ and $t_n = n \cdot h$, which represents the following approximations:

$$z_K(t_n) \approx \widetilde{z}_K(t_n) := X_n; \quad \dot{z}_K(t_n) \approx \widetilde{\dot{z}}_K(t_n) := Y_n,$$

$$z_K(t) \approx \widetilde{z}_K(t) := X_{n-1} + (t - t_{n-1}) \cdot Y_{n-1}; \; t \in [t_{n-1}, t_n],$$

$$\int_0^{t_n} L_z(s, P(s), z_K(s), \dot{z}_K(s))\mathrm{d}s \approx I_n := \int_0^{t_n} L_z(s, P(s), \widetilde{z}_K(s), \widetilde{\dot{z}}_K(s))\mathrm{d}s.$$

In general, the construction of $\dot{z}_K$ must be carried out by constructing and successively concatenating the extremal arcs and boundary arcs until completing the interval $[0, T]$.

Step 2 Construction of a sequence $\{K_j\}_{j\in\mathbb{N}}$ such that $z_{K_j}(T)$ converges to b (the adapted shooting method).

Varying the coordination constant K, we would search for the extremal that fulfils the second boundary condition $z_K(T) = b$. The procedure is similar to the shooting method used to resolve a two-point boundary value problem (TPBVP). A number of methods exist for solving these problems, including shooting, collocation and finite difference methods. Among the shooting methods [15], the Simple Shooting Method (SSM) and the Multiple Shooting Method (MSM) appear to be the most widely known and used methods. We implemented a SSM and obtained good results. Effectively, we may consider the function $\varphi(K) := z_K(T)$ and calculate the root of $\varphi(K) - b = 0$, which may be realized approximately using elemental procedures. The secant method was used in the present paper, and the algorithm shows a rapid convergence to the optimal solution for a wide range of $K_{\min}$ and $K_{\max}$.

5. Application to a real hydrothermal system

A computer program was written (using the Mathematica package) to apply the results obtained in this paper to a real power system. As an example, we shall use the hydrothermal system that the electricity company HC [9] has in Asturias (Spain), which is made up of two classic thermal plants:

Aboño (with two groups of 360 and 543 of power (MW) respectively) and *Soto* (with two groups of 254 and 350 of power (MW), respectively) and nine hydro-plants. For our optimization problem, we shall only use the three variable-head (the generation is function of z and $\dot{z}$) hydro-plants of the utility company HC: Salime, Tanes (pumped-storage) and La Barca. We do not consider the remaining hydro-plants, because they are run-of-river type (without reservoir) and power generation is not controllable. Let us see the models of different subsystems used in our study. For the cost functions, we use a second-order polynomial. The data on the plants is summarized in Table 1. The units for the coefficients are: α_i in (€/h), β_i in (€/h.MW), γ_i in (€/h.MW^2) (we have included the cost of CO_2 emissions) and the technical limits for thermal generation $P_{i,\min}$ and $P_{i,\max}$ in (MW). We construct the equivalent thermal plant as we saw in ref. [2], obtaining:

$$\Psi(P(t)) = 11188.7 + 56.761P(t) + 0.0056812P(t)^2; \quad \text{with } 200 \le P(t) \le 1507.$$

The hydro-network has the three hydro-plants on different rivers, so the rate of discharge at the upstream plant does not affect the behaviour at the downstream plants: the hydraulic system has no hydraulic coupling. We use a variable head model and the i-th function of effective hydraulic generation H_i (for a conventional hydro-plant) is given by

$$H_i(t, z_i(t), \dot{z}_i(t)) = A_i(t)\dot{z}_i(t) - B_i\dot{z}_i(t)z_i(t) - C_i\dot{z}_i^2(t); \quad \dot{z}_i(t) \ge 0,$$

where $A_i(t)$, B_i and C_i are the coefficients: $A_i(t) = 1/G_i B_{y_i}(S_{0i} + t \cdot i_i)$; $B_i = B_{y_i}/G_i$; $C_i = B_{t_i}/G_i$. For the pumped-storage plant, H_i is defined piecewise, taking in the pumping zone:

$$H_i(t, z_i(t), \dot{z}_i(t)) = M \cdot \left[A_i(t)\dot{z}_i(t) - B_i\dot{z}_i(t)z_i(t) - C_i\dot{z}_i^2(t)\right]; \quad \dot{z}_i(t) < 0.$$

The parameters that appear in this formula are: the efficiency G in (m^4/h.MW), the natural inflow i in (m^3/h), the initial volume S_0 in (m^3), and the coefficients B_y in (m^{-2}) and B_t in (hm^{-2}). The data on the hydro-plants is summarized in Table 2. The minimum and maximum effective hydraulic generation $H_{\min}$ and $H_{\max}$ are in (MW) and the efficiency of Tanes hydro-plant is $M = 1.15$.

Let us consider the construction of the scenario structure for the day-ahead market problem faced by the company HC in the Spanish spot market. In particular, the market session of 15 February 2006 is considered as the current session. The past market sessions [6] that are considered relevant range from 1 February to 14 February. Table 3 presents the results of the clustering analysis performed on this range of days. The classification provided by the S-means algorithm for $S = 4$ (four clusters) is presented below.

Table 1. Coefficients of the thermal plants.

Plant i	α_i	β_i	γ_i	$P_{i\,\min}$	$P_{i\,\max}$
(Aboño 1)	3683.49	52.863	0.03975	50	360
2 (Aboño 2)	2231.34	62.526	0.00333	50	543
3 (Soto 2)	233.16	63.831	0.00858	50	254
4 (Soto 3)	4846.05	50.028	0.04977	50	350

Table 2. Hydro-plant coefficients.

Plant i	G	b	i	S_0	B_y	B_T	$H_{\min}$	$H_{\max}$
1 (Salime)	519840	6×10^6	133200	239.5×10^6	4.34079×10^{-7}	2.94×10^{-5}	0	112
2 (Tanes)	337542	5×10^6	21600	25.3×10^6	30.6555×10^{-7}	3.12×10^{-5}	-100	123
3 (La Barca)	363950	3×10^6	111600	25.2×10^6	26.1709×10^{-7}	2.35×10^{-5}	0	57.7

Table 3. Clustering analysis.

Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Day	W	Th	F	Sa	S	M	T	W	Th	F	Sa	S	M	T	W
Cluster	4	4	4	1	2	3	4	4	4	4	1	2	3	4	4

It has been obtained the four day types provided by the clustering analysis: Cluster 1 and Cluster 2 corresponds to low-price days (Saturdays and Sundays, respectively), Cluster 3 includes one type of weekday: Mondays, and Cluster 4 comprises the other type of weekdays. This analysis suggests considering eight scenarios (eight realizations) for the day-ahead market problem faced by the company on 15 February.

We consider short-term hydrothermal scheduling (24 h) with an optimization interval [0, 24] and we consider a discretization of 24 subintervals. The total optimal hydro and thermal power generation for the company HC are shown in Figure 1a and b respectively. The eight scenarios considered are presented in both figures.

The solution yields the optimal offers that the company must submit to each of the day-ahead market auctions. Figure 2a shows the offers corresponding to the fourth auction for the total optimal thermal-power, and for the eight possible realizations. The eight quantities and eight prices for that hour constitute the offer curve (nondecreasing) that the company must submit to that auction.

These results can be easily analyzed. Figure 2a shows that the offer curve obtained for the 4th hourly auction is quite flat, thus making the company rather uncertain about the amount of energy that it will finally sell. This is confirmed by Figure 1b, in which the company's eight possible levels of sales for the 4th hour are very different. However, it is not possible to construct an offer curve (nondecreasing) for the company's optimal hydro-power. Figure 2b shows the offers

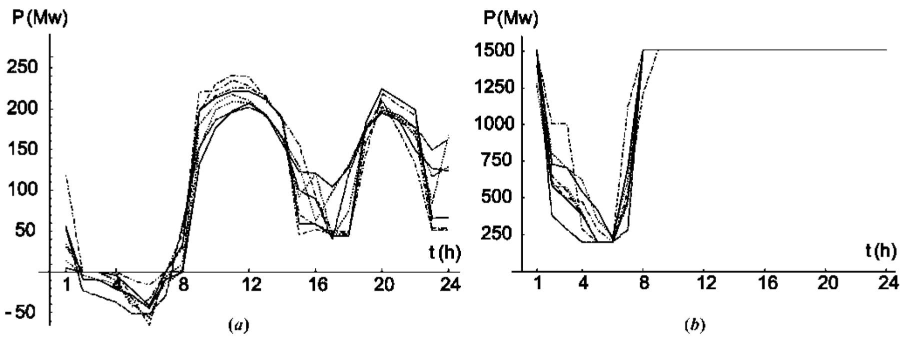

Figure 1. (a) Optimal hydro-power. (b) Optimal thermal-power.

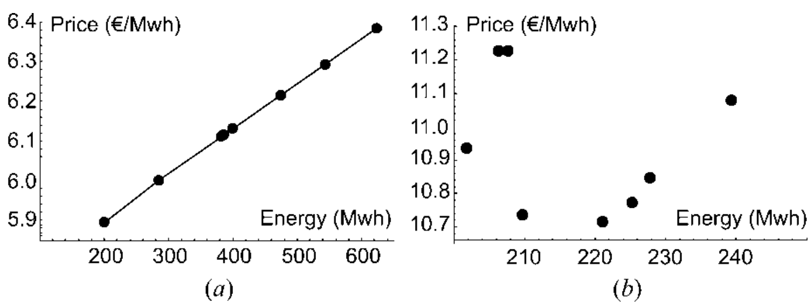

Figure 2. (a) Thermal-offers. (b) Hydro-offers.

corresponding to the 12th auction for the total optimal hydro-power, and for the eight possible realizations. It is easy to understand that this behaviour is due to the inter-temporal constraints for the hydro-plants, besides the pumped-storage character of some of the hydro-plants (the optimal hydro-solution of one of the auctions influences the rest of the auctions). Therefore, we suggest that the optimal offers that the company must submit, for the hydro-plants, must be a half value of the optimal hydro-power generation that we present in Figure 1a.

6. Conclusions

In this paper, we have proposed a new formulation of the single-firm optimization problem that is valid under deregulation and which constitutes, to our knowledge, the first work in the literature in which the Nonsmooth Maximum Principle is applied to this hydrothermal problem. The advantage of this infinite-dimensional technique compared to previous ones lies in the possibility of obtaining theoretical results whose implementation is feasible regardless of the size of the problem. The power generation system of the marketplace has been modelled to a high degree of detail, paying special attention to the hydraulic subproblem, including pumped-storage plants and variable head plants. Our model takes into account uncertainty on the spot market price of electricity. The solution of this problem allows us to derive optimal offers for a generation company. The approach is suitable for real-size systems, as shown in the example.

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