International Journal of Computer Mathematics, 2013
Vol. 90, No. 10, 2147–2160, http://dx.doi.org/10.1080/00207160.2013.792922

Real-time optimization of wind farms and fixed-head pumped-storage hydro-plants

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(*Received 7 September 2012; revised version received 10 January 2013; accepted 30 March 2013*)

Renewable energies and, in particular, wind power have come to the forefront in the electricity market in recent years. The main drawback of wind power generation, however, is the major difficulty in forecasting its production. For this reason, when wind farms go to the market, they are very often made to pay penalties for the deviations between forecasting and actual production. In this paper we shall analyse whether real-time compensation of wind power plant deviation penalties is profitable by means of the coordinated optimization of the wind power plant with a large pumped-storage hydro-plant. We shall model the pumped storage plant in great detail and make use of optimal control techniques to carry out the optimization. We shall analyse the most relevant recent papers on the subject and compare them with our technique. We shall also analyse another possible solution based on compensation carried out a posteriori, instead of in real time.

Keywords: optimal control; pumped storage plant; wind farm

2010 AMS Subject Classifications: 49J52; 49M05

1. Introduction

Currently, all countries are making a major effort to increase their pool of renewable energies. Environmental motivations are logically accompanied by those of an economic nature. Within the framework of renewable energies worldwide, wind power occupies a preferential place. In this context, at the end of 2011 Spain was the fourth country in the world in terms of wind power facilities, after the USA, Germany and China. According to the Spanish Wind Energy Association (Spanish acronym, AEE), this type of energy covered 15.75% of the country's electricity demand in 2011. Wind power was the third technology in the electricity system, after gas and nuclear power, and even occupied first place in March 2011.

A major part of this boom is due to the new regulations [21] that allow wind farms to go to the market to sell the energy generated by their facilities. If wind farms offer in the pool, they will prepare their offers and schedule their power production. However, a major problem exists: the unpredictability of wind farm production. Forecasting errors lead to the wind farm incurring financial losses, known as deviation penalties. An excellent review of the history of wind power short-term prediction can be seen in [8,12]. In the present paper, we shall not study forecasting, but shall rather attempt to mitigate its errors. When faced with this situation, wind farms have several available options: they can try to offer on the intraday spot markets; they can pay the

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associated penalties; or they can try to store wind power energy in some way. Diverse methods have also been proposed to store this energy [22]. In this paper we focus on combined use of a wind farm with pumped-storage plants.

Some authors have investigated the economic viability of the operation of a wind park cooperating with a micro pumped-storage hydro-plant, like, for example, [14,15]. In [4,5], a mini hydro-power plant is considered in the Portuguese market. Considerations about the optimal size of the wind farm and the hydro-pumped storage plant were analysed in [7]. A similar study is reported in [1], in which the plant consists of a wind farm and a pumped-storage unit, which absorbs almost the entire wind production to elevate water. The particular situation that island systems present, where meeting demand is the priority, is studied, for example, in [3]. Other studies such as Matevosyan and Soder [16] and Matevosyan *et al.* [17] focus on avoiding congestion on the adjacent transmission lines in areas with limited export capability.

Previous studies exclusively employ storage ability to compensate for wind power imbalances. However, this approach is not representative for large pumped-storage plants in power systems, which is the case we shall study in this paper. One of the techniques used for large pumped-storage plants [19,20] is to calculate the optimal amount of spinning reserve that the system operator should provide so as to be able to respond to errors in forecasts. In [13], the hydro-plant offers a reserve to a wind power producer for managing power imbalances. These authors simplify the problem assuming that the intervals of generation and pumping of the hydro-plant obtained in the base schedule are respected when operating jointly. The combined operation of wind farms and a pumped-storage hydro-plant is also analysed in [11]. A number of simplifications are introduced in the hydraulic problem: the natural inflows in the reservoirs are not considered and the net head dependency of the production is treated in a simplified way.

The present paper aims to calculate the optimal operation of a large pumped-storage plant, simultaneously pursuing two goals: to maximize revenue in conventional operations in the day-ahead market and to coordinate with the wind power producer with the aim of partially compensating for wind power imbalances. In this paper we shall consider a large capacity pumped-storage working jointly with a wind farm adjacent to its facilities. We shall consider them to be a single unit (a *wind-hydro power plant*), as it should be borne in mind that, according to the rules of the Spanish electricity market, different units cannot make joint offers. Moreover for a large capacity reservoir, it is practical to assume that the effective head is constant over the optimization interval and here the fixed-head hydro-plant model is defined.

Two different joint configurations for the resulting joint-unit formed by the pumped-storage plant and the wind farm are considered. In the first (un-coordinated operation), the pumped-storage plant does not compensate for the errors due to forecasting wind power. In the second (coordinated operation), we shall attempt to compensate for these errors in real time. We shall see in this paper that the fact that the pumped-storage plant is a fixed-head plant will mean that the optimal solution is of a very special type: bang-singular-bang. This will have crucial consequences in coordinated operation and we shall present a qualitative study of the real-time compensation of forecasting errors.

The following assumptions are made in this paper: the spot prices are deterministic, the planning is based on the price taking assumption, only the planning for the daily market is considered, and we shall not take into account offers in the intraday spot markets. The main contributions of this study with respect to previous papers are that the optimization is performed in a realistic market environment and the optimization algorithm provides the optimal bids that the storage plant should submit to the day-ahead market. Furthermore, we present a qualitative study on the compensation of forecasting errors in real time.

The paper is organized as follows. First of all, Section 2 presents the description of the problem and a model overview. The mathematical optimization is then presented in Section 3. In Section 4, we carry out a qualitative study to demonstrate whether, when faced with a deviation in wind power generation, it may be of interest to the pumped-storage plant to offset the wind power deviation penalties to obtain a higher joint profit. In view of the result obtained in this study, we shall propose a second solution in Section 5: to employ the over-generation deviations of the wind power plant to pump water into the upper reservoir of the pumped-storage plant, thus increasing profits. A realistic example case is then presented in Section 6. Finally, concluding remarks are given in Section 7.

2. Problem description and model overview

Spanish activity regulations [21] have been used as a reference model for the market. The day-ahead market in the Spanish wholesale electricity market is organized as a set of 24 simultaneous hourly auctions. The simple bid format consists of a pair of (hourly) values: quantity $q(\mathrm{MW}h)$ and price $p(\mathrm{euro}/\mathrm{MW}h)$.

The problem we shall solve is the one faced by a wind-hydro power plant when preparing its offers for the day-ahead market. Hydro power and wind power planning for the coming day is assumed to be performed at 10 am the day before. This basic scheduling, with plants working independently, is based on the volume of water $b(\mathrm{m}^3)$ that must be used and on the best forecast of wind power generation available each hour $W^{\mathrm{f}}(t)(MW)$. Unfortunately, wind power forecasts within a 14–38 h time horizon (which is the time horizon we must manage, as we close our offers at 10 a.m. the day before) are usually highly inaccurate and hence incur deviation penalties.

In this paper, two different joint configurations for the resulting joint-unit formed by the pumped-storage plant and the wind farm are considered. In the first, the pumped-storage plant does not compensate for the errors due to forecasting wind power, i.e. un-coordinated operation. In the second, we shall attempt to compensate for these errors in real time, i.e. coordinated operation. We shall assume that minutes before the actual operation we have precise knowledge of the wind (the persistent model is virtually insuperable a few minutes before the time horizon) and hence there is no uncertainty.

As regards the pumped-storage plant, we shall model it in great detail without any additional simplifications. For a large capacity reservoir, the effective head is constant over the optimization interval and we define here the fixed-head hydro-plant model. In plants of this type, the active power generated, $P(MW)$, is represented by the linear equation: $P(z'(t)) = Az'(t)$, where A represents the efficiency and diverse parameters related to the geometry of the hydro-plant [9], and $z'(\mathrm{m}^3/\mathrm{s})$ is the rate of water discharge. Taking into account the conversion losses of the pumping process, we must therefore introduce the efficiency, η, in the model. The function P is thus defined piecewise as:

$$P(z') := \begin{cases} A \cdot z' & \text{if } z' \geq 0 \\ \eta \cdot A \cdot z' & \text{if } z' < 0 \end{cases} \tag{1}$$

with $\eta > 1$. We consider $z'(t)$ to be bounded by technical constraints:

$$q_{\min} \leq z'(t) \leq q_{\max}, \quad \forall t \in [0, T] \tag{2}$$

and we assume that b is the volume of water that must be discharged over the entire optimization interval $[0, T]$, so:

$$z(0) = 0, \quad z(T) = b. \tag{3}$$

3. Optimization of fixed-head pumped-storage plant

In a previous paper [2] by the authors, we presented an algorithm that allows the optimal solution of a fixed-head pumped-storage plant to be obtained. The objective function is given by hydraulic profit over the optimization interval, $[0, T]$. Profit is obtained by multiplying the hydraulic production of the pumped-storage hydro-plant by the clearing price, $\pi(t)$, at each hour, t. An optimal control problem can thus be mathematically formulated as follows:

$$\begin{gathered} \max_{(u,z)} \int_0^T L(t, z(t), u(t))\,\mathrm{d}t = \max_{(u,z)} \int_0^T \pi(t)P(u)\,\mathrm{d}t, \\ z' = u; \quad z(0) = 0, \quad z(T) = b; \quad u_{\min} \le u(t) \le u_{\max}. \end{gathered} \tag{4}$$

A standard Lagrange type optimal control problem can be formulated as:

$$\max_{u(t),z(t)} \int_0^T L(t, z(t), u(t))\,\mathrm{d}t \tag{5}$$

subject to satisfying:

$$z'(t) = f(t, z(t), u(t)); \quad z(0) = z_0; \quad z(T) = z_T \tag{6}$$

$$u(t) \in U(t), \quad 0 \le t \le T, \tag{7}$$

where L is an objective function, $z = (z_1(t), \ldots, z_n(t)) \in \mathbb{R}^n$ is the state vector, with initial conditions z_0 and final conditions z_T, $u = (u_1(t), \ldots, u_m(t)) \in \mathbb{R}^m$ is the control vector, U denotes the set of admissible control values, and t is the operation time that starts from 0 and ends at T. Let H be the Hamiltonian function associated with the problem

$$H(t, z, u, \lambda) = L(t, z, u) + \lambda \cdot f(t, z, u), \tag{8}$$

where $\lambda = (\lambda_1(t), \ldots, \lambda_n(t)) \in \mathbb{R}^n$ is called the costate vector. The classical approach involves the use of Pontryagin's Minimum Principle, which results in a two-point boundary value problem. In order for $u \in U$ to be optimal, a non-trivial function λ must necessarily exist, such that for almost every $t \in [0, T]$:

$$z' = H_\lambda = f; \quad z(0) = z_0; \quad z(T) = z_T, \tag{9}$$

$$\lambda' = -H_z, \tag{10}$$

$$H(t, z, u, \lambda) = \max_{v(t)\in U} H(t, z, v, \lambda). \tag{11}$$

We now consider the case of control appearing linearly. The Hamiltonian is linear in u and the optimality condition (maximize H w.r.t. u) leads to:

$$u^*(t) = \begin{cases} u_{\max} & \text{if } H_u > 0, \\ u_{\text{sing}} & \text{if } H_u = 0, \\ u_{\min} & \text{if } H_u < 0. \end{cases} \tag{12}$$

The function $\Phi(z, \lambda) \equiv H_u$ is called the switching function. The times when the solution switches from $u_{\max}$ to $u_{\min}$ or vice-versa are called switch times.

For the optimal control Problem (4), we define the Hamiltonian in normal form:

$$H(t, z, u, \lambda) := L(t, z, u) + \lambda u = \pi(t)P(u) + \lambda u \tag{13}$$

and the resulting Hamiltonian, H, is linear in the control variable, u. It is well known [23] that when the Hamiltonian is linear in u, the optimality condition leads to the optimal u^* being undetermined if the switching function $\Phi(z,\lambda) \equiv H_u = 0$. An added complication arises in our problem: the Hamiltonian is defined piecewisely and the derivative of H with respect to u (H_u) presents discontinuity at $u = 0$. When non-differentiable objective functions arise in optimization problems, the generalized (or Clarke's) gradient [7] must be considered. Based on the above theoretical results, in [2] we determined the *bang-singular-bang* (*b-s-b*) *optimal solution*:

$$u^*(t) = \begin{cases} u_{\max} & \text{if } A \cdot \pi(t) > -\lambda_0, \\ u_{\text{sing}} = 0 & \text{if } -\lambda_0 \in [A \cdot \pi(t), \eta \cdot A \cdot \pi(t)], \\ u_{\min} & \text{if } \eta \cdot A \cdot \pi(t) < -\lambda_0. \end{cases} \tag{14}$$

The optimization algorithm in [2] comprises the following steps: First, $\pi(t)$ must be interpolated to obtain a continuous function. A piecewise linear interpolation has been used in this work. Second, for a given λ, we have to determine the switching times: $t_1, t_2, \ldots$ These instants are calculated solving the equations

$$A \cdot \pi(t) = -\lambda; \quad \eta \cdot A \cdot \pi(t) = -\lambda. \tag{15}$$

Finally, the optimal value λ_0 must be determined in order for: $z_\lambda(T) = b$. To calculate an approximate value of the optimal value λ_0, we propose an iterative method (like, e.g. bisection or the secant method). In this paper the secant method was used to calculate the approximate value of λ for which

$$\text{Error} = |z_\lambda(T) - b| < \text{tol}. \tag{16}$$

The secant method has provided satisfactory results using these initial values:

$$\lambda^{\min} = \min A\pi(t); \quad \lambda^{\max} = \max \eta A\pi(t). \tag{17}$$

The aforementioned algorithm interpolates $\pi(t)$ and works with a continuous function. Thus, by adjusting the switching times, it is capable of achieving the final volume b to discharge with the desired precision.

However, generating companies must in fact present offers in the day-ahead market for each of the 24 h of the following day. This means that the aforementioned algorithm cannot be applied in the form it was developed. We therefore propose the following modification in this paper: we shall convert a continuous variable into a discrete variable. We shall lose an essential feature in this conversion: we shall no longer be able to achieve any final volume of water precisely. In fact, the volume discharged in the b-s-b solution must belong to the set of M possible values: $\Omega = \{b_1, b_2, \ldots, b_M\}$. We shall calculate this set Ω by simply performing a sweep of the variable λ in the aforementioned algorithm, considering

$$\lambda_{\min} = \min[A \cdot \pi(t)], \quad \text{and} \quad \lambda_{\max} = \max[\eta \cdot A \cdot \pi(t)] \tag{18}$$

thereby obtaining the set of b-s-b solutions for the given plant and for the stated price, $\pi(t)$. The plant operator therefore only needs choose in $\Omega = \{b_i\}_{i=1}^M$ the nearest value, without exceeding the available volume, b ($b_{\text{sol}} < b < b_{\text{sol}+1}$). In this case, b_{sol} is the discharged volume corresponding to the optimal b-s-b solution.

4. Qualitative analysis of real-time optimization

In view of the above results, we shall now analyse the influence of the fact that the solution for the pumped-storage plant is b-s-b on the decisions to be taken in real time optimization. Let us assume we have obtained the solution for a certain λ_{sol} (calculated by aiming at a certain final volume, b_{sol}). We know the price, $\pi_{\text{turb}} = \lambda_{\text{sol}}/A$, above which it is of interest to discharge water; hence the instants at which the plant will discharge water are those whose price verifies: $\pi(t) > \pi_{\text{turb}}$. We know the price, $\pi_{\text{pump}} = \lambda_{\text{sol}}/(\eta \cdot A)$, below which it is of interest to pump water; hence the instants at which the plant will pump water are those whose price verifies: $\pi(t) < \pi_{\text{pump}}$. It is obvious that the instants at which the plant is not operating are those whose price verifies: $\pi_{\text{pump}} \leq \pi(t) \leq \pi_{\text{turb}}$. In view of these results, it is obvious that, between the instants of pumping (t_{pump}), stoppage (t_{stop}) and discharging water (t_{turb}), the following relations exist between the prices:

$$\pi(t_{\text{pump}}) < \pi(t_{\text{stop}}); \quad \pi(t_{\text{stop}}) < \pi(t_{\text{turb}}), \tag{19}$$

$$\pi(t_{\text{turb}}) > \eta \cdot \pi(t_{\text{pump}}). \tag{20}$$

Furthermore, two instants of stoppage, must verify that:

$$\pi(t^1_{\text{stop}}), \quad \pi(t^2_{\text{stop}}) \in \left[\frac{\lambda_{\text{sol}}}{\eta \cdot A}, \frac{\lambda_{\text{sol}}}{A}\right]. \tag{21}$$

When the plant operator prepares its offer for the day-ahead market for day F, this solution obtained for the pumped-storage plant, assuming the market prices and available water to be known, is the one that it will offer, seeing as it maximizes profits. The wind power plant will offer according to the best forecast for wind power production available at 10 h the day before, $F - 1$. However, when day F arrives, deviations will almost certainly be produced between the actual wind power production, $W^{\text{r}}(t)$, and the forecasted production, $W^{\text{f}}(t)$. In this context, we shall pose the following question: when faced with a deviation in wind power generation at the instant t, might it be of interest to the pumped-storage plant to modify its behaviour in real time (i.e. at t) so as to compensate for the deviation penalties of the wind farm and thus achieve a greater joint profit?

Let us assume in all cases that the deviations are against the system. Let us call $d(t) = W^{\text{r}}(t) - W^{\text{f}}(t)$ the deviation of the wind farm at the instant t, both for surpluses and shortages, and we shall analyse which of the two options is more profitable:

(1) Un-coordinated operation: assume deviation penalties without modifying how the system behaves, or
(2) Coordinated operation: modify, when possible, the behaviour of the pumped-storage plant to compensate in real time for the total deviation produced by the wind power plant.

In either of the two situations, we shall analyse the income or expenses resulting from the difference between the forecasted power and the actual power we take to the market; i.e. the deviation. Income will derive from an increase in power generation with respect to the forecast. Let us denote by $p^+(t)$ the price the market pays for the over-generation deviation (which will be a certain fraction s of the market price). Expenses will derive from a negative increase in power generation with respect to the forecast. In this case, we shall take into account the fact that we shall not be paid for the power we do not take to the market and that we shall also incur a penalty for not fulfilling what was agreed on. Let us denote by $p^-(t)$ the price we must pay for the under-generation penalty (which will be a certain fraction, l, of the market price).

We denote by $\mathbb{D}$ the income or expenses resulting from the deviation produced in the wind farm in the un-coordinated case, and by D the income or expenses resulting from the deviation in the coordinated case. It should be borne in mind that in the latter case the action taken on the pumped-storage hydro-plant at instant t will mean the fulfillment of the forecast of power sent to the market at that instant and a modification at another instant t^* of the behaviour of the plant, as we shall impose the condition that the volume employed, b_{sol}, is the same at the end of the interval. Furthermore, the modification at t^* must be planned without yet having any information on wind power generation, which we shall assume follows the established plan.

It is obvious that the modifications will be more profitable if the following condition is fulfilled:

$$\mathbb{D} < D. \tag{22}$$

Let us first consider that the deviation at instant t is due to surplus power, $d(t) > 0$. We shall analyse the two possibilities in detail:

(1) *Un-coordinated*. On this occasion, the deviation, $d(t)$, in wind power will produce income in this situation of:

$$\mathbb{D}^w(t) = p^+(t) \cdot d(t) = s \cdot \pi(t) \cdot d(t). \tag{23}$$

(2) *Coordinated*. Let us see the possible modifications to carry out on the hydro-plant, bearing in mind that, if it was already pumping, as the solution is of the b-s-b type, it will not be able to modify its behaviour. Hence, action may only be carried out to modify its behaviour if it was stopped or discharging water:

(a) If it was stopped, it will use the over-generation from the wind power plant, $d(t)$(MW), to pump water (at zero cost). The amount of water pumped at t which will then be used is: $d(t)/\eta \cdot A$. We must find an instant t^* at which it is of interest to the pumped-storage plant to discharge this water. At t^*, as the solution is of the b-s-b type, if it was discharging water, the turbines cannot be put to greater use. Hence, the hydro-plant will be able to act at t^* in only two cases:

(a1) If it was stopped and then started to discharge water, it would be paid for what it generates with a certain penalty:

$$D^h(t^*) = s \cdot \pi(t^*) \cdot \frac{d(t)}{\eta}. \tag{24}$$

Using Equation (22),we know that this action is profitable if:

$$s \cdot \pi(t) \cdot d(t) < s \cdot \pi(t^*) \cdot \frac{d(t)}{\eta} \Longrightarrow \pi(t^*) > \eta \cdot \pi(t). \tag{25}$$

However, this (with t and t^*: t_{stop}) is impossible by condition (21).

(a2) If it was pumping and then began to pump a lesser amount, seeing as it has at its disposal the water pumped at instant t, $d(t)/\eta \cdot A$, the income (equivalent to what we stop paying), though penalized for consuming less than the forecasted amount, is:

$$D^h(t^*) = s \cdot \pi(t^*) \cdot d(t). \tag{26}$$

By Equation (22), this modification will be of interest if:

$$s \cdot \pi(t) \cdot d(t) < s \cdot \pi(t^*) \cdot d(t) \Longrightarrow \pi(t^*) > \pi(t). \tag{27}$$

However, this (with t: t_{stop} and t^* : t_{pump}) is impossible by condition (19).

(b) If it was discharging water, it will produce less power to compensate for the over-generation of the wind farm, $d(t)$(MW). The amount of water that the hydro-plant ceases to consume at the instant t, due to having to generate less power, and which must then be used, is: $d(t)/A$. We need to find an instant t^* at which it is in the interest of the pumped-storage plant to consume this water. At t^*, action can once more be taken at the hydro-plant only if it is not operating or pumping:

(b1) If it was stopped and then started discharging water, it will be paid for what it discharges (which was not scheduled) incurring a certain penalty:

$$D^h(t^*) = s \cdot \pi(t^*) \cdot d(t). \tag{28}$$

By Equation (22), it will be of interest to do so if:

$$s \cdot \pi(t) \cdot d(t) < s \cdot \pi(t^*) \cdot d(t) \Longrightarrow \pi(t^*) > \pi(t). \tag{29}$$

However, this (with $t^* : t_{\text{stop}}$ and $t : t_{\text{turb}}$) is impossible by condition (19).

(b2) If it was pumping, it will pump a lesser amount and the income (equivalent to what it ceases to pay), though penalized for consuming less than scheduled, is:

$$D^h(t^*) = s \cdot \pi(t^*) \cdot \eta \cdot d(t). \tag{30}$$

Using Equation (22), this action will be profitable if:

$$s \cdot \pi(t) \cdot d(t) < s \cdot \pi(t^*) \cdot \eta \cdot d(t) \Longrightarrow \pi(t) < \eta \cdot \pi(t^*). \tag{31}$$

However, this (with $t^* : t_{\text{pump}}$ and $t : t_{\text{turb}}$) is impossible by condition (20).

We shall now consider that the deviation at the instant t is, by default, $d(t) < 0$.

(1) *Un-coordinated*. The default deviation at the wind power plant will result in not receiving the income corresponding to the amount of non-generated power and a penalty for not complying with the forecast. The expenses ($-$ sign) with respect to the forecast are:

$$\mathbb{D}^w(t) = -\pi(t) \cdot d(t) - p^-(t) \cdot d(t) = -(1+l) \cdot \pi(t) \cdot d(t). \tag{32}$$

(2) *Coordinated*. The hydro-plant will be able to act at t only if it was stopped or pumping:

(a) If it was stopped, it will discharge the default wind power, $d(t)$(MW). The amount of water discharged at t and which the hydro-plant will then have to cease using or recovering is: $d(t)/A$. We must find an instant t^* when it is not in the interest of the pumped-storage plant to use or recover this amount of water. The hydro-plant will only be able to act at t^* if it was stopped or discharging water:

(a1) If it was stopped and then starts to pump at the instant t^*, it will pay for what it pumps and also a certain penalty, as this action was not scheduled:

$$D^h(t^*) = -(1+l) \cdot \pi(t^*) \cdot \eta \cdot d(t). \tag{33}$$

By Equation (22), it will be of interest to do so if:

$$-(1+l) \cdot \pi(t) \cdot d(t) < -(1+l) \cdot \pi(t^*) \cdot \eta \cdot d(t) \Longrightarrow \pi(t) > \eta \cdot \pi(t^*). \tag{34}$$

However, this (with t and $t^* : t_{\text{stop}}$) is impossible by condition (21).

The remaining cases can be treated similarly, obtaining the same result in them all:

Conclusion: no real-time modification is of interest.

Note. It should be stressed that if the plant is of the variable-load and not of the fixed-head type, the above conclusions are not at all applicable, as the solution is no longer of the b-s-b type in this case.

5. A posteriori optimization of a wind-hydro plant

Subsequent to the above study, we posed the question as to whether it is possible to model the functioning of the wind-hydro power plant so as to operate in a coordinated manner a posteriori and thus improve profits. We shall not make real-time compensations for under-generation deviations in wind power. We shall however compensate for over-generation deviations in wind power. We shall attempt to use the surplus wind power generated on day F to pump water, thereby avoiding penalties for over-generation on day F, and subsequently use this water, b^*, in the hydro-plant by discharging it on the following day, $F+1$ (Figure 1).

We shall consider our hydro-plant as being able to function with a dual flow, pumping and discharging water at the same. Only in those cases in which the original schedule of the plant was to pump shall we find it impossible to act. Furthermore, as we are working for the day-ahead market, we shall eliminate all the uncertainty associated with the process.

The total profit B over the optimization interval $[0, T]$ is:

$$B = \int_0^T (\pi^{F+1}(t)P^{F+1}(t) + \pi^F(t)W^F(t) - C^F(t))\mathrm{d}t. \tag{35}$$

Profit B is revenue minus cost. Revenue is obtained by multiplying the hydraulic production, $P(t)$, and the wind power production, $W(t)$, by the clearing price, $\pi(t)$, at each hour, t. The sole cost in our system is the cost of deviation penalties $C(t)$. Accordingly, and in order for the comparison to be rigorous, wind power production is considered to be sold to the market on day F, and that of the hydro-plant on day $F+1$. We shall use superscripts to denote the day under consideration. In un-coordinated operation we shall have that $z(T) = b_{\mathrm{sol}}$. In the coordinated configuration, the profit obtained shall have to take into account the reduction in deviation penalties, $C(t)$, and the increase in the volume of water available: $z(T) = b_{\mathrm{sol}} + b^*$.

Note. It should be stressed that the new volume to consume, $z(T) = b_{\mathrm{sol}} + b^*$, will almost certainly not belong to the set of possible solutions $\Omega = \{b_1, b_2, \dots, b_M\}$. To adjust the precise volume, we shall seek the two consecutive solutions of Ω : b_n and b_{n+1} such that

$$b_n < b_{\mathrm{sol}} + b^* < b_{n+1}. \tag{36}$$

The difference between the two solutions (assuming there are no ties) is reduced to the hour the plant takes to stop pumping (when operating) or start pumping (when stopped). We would take

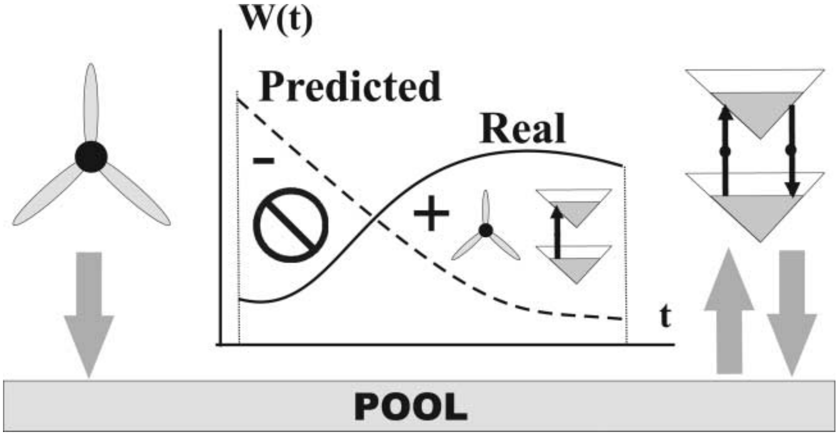

Figure 1. Configuration.

the unused volume, $b_{\text{sol}} + b^* - b_n$, and use it or stop pumping (depending on the case) in that hour it takes to change operation. Considering possible ties between times does not mean adding any complication to the proposed scheme. That is, the solution now obtained is no longer purely b-s-b; we shall call it *quasi-b-s-b*.

To illustrate the behaviour of this solution, we shall now consider an example of a wind-hydro power plant and compare the un-coordinated and coordinated configurations. We shall see that a profit may be obtained in the latter case.

6. Example

Spain is one of the few countries that requires the communication of forecasts for renewable energy generation. Furthermore, and this is exclusive to the Spanish case, the cost of deviations is charged to the producer. According to Spanish regulations, the sense of deviations in the system as a whole and of each producer is a determining factor, as a penalty is only incurred when the senses of both deviations do not coincide (contrary deviations). In general, the cost of over-generation is greater than the cost of under-generation. It is usual (and fairly close to reality) to consider that the cost of contrary over-generation is obtained for $s = 0.6$, and hence: $p^+(t) = 0.6\pi(t)$ and that the cost of contrary under-generation is obtained for $l = 0.15$ and hence: $p^-(t) = 0.15\pi(t)$.

A program was written using the Mathematica package to apply the results obtained in this paper to an example of a wind-hydro power plant made up of one fixed-head pumped-storage hydro-plant and a wind farm. The hydraulic model consider $A = 0.0000253641$. We consider an efficiency $\eta = 1.25$ and a restriction on the volume $b = 15 \cdot 10^6(\text{m}^3)$. We shall also consider the technical constraints: $q_{\min} = -1.41933 \cdot 10^6$ (m^3/h); $q_{\max} = 1.97129 \cdot 10^6$ (m^3/h). With the efficiency η, these constraints respectively correspond to $P_{\min} = -45$; $P_{\max} = 50$ (MW).

In this paper, we focus on the problem that a generation company faces when preparing its offers for the day-ahead market. Thus, the classic optimization interval of $T = 24$ h was considered. The clearing price (Table 1), $\pi(t)$ (euros/h $\cdot$ MW), corresponding to one day, was taken from the Spanish electricity market [18]. For the sake of simplicity, we shall also assume that the prices are equal on both days: $\pi^{F+1}(t) = \pi^F(t)$.

After the sweep of the variable λ in the aforementioned algorithm and calculating Ω, the closest value without exceeding the available volume b was $b_{\text{sol}} = 14.0356\ 10^6$ (m^3), with $b_{\text{sol}+1} = 16.0069\ 10^6$ (m^3). The optimal b-s-b solution obtained can be seen in Figure 2, while the profit obtained was 41144.5 (euros). As regards wind power generation, we shall consider a wind farm adjacent to the pumped-storage plant with a rated output of $W^n = 30$ (MW). Although the relative size of the two components of the wind-hydro power plant is important, we shall not address this aspect in detail in the present paper. The offer made by the power plant is based on the forecast of the previous day.

However, the forecasts provided by a short-term wind power prediction are uncertain. To model this uncertainty, a Beta probability density function (PDF) will be used, as proposed by several

Table 1. The clearing price, $\pi(t)$.

t	$\pi(t)$	t	$\pi(t)$	t	$\pi(t)$	t	$\pi(t)$
1	76.93	7	69.47	13	104.08	19	90.00
2	68.20	8	75.79	14	100.00	20	106.89
3	64.20	9	105.90	15	80.50	21	103.00
4	60.00	10	106.50	16	78.23	22	100.00
5	55.01	11	110.00	17	76.93	23	76.93
6	56.28	12	108.46	18	76.93	24	76.93

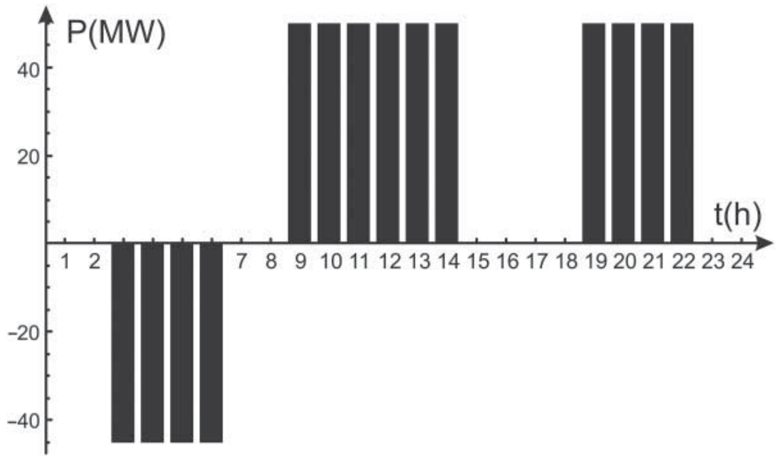

Figure 2. Optimal b-s-b solution.

authors [10]. Heuristic PDFs support this assumption, although this is still an open field for research. However, this is not the purpose of the present study. The analytic expression of the Beta PDF is:

$$f(x;\alpha,\beta) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \tag{37}$$

where $B(\alpha, \beta)$ is the Beta function and α, β are parameters related to the average of the distribution, μ, and the variance, σ^2. In our case, the average of the distribution will be the predicted power (p.u.) at the time of interest:

$$\mu = \frac{W^{\mathrm{f}}}{W^{n}}, \tag{38}$$

where W^{f} is the forecasted power, in MW, and W^n the rated output, in MW. The variance, σ^2, is calculated through the standard deviation, σ, which will depend, for each forecast horizon, on the power generated by the wind farm with respect to its rated output. This dependence has been obtained heuristically for some wind farms. Although there are wide variations, an approximation by means of a quadratic curve will provide realistic results. Moreover, this standard deviation, σ, increases with the forecast horizon, h. In the example, we shall consider the maximum value of the standard deviation for the instant $T = 24$ (i.e. 38 h after the forecast) to be:

$$\sigma = -0.79257\mu^2 + 0.77991\mu + 0.042078. \tag{39}$$

We now use the average, μ, and the variance, σ^2, to calculate the parameters α, β in the following way:

$$\alpha = \mu\left(\frac{\mu(1-\mu)}{\sigma^2} - 1\right); \quad \beta = (1-\mu)\left(\frac{\mu(1-\mu)}{\sigma^2} - 1\right). \tag{40}$$

One hundred Monte-Carlo simulations were performed considering the stochastic characteristics of the wind power. The problem was solved for each of these simulations. This enables us to obtain a view of the set of operational solutions considering the stochastic characteristics of the wind power. Figure 3 shows only 10 of the calculated probable scenarios of actual power generation $W^{\mathrm{r}}(t)$ (MW) and predicted wind power $W^{\mathrm{f}}(t)$ (MW). So as not to overcomplicate this study, we shall assume that all the deviations are against the system. To better understand the way the method works, we next present the results obtained in one of the scenarios: scenario 10 (Figure 3). The

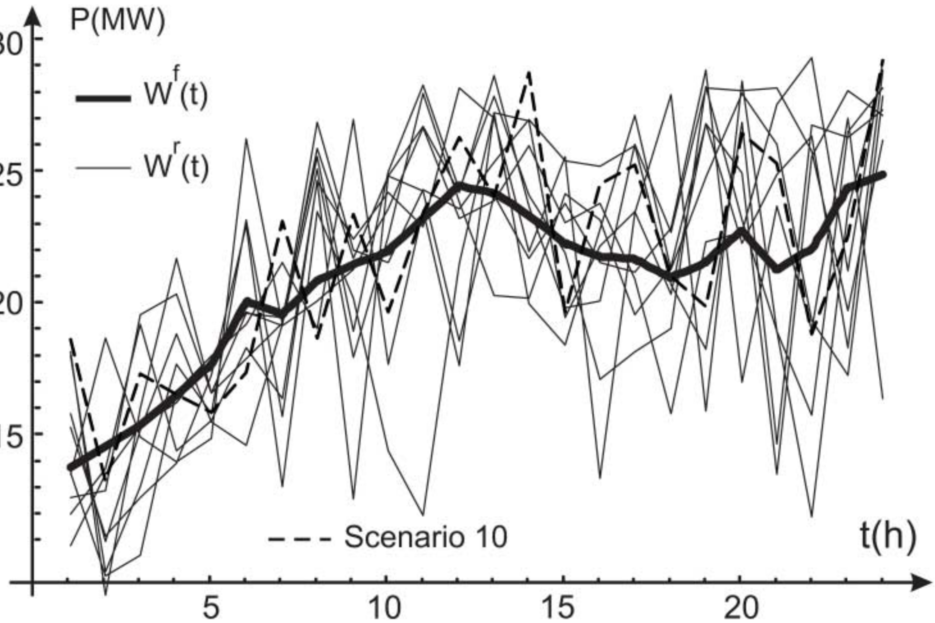

Figure 3. Possible scenarios.

Table 2. Comparison of profits (euros) in scenario 10.

	Un-coordinated	Coordinated
Hydraulic	41144.5	43472.6
Wind	43533.9	41603.4
Total	84678.3	85076

algorithm runs very quickly. For scenario 10, seven iterations were needed, the CPU time required by the program being 1.1 s on a personal computer (Pentium IV/2 GHz). The secant method was used to calculate the approximate value of λ for which

$$\text{Error} = |z_\lambda(T) - b| < \text{tol} \tag{41}$$

with tol $= 50$ (m^3).

The benefits of un-coordinated operation can be seen in Table 2. We shall now analyse the results obtained when operating in a coordinated way: the pumped-storage plant was able to assume the over-generation deviations of the wind farm produced at hours: 1, 7, 9, 11, 12, 14, 16, 17, 18, 20, 21 and 24; it could not offset those at hours 3 and 4 as its b-s-b solution (Figure 2) was already that of pumping. The total amount of water pumped at these instants was $b^* = 1.14025 \cdot 10^6$ (m^3). This water is used by the hydro-plant on day $F + 1$, producing at hour 15 (before it was stopped) a total of 28.92 (MW). Logically, the hydraulic power profit from the coordinated operation rose, while the wind power profit decreased as the over-generation was used to pump water. However, the overall sum of profits presents an improvement of 0.47% for coordinated operation.

The mean of the 100 scenarios shows an increase in profit of 0.41%. Although this figure does not seem very remarkable, the large number of factors that influence the result should be borne in mind: the deviation factor imposed by current legislation, the non-compensation of deviations by default, the impossibility of compensating for over-generation deviations if these coincide with instants of pumping and hence if the basic schedule includes pumping or not, being strongly related to the water available at the plant, etc.

One particular detail: in scenario 10, it suffices to change the penalty of $s = 0.6$ to $s = 0.4$ for the increase in profit to become 1.24%. This shows how important it is to have a flexible tool such as the one proposed here which is able to take into consideration the numerous variables that exert an influence and which allows us to obtain the solution easily. The results also reveal a possible strategy when preparing the offer: it may be of interest to always under-forecast, as over-generation deviations may be offset (in some cases).

7. Conclusions

In this paper we have presented a tool to design the optimal configuration of a wind-hydro plant. When the pumped-storage hydro-plant is of the fixed-head type, the bang-singular-bang solution presents a very notable feature: it is not profitable from an economic viewpoint to carry out any modification in real time of wind power deviations. However, a posteriori compensation of over-generation deviations on the part of the wind farm do produce a profit.

From both the economic point of view and that of the functioning of the electricity market, there are several reasons for designing wind-hydro power plants such as the one proposed in this paper. On the one hand, wind farms cause serious problems in the regulation of the electricity grid due to their unpredictability. A wind farm equipped with a system for accumulating power such as the one proposed here would be able to collaborate better in such regulation. On the other hand, the regulations governing the electricity market are continually changing. The best way to be prepared for these changes is to have an efficient, flexible schedule that allows the optimal solution to be obtained in any market scenario.

References

[1] J.S. Anagnostopoulos and D.E. Papantonis, *Pumping station design for a pumped-storage wind-hydro power plant*, Energy Convers. Manage. 48 (2007), pp. 3009–3017.

[2] L. Bayon, J.M. Grau, M.M. Ruiz, and P.M. Suarez, *Optimization of a pumped-storage fixed-head hydro-plant: The bang-singular-bang solution*, Math. Probl. Eng. 2011 (2011), pp. 1–11.

[3] C. Bueno and J.A. Carta, *Wind powered pumped hydro storage systems, a means of increasing the penetration of renewable energy in the Canary Islands*, Renew. Sustain. Energy Rev. 10 (2006), pp. 312–340.

[4] E.D. Castronuovo and J.A.P. Lopes, *Wind and small-hydro generation. An optimisation approach for daily integrated operation*, EWEA Conference, Madrid, Spain 2003.

[5] E.D. Castronuovo and J.A.P. Lopes, *On the optimization of the daily operation of a wind-hydro power plant*, IEEE Trans. Power Syst. 19 (2004), pp. 1599–1606.

[6] .D. Castronuovo and J.A.P. Lopes, *Optimal operation and hydro storage sizing of a wind-hydro power plant*, Int. J. Electr. Power Energy Syst. 26 (2004), pp. 771–778.

[7] F.H. Clarke, *Optimization and Nonsmooth Analysis*, Wiley-Interscience, New York, 1983.

[8] A. Costa, A. Crespo, J. Navarro, G. Lizcano, H. Madsen, and E. Feitosa, *A review on the young history of the wind power short-term prediction*, Renew. Sustain. Energy Rev. 12(6) (2008), pp. 1725–1744.

[9] M.E. El-Hawary and G.S. Christensen, *Optimal Economic Operation of Electrical Power Systems*, Academic Press, New York, 1979.

[10] A. Fabbri, T. Gomez San Roman, J. Rivier, and V.H. Mendez Quezada, *Assessment of the cost associated with wind generation prediction errors in a liberalized electricity market*, IEEE Trans. Power Syst. 20(3) (2005), pp. 1440–1446.

[11] J. Garcia-Gonzalez, R. de la Muela, L. Santos, and A. Gonzalez, *Stochastic joint optimization of wind generation and pumped-storage units in an electricity market*, IEEE Trans. Power Syst. 23 (2008), pp. 460–468.

[12] G. Giebel, R. Brownsword, G. Kariniotakis, M. Denhard, and C. Draxl, *The State of the Art in Short-Term Prediction of Wind Power*. Availale at http://www.prediktor.dk/.

[13] A. Jaramillo, E.D. Castronuovo, I. Sanchez, and J. Usaola, *Optimal operation of a pumped-storage hydro plant that compensates the imbalances of a wind power producer*, Electr. Power Syst. Res. 81 (2011), pp. 1767–1777.

[14] J.K. Kaldellis and K.A. Kavadias, *Optimal wind hydro solution for Aegean sea islands' electricity demand fulfilment*, J. Appl. Energy 70 (2001), pp. 333–54.

[15] J.K. Kaldellis, K.A. Kavadias, and D.S. Vlachou, *Electricity load management of APS using wind-hydro solution*, Wind Power 2002 Conference, Athens, Greece, 2002.
[16] J. Matevosyan and L. Soder, *Short-term hydropower planning coordinated with wind power in areas with congestion problems*, Wind Energy 10 (2007), pp. 195–208.
[17] J. Matevosyan, M. Olsson, and L. Soder, *Hydropower planning coordinated with wind power in areas with congestion problems for trading on the spot and the regulating market*, Electr. Power Syst. Res. 79 (2009), pp. 39–48.
[18] S.A. OMEL. Available at http://www.omel.es/.
[19] M.A. Ortega-Vazquez and D.S. Kirschen, *Optimizing the spinning reserve requirements using a cost/benefit analysis*, IEEE Trans. Power Syst. 22(1) (2007), pp. 24–33.
[20] M.A. Ortega-Vazquez and D.S. Kirschen, *Estimating the spinning reserve requirements in systems with significant wind power generation penetration*, IEEE Trans. Power Syst. 24 (2009), pp. 114–124.
[21] RD 661/2007. Available at www.boe.es
[22] E. Spahic, G. Balzer, B. Hellmich, and W. Munch, *Wind energy storages – possibilities*, Proc. Power Tech. 2007 IEEE Lausanne, 2007, pp. 615–620.
[23] R. Vinter, *Optimal Control, Systems & Control: Foundations & Applications*, Birkhäuser Boston, Inc., Boston, MA, 2000.