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# A comparative economic study of two configurations of hydro-wind power plants

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ABSTRACT

This paper presents a comparison between two options for operating a wind park in combination with a pumped-storage hydro-plant. First, we analyze the behavior of a wind farm that goes to the electricity market having previously forecasted the wind speed, while accepting the deviation penalties these forecasts may incur. Second, we study the possibility of the wind farm not going to the market individually, but as part of a hydro-wind power plant. Considerations about the optimal size of the wind farm and the hydro-pumped storage plant have previously been analyzed in the literature. However, most of these papers do not analyze dynamic considerations. The dimensioning of the system is not studied in our paper; its main feature will instead be the consideration of the dynamic optimal control problem. The use of Pontryagin's Maximum Principle allows us to obtain a very efficient optimization algorithm. The hydro-plant is modelled in great detail, using a variable-head model for the pumped hydro-plant. Our study provides a useful tool for electricity companies. The algorithm is able to solve the problem of the day-ahead market, a problem that is not solved in other studies.



## 1. Introduction

Wind energy has developed rapidly worldwide and Spain is actually a unique case within the global context. It is now, in fact, the foremost country in the world where wind power has become the leading electricity generating technology over an entire year (2013). At the end of 2012, the total capacity of existing wind farms in Spain exceeded 23,000 MW. This boom is mainly due to new regulations [1] that allow wind farms to sell the energy generated by their facilities on the market. If wind farms offer in the pool, they will prepare their offers and schedule their power production. However, a major problem exists: the unpredictability of wind farm production. Forecasting errors lead to the wind farm incurring financial losses, known as deviation penalties. Under the deregulated electricity markets, energy imbalance charges based on market prices provide appropriate incentives for accurate wind forecasting or for developing new techniques enabling wind power to be harnessed more efficiently, such as the technique proposed in this paper, namely hydro-wind power plants.

Pumped hydro energy storage is a well-known, commercially acceptable technology for electricity storage. Ref. [2] presents a review of existing pumped hydro energy storage capacities worldwide. Several authors have analyzed the economic viability of operating a wind park in combination with a micro pumped-storage hydro-plant. Ref. [3] presents the case of Aegean sea islands, while a mini hydro-power plant is considered for the Portuguese market in Ref. [4]. The very particular situation that isolated island systems present, where meeting demand is the absolute priority, is studied, for example, in Ref. [5]. Other studies, such as [6], focus on avoiding congestion on the adjacent transmission lines in areas with limited export capability. Studies that exclusively employ storage capacity to compensate for wind power imbalances are [7,8], in which the plant consists of a wind farm and a pumped-storage unit that absorbs almost the entire wind production to elevate water. When large pumped-storage plants are considered [9], uses a technique based on calculating the optimal amount of spinning reserve that the system operator should provide so as to be able to respond to errors in forecasts. In Ref. [10], the hydro-plant also offers a reserve for managing power imbalances. However, the problem is simplified assuming that the intervals of generation and pumping of the hydro-plant obtained in the base schedule are respected when operating jointly.

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Recently, several new operating algorithms have been put forward for the Wind Powered Pumped Storage System (WP-PSS). Ref. [11] presents an interesting study in which the dimensioning of the WP-PSS is performed with a software tool developed by the Wind Energy and Power Plants Synthesis Laboratory. In Ref. [12], the same algorithm for the WP-PSS is applied to an offshore wind park. Ref. [13] presents a study comparing the operating of a hybrid wind-hydro system including a conventional hydro-plant with a reservoir and a pumped storage hydro-plant. This comparative study was carried out based on the adaptation of HOMER software.

The aforementioned papers basically carry out an energy balance of the hybrid system under study. The dimensioning, siting and economic evaluation of the proposed system is obtained, but without dynamic considerations. This will be precisely one of the main features of our paper: the consideration of the dynamic optimal control problem. The dynamic problem is addressed in Ref. [14], where the optimization model is formulated as a two-stage stochastic programming problem. A number of simplifications are introduced in the hydraulic problem: natural inflows into the reservoirs are not considered and the net head dependency of production is treated in a simplified way. However, these approaches are not representative of medium pumped-storage plants in power systems, which is the case we shall study in this paper.

Focusing on the field of optimal control, only a few related papers can be found in the literature. Ref. [15] includes the development of an optimal control strategy and the model is solved via stochastic dynamic programming. Ref. [16] presents an optimal control strategy based on Dynamic Programming. The study requires a careful approach to system modeling aimed at reducing its complexity and the consequent computational load. In this respect, our paper presents several novelties: (i) the use of Pontryagin's Maximum Principle to obtain the corresponding optimization algorithm; (ii) the detailed modeling of the hydro-plant, using a variable head model, the most suitable one for the usual design size of a hydro-wind plant employed in Spain; and (iii) the algorithm is simple and efficient, allowing the incorporation of numerous parameters.

In two previous papers [17,18], the authors addressed the combined optimization of a pumped-storage hydro-plant and a wind farm. In both papers, the authors considered a fixed-head model for the pumped-storage hydro-plant. The model considers the relationship between the active power generated by a hydro-plant, $P$, and the rate of water discharge, $\dot{z}$, to be linear. This is the most suitable model when very large hydro-plants are considered, but is not the most suitable one for the usual design size of a hydro-wind plant employed in Spain.

Furthermore, weather forecasting was carried out in both papers with certain simplifications. In Ref. [18], we assumed that the deviations are a certain % of the wind power over the optimization interval and presented diverse cases, from low deviations (40% on average) to high deviations (80% on average), but did not carry out prior studies to calculate them. In Ref. [17], we modeled the uncertainty of wind power prediction using a Beta probability density function, performing several Monte-Carlo simulations and considering the stochastic characteristics of wind power.

In the present paper, we make three substantial improvements to the models cited above. First, we consider variable-head hydro-plants, a much more realistic model for the case in hand, with medium-sized hydro-plants. This change in model means that the mathematical techniques employed will be very different, giving rise to a different optimization algorithm. Second, we consider a different hydro-wind power plant configuration to those previously presented in Refs. [17,18]. We consider this configuration to be much more appropriate. Third, we do actually carry out a prior study in the present paper to obtain our own data on the errors that arise when forecasting wind power. One of the aims of this study is to evaluate the applicability of ARIMA models to the time series of hourly average wind speed. The analysis was carried out using data collected on an off-shore station located off Cape Peñas, Asturias (Spain) [19]. Moreover, we review the main functions available for time series analysis and forecasting included in the R language for statistical computing [20]. Bearing in mind all these considerations, the aim of the paper is to compare two methods of harnessing wind power.

The paper is organized as follows. Section 2 presents an introduction to several forecasting methods and describes the ARIMA model in detail. Section 3 comprises a brief presentation of the Spanish market, highlighting the role of wind power generators. In Section 4, we present the results obtained using the ARIMA models in the R package in several tests. Section 5 summarizes the optimization algorithm that leads to the optimal solution for the hydro-plant. We approach the problem as an Optimal Control Problem, using Pontryagin's Maximum Principle to obtain the corresponding optimization algorithm. Section 6 presents a comparison between the behavior of forecasting and a hydro-wind power plant. Finally, Section 7 presents the main conclusion of the paper.

## 2. Forecasting wind speed methods

High-precision wind speed forecasts are needed to use wind energy effectively. Many methods have been developed to address the problem of wind speed forecasting. Without any pretensions to being exhaustive, it is possible to divide these methods into three major categories. The first comprises physical methods, which use a considerable number of physical considerations to achieve the best forecasting accuracy. The second category is composed of statistical methods, like the ARIMA model, which aim to find the relationship with on-line measured data. Finally, there are hybrid models which are a mix of physical and statistical techniques. We refer the reader to the excellent papers by Refs. [21–24] reviewing the different methods used to forecast wind speed.

Focussing on statistical methods, a forecasting model based on time series aims to extrapolate the future values that a variable will take using the knowledge and analysis of the past values of the variable. This is the approach proposed in Box-Jenkins models [25]. These models can be divided as follows: the autoregressive (AR), moving average (MA), autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) model.

In recent years, some new studies based on ARIMA models have been conducted to forecast wind speed. A comparison between one-step predictions by ARIMA and decomposed ARIMA is made in Ref. [26]. H. Liu et al. [27] did not use the ARIMA model to forecast wind speed directly, but adopted it to choose the best parameters for Artificial Neural Networks (ANN) and Kalman components. H.P. Liu et al. [28] presented a hybrid ARMA-GARCH method to forecast a series of hourly wind speed data.

Let us briefly see what the ARIMA model comprises. Box and Jenkins [25] introduced the ARIMA ($p$,$d$,$q$) class of processes which have been applied ever since to a wide variety of time series forecasting applications. Basically, the forecast of wind speed in the ARIMA models depends not only on the values it has had in the more or less recent past according to the autoregressive component, but may also be a function of the residuals of past forecasts, corresponding to previous hours to the one being forecasted. The general methodology of the Box-Jenkins approach involves: model identification, parameter estimation and diagnostic checking followed by forecasting. Contreras el al. [29] provides an illustration of these steps in which ARIMA models are employed to predict next-day electricity prices.

### 2.1. Model identification

For a stationary time series, the ARIMA ($p$,$d$,$q$) model, which takes into account past data on wind speed, prediction errors and residuals, is represented as [25]:

$$x_t = \sum_{i=1}^{p} \varphi_i x_{t-i} - \sum_{j=1}^{q} \theta_j a_{t-j} + a_t \tag{1}$$

where $x$ is the time series of wind speed data and $x_{t-i}$ is the $(t-i)$th data, $a_t$ is a random time series of white noise (random uncorrelated variables with an average value of zero and variance $\sigma_a^2$), $\varphi_i$ are the autoregressive and $\theta_j$ the moving average parameters. The first order difference can be expressed as:

$$y_t = x_{t+1} - x_t \tag{2}$$

If $y_t$ is stationary, d = 1; otherwise, it needs to differ another $d-1$ times until stationarity is achieved.

### 2.2. Parameter estimation

The purpose of parameter estimation is to choose a suitable $p$ and $q$ in Eq. (1) to obtain the most accurate $x_t$. It can be roughly estimated by inspecting both the auto-correlated function (ACF) and the partial auto-correlated function (PACF). In this paper, however, we use the forecast package for R [30]. This package includes an automatic univariate forecasting method [31] via the *auto.arima {forecast}* function, which obtains the best fit ARIMA model to univariate time series.

### 2.3. Evaluation criteria

There are many error measures to test the efficiency of the forecast. Some authors use persistence models for the sake of comparison. The simplest way to forecast the wind is to use persistence. This method uses the simple assumption that the wind speed at time t + x is the same as it was at time $t$. Despite its simplicity, it is difficult to improve on the goodness of the prediction of this method in short-term forecasting horizons (less than 4 h) by means of other techniques.

Other typical measures of accuracy include RMSE, MAE, MAPE and NMAPE. We shall use MAPE. The mean absolute percentage error (MAPE) is defined as [32]:

$$MAPE = \frac{100}{n} \cdot \sum_{t=1}^{n} \left| \frac{A_t - F_t}{A_t} \right| [\%] \tag{3}$$

where $n$ is number of data, $A_t$ is the actual value and $F_t$ is the forecast value. The absolute value in this calculation is summed for every fitted or forecasted point in time and divided again by the number of fitted points, $n$. As we shall see, a key issue is that of knowing the time horizon our forecasts will need to cover. To do so, the next section describes how the Spanish electricity market works.

## 3. Spanish electricity market

The Spanish electricity market comprises a set of transactions organized in several sessions: the day-ahead and intraday markets, short-term trades, and the Technical Operating Procedures of the System. Seeing as a detailed explanation of how the market operates falls completely beyond the scope of this paper, we shall focus on those explanations which constitute the true subject of the paper: the *spot market* and *intraday market*, as these fix the forecast horizon. The reader is referred to [33] for a more detailed explanation.

The aim of the spot market is to carry out electricity transactions for the day after, $D$, via the issuing of offers to sell and bids to buy electrical power by market traders. The market operator matches the transactions and the spot market closing time is 12 h on day D − 1. Within the spot market process, a *feasible spot scheduling* is obtained by incorporating the necessary modifications for solving the technical constraints resulting from security considerations.

The aim of the intraday market, on the other hand, is to deal with adjustments to the *feasible spot scheduling*. All intraday market sessions take place after the spot market and enable the previous market programme to be adjusted with more up-to-date forecasts regarding generation or consumption. The reasons why traders may wish to change their generation or consumption forecast include faults in the grid or, as in the case under study here, a change in the forecast of wind power production. The intraday market is currently divided up into six sessions according to the time distribution shown in Table 1. As can be seen, the horizons that our wind forecasts need to cover range from the 12–36 hours, which the spot market has to cover, to the 3–12 h of the last intraday session.

Furthermore, it should be borne in mind that if wind farms offer in the pool, they will prepare their offers taking into consideration the unpredictability of wind farm production. Forecasting errors lead to the wind farm incurring financial losses, known as deviation penalties [1]. Let us now take a look at the functioning of the spot market and the intraday market.

The spot market in the Spanish wholesale electricity market is organized as a set of twenty-four simultaneous hourly auctions. The simple bid format consists of a pair of (hourly) values: quantity, q(t)(MWh), and price, π(t)(euro/MWh). We need to analyze the income or expenses resulting from the difference between the forecasted power and the actual power we take to the market; i.e., the deviation. Let us denote by π+(t) the price the market pays for the over-generation deviation (which will be a certain fraction $s < 1$ of the market price, $\pi$). In the case of under-generation, we shall take into account the fact that we shall not be paid for the power we do not take to the market and that we shall also incur a penalty for not fulfilling what was agreed on. Let us denote by $\pi^-(t)$ the price we must pay for the under-generation penalty (which will be a certain fraction $l < 1$ of the market price). It is usual (and close to reality) to consider that the cost of contrary over-generation is obtained for $s = 0.6$, and that the cost of contrary under-generation is obtained for $l = 0.15$, and hence:

$$\pi^+(t) = 0.6\pi(t); \; \pi^-(t) = 0.15\pi(t) \tag{4}$$

The operation of intraday markets should also be carefully analyzed. When wind power is involved in the intraday market in order to adjust the programmed power supply, this occurs in an increasingly closer time interval to the actual production time with the aim of avoiding any deviation. This allows forecasts to be further adjusted. However, participating in intraday markets involves adding the uncertainty of the new clearing price to the

**Table 1**
Intraday markets.

| Session | I 1 | I 2 | I 3 | I 4 | I 5 | I 6 |
|---|---|---|---|---|---|---|
| Opening | 17:00 | 21:00 | 01:00 | 04:00 | 08:00 | 12:00 |
| Close | 18:45 | 21:45 | 01:45 | 04:45 | 08:45 | 12:45 |
| Scheduling horizon | 27 h | 24 h | 20 h | 17 h | 13 h | 9 h |
| Hourly periods | (22–24) | (1–24) | (5–24) | (8–24) | (12–24) | (16–24) |

uncertainty of wind power, as it is necessary to ensure that the savings from decreasing the deviation penalty offset the sale of this energy at the new price resulting from the intraday market. That is, gains or losses are obtained in the final compensation in the intraday market not only in terms of wind forecasting, but also due to the difference in price between the day-ahead market and the intraday market sessions taking place at the same time. The problem, as we see it, is complex. Accordingly, we shall not take uncertainty in the market price into consideration when comparing the two proposed configurations in Section 6.

## 4. Numerical results of forecasting with the R package

Let us now see a summary of the results obtained using the forecasting functions implemented in the R package. We performed 3 different tests with data collected from the off-shore station located on a sea buoy off Cape Peñas (Asturias-Spain) [19].

(I) Forecast with a 1-h horizon

This first test focusses on forecasting the wind speed with a time horizon of 1 h. This has no real application to the case under study in this paper of the Spanish electricity market, as the minimum time horizon (for intraday 6) is 3 h, as we have already seen. We present it, however, for the sake of comparison with the other cases. 23 h from day D were used to calculate the speed at hour 24. 50 cases were analyzed, taken from the months of May and June 2011. The Mean Absolute Percentage Error (MAPE) of the 50 cases was: 16.82%.

This value may be considered very good according to the numerous comparative studies carried out recently. For example [32], compares four methods for wind power forecasting (an EPSO-based hybrid method, RBF neural network-based method, persistence method and back propagation neural network method). Values vary considerably depending on the season, ranging between 12.38%, obtained using the EPSO-based hybrid method, to 35.42%, obtained by the persistence method. Ref. [34] in turn presents a comparison of five methods: the persistence method, back propagation neural network (BPNN) method, compact wavelet neural network (cWnn) method, loose wavelet neural network (LWNN) method and a combination model named LCWNN. Values range from 36.65%, obtained using the persistence method, to 11.82%, obtained by the best method, LCWNN. Note that the comparison between forecasting made in different geographical areas is not very reliable, as the characteristics of the place exert a major influence. These results are hence provided for illustrative purposes only. As our data originates from an area where no previous studies have been carried out, further studies will be needed to obtain a more reliable comparison.

(II) Forecasting the average daily wind speed

In this second test, we forecast the average wind speed for day *D*, using the average speeds of the previous days, until day $D-1$, for this purpose. We thus cover the time horizons of the majority of intraday markets in Spain. Average wind speeds for every day of the month, from May to December 2011 (cases 1–8 resp.), were used, forecasting the average wind speed on the first day of the following month for each of these cases. The results are given in Table 2, where APE is the Absolute Percentage Error.

The MAPE of the 8 cases was: 46.47%. As can be seen, the results in this case are not good. We shall thus consider this way of forecasting unsuitable.

(III) Forecasting with a 24-h horizon

**Table 2**
Average daily wind speed forecast.

| Case | Forecast | Actual | Deviation | APE |
|---|---|---|---|---|
| 1 | 5.1 | 6.5 | 1.4 | 21.54 |
| 2 | 4.9 | 9.2 | 4.3 | 46.74 |
| 3 | 5.1 | 2.5 | 2.6 | 104 |
| 4 | 4.3 | 2.1 | 2.2 | 104.76 |
| 5 | 4 | 3 | 1 | 33.33 |
| 6 | 3.7 | 3.6 | 0.1 | 2.78 |
| 7 | 3.7 | 6.7 | 3 | 44.78 |
| 8 | 7.4 | 6.5 | 0.9 | 13.85 |

The aim of this third test is to forecast the wind speed for each of the 24 h of day *D*, using the data from the 24 h of day $D-1$. 30 cases from the same months mentioned above were used. Once again, the aim is to obtain forecasts that cover the time horizons of the intraday markets (at least 12–16). The results are given in Table 3.

It can be seen that the MAPE of the tests is highly variable, being too high on some days to be considered satisfactory. We also draw attention to the fact that, in 16 of the aforementioned cases, the automatically adjusted model corresponds to ARIMA (0,1,0), which corresponds to a random walk: $xt = xt - 1 + at$. Fig. 1 presents cases 3 and 4 so that the reader may to get an idea of the results obtained presented in R.

Finally, we also need to consider another fact: forecasts of wind power are derived from forecasts of wind speed. In traditional models, the power in the wind is proportional to the surface area of the rotor being swept by the wind, the cube of the wind speed, and the air density (which varies with altitude).

However, only a fraction of this power can actually be harnessed. For the purpose of simulating power generation, we need to consider the efficiency of the turbine-generator unit. The performance of a wind turbine is expressed by its power curve, which shows the relationship between the power output of the turbine at different wind speeds, from cut-in to cut-off. Bearing in mind that the efficiency of the turbine is a function of wind speed, the generated power is more or less proportional to the wind speed in its range from cut-in to cut-off [35].

We therefore pose the following question: Is it in the interest of wind farms to go to the market? To obtain the answer to this question, the following sections present the comparison with a hydro-wind power plant.

## 5. Hydro-wind power plant. Optimization of a variable-head hydro-plant

As stated in the Introduction, in this paper we shall improve the modeling of the hydro-plant by considering variable-head hydro-plants, a much more realistic model for the case of medium-sized hydro-plants. We shall also consider a new configuration for the

**Table 3**
Forecasting with a 24-h horizon.

| Case | MAPE | Case | MAPE | Case | MAPE |
|---|---|---|---|---|---|
| 1 | 24.55 | 11 | 41.32 | 21 | 70.01 |
| 2 | 5.57 | 12 | 31.7 | 22 | 28.21 |
| 3 | 21.23 | 13 | 19.79 | 23 | 18.92 |
| 4 | 17.74 | 14 | 34.51 | 24 | 39.27 |
| 5 | 24.14 | 15 | 44.34 | 25 | 21.62 |
| 6 | 51.3 | 16 | 18.55 | 26 | 67.97 |
| 7 | 52.31 | 17 | 22.13 | 27 | 19.54 |
| 8 | 31.6 | 18 | 45.74 | 28 | 16.24 |
| 9 | 66.1 | 19 | 35.85 | 29 | 12.6 |
| 10 | 10.44 | 20 | 18.56 | 30 | 12.15 |

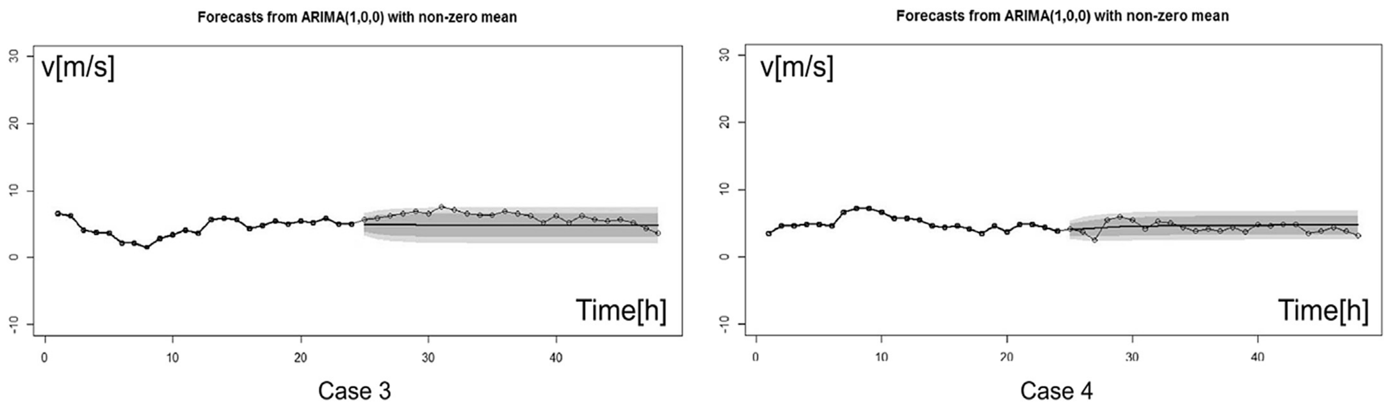


**Fig. 1.** Cases 3 and 4 of forecasting with a 24-h horizon.

hydro-wind plant. In Ref. [17], we only compensated for over-generation deviations in wind power. We used the surplus wind power generated on day $D$ to pump water, thereby avoiding penalties for over-generation on day $D$, subsequently using this water in the hydro-plant by discharging it on the following day, D + 1. However, under-generation penalties were not offset. In Ref. [18], we proposed a configuration in which the pumped-storage hydro-plant could choose to pump at its optimum operating configuration. This meant that, in those cases in which the original schedule of the hydro-plant was to pump, it would be impossible to take any action involving the wind farm.

### 5.1. Configurations

Fig. 2 shows the two configurations we shall be comparing. In configuration (a), the wind farm sells the energy it produces on the market. In configuration (b), the wind farm does not sell electricity on the market, but uses the generated power to pump water to the upper reservoir of the pumping station. To avoid the drawbacks of the configurations presented in Refs. [17,18], the hydro-plant in configuration (b) now only pumps water to harness wind power, but not when functioning optimally. We shall consider our hydro-plant to be able to function with a dual flow, pumping and discharging water at the same time.

### 5.2. Hydro-plant model

In a variable-head model (following [36]) the hydro-plant's hydraulic generation, $P$ in (MW), is a function of $z(t)$ in $(m^3)$, the volume discharged up to the instant $t$, and $\dot{z}(t)$ in $(m^3/h)$, the rate of

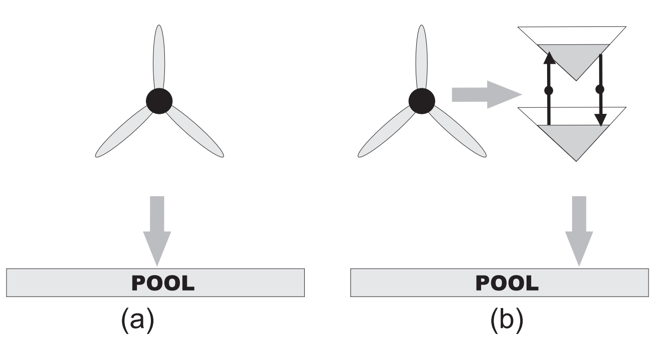


**Fig. 2.** Two configurations.

water discharge at the instant $t$, if $\dot{z} \geq 0$ :

$$P(t,z(t),\dot{z}(t)) := A(t)\cdot\dot{z}(t) - B\cdot z(t)\cdot\dot{z}(t) - C\cdot\dot{z}^2(t) \tag{5}$$

with:

$$A(t) = \frac{B_y(S_0 + i\cdot t)}{G};\ B = \frac{B_y}{G};\ C = \frac{B_T}{G} \tag{6}$$

The parameters that appear in this formula are: the efficiency, $G$, in (m$^4$/h. MW), the natural inflow, $i$, in $(m^3/h)$, the initial volume, $S_0$, in $(m^3)$, and the coefficients $B_y$ in $(m^{-2})$ and $B_T$ in (h m$^{-2}$), parameters that depend on the geometry of the reservoir. As just explained, to model optimal functioning we shall only admit non-negative volumes of water, $z(t) \geq 0$, and rates of water discharge, $\dot{z}(t) \geq 0$. That is, the plant behaves like a conventional power plant, with pumping simply being a tool to harness wind energy.

When the plant harnesses wind power to pump water, we need to take into account the conversion losses of the pumping process and must therefore introduce a *pump efficiency* factor, $\eta$, in the model. In general, a maximum of 75% or 85% of the electrical energy used to pump the water into the elevated reservoir can be regained, i.e., $\eta\in[1.15,1.30]$, approximately. Thus, when pumping water, the hydro-plant satisfies:

$$P(t,z(t),\dot{z}(t)) := \eta\cdot\frac{B_y S_0}{G}\cdot\dot{z}(t) \text{ if } \dot{z}<0 \tag{7}$$

as the influence of the natural flow, $i$, and parameters $B$ and $C$ relating to the volume of water vanish.

### 5.3. Statement of the problem and optimal solution

If we assume that $b$ is the volume of water that must be discharged during the entire optimization interval $[0,T]$, the following boundary conditions will have to be fulfilled:

$$z(0) = 0,\ z(T) = b \tag{8}$$

and we also consider $P$ to be bounded by technical restrictions:

$$P_{\min} \leq P(t,z(t),\dot{z}(t)) \leq P_{\max},\ \forall t\in[0,T] \tag{9}$$

Hence, the objective function faced by the hydro-plant is given by revenue during the optimization interval $[0,T]$, where revenue is obtained by multiplying the hydraulic production by the clearing market price, $\pi(t)$, at each hour $t$:

$$\max_z F(z) = \max_z \int_0^T L(t,z(t),\dot{z}(t))\mathrm{d}t = \max_z \int_0^T \pi(t)P(t,z(t),\dot{z}(t))\mathrm{d}t \quad (10)$$

To obtain the optimal solution, the problem is formulated in this paper within the framework of Optimal Control Theory [37]. We present the problem considering the control variable $u(t) = P(t,z(t),\dot{z}(t))$ and the state variable to be $z(t)$. The optimal control problem is thus:

$$\max_{u(t)} \int_0^T L(t,z(t),u(t))\mathrm{d}t \quad (11)$$

with:

$$\begin{cases} \dot{z} = f(t,z,u),\ \dot{z} \geq 0 \\ z(0) = 0,\ z(T) = b,\ z \geq 0 \\ u(t) \in \{x \mid P_{\min} \leq x \leq P_{\max}\} \end{cases} \quad (12)$$

The state equation $\dot{z} = f(t,z,u)$ can be explicitly defined by simply assuming that the function of hydraulic generation $P(t,z,\dot{z}) : \Omega_P = [0,T] \times \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is strictly increasing with respect to the rate of water discharge, $\dot{z}$, i.e., $P_{\dot{z}} > 0$. Real models meet this constraint: it means the higher rate of water discharge, the greater the power. Let us term the coordination function of $z \in \Omega$ the function in $[0,T]$, defined as follows:

$$\mathbb{Y}_z(t) = L_{\dot{z}}(t,z(t),\dot{z}(t)) \cdot e^{-\int_0^t \frac{P_z(s,z(s),\dot{z}(s))}{P_{\dot{z}}(s,z(s),\dot{z}(s))}\mathrm{d}s} \quad (13)$$

Using Pontryagin's Maximum Principle [37], the following theorem is easily proved (see, for example, [38]):

**Theorem**. *If $z^*$ is a solution of our problem, then $\exists K \in \mathbb{R}^+$ such that*:

$$\mathbb{Y}_{z^*}(t) is \begin{cases} \leq K & if\ P(t,z^*(t),\dot{z}^*(t)) = P_{\min} \\ = K & if\ P_{\min} < P(t,z^*(t),\dot{z}^*(t)) < P_{\max} \\ \geq K & if\ P(t,z^*(t),\dot{z}^*(t)) = P_{\max} \end{cases} \quad (14)$$

On the basis of this theorem, the optimization algorithm that leads to the determination of the optimal solution for the hydro-plant is easy to construct. To obtain the optimum operating conditions of the hydro-plant, we shall use the following coordination equation:

$$\mathbb{Y}_z(t) = K,\ \forall t \in [0,T] \quad (15)$$

The problem will thus consist in finding for each $K$ the function $z_K$ that satisfies $z_K(0) = 0$ and the conditions of the theorem, and from among these functions, the one that gives rise to an admissible function ($z_K(T) = b$). From the computational point of view, the construction of $z_K$ can be performed using the same procedure as in the shooting method, employing a discretized version of the coordination equation. The exception is that, at the instant when the values obtained for $z$ and $\dot{z}$ do not obey the constraints, we force the solution $z_K$ to belong to the boundary until the moment when the conditions of leaving the domain (established in the theorem) are fulfilled.

It should be stressed here that the result obtained in this paper by modeling the hydro-plant as a variable-load plant is totally different from that studied in Refs. [17,18], in which a fixed-load model was employed. As the control, $u$, is linear in the latter model, this leads to bang-bang type solutions and also to a different optimization algorithm.

## 6. Comparison between the two configurations

With the aid of the algorithm presented above, we are now in a position to analyze the combined optimization of a pumped-storage hydro-plant and a wind farm. In this section, we shall analyze whether it is in the interest of wind farms to go to the market. A program was written using the Mathematica package to apply the results obtained in this paper to an example of variable-head hydro-plant. The hydro-plant data ($G$ in ($m^4$/h. MW),i in ($m^3$/h) ), $S_0$ in ($10^8 m^3$), $B_y$ in ($10^{-8} m^{-2}$ and $B_T$ in ($10^{-7} hm^{-2}$)), corresponding to the previous model, are summarized in Table 4.

The final available volume in the hydro-plant, b($m^3$), will be function of the energy in (MW*h*) obtained from wind power, and the pump efficiency of the hydro-plant, $\eta$, and will be calculated next. We shall assume that the hydro-plant will discharge the water obtained from the natural watercourse, *i*, plus the water that is pumped. We shall also study the pump efficiency of the hydro-plant, $\eta$, considering it to vary over the interval $\eta \in [1.15,1.30]$, values close to reality.

The hydro-wind power plant must be properly designed; i.e., the size of the wind farm and hydro-plant should be similar. In this example, we have considered a maximum value of 45(MW). With these dimensions, the variable-load model is the most suitable model for the hydro-plant, as the generated power, *P*, depends not only on the rate of water discharge, $\dot{z}$, but also on the effective height [36], a ratio introduced in the model given by Eq. (5) via the volume of water, *z*. In this study, we shall therefore consider the following technical constraints: $P_{\min} = 0$; $P_{\max} = 45$.

In view of the rules governing the Spanish electricity market, an optimization interval of T = 24 h. was considered, with a discretization of 24 subintervals. There is no point in increasing the discretization, as the bids that must be submitted are hourly bids.

As we have seen, our problem depends on several variables. When presenting the results, we have fixed the two which have the least influence on the results: the market price, $\pi(t)$ and the actual wind production, $W^a(t)$. It is thus easier to analyze the two key variables when choosing the most suitable configuration: namely, the pump efficiency of the hydro-plant, $\eta$, which is a design value specific to each plant, and especially the deviation, $d_P$, between the actual, $W^a(t)$, and forecasted wind power production, $W^f(t)$.

Table 5 presents the clearing price, $\pi$(*euro*/*h* MW) (corresponding to one actual day from the Spanish market), the optimal hydro power, *P*(MW), in the particular case in which $\eta = 1.15$, (b = 1.9979,107 ($m^3$)) and the actual wind power production, $W^a$(MW), for t = 1,…,24 (h). Fig. 3 presents the actual wind power production, $W^a$, while Fig. 4 presents the clearing price, $\pi$, and the optimal hydro-power, *P*.

It should be stressed that, in configuration (b), we consider the water pumped thanks to wind power, *b*, on day *D* to be used in the hydro-plant on day D + 1. In this way, the problem of the unpredictability of the wind is avoided completely. Accordingly, and in order for the comparison to be rigorous, the wind power production in configuration (a) is considered to be sold to the market on

**Table 4**
Hydro-plant coefficients.

| $G$ | $i$ | $S_0$ | $B_y$ | $B_T$ |
|---|---|---|---|---|
| 319,840 | 133,200 | 2.395 | 2.89,386 | 2.94 |

**Table 5**
Wind power production, optimal solution ($\eta = 1.15$) and clearing price.

| $t$ | $W^a(t)$ | $P(t)$ | $\pi(t)$ |
|---|---|---|---|
| 1 | 13.9 | 0 | 76.93 |
| 2 | 14.7 | 0 | 68.20 |
| 3 | 15.5 | 0 | 68.20 |
| 4 | 16.6 | 0 | 60.00 |
| 5 | 17.8 | 0 | 55.01 |
| 6 | 20.2 | 0 | 56.28 |
| 7 | 19.7 | 0 | 69.47 |
| 8 | 21.0 | 0 | 75.79 |
| 9 | 21.6 | 43.35 | 105.90 |
| 10 | 22.1 | 43.97 | 106.50 |
| 11 | 23.4 | 45 | 110.00 |
| 12 | 24.6 | 45 | 108.46 |
| 13 | 24.3 | 39.49 | 104.08 |
| 14 | 23.5 | 32.86 | 100.00 |
| 15 | 22.4 | 0 | 80.50 |
| 16 | 21.9 | 0 | 78.23 |
| 17 | 21.8 | 0 | 76.93 |
| 18 | 21.1 | 0 | 76.93 |
| 19 | 21.7 | 13.98 | 90.00 |
| 20 | 22.9 | 43.31 | 106.89 |
| 21 | 21.4 | 37.66 | 103.00 |
| 22 | 22.2 | 32.99 | 100.00 |
| 23 | 24.5 | 0 | 76.93 |
| 24 | 25 | 0 | 76.93 |

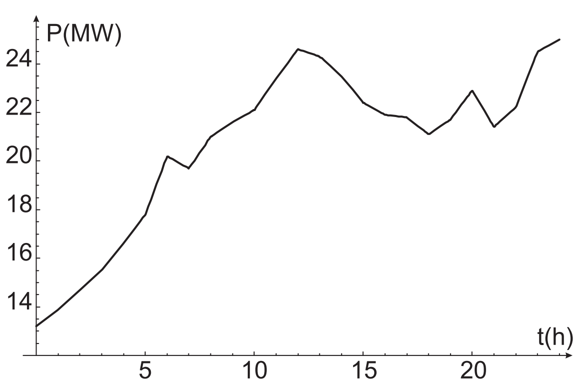


**Fig. 3.** Actual wind power production, $W^a$.

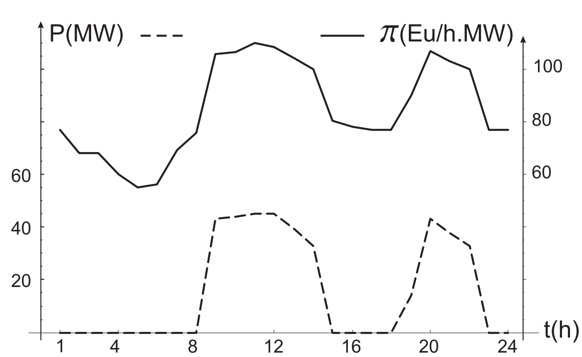


**Fig. 4.** Clearing Price and Optimal hydro-power, $P$.

day $D$, and we must assume that the prices are equal on both days: $\pi_{D+1}(t) = \pi_D(t)$.

The solution may be straightforwardly constructed taking into account the above theorem. The secant method was used to calculate the approximate value of $K$ for which:

$$z_K(T) - b = 0 \tag{16}$$

obtaining K = 0.0.001874403592475033. The algorithm runs very quickly, the CPU time employed for both models being 0.7 sec on a personal computer (Intel Core 2/2.66 GHz). The convergence of the numerical solution is also shown in Fig. 5. We achieve the prescribed tolerance: tol = 10−2, in only 9 iterations. For the convergence of the algorithm, the error has been considered as the difference (in absolute value) between the value of the available volume, $b(m^3)$, and the volume $z_K(24)(m^3)$ in each $K$-th iteration:

$$Error = |z_K(24) - b| < tol \tag{17}$$

Given the physical meaning of $b$, the considered value of $tol$ is completely realistic.

With this actual wind power production, $W^a$, the final available volume in the hydro-plant, $b(m^3)$, is a function of $\eta$, and we obtain the results presented in Table 6.

For the example presented here, the minimum constraint has an influence in all cases, as the available volume, $b$, is not sufficient for the hydro-plant to generate power at all times, working only at those times with the highest prices. However, the maximum constraint only exerts an influence in a few cases (as presented previously in Table 5 and Fig. 4), namely when the available volume, $b$, allows it to do so.

We are now in a position to present the influence of the pump efficiency of the hydro-plant, $\eta$, and the deviation, $d_W$, between the actual, $W^a(t)$, and forecasted wind power production, $W^f(t)$. To define the deviation, $d_W$, we shall consider the APE (Absolute Percentage Error) [32]:

$$APE = \frac{\left|W^a(t) - W^f(t)\right|}{W^a(t)} \cdot 100 \Rightarrow \mathrm{d}_W(t) = \frac{APE}{100} \tag{18}$$

from which the MAPE used previously follows straightforwardly. We shall denote by $b_W^+$ the profits obtained by the wind farm in configuration (a) in the case of over-generation. According to the rules of the Spanish market explained in Section 3, this is obtained straightforwardly in the case of over-generation, with a deviation, $d_W$:

$$W^f(t) = W^a(t) \cdot (1 - \mathrm{d}_W(t)) \tag{19}$$

$$b_W^+ = \sum_{t=1}^{24} \left[ W^a(t) \cdot (1 - \mathrm{d}_W(t)) \cdot \pi(t) + \mathrm{d}_W(t) \cdot W^a(t) \cdot \pi^+(t) \right] \tag{20}$$

We shall likewise denote by $b_W^-$ the profits obtained by the wind farm in configuration (a) in the case of under-generation, obtaining:

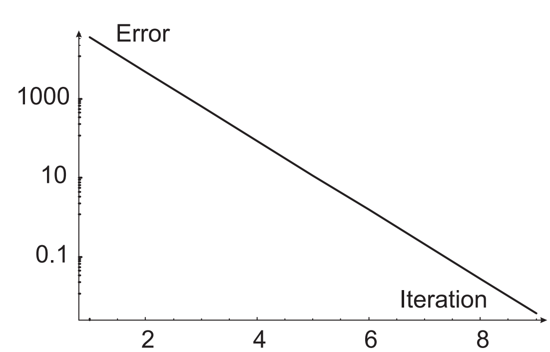


**Fig. 5.** Convergence of the algorithm.

**Table 6**
Final volume available.

| $\eta$ | 1.15 | 1.20 | 1.25 | 1.30 |
|---|---|---|---|---|
| $b(10^7 m^3)$ | 1.99,799 | 1.91,474 | 1.83,815 | 1.76,746 |

$$W^f(t) = W^a(t) \cdot (1 + \mathrm{d}_W(t)) \tag{21}$$

$$b_W^- = \sum_{t=1}^{24} \left[ W^a(t) \cdot \pi(t) - \mathrm{d}_W(t) \cdot W^a(t) \cdot \pi^-(t) \right] \tag{22}$$

where $\pi^+(t)$ and $\pi^-(t)$ are given by Eq. (4):

$$\pi^+(t) = 0.6\pi(t);\ \pi^-(t) = 0.15\pi(t) \tag{23}$$

Given $W^a$, these profits, $b_W^+$ and $b_W^-$, are exclusively a function of $d_W$. For the sake of simplicity, we shall assume, without loss of generality, that $\mathrm{d_W(t)} = \mathrm{d_W} \equiv$ cte throughout the entire interval [1,24]. In some cases [18], the authors simplify the study considering deviation penalties to be 27.5% of the clearing price on the day-ahead market (regardless of over- or under-generation). This approximation is reasonable, but only for a certain range of $d_W$. We shall accordingly consider the above formulations in this paper, which correspond to the actual market. We are thus able to analyze the different behavior of under- and over-generation in detail.

On the other hand, given the pump efficiency of the hydro-plant, $\eta$, we shall denote by $b_H$ (*euro*) the profits obtained by the hydro-plant in configuration (b). This value is obviously independent of $d_W$, as it is based on the available water, $b$, a function of the actual wind power production, $W^a$. The results obtained in different simulations are shown in Tables 7–9. The last table shows the relative gain $Ga$(%) between the different profits:

$$Ga = \frac{b_H - b_W}{b_W} \cdot 100 \tag{24}$$

As can be seen, under-generation is penalized less than over-generation: it is easier for the market operator (OMIE) to correct an under-deviation than an over-deviation. Hence, configuration (a) is better than (b) in most of the simulations, though with modest gains ($\simeq$1–10%). However, the result is the contrary in the case of over-generation; in addition to the fact that the gain in configuration (b) is also much higher ($\simeq$10–35%). Of course, the wind farm does not know the sense of its error in advance, otherwise it would correct it. As can be seen, both configuration options appear interesting, although the choice of the hydro-wind power plant offers more advantages under the most usual working conditions. The system is highly sensitive to numerous factors and each company must carefully assess particular situations that may result in variations in the optimal configuration.

## 7. Conclusions

Weather forecasting in general is of major economic and social interest, not only in the field of wind power, but also in other fields. These include crop improvement and care, forecasts related to tourism or road traffic flows, and even the prevention of natural disasters. In this paper, we have studied the application of ARIMA models for hourly wind speed forecasting. To this end, we have employed the R package and the auto.arima automatic adjustment function to choose the most suitable model parameters. The results show that ARIMA models seem to constitute a very suitable tool for very short-term forecasting (1–2 h). The R package is found to be an extremely useful tool within this context, allowing the less experienced user to carry out studies easily. However, ARIMA does not seem to be a very appropriate tool for the medium-term horizons (12–36 h) required within the context of the Spanish electricity market spot (day-ahead) and intraday markets. Moreover, the possibility of resorting to intraday markets does not seem to be the solution either, as their uncertainty is greater than that of the spot market. The uncertainty of the clearing price is now added to that of the wind, as it is necessary to ensure that the savings from decreasing the deviation penalty offset the sale of this energy at the new price resulting from the intraday market.

For all these reasons, the comparison with other methods of incorporating wind power in the electricity market seems reasonable. This is the context within which we have analyzed in this paper whether it is in the interest of wind farms to go to the market or not. There are a number of factors that influence the final result, such as: the pump efficiency of the hydro-plant, the volume of water available, deviation penalties, and wind power production. These can be summarized by concluding that both configuration options appear interesting, although the choice of the hydro-wind power plant offers more advantages under the most usual working conditions. The system is highly sensitive to numerous factors and each company must carefully assess particular situations that may result in variations in the optimal configuration. Our study provides a useful, simple and efficient tool for taking such a decision.

**Table 7**
Influence of $d_W$ on the profits obtained, $b_W^+$ and $b_W^-$.

| $d_W$ | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 |
|---|---|---|---|---|---|---|
| $b_W^+(euro)$ | 36,467 | 34,730 | 32,994 | 31,257 | 29,521 | 27,784 |
| $b_W^-(euro)$ | 40,808 | 40,157 | 39,506 | 38,854 | 38,203 | 37,552 |

**Table 8**
Influence of $\eta$ on the profit, $b_H$.

| $\eta$ | 1.15 | 1.20 | 1.25 | 1.30 |
|---|---|---|---|---|
| $b_H(euro)$ | 39,566 | 38,167 | 36,860 | 35,638 |

**Table 9**
Relative gain $Ga$(%): (i)with $b_W^+$; (ii) with $b_W^-$.

| $(i) d_W \backslash \eta$ | 1.15 | 1.20 | 1.25 | 1.30 |
|---|---|---|---|---|
| 0.40 | 8.49 | 4.66 | 1.07 | −2.27 |
| 0.50 | 13.92 | 9.89 | 6.13 | 2.61 |
| 0.60 | 19.91 | 15.67 | 11.71 | 8.01 |
| 0.70 | 26.58 | 22.10 | 17.92 | 14.01 |
| 0.80 | 34.02 | 29.28 | 24.86 | 20.72 |
| 0.90 | 42.40 | 37.36 | 32.66 | 28.26 |
| $(ii) d_W \backslash \eta$ | 1.15 | 1.20 | 1.25 | 1.30 |
| 0.40 | −3.04 | −6.47 | −9.67 | −12.6 |
| 0.50 | −1.47 | −4.95 | −8.20 | −11.2 |
| 0.60 | 0.15 | −3.38 | −6.69 | −9.78 |
| 0.70 | 1.83 | −1.77 | −5.13 | −8.27 |
| 0.80 | 3.56 | −0.09 | −3.51 | −6.71 |
| 0.90 | 5.36 | 1.63 | −1.84 | −5.09 |

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