Mathematical modelling of the combined optimization of a pumped-storage hydro-plant and a wind park

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1 Introduction

The new Spanish regulations (BOE [1]) allows wind farms to go to the market to sell the energy generated. In the case of over- or undersupply, other producers must reduce or increase their production to resolve the so-called deviation, thereby incurring financial losses. Faced with this situation, wind farms have several options. In this paper we consider one promising method: the combined optimization of a pumped-storage hydro-plant and a wind park and we present a tool to design the optimal configuration.

2 Mathematical optimization

When pumped-storage plants are considered, the fixed-head hydro-plant model for the active power generated P is defined piecewise as:

$$P(z') := \begin{cases} A \cdot z' & \text{if} \quad z' \geq 0 \\ \eta \cdot A \cdot z' & \text{if} \quad z' < 0 \end{cases} \tag{1}$$

where z' is the rate of water discharge, and η is the efficiency ([2]). Let b be the volume of water that must be discharged over the optimization interval

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$[0, T]$. So, the following boundary conditions will have to be fulfilled:

$$z(0) = 0, \; z(T) = b \tag{2}$$

Besides the previous statement, we consider $z'(t)$ to be bounded by technical constraints

$$q_{\min} \leq z'(t) \leq q_{\max}, \quad \forall t \in [0, T] \tag{3}$$

In this section we focus on the new short-term problem faced by a generation company in a deregulated electricity market when preparing its offers for the day-ahead market. In our problem, the objective function is given by hydraulic profit over the optimization interval, $[0, T]$. Profit is obtained by multiplying the hydraulic production of the pumped-storage hydro-plant by the clearing price, $\pi(t)$, at each hour, t. Taking our objective functional $F(z)$ in continuous time form, a standard Lagrange-type Optimal Control problem can be mathematically formulated as follows:

$$\begin{array}{l} \displaystyle\max_{(u,z)} \int_0^T L(t, z(t), u(t))dt = \max_{(u,z)} \int_0^T \pi(t)P(u)dt \\ z' = u; \quad z(0) = 0, z(T) = b; \quad u_{\min} \leq u(t) \leq u_{\max} \end{array} \tag{4}$$

3 Optimization algorithm

For the Optimal Control problem (4), the resulting Hamiltonian, H, is linear in the control variable, u, and results in an optimal singular/bang-bang control policy. In general, the application of Pontryagin's Maximum Principle [3] is not well suited for computing singular control problems as it fails to yield a unique value for the control. In our problem, however, an added complication arises: the Hamiltonian is defined piecewisely and the derivative of H with respect to u presents discontinuity at $u = 0$, which it is the border between the power generation zone (positive values of u) and the pumping zone (negative values of u). When non-differentiable objective functions arise in optimization problems, the generalized (or Clarke's) gradient ([4]) must be considered.

On the basis of the above theoretical results, in [5] we determined the optimal solution: the bang-singular-bang segments and the boundary on which the solution is situated:

$$u^*(t) = \begin{cases} u_{\max} & \text{if} \quad A \cdot \pi(t) > -\lambda_0 \\ u_{\text{sing}} = 0 & \text{if} \quad -\lambda_0 \in [A \cdot \pi(t), \eta \cdot A \cdot \pi(t)] \\ u_{\min} & \text{if} \quad \eta \cdot A \cdot \pi(t) < -\lambda_0 \end{cases} \tag{5}$$

The algorithm that leads to the optimal solution comprises the following steps: (i) First, for a given λ, we have to determine the switching times: $t_1, t_2, ...$ These instants are calculated solving the equations

$$A \cdot \pi(t) = -\lambda; \quad \eta \cdot A \cdot \pi(t) = -\lambda \tag{6}$$

(ii) Second, the optimal value, λ_0, must be determined in order for:

$$z_\lambda(T) = \sum_{i=1}^{N_u} \delta_i^u \cdot q_{\max} + \sum_{i=1}^{N_l} \delta_i^l \cdot q_{\min} = b \tag{7}$$

δ_i^u and δ_i^l being the duration of the i-th bang-bang segment in the upper and lower bound respectively, N_u and N_l the number of such segments, and $z_\lambda(T)$ the final volume obtained for each λ. (iii) To calculate an approximate value of λ_0, we propose a classic iterative method (like, for example, bisection or the secant method).

4 Combined optimization

With the aid of the algorithm presented in the previous section, we are now in a position to analyze the combined optimization of a pumped-storage hydro-plant and a wind farm. In this section we shall analyze whether it is in the interest of wind farms to go to market. To do so, we address two configurations (see Figure 1):

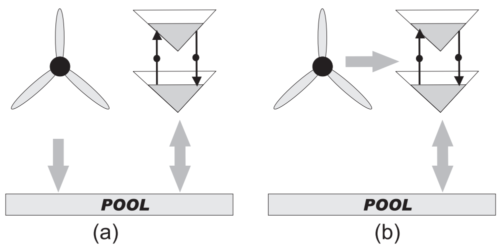

Figure 1. Two configurations.

(a) The wind farm and the pumped-storage hydro-plant work independently, each selling the energy it produces on the market.

(b) The wind farm does not sell energy on the market, but uses the generated power to pump water to the upper reservoir of the pump-plant.

There are a number of factors that influence the final result, like for example: the efficiency of the hydro-plant η, the volume of water available b, deviation penalties, or wind power production. The first two factors have already been analysed in [5]. We shall now analyse the last two factors, which correspond to the wind farm. In order for our study to reflect a broad range of possibilities, we propose two scenarios: (i) Low wind power production in peak hours; and (ii) High wind power production in peak hours.

5 Results and conclusions

A program was written using the Mathematica package to apply the results obtained in this paper to an example of a hydraulic system made up of one fixed-head pumped-storage hydro-plant and a wind farm. The problem studied in this paper analyses the convenience, or not, of the wind farms going to market. In conclusion, both configurations (a) and (b) seem interesting options. But, the system is highly sensitive to numerous factors and each company must carefully assess particular situations that may result in variations in the optimum configuration. Our algorithm allows the optimal solution to be obtained easily and the obtained results provide real-time information to determine which configuration is preferable in each specific real situation of the electricity market.

References

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