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Real-time optimization of wind farms and fixed-head pumped-storage hydro-plants

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Abstract

In this paper we analyze whether real-time compensation of wind power plant deviation penalties is profitable by means of the coordinated optimization of the wind power plant with a pumped-storage hydro-plant. We shall make use of optimal control techniques to carry out the optimization. We shall also analyze another possible solution based on compensation carried out *a posteriori*, instead of in real time.

Key words: Optimal Control, Pumped-Storage Plant, Wind Farm
MSC 2000: 49J52, 49M05

1 Introduction

The new regulations allow wind farms to go to the market to sell the energy generated by their facilities. If wind farms offer in the pool, they will prepare their offers and schedule their power production. However, a major problem exists: the unpredictability of wind farm production. Forecasting errors lead to the wind farm incurring financial losses, known as deviation penalties. Diverse methods have also been proposed to store this energy [1]. In this paper we focus on combined use of a wind farm with pumped-storage plants.

Some authors ([2], [3]) have researched the operation of a wind farm cooperating with a micro-hydroelectric power plant and a pumped-storage hydro-plant. Previous studies exclusively employ the storage ability to compensate for wind power imbalances. However, this approach is not representative for large pumped-storage plants in power systems. One of the techniques used for large pumped-storage plants ([4], [5]) is to calculate the optimal amount of spinning reserve that the system operator should provide so as to be able to respond to errors in forecasts. The combined operation of wind farms and a pumped-storage hydro-plant is also analyzed in [6].

The present paper aims to calculate the optimal operation of the pumped-storage plant, simultaneously pursuing two goals: to maximize revenue in conventional operations in the day-ahead market and to coordinate with the wind power producer with the aim of partially compensating for wind power imbalances. In this paper we shall consider a large capacity pumped-storage working jointly with a wind farm adjacent to its facilities. We shall consider them to be a single unit (a *wind-hydro power plant*). Two different joint configurations for the resulting joint-unit formed by the pumped-storage plant and the wind farm are considered. In the first (uncoordinated operation), the pumped-storage plant does not compensate for the errors due to forecasting wind power. In the second (coordinated operation), we shall attempt to compensate for these errors in real time. We shall see in this paper that the fact that the pumped-storage plant is a fixed-head plant will mean that the optimal solution is of a very special type: bang-singular-bang. This will have crucial consequences in coordinated operation and we shall present a qualitative study of the real-time compensation of forecasting errors. In view of the result obtained in this study, we shall propose a second solution: to employ the over-generation deviations of the wind power plant *a posteriori* to pump water into the upper reservoir of the pumped-storage plant, thus increasing profits. Finally, we present a realistic example.

2 Problem description and model overview

The day-ahead market in the Spanish wholesale electricity market is organized as a set of twenty-four simultaneous hourly auctions. The simple bid format consists of a pair of (hourly) values: quantity $q\,(MWh)$ and price $p\,(euro/MWh)$. The problem we shall solve is the one faced by a wind-hydro power plant when preparing its offers for the day-ahead market. This basic scheduling, with plants working independently, is based on the volume of water $b\left(m^3\right)$ that must be used and on the best forecast of wind power generation available each hour $W^f(t)(MW)$. Unfortunately, wind power forecasts within a $14-38$ hour time horizon are usually highly inaccurate and hence incur deviation penalties.

As regards the pumped-storage plant, we shall model it in great detail without any additional simplifications. For a large capacity reservoir, the effective head is constant over the optimization interval and here the fixed-head hydro-plant model is defined. In plants of this type, the active power generated, $P(MW)$, is represented by the linear equation: $P(z'(t)) = Az'(t)$, where A represents the efficiency and diverse parameters related to the geometry of the hydro-plant (see [7]) and $z'(m^3/s)$ is the rate of water discharge. Taking into account the conversion losses of the pumping process, we must therefore introduce the efficiency, η, in the model.

We consider $z'(t)$ to be bounded by technical constraints: $q_{\min} \leq z'(t) \leq q_{\max}, \forall t \in [0,T]$

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and we assume that b is the volume of water that must be discharged over the entire optimization interval $[0, T]$, so: $z(0) = 0$, $z(T) = b$. The function P is thus defined piecewise as:

$$P(z') := \begin{cases} A \cdot z' & \text{if} \quad z' \geq 0 \\ \eta \cdot A \cdot z' & \text{if} \quad z' < 0 \end{cases} \tag{1}$$

3 Optimization of a fixed-head pumped-storage plant

In a previous paper [8] by the authors, we presented an algorithm that allows the optimal solution of a fixed-head pumped-storage plant to be obtained. The objective function is given by hydraulic profit over the optimization interval, $[0, T]$. Profit is obtained by multiplying the hydraulic production of the pumped-storage hydro-plant by the clearing price, $\pi(t)$, at each hour, t. An Optimal Control problem can thus be mathematically formulated as follows:

$$\begin{aligned} &\max_{(u,z)} \int_0^T L(t, z(t), u(t))dt = \max_{(u,z)} \int_0^T \pi(t)P(u)dt \\ &z' = u; \quad z(0) = 0, z(T) = b; \quad u_{\min} \leq u(t) \leq u_{\max} \end{aligned} \tag{2}$$

For the Optimal Control problem (2), we define the Hamiltonian in normal form:

$$H(t, z, u, \lambda) := L(t, z, u) + \lambda u = \pi(t)P(u) + \lambda u \tag{3}$$

and the resulting Hamiltonian, H, is linear in the control variable, u. It is well known [9] that when the Hamiltonian is linear in u, the optimality condition leads to the optimal u^* being undetermined if the switching function $\Phi(x, \lambda) \equiv H_u = 0$. An added complication arises in our problem: the Hamiltonian is defined piecewisely and the derivative of H with respect to u (H_u) presents discontinuity at $u = 0$. When non-differentiable objective functions arise in optimization problems, the generalized (or Clarke's) gradient (see [9]) must be considered. Based on the above theoretical results, in [8] we determined the *bang-singular-bang (b-s-b) optimal solution*:

$$u^*(t) = \begin{cases} u_{\max} & \text{if} \quad A \cdot \pi(t) > -\lambda_0 \\ u_{\text{sing}} = 0 & \text{if} \quad -\lambda_0 \in [A \cdot \pi(t), \eta \cdot A \cdot \pi(t)] \\ u_{\min} & \text{if} \quad \eta \cdot A \cdot \pi(t) < -\lambda_0 \end{cases} \tag{4}$$

The previous algorithm interpolates $\pi(t)$ and works with a continuous function. Thus, by adjusting the switching times, it is capable of achieving the final volume b to discharge with the desired precision. However, generating companies must in fact present offers in the day-ahead market for each of the 24 hours of the following day. That is, we need to convert a continuous variable into a discrete variable. We shall lose an essential feature in this conversion: we shall no longer be able to achieve any final volume of water precisely.

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In fact, the volume discharged in the b-s-b solution must belong to the set of M possible values: $\Omega = \{b_1, b_2, ..., b_M\}$. The plant operator therefore only needs choose in $\Omega = \{b_i\}_{i=1}^{M}$ the nearest value, without exceeding the available volume, b ($b_{sol} < b < b_{sol+1}$). In this case, b_{sol} is the discharged volume corresponding to the optimal b-s-b solution.

4 Qualitative analysis of real-time optimization

In view of the above results, we shall conduct a qualitative study on the b-s-b solution. Let us assume we have obtained the solution for a certain λ_{sol} (calculated by aiming at a certain final volume, b_{sol}). We can know the price, π_{turb}, above which it is of interest to discharge water, and we can know the price, π_{pump}, below which it is of interest to pump water. It is shown that, between the instants of pumping (t_{pump}), stoppage (t_{stop}) and discharging water (t_{turb}), the following relations exist between the prices:

$$\pi(t_{pump}) < \pi(t_{stop}); \quad \pi(t_{stop}) < \pi(t_{turb}); \; \pi(t_{turb}) > \eta \cdot \pi(t_{pump}) \tag{5}$$

Furthermore, between two instants of stoppage, it is verified that:

$$\pi(t_{stop}^{1}), \pi(t_{stop}^{2}) \in \left[\frac{\lambda_{sol}}{\eta \cdot A}, \frac{\lambda_{sol}}{A}\right] \tag{6}$$

When the plant operator prepares its offer for the day-ahead market for day D, this solution obtained for the pumped-storage plant, assuming the market prices and available water to be known, is the one that it will offer, seeing as it maximizes profits. The wind power plant will offer according to the best forecast for wind power production available at 10 hours the day before, $D-1$. However, when day D arrives, deviations will almost certainly be produced between the actual wind power production, $W^r(t)$, and the forecasted production, $W^f(t)$. In this context, we shall pose the following question: when faced with a deviation in wind power generation at the instant t, might it be of interest to the pumped-storage plant to modify its behavior in real time (i.e. at t) so as to compensate for the deviation penalties of the wind farm and thus achieve a greater joint profit?

Let us call $d(t) = W^r(t) - W^f(t)$ the deviation of the wind farm at the instant t, $p^+(t)$ the price the market pays the over-generation deviation (which will be a certain fraction s of the market price) and $p^-(t)$ the price we must pay for the under-generation penalty (which will be a certain fraction l of the market price). Let us assume in all cases that the deviations are against the system. We shall analyze in detail the two possibilities for the deviations, $d(t)$, of the wind power plant: 1) the over-generation deviation, and 2) the under-generation deviation.

Let us now consider the first case. 1) If the wind farm presents an over-generation deviation, the hydro-plant will be able to act at t in only two cases: 1a) If it was stopped, it will use the over-generation from the wind power plant to pump water; 1b) If it was

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discharging water, it will produce less power to compensate for the over-generation of the wind farm. If it was already pumping, as the solution is of the b-s-b type, it will not be able to act. Let us analyze sub-case 1a). At instant t, the hydro-plant was stopped and pumped $d(t)(MW)$ at zero cost. The amount of water pumped at t which will then be used is: $d(t)/\eta.A$. With this modification, the deviation in wind power generation does not produce any profit at t, and we must find an instant t^* at which it is of interest to the pumped-storage plant to discharge this water. At t^*, the hydro-plant may be stopped (sub-case 1a1) or pumping (sub-case 1a2), seeing that, as the solution is of the b-s-b type, if it was discharging water, the turbines cannot be put to greater use. It should be borne in mind that this action will mean a change in its scheduling and will hence result in a penalty; in this case, for over-generation.

We shall analyze all the other cases in a similar way to this case and shall see in a detailed manner that the conditions that must be fulfilled for the real-time modification to be of interest can never be given by the conditions (5) and (6). Conclusion: *no real-time modification is of interest.*

5 *A posteriori* optimization of a wind-hydro power plant

Subsequent to the above study, we posed the question as to whether it is possible to model the functioning of the wind-hydro power plant so as to operate in a coordinated manner *a posteriori* and thus improve profits. We shall not make real-time compensations for under-generation deviations in wind power. We shall however compensate for over-generation deviations in wind power. We shall attempt to use the surplus wind power generated on day D to pump water, thereby avoiding penalties for over-generation on day D and subsequently use this water in the hydro-plant by discharging it on the following day $D+1$. Furthermore, as we are working for the day-ahead market, we shall eliminate all the uncertainty associated with the process.

$$B = \int_0^T \left(\pi^{D+1}(t)P^{D+1}(t) + \pi^D(t)W^D(t) - C^D(t)\right) dt \tag{7}$$

The total profit over the optimization interval $[0,T]$ is revenue minus cost. Revenue is obtained by multiplying the hydraulic production, $P(t)$, and the wind power production, $W(t)$, by the clearing price, $\pi(t)$, at each hour, t. The unique cost in our system is the cost of deviation penalties, $C(t)$. Accordingly, and in order for the comparison to be rigorous, the wind power production is considered to be sold to the market on day D and that of the hydro-plant on day $D+1$. We shall use superscripts to denote the day under consideration. In uncoordinated operation, we shall have that $z(T) = b_{sol}$. In the coordinated configuration, the profit obtained shall have to take into account the reduction in deviation penalties, $C(t)$, and the increase in the volume of water available: $z(T) = b_{sol} + b^*$. To illustrate

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the behavior of this solution, we shall consider an example of a wind-hydro power plant and compare the uncoordinated and the coordinated configurations. We shall see that it is possible to obtain profit in the latter case.

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