Minimizing the emissions of precursors of aerosols (SO₂ and NO_x) in hydrothermal systems

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Electric power systems are traditionally operated in such a way that the total fuel cost is minimized regardless of the emissions produced. Increasing requirements aimed at environmental protection have given rise to the need for alternative strategies. It is well known that, for our hydrothermal problem, the most important aerosol precursors are: sulphur dioxide (SO₂), which is then transformed in small sulphate particles and, to a lesser degree, nitrous oxide emissions (NO_x), which will lead to the formation of nitrate particles (Tsuang, 2003). This paper presents an environmental dispatch algorithm in a hydrothermal system and addresses the problem of minimization of emissions of SO₂ and NO_x caused by the operation of thermal plants.

Several models have been used to represent the emissions function. In this paper we adopt a quadratic model for both emissions:

$$E(P) = \alpha + \beta P + \gamma P^2$$

where P is the power generated and the parameters were computed via the least square criteria from several tests at Aboño thermal plant (Spain). The unit emission of E(P) is calculated knowing:

The Net Consumption NC, obtained by multiplying the net specific consumption curve of the plant by P.
The average amount of coal EQ consumed at the plant.

- The concentration C of the pollutant as a function of the power P, which is obtained by continuous measurement at the discharge point of the stack.

- The production of smoke *S* for the average amount of coal consumed by the group.



Figure 1. Approximation to the function E(P).

Given that the power at which this pollution is emitted is known, it is possible to build point graphs like those in Fig. 1, and to perform the least square approximation over these. Data was collected over several weeks, for both SO₂ and NO_x, and was comprised of some 150 measurements per week, the r^2 (correlation ratio) obtained always being higher than 0.96.

The hydrothermal system considered contains m thermal and n hydro plants. The problem of optimum emissions in a hydrothermal system consists (Bayón *et al.*, 2002) in minimizing the functional

$$J = \int_0^T \sum_{i=1}^m \left[\alpha_i + \beta_i P_i(t) + \gamma_i P_i^2(t) \right] dt$$

during the optimization interval [0, T]. The following equilibrium equation of active power will have to be fulfilled

$$\sum_{i=1}^{m} P_{i}(t) + \sum_{i=1}^{n} H_{i}\left(t, z_{i}(t), z_{i}'(t)\right) = P_{d}(t), \forall t \in [0, T]$$

where $P_d(t)$ is the power demand and $H_i(t, z_i(t), z_i'(t))$ is the power contributed to the system at the instant *t* by the *i*-th hydro-plant, $z_i(t)$ being the volume that is discharged up to the instant *t*, and $z_i'(t)$ the rate of water discharge at the instant *t*. If we assume that b_i is the volume of water that must be discharged during the optimization interval by the *i*-th hydro-plant, the following boundary conditions will have to be fulfilled: $z_i(0) = 0, z_i(T) = b_i$.

An optimal control technique (Clarke, 1983) is applied and Pontryagin's theorem is employed. The algorithm proposed is easily implemented using the Mathematica © Package and is applied to a sample system to illustrate the results obtained.

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