A new Algorithm for the Optimization of a Simple Hydrothermal Problem

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1. Introduction

A hydrothermal system is made up of hydraulic and thermal power plants that must jointly satisfy a certain demand in electric power during a definite time interval. Thermal plants generate power at the expense of fuel consumption (which is the object of minimization), while hydraulic plants obtain power from the energy liberated by water that moves a turbine; there being a limited quantity of water available during the optimization period. In prior studies [1-2], it was proven that the problem of optimization of the fuel costs of a hydrothermal system with m thermal power plants without transmission losses may be reduced to the study of a hydrothermal system made up of one single thermal power plant, called the thermal equivalent. In the present paper, we consider a simple hydrothermal system with one hydraulic power plant and m thermal power plants without transmission losses that have been substituted by their thermal equivalent. Under these conditions, we present the problem from the Electrical Engineering perspective to then go on to resolve the mathematical problem thus formulated. We will call this problem: the $H_1 - T_1$ Problem.

2. Hydrothermal Statement of the $H_1 - T_1$ Problem

The problem consists in minimizing the cost of fuel needed to satisfy a certain power demand during the optimization interval $[0, T]$. Said cost may be represented by the functional

$$F(P(t)) = \int_0^T \Psi(P(t))dt$$

where Ψ is the function of thermal cost of the thermal equivalent and $P(t)$ is the power generated by said plant. Moreover, the following equilibrium equation of active power will have to be fulfilled

$$P(t) + H(t, z(t), z'(t)) = P_d(t), \ \forall t \in [0, T]$$

where $P_d(t)$ is the power demand and $H(t, z(t), z'(t))$ is the power contributed to the system at the instant t by the hydraulic plant, being: $z(t)$ the volume that is discharged up to the instant t (in what follows, simply volume) by the plant, and $z'(t)$ the rate of water discharge of the plant at the instant t.

Taking into account the equilibrium equation, the problem reduces to calculating the minimum of the functional

$$F(z) = \int_0^T \Psi\left(P_d(t) - H\left(t, z(t), z'(t)\right)\right) dt$$

If we assume that b is the volume of water that must be discharged during the entire optimization interval, the following boundary conditions will have to be fulfilled

$$z(0) = 0, \;\; z(T) = b$$

The classic studies dealing with hydrothermal optimization employ concrete models both for the function of thermal cost Ψ, as well as for the function of effective hydraulic generation H. Hence, if the model changes, the algorithms obtained are not valid. The study of optimal conditions for the functioning of a hydrothermal system constitutes a complicated problem which has attracted significant interest in recent decades. Various techniques have been applied to solve the problem, such as functional analysis techniques [3] or Ritz's method [4]. Such a variety of the mathematical models forces us to undertake a general study of the problem. The algorithms obtained with this study should be extensible to a large set of hydrothermal problems.

One of the main contributions of this paper is that the method is valid for any model of power plants, since we will try to consider the functions P_d, Ψ and H as general as possible without any restrictions, except those that are natural for problems of this type.

3. Variational Statement of the $H_1 - T_1$ Problem

We will call $H_1 - T_1$ the problem of minimization of the functional

$$F(z(t)) = \int_0^T L(t, z(t), z'(t))dt$$

with L of the form

$$L(t, z(t), z'(t)) = \Psi(P_d(t) - H(t, z(t), z'(t)))$$

over the set

$$\Theta_b = \{z \in \widehat{C}^1[0, T] \; / \; z(0) = 0, \, z(T) = b, \, z'(t) \geq 0 \, \wedge \, H(t, z(t), z'(t)) \leq P_d(t)\}$$

So the problem involves inequality non-holonomic constraints.

Variational problems in which the derivatives of the admissible functions must be comprehended between two curves have traditionally been dealt with by recurring to diverse techniques. Clarke [5] deals with necessary conditions for problems in the calculus of variations that incorporate inequality constraints of the form $f(x, x') \leq 0$. Loewen and Rockafellar [6] consider the classic Bolza problem in the calculus of variations, incorporating endpoint and velocity constraints through infinite penalties.

It can be seen that all these studies use very diverse techniques, but to date a simple theory has still to be found that develops the stationary function for variational problems with the constraint for admissible functions of the above type. Moreover, these studies do not generate algorithms that are easily implementable and that resolve the problem we have presented.

In this paper, we have developed a much simpler theory than previous ones that solves the $H_1 - T_1$ problem. The development is hence self-contained and extremely basic, and also enables the construction of the optimal solution.

4. Construction of the Optimal Solution. Algorithm

Firstly, we demonstrate the result that we have denominated the main coordination theorem, which will enable us to find the optimum solution.

Theorem. (The main coordination theorem)

Let

$$¥_q(x) = \int_0^x L_z(t, q(t), q'(t))dt - L_{z'}(x, q(x), q'(x)) \tag{1}$$

If $q \in C^1$ *is a solution of the problem, then* $\exists K$ *such that:*

i) If $q'(t) > 0$ *and* $H(t, q(t), q'(t)) < P_d(t)$ *(*t *is not a boundary point)* $\Longrightarrow ¥_q(t) = K$.

ii) If $q'(t) = 0 \Longrightarrow ¥_q(t) \leq K$.

iii) If $H(t, q(t), q'(t)) = P_d(t) \Longrightarrow ¥_q(t) \geq K$.

This theorem allows us to elaborate the optimization algorithm that leads to determination of the optimal functioning of the hydroplant and of the whole hydrothermal system.

Algorithm

If we did not have inequality restrictions, the solution could be constructed by means of the shooting method. We use the same framework in the present case, but the variation of the initial condition for the derivative, which now need not make sense, is substituted by the variation of the coordination constant K.

The problem will consist in finding for each K the function Q_K which satisfies $Q_K = 0$ and the conditions of the main coordination theorem, and from among these functions, the one which generates an admissible function ($Q_K(T) = b$).

From the computational point of view, the construction of Q_K can be performed with the use of a discretized version of equation

$$¥_q(t) = K$$

The exception is that at the instant when the values obtained for z and z' do not obey the restrictions, we force the solution Q_K to belong to the boundary until the moment when the conditions of leaving the domain (established in the Theorem) are fulfilled.

5. Example

Finally, we present a example employing the Algorithm realized to this end with the "Mathematica" package. The program developed is very simple and easy to use.

6. Conclusions

From the Engineering perspective, one of the main contributions of this paper is that the algorithm carried out is independent of the models used both for thermal and for hydraulic power plants, in contrast to the majority of methods in this field, which use concrete models. What is more, we have obtained a very simple method that enables us to find an optimal solution in the presence of inequality constraints, and which requires very little computational effort.

From the mathematical point of view, we have also obtained notable results. The main contribution of this paper is a property of the extremals in variational problems with non-holonomic constraints. Said property permits the solution to be constructed by means of a method inspired by the shooting method that is much simpler than those employed up until now for resolving this type of problem.

The Algorithm presents many advantages. First of all, to run the method one does not have to start from specially selected initial values. Moreover, it shows a rapid convergence to the optimal solution, and due to the simplicity of the operations to be performed in this method, its realization does not take much time.

7. References

[1] M. E. El-Hawary and G. S. Christensen, "Optimal economic operation of electric power systems", Academic Press, New York, 1979.

[2] L. F. Bayón, J. M. Grau and P. M. Suárez, A New Formulation of the Equivalent Thermal in Optimization of Hydrothermal Systems, *Math. Probl. Eng. (2002)*, Accepted.

[3] L. F. Bayón, P. M. Suárez, J. A. Otero and B. Dugnol, Fuel cost optimization of hydro-thermal systems, "Approximation and Optimization", Vol. 8, 631-650, Frankfurt, 1995.

[4] L. F. Bayón and P. M. Suárez, Multiple Objective Optimization of Hydro-Thermal Systems Using Ritz´s Method, *Math. Probl. Eng.*, Vol. 5 (2000), 379-396.

[5] F. H. Clarke, Inequality constraints in the calculus of variations, *Canad. J. Math.* 29, no. 3 (1977), 528–540.

[6] P. D. Loewen and R. T. Rockafellar, New necessary conditions for the generalized problem of Bolza, *SIAM J. Control Optim.* 34, no. 5 (1996), 1496–1511.