A New Mathematical Model and Algorithm for a Complex Hydrothermal Problem

Bayón, L.; Grau, J.M. and Suárez, P.
University of Oviedo. Department of Mathematics. E.U.I.T.I. Spain.

November 20, 2002

Key Words: Calculus of Variations, Optimization, Hydrothermal System. **MSC:** 49K30

1. Introduction

In a previous paper [6], the authors expound a classic problem of calculus of variations for a simple hydrotermal system that accounts for m thermal stations without transmission losses and one hydroplant. Once the thermal equivalent [7] has been found, the equilibrium equation of active power may be eliminated. On the one hand, this constraint generates unknown multiplier functions and, on the other, complicates the calculus of the optimal solution.

In hydrotermal systems with transmission losses we cannot solve for the thermal power in the equilibrium equation and thus eliminate the constraint. To avoid this problem, in this paper we develop a valid method for complex hydrothermal systems that accounts for m thermal stations with transmission losses and n hydroplants.

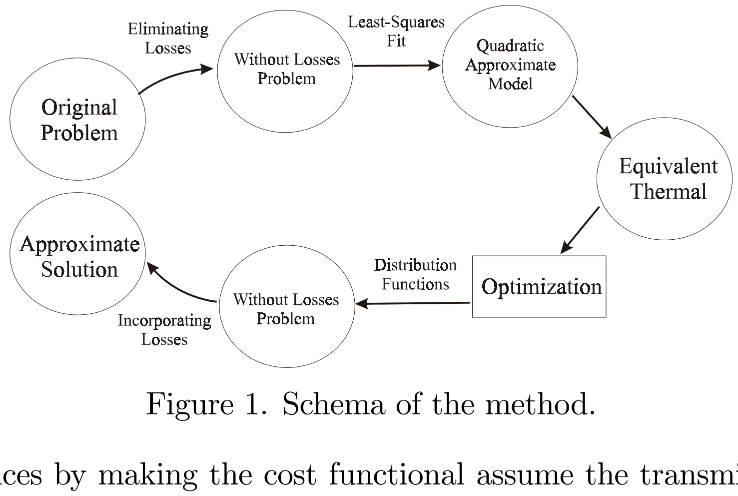

Figure 1. Schema of the method.

The method commences by making the cost functional assume the transmission losses. It hence becomes a non-quadratic functional, which complicates the construction of the thermal equivalent. We therefore approximate said functional by means of Least-Squares fit to a new quadratic functional and thus obtain the thermal equivalent from the latter. It is thus possible to newly eliminate the equilibrium equation constraint. We then go on to generalize the method employed in [6] to obtain the optimal solution to the case of n hydroplants. Finally, we recover the optimal solution with the technique developed in this paper (Fig. 1).

2. Description of the Problem

Let us assume that a hydrothermal system consists of m thermal plants and n hydroplants.

The cost function of the i-th thermal plant: $F_i : D_i \subset \mathbb{R} \longrightarrow \mathbb{R}$.

The function of effective contribution of the i-th thermal plant: $\phi_i : D_i \to \mathbb{R}$.

The function of losses of the i-th thermal plant: $l_i(x) = x - \phi_i(x)$.

The function of effective hydraulic contribution: $H : \Omega_H \longrightarrow \mathbb{R}$.

The power demand: $P_d : [0,T] \longrightarrow \mathbb{R}$. *The optimization interval* $[0,T]$.

The vector of admissible volumes: $\overrightarrow{b} = (b_1, \ldots, b_n) \in \mathbb{R}^n$.

The generalized hydrothermal problem $\Gamma \equiv \mathrm{H}_n\text{-}\mathrm{T}_m\{P_d, \{F_i, \phi_i\}_1^m, H, \overrightarrow{b}\}$:

$$\min J(y_1, \ldots, y_m, z_1, \ldots, z_n) = \min \int_0^T \sum_{i=1}^{m} F_i(y_i(t))dt \ \text{ satisfying:}$$

(1) the equilibrium eq.: $\displaystyle\sum_{i=1}^{m} \phi_i(y_i(t)) + H(t, z_1(t), \ldots, z_n(t), z_1'(t), \ldots, z_n'(t)) = P_d(t),\ \forall t \in [0,T]$

(2) the restrictions on the admissible volume: $z_i(0) = 0,\ \ z_i(T) = b_i\ \ (i = 1, \ldots, n)$

3. Elimination of the concept of losses and Construction of Equivalent Thermal

From the mathematical point of view, the concept of transmission losses is superfluous and can be circumvented by means of an adequate modification of the cost functions for the thermal plants.

Theorem. *Consider the problems*

$$\Gamma \equiv \mathrm{H}_n\text{-}\mathrm{T}_m\{P_d, \{F_i, \phi_i\}_1^m, H, \overrightarrow{b}\} \ \ \textit{and} \ \ \widetilde{\Gamma} \equiv \mathrm{H}_n\text{-}\mathrm{T}_m\{P_d, \{(F_i \circ \phi_i^{-1}), Id_i\}_1^m, H, \overrightarrow{b}\}$$

with bijective functions of effective contribution $\phi_i : D_i \to \widetilde{D}_i$. *Then:*

I) $(P_1, \ldots, P_m, \overrightarrow{Q})$ *is a solution of* Γ *if* $(\phi_1 \circ P_1, \ldots, \phi_m \circ P_m, \overrightarrow{Q})$ *is a solution of* $\widetilde{\Gamma}$.

II) the functions of losses in problem $\widetilde{\Gamma}$ *are identically equal to zero.*

Example. The cost function that has systematically been used is a second-order polynomial: $F_i(x) = \alpha_i + \beta_i x + \gamma_i x^2$. It is also usual to consider the function of losses $l_i(x) = b_{ii} \cdot x^2$, (Kirchmayer's formula, where b_{ii} is termed the *coefficient of losses*).

Once the losses have been incorporated in the functionals $\widetilde{F}_i(x) = (F_i \circ \phi_i^{-1})(x)$, we approximate by means of second degree polynomials, and obtain: $\widetilde{F}_i(x) \simeq \alpha_{wl} + \beta_{wl} x + \gamma_{wl} x^2$. Now we substitute the problem with m thermal plants $(H_{\mathbf{n}} - T_{\mathbf{m}})$ with an equivalent problem $(H_{\mathbf{n}} - T_{\mathbf{1}})$ with a single thermal plant: the *equivalent thermal plant* [7].

So, the variational problem has a constraint (the equilibrium equation): $y(t) + H(t, \overrightarrow{Z}(t), \overrightarrow{Z'}(t)) = P_d(t)$, $\forall t \in [0, T]$ which can be omitted, together with the unknown function $y(t)$, thus yielding the classic variational problem of minimizing the functional:

$$J(\overrightarrow{Z}) = J(z_1, \ldots, z_n) = \int_0^T \Psi\left(P_d(t) - H(t, \overrightarrow{Z}(t), \overrightarrow{Z'}(t))\right) dt$$

$$\text{with the boundary conditions: } \overrightarrow{Z}(0) = \overrightarrow{0}, \overrightarrow{Z}(T) = \overrightarrow{b}$$

The value of the unknown $y(t)$ that disappears in the new formulation can be recovered once the values of the other unknowns $\overrightarrow{Z}$ are established. To define the partial contribution of each of these, we use the distribution functions [7], obtained from Ψ. Once the distribution has been carried out and the optimal power without losses found: $P_1, P_2, \ldots, P_m$, we obtain the optimal power solution of the original problem: $\phi_1^{-1} \circ P_1, \ldots, \phi_m^{-1} \circ P_m$.

Test. Next, we present a Test that demonstrates how the method developed contributes an approximate solution that is almost coincident with the original problem. As an example, we shall use the thermal system of Asturias (Spain).

4. The $(H_{\mathbf{n}} - T_{\mathbf{1}})$ Problem. Constructing the Solution

The problem of optimization of a hydrothermal system which involves various hydroplants is highly complicated. One should not forget that the associated variational problem is related to solving a boundary-value problem for a system of differential equations. We have developed an algorithm of its numerical resolution prompted by the so-called method of cyclic coordinate descent. We will now see how a problem of the type $(H_{\mathrm{n}} - T_1)$ could be solved under certain conditions if we start out from the resolution of a sequence of problems of the type $(H_1 - T_1)$ [6].

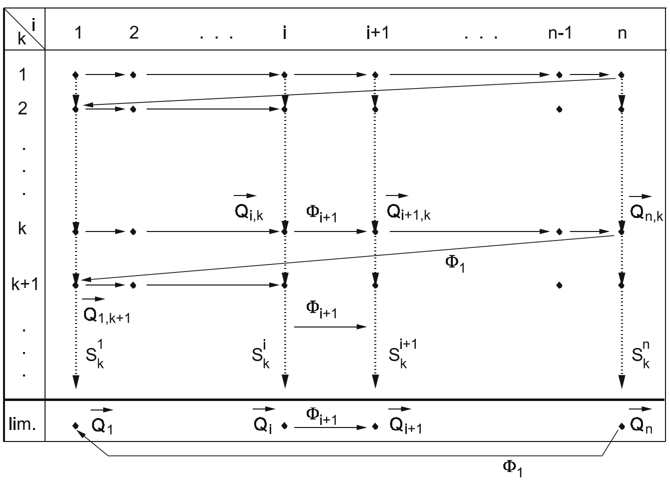

Figure 2. Descending subsequences.

We define $\Phi_i(z_1, ..., z_i, ..., z_n) = (z_1, ..., \widetilde{z}_i, ..., z_n)$, with $(z_1, ..., \widetilde{z}_i, ..., z_n)$ a vector that provides the minimum of J on fixing all the components except for the i-th.

Theorem. *If the functional J is convex in Ω, and, in certain topology:*

i) the mappings Φ_i are continuous, $\forall i = 1, \ldots, n$ and ii) the descending subsequences (Fig. 2) S_k^i are convergents, $\forall i = 1, \ldots, n$, then every descending subsequence converges to a solution of the problem.

The solution of the problem will be constructed as the limit of a minimizing sequence. If the conditions of the theorem are fulfilled, this sequence provides us with an approximation of the solution. Beginning with some admissible $\overrightarrow{Q^0} = (z_1, \ldots, z_n)$, we construct a sequence via successive and iterative applications of $\Phi_1, \Phi_2, \ldots, \Phi_n$. The application of every Φ_i involves solving a problem of the type $(H_1 - T_1)$. If we set $\Phi = (\Phi_n \circ \Phi_{n-1} \circ \cdots \circ \Phi_2 \circ \Phi_1)$, the solution will be: $\lim_{k\to\infty} \Phi^k \left(\overrightarrow{Q^0}\right)$. From the algorithmic and computational point of view, we get the iteration process that at each stage calculates the optimal functioning of a hydraulic power station, while the behavior of the rest of the stations is assumed fixed.

5. Example. A computer program (Mathematica) was written to apply the results obtained in this paper to a real power system.

6. Conclusions

In this paper, we have developed a valid method for complex hydrothermal systems that notably simplifies their optimization. A test is used to prove that the error made in the approximate solution is insignificant, since the fuel cost is practically the same. A major advantage of our method with respect to those previously employed is that it reduces the optimization of a system with m thermal plants and n hydraulic plants to the resolution of a succession of problems with one thermal plant and one hydraulic plant, $(H_1 - T_1)$, a problem easily resolved from the computational viewpoint. We elaborate an algorithm that presents several advantages with respect to classic algorithms, such as: ease of implementation, rapid convergence, the convergence depends very little on the initial values, and minimum memory requirements (the program was developed on a PC with the Mathematica package).

7. References

[1] M. E. El-Hawary and G. S. Christensen, *Optimal economic operation of electric power systems*, Ed. Academic Press 1979.

[2] L. Bayón and P. Suárez, *Multiple Objective Optimization of Hydro-Thermal Systems Using Ritz´s Method*, Mathematical Problems in Engineering, Vol. 5, pp. 379-396, USA, 2000.

[3] X. Ernan, X. Guan, R. Li and X. Jiaotong, *Scheduling hidrothermal power systems with cascaded and head-dependent rerservoirs*, IEEE Trans. on Power Systems, Vol. 14, pp. 1127-1132, 1999.

[4] E. S. Ibraham, *Dynamic hydro-thermal economic dispatch with water availability constraints and variable water head*, Computers & Electrical Engineering, Vol. 23, no. 1, pp. 15-23, 1997.

[5] L. Bayón, J. Grau, J. Otero, M. Ruíz and P. Suárez, *Uso del Calculo Variacional y el metodo de Han-Powell para la resolución de la coordinación hidrotérmica y el óptimo flujo de carga en un sistema hidrotérmico*, Proceedings IEEE-ANDESCON'99, Vol. 1, pp. 268-291, Venezuela, 1999.

[6] L. Bayón, J. Grau and P. Suárez, *A new Algorithm for the Optimization of a Hydrothermal Simple Problem*, Proceedings CMMSE 2002, Spain, 2002.

[7] L. Bayón, J. Grau and P. Suárez, *A new formulation of the equivalent thermal in optimization of hydrothermal systems*, Mathematical Problems in Engineering, Aceptado, 2002.