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A Environmentally Constrained Economic Dispatch: CFBC Boilers in the Day-ahead Market

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Abstract

This paper presents an environmentally constrained economic dispatch algorithm in a hydrothermal system within the framework of a competitive and deregulated electricity market. The optimization problem of one firm in a competitive market is described, whose objective function can be defined as its profit maximization, and we consider that the thermal plants are constrained to technical and environmental restrictions. An optimal control technique is applied and Pontryagin's theorem is employed. The algorithm proposed is implemented using the Mathematica © Package and is applied to a sample system.

Key words: Emissions, Optimization, Control Problem, Hydrothermal
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1 Introduction

The electricity supply industry is undergoing a major restructuring process [1]. Traditional centralized regulation is being replaced by a competitive deregulated framework. This is the case for Spanish utilities since January 1st, 1998. In this paper the new short-term problems that are faced by a generation company in a deregulated electricity market are addressed and an optimization algorithm is proposed.

On the other hand, during recent decades pulverised coal combustion (PCC) power plants have constituted the dominating technology among *Coal Power Generation Technologies*. Worldwide the majority of these PCC plants have no emission control equipment other than some particulate removal systems. The technology for generating electricity from coal is undergoing change due to continued demand for cleaner power production. More efficient and cleaner power generation technologies [2] that can enable utilities to meet future environmental requirements while containing electricity costs will be the leading candidates for the next few decades to come. Fluidised bed combustion (FBC) has emerged as a viable alternative, presenting significant advantages over the conventional firing system and offering multiple benefits – compact boiler design, fuel flexibility, higher combustion efficiency and reduced emission of noxious pollutants such as SO_x and NO_x.

Traditionally, power generating plants have been dispatched following minimum fuel cost criteria (economic dispatch or optimal load flow) without considering the pollution produced. However, due to the ever increasing requirements of environmental regulations and social awareness, the opening up of these types of alternative strategies is becoming fundamental. This is why diverse measures have been adopted recently in the reduction of pollution in electric systems: Addition of anti-pollutant equipment, change in fuels and economic/environmental operating strategies. This paper focuses on this third group of decisions. Numerous strategies exist [3] with the common goal of reducing the pollutant emissions of thermal power generation: minimization of total emissions (also known as emission dispatch) [4], minimization of the weighted sum of cost and emissions [5] and minimization of the cost with environmental constraints [6]. This is the typical economic dispatch, but maximum emissions are included among the operating constraints. This dispatch is called *Environmentally Constrained Economic Dispatch* (ECED). These are more realistic studies, as the majority of regulations concerning environmental matters take the form of maximum pollution constraints. This paper develops an ECED for a system that considers both thermal and hydro power plants, all within the framework of the new competitive deregulated electricity market and will analyze the role of the circulating FBC (CFBC) plants in detail.

2 Environmental Characteristics of the CFBC

The major advantage of the CFBC lies in the area of pollution control.

Nitrogen oxides are formed during the combustion of coal; these oxides are normally abbreviated as NO_x. Of these, 90% is NO, N_2O constitutes an insignificant part, representing ppm, and the rest is NO_2. NO_x are partially formed by the nitrogen in the air (called thermal NO_x) and partially by the nitrogen bound in the coal (fuel NO_x). The reactions involving thermal NO_x are only significant at high temperatures ($> 1200°C$) and, since the combustion temperature in a CFBC boiler is below 900°C, this extra NO_x is avoided. Thus, the emissions of NO_x from a fluidised bed are lower than in high-temperature combustion technologies. Fuel NO_x may be reduced with the aid of the CO present in the combustion gases that react with the nitrogen oxides. This fact is used in phase combustion, where the combustion in the first phase takes place under sub-stoichiometric conditions, resulting in CO being formed. Final combustion takes place in the following phase, once the remaining air has been added as secondary air.

Other of the main advantages of CFBC is the possibility of reducing the sulphur dioxide (SO_2) formed during combustion from the sulphur content of the fuel by adding a cheap absorbent material to the bed such as limestone ($CaCO_3$) or dolomite ($CaCO_3 \cdot MgCO_3$). If limestone is added to the bed, this undergoes a transformation called *calcination* to then form calcium oxide (CaO) according to the endothermic reaction:

$$CaCO_{3(s)} \to CaO_{(s)} + CO_2 - 183KJ/gmol \tag{2.1}$$

The calcium oxide (a porous product) reacts with the SO_2 and oxygen to form calcium sulphate ($CaSO_4$) according to the exothermic reaction:

$$CaO_{(s)} + SO_{2(g)} + \frac{1}{2}O_{2(g)} \to CaSO_{4(s)} + 486KJ/gmol \tag{2.2}$$

We term (2.1) and (2.2): *calcination, followed by sulphation. Direct sulphation* may also be considered:

$$CaCO_{3(s)} + \frac{1}{2}O_{2(g)} + SO_{2(g)} \to CaSO_{4(s)} + CO_2 \tag{2.3}$$

These reactions are optimum at temperatures of around 850°C and this is one of the reasons why the operating temperatures in CFBC boilers are normally around 850°C. The bed temperature is below the softening point of the ashes, which means that the formation of slag is an inexistent phenomenon.

3 Statement of the Hydrothermal Problem

In this section the optimization problem of one company is described, the objective function of which can be defined as its *profit maximization*. Let us assume that our hydrothermal system accounts for one hydro-plant and m thermal plants.

Let $\Psi_i : D_i \subseteq \mathbb{R} \longrightarrow \mathbb{R}$ $(i = 1, \ldots, m)$ be the cost functions $(Euro/h)$ of the m thermal plants. In general, the cost functions of the thermal plants are second-order polynomials

$$\Psi_i(P_i(t)) = \alpha_i + \beta_i P_i(t) + \gamma_i P_i^2(t); i = 1, ..., m \tag{3.1}$$

where $P_i(MW)$ is the power generated, and we consider the thermal plants to be constrained by *technical restrictions* of the type

$$P_{i\min}^{Tch} \leq P_i(t) \leq P_{i\max}^{Tch}; i = 1, ..., m, \forall t \in [0, T] \tag{3.2}$$

$[0, T]$ being the optimization interval.

On the other hand, several models have been used to represent the emissions function [3] of thermal plants. In [4] we construct a quadratic model for both emissions (SO_2 and NO_x) and calculate

$$E_i(P_i(t)) = \varepsilon_i P_i(t) + \sigma_i P_i^2(t); i = 1, ..., m \tag{3.3}$$

where $E_i(mg/Nm^3)$ is the pollutant emission (6% O_2) and $P_i(MW)$ is the power generated, the parameters being computed via the least square criteria from several tests at thermal plants. Our problem considers the economic dispatch but also includes maximum emissions among the operating constraints: Environmentally Constrained Economic Dispatch (ECED). Spain decided to formulate a national plan for reducing emissions, a decision that figures expressly in *Royal Decree 430/2004*. Said *National Plan for Reducing Emissions from existing large combustion plants (LCP)* (Nov 2005) has been recently drawn up and contains the Emission Limit Values (ELV) for SO_2 and NO_x (in mg/Nm^3) for each plant, applicable to the period 2008-2015. Knowing the curve for each plant (3.3), and imposing the ELV of said plant, we immediately obtain *environmental restrictions* of the type

$$P_{i\min}^{Env} \leq P_i(t) \leq P_{i\max}^{Env}; i = 1, ..., m, \forall t \in [0, T] \tag{3.4}$$

In prior studies [7], it was proven that the problem of optimization of the fuel cost of a hydrothermal system with m thermal plants (with restrictions of the type (3.2) or (3.4)) may be reduced to the study of a hydrothermal system made up of one single thermal plant, called the *thermal equivalent*. We shall denote as the equivalent minimizer of $\{\Psi_i\}_1^m$, the function $\Psi : D_1 + \cdots + D_m \to \mathbb{R}$ defined by

$$\Psi(P(t)) = \min \sum_{i=1}^{m} \Psi_i(P_i(t)) \tag{3.5}$$

with $P(t)$ the power generated by said thermal equivalent.

Throughout the paper, no *transmission losses* will be considered, a crucial aspect when addressing the optimization problem from a centralised viewpoint. From the perspective of a generation company, and within the framework of the new electricity market, said lossesare not relevant, since the generators currently do not participate in the sharing out of losses, thus receiving payment for all the energy they generate in power plant bars.

Let $H(t, z(t), \dot{z}(t))$ be the function of *effective hydraulic contribution*, i.e. the power contributed to the system at the instant t by the hydro-plant, $z(t)$ being the volume that is discharged up to the instant t by the plant, and $\dot{z}(t)$ the rate of water discharge of the plant at the instant t. If we assume that b is the volume of water that must be discharged during the entire optimization interval $[0, T]$, the following boundary conditions will have to be fulfilled

$$z(0) = 0, z(T) = b \tag{3.6}$$

Besides the previous statement, we consider $H(t, z(t), \dot{z}(t))$ to be bounded by technical restrictions

$$H_{\min} \leq H(t, z(t), \dot{z}(t)) \leq H_{\max}, \forall t \in [0, T] \tag{3.7}$$

In our problem the *objective function* is given by revenue minus cost during the optimization interval $[0, T]$

$$F(P, z) = \int_0^T [p(t)(P(t) + H(t, z(t), \dot{z}(t))) - \Psi(P(t))] \, dt \tag{3.8}$$

Revenue is obtained by multiplying the total production (thermal and hydraulic) of the company by the clearing price $p(t)$ in each hour t. The cost is given by Ψ, the cost function of the thermal equivalent, where $P(t)$ is the power generated by said plant. With the previous statement, our objective functional in *continuous time form* is

$$\max_{P,z} F(P, z) = \max_{P,z} \int_0^T L(t, P(t), z(t), \dot{z}(t)) dt \tag{3.9}$$

with $L(t, P(t), z(t), \dot{z}(t)) = p(t)(P(t) + H(t, z(t), \dot{z}(t))) - \Psi(P(t))$, on the set

$$\Omega = \left\{ z \in \widehat{C}^1[0, T] \middle| \begin{array}{c} z(0) = 0, z(T) = b \\ H_{\min} \leq H(t, z(t), \dot{z}(t)) \leq H_{\max}, \forall t \in [0, T] \end{array} \right\} \tag{3.10}$$

4 Optimal Solution

We shall focus in the present paper on the development of the applications of Optimal Control Theory (OCT) to this problem. If z satisfies Euler's equation for the functional F, we have that, $\forall t \in [0, T]$, Euler's equation is fulfilled

$$L_z(t, P(t), z(t), \dot{z}(t)) - \frac{d}{dt} L_{\dot{z}}(t, P(t), z(t), \dot{z}(t)) = 0 \tag{4.1}$$

If we divide Euler's equation by $L_{\dot{z}}(t, P(t), z(t), \dot{z}(t)) > 0$, $\forall t$, and integrate, we have that

$$L_{\dot{z}}(t, P(t), z(t), \dot{z}(t)) \cdot \exp\left[-\int_0^t \frac{H_z(s, z(s), \dot{z}(s))}{H_{\dot{z}}(s, z(s), \dot{z}(s))} ds\right] = \\ = L_{\dot{z}}(0, P(0), z(0), \dot{z}(0)) = K \in \mathbb{R}^+, \forall t \in [0, T]$$

We shall call this relation the *coordination equation* for $z(t)$, and the positive constant $K \in \mathbb{R}^+$ will be termed the *coordination constant* of the extremal. Let us term the *coordination function* of $z \in \Omega$ the function in $[0, T]$, defined as follows

$$\mathbb{Y}_z(t) = L_{\dot{z}}(t, P(t), z(t), \dot{z}(t)) \cdot \exp\left[-\int_0^t \frac{H_z(s, z(s), \dot{z}(s))}{H_{\dot{z}}(s, z(s), \dot{z}(s))} ds\right] \tag{4.2}$$

with $L_{\dot{z}}(t, P(t), z(t), \dot{z}(t)) = p(t) H_{\dot{z}}(t, z(t), \dot{z}(t))$. Let us now see the fundamental result, which enables us to characterize the extremals of the problem and which is also the basis for elaborating the optimization algorithm that leads to determination of the optimal solution of the hydrothermal system. We present the problem considering the *state variables* to be $z(t)$ and $P(t)$ and the *control variables* $u_1(t) = H(t, z(t), \dot{z}(t))$ and $u_2(t) = \dot{P}(t)$. Moreover, as $H_{\dot{z}} > 0$, the equation

$$u_1(t) - H(t, z(t), \dot{z}(t)) = 0 \tag{4.3}$$

allows the *state equation* $\dot{z} = f(t, z, u_1)$ to be explicitly defined and we obtain

$$f_z = -\frac{H_z}{H_{\dot{z}}}; \; f_{u_1} = \frac{1}{H_{\dot{z}}} \tag{4.4}$$

The *optimal control problem* is thus:

$$\max_{u_1(t), u_2(t)} \int_0^T L(t, P(t), u_1(t)) dt \quad \text{with} \quad \begin{cases} \dot{z} = f(t, z, u_1) \\ \dot{P} = u_2 \\ z(0) = 0, \quad z(T) = b \\ u_1(t) \in \{x \mid H_{\min} \le x \le H_{\max}\} \end{cases}$$

We shall use Pontryagin's Minimum Principle (PMP) [8] as the basis for proving this theorem.

Theorem 1 (Theorem of Coordination). *If* $(z^*, P^*) \in (\widehat{C}^1, C^1)$ *is a solution of our problem, then* $\exists K \in \mathbb{R}^+$ *such that:*

$$\mathbb{Y}_{z^*}(t) \quad is \quad \begin{cases} \le K & if \quad H(t, z^*(t), \dot{z}^*(t)) = H_{\min} \\ = K & if \quad H_{\min} < H(t, z^*(t), \dot{z}^*(t)) < H_{\max} \\ \ge K & if \quad H(t, z^*(t), \dot{z}^*(t)) = H_{\max} \end{cases}$$

and

$$\dot{\Psi}\left(P^*(t)\right) = p(t)$$

Note. It is very important to stress that the problem is thus easily broken down into the two sub-problems: Thermal and Hydro. To obtain the optimum operating conditionsof the hydro-plant, we shall use the coordination equation

$$\mathbb{Y}_z(t) = K, \forall t \in [0, T] \tag{4.5}$$

The problem will consist in finding for each K the function z_K that satisfies $z_K(0) = 0$ and the conditions of Theorem 1, and from among these functions, the one that gives rise to an admissible function ($z_K(T) = b$). From the computational point of view, the construction of z_K can be performed using the same procedure as in the shooting method, with the use of a discretized version of the coordination equation (4.5). The exception is that at the instant when the values obtained for z and $\dot{z}$ do not obey the constraints, we force the solution z_K to belong to the boundary until the moment when the conditions of leaving the domain (established in Theorem 1) are fulfilled.

To calculate the optimum power $P(t)$ of the thermal plant, we solve the equation

$$p(t) = \dot{\Psi}\left(P(t)\right), \forall t \in [0, T] \tag{4.6}$$

The distribution among the thermal plants is immediate by means of the definition of the thermal equivalent, imposing the corresponding constraints (3.2) or (3.4) for each one of the power plants.

5 Example

A computer program was written using the Mathematica © package to apply the results obtained in this paper to a hydrothermal power system. In order to consider an example close to reality, we focused on a thermal system from Asturias (Spain). We consider a conventional 550 MW PCC plant belonging to the company *HC*, *Aboño II,* which was studied by the authors [4] and whose pollutant emissions were modelled, as well as another 50 MW CFBC plant belonging to the company *Hunosa*, *La Pereda*, which presents much more favorable environmental advantages compared to the former plant. The idea underlying this paper is to compare the two technologies. Therefore, given the small size of La Pereda power plant, it was decided to take as an example the two CFBC plants that currently constitute a reference worldwide: *Jacksonville* (USA), generating 300 MW, and *Gardanne* (France), generating 250 MW. Using these two plants, we construct the equivalent CFBC plant, as we saw in [7], obtaining the parameters that are summarized in Table I.

Table I: Coefficients of the thermal plant.

Plant	α_i	β_i	γ_i	$P_{i\max}^{Tch}$	$P_{i\max}^{Env}$
1 (PCC)	1615.35	36.676	0.03659	550	100
2 (CFBC)	1724.55	40.072	0.03511	550	550

We consider $P_{i\min}^{Tch} = P_{i\min}^{Env} = 0$. To calculate $P_{1\max}^{Env}$ for the PCC plant, we took the ELV published in the National Plan for Reducing Emissions from existing LCP as reference. The *Aboño II* plant was assigned (from 2008 to 2015): $484(mg/Nm^3)$ of SO_2 and $437(mg/Nm^3)$ of NO_x. This means a reduction in SO_2 of 83% and a reduction in NO_x of 44%. With these data, we obtain $P_{1\max}^{Env} = 100MW$, a restriction that must be complied with from the year 2008 on. For the CFBC plant, we took the pollutant emissions published for Jacksonville: $90(mg/Nm^3)$ of NO_x and $140(mg/Nm^3)$ of SO_2 and for Gardanne: $240(mg/Nm^3)$ of NO_x and $30(mg/Nm^3)$ of SO_2. With these data, our equivalent CFBC does not exceed the ELV in any case, with which we have $P_{2\max}^{Env} = 550$, which is hence equal to the technical restriction.

The hydrothermal system also considers one hydro-plant. We shall use the *Salime* plant in Asturias (Spain), which also belongs to an HC company. We use a *variable-head* model and the hydro-plant's effective hydraulic generation H (without transmission losses) is a function of $z(t)$ and $\dot{z}(t)$

$$H(t, z(t), \dot{z}(t)) := A(t) \cdot \dot{z}(t) - B \cdot z(t) \cdot \dot{z}(t) - C \cdot \dot{z}^2(t)$$
$$A(t) := \frac{B_y}{G}(S_0 + t \cdot i);\ B = \frac{B_y}{G};\ C = \frac{B_T}{G}$$

The hydro-plant data are summarized in Table II.

Table II. Hydro-plant coefficients.

G	b	i	S_0	B_y	B_T	$H_{\max}$
519840	$6 \cdot 10^6$	133200	$239.5 \cdot 10^6$	$4.34079 \cdot 10^{-7}$	$2.94 \cdot 10^{-5}$	112

For this system we analyze two systems: the one formed by the PCC plant and the hydro-plant and the one formed by the equivalent CFBC plant and the hydro-plant. In both cases we shall carry out two studies: the Economic Dispatch (ED) with technical restrictions, in which we shall maximize the profit for a given price and the Environmentally Constrained Economic Dispatch (ECED), which includes among the operating constraints those referring to maximum emissions. The obtained results are shown below.

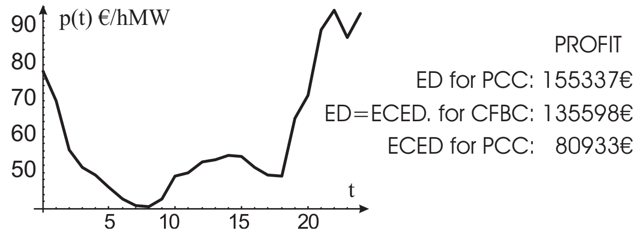

Fig. 1. Clearing price $p(t)$ and profit.

In the figures, we use the terms $P(t)$ to denote the optimal power for the thermal plant and $H(t)$ for the hydro-plant. The clearing price $p(t)$ corresponding to the 5th February 2006 (Sunday) for the Spanish electricity market and the profit obtained for all the cases are presented in Figure 1. Figures 2 and 3 show the Economic Dispatch with technical restrictions for the two systems. Comparing the two cases, we see that the PCC plant is more profitable, as expected given its lower electricity cost. With the new regulations, it is necessary to impose environmental restrictions; Figures 3 and 4 clearly show that the equivalent CFBC plant is more profitable in this case. It is obvious that the CFBC plant solution is the same in the two dispatches, as it is likewise evident that the hydro-plant solution is the same in both cases, since, as we saw in Theorem 1, its functioning is independent of the behavior of the thermal power plant.

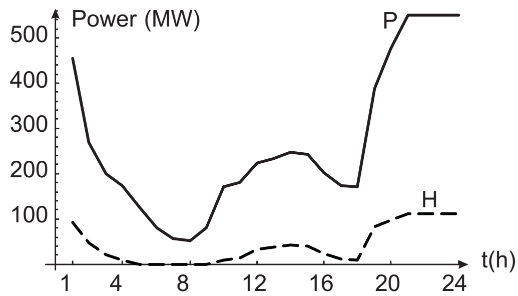

Fig. 2. PCC plant with technical restrictions.

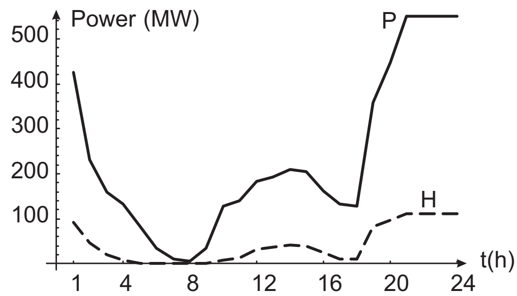

Fig. 3. CFBC plant with technical or environmental restrictions.

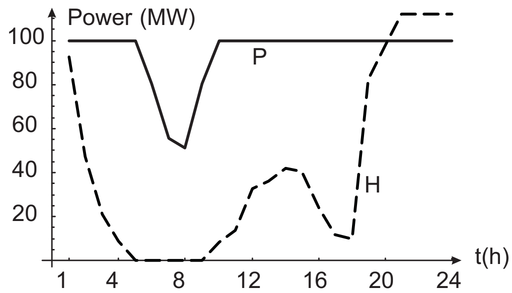

Fig. 4. PCC plant with environmental restrictions.

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