A Sufficient Condition for Minimum in a Hydrothermal Problem with Pumped-storage Plants

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ABSTRACT

Variational problems in which the Lagrangian L of the functional $J(z) := \int_0^T L(t, z(t), z'(t))dt$ presents discontinuities are well known. In this paper we present a sufficient condition for minimum of a functional $J(z) := \int_0^T L(t, z'(t))\ dt$ in the case in which the function L is continuous but not of class C^1. This situation arises in problems of optimization of hydrothermal systems with pumped-storage plants. In such problems, the function $\Upsilon_t(z') := L_{z'}(t, z')$ is discontinuous in $z' = 0$, which is the borderline point between the power generation zone ($z' > 0$) and the pumping zone ($z' < 0$).
The main contribution of this paper, from mathematical point of view, is that the demonstration of the condition solely requires classical Calculus of Variations tools, and it´s not necessary the introduction of subdifferentials. Moreover, from hydrothermal point of view, it provides a simple algorithm for resolving the problem of hydrothermal optimization: the construction of the function that provides the minimum value of J in $A := \{z \in KC^1[0,T] \mid z(0) = a \wedge z(T) = b\}$. Said algorithm was implemented using the Mathematica package. We resolve several real problems of hydrothermal optimization that involve pumped-storage plants. Finally, we have studied the factors that contribute that the optimum mode of functioning consists in not using the hydroplant to generate or to pump.

Keywords: Calculus of Variations, Optimization, Hydrothermal systems, Pumped-Storage plants.