An Economic Dispatch Algorithm of Combined Cycle Units

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1 Introduction

The economic dispatch (ED) problem is defined as that of finding an optimal distribution of system load to the generators in order to minimize the total generation cost while satisfying the total demand and generating-capacity constraints. The different combinations of combustion turbines and steam turbines in a combined cycle (CC) unit produce multiple states and each state has its own unique cost curve. Therefore, in performing ED, we need to be able to shift between these cost curves. However, there is another, more serious problem: the cost curve is not convex for some of these states. As a result, ED of CC units must use special techniques like, for example: Complete Enumeration, Merit Order Loading, Genetic Algorithm (GA), Evolutionary Programming (EP), and Particle Swarm (PS) (see [1] and [2]). The three former techniques (GA, EP, and PS) involve a stochastic searching mechanism, and it should be noted that these heuristic methods do not always guarantee obtaining the globally optimal solution: they only provide an approximate solution for the non-convex optimization problem. In this paper we present a new technique for solving the ED problem of CC units. The technique, de-

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veloped to find the global solution, is based on the calculation of the Infimal Convolution (IC).

2 Mathematical Modelling of CC units

The literature [3] provides several alternatives to model CCs in electricity markets: Aggregated model, Pseudo unit model, Configuration-based model, and Physical unit model. The last two models are the most accurate ones. In general, the physical unit model is more suitable for power flow and network security analysis. However, the configuration-based model is more suitable for bid/offer processing and dispatch scheduling.

This paper focuses on the ED problem that a generation company with CC units faces when preparing its offers for the day-ahead market. We hence consider the configuration-based model. Each state has its own cost curve; for some states, this curve is not convex. The most widely-used model to represent the non-convexity of cost curves is a piecewise linear cost function ([1], [2]). This is the most flexible model and allows a greater approximation to reality. In the present paper, we shall use piecewise linear cost functions to represent the states of a CC unit.

3 Algorithm of Optimization

The classic ED problem can be described as an optimization problem:

$$\min \sum_{i=1}^{N} F_i(P_i) \text{ subject to: } \sum_{i=1}^{N} P_i = P_D;\ P_{i\,\min} \leq P_i \leq P_{i\,\max},\ \forall i = 1, ..., N$$

where $F_i(P_i)$ is the fuel cost function of the i-th unit, P_i is the power generated by the i-th unit, P_D is the system load demand, $P_{i\,\min}$ and $P_{i\,\max}$ are the minimum and maximum power outputs of the i-th unit, and N is the number of units. To solve this problem, we have designed an algorithm based on the mathematical concept of IC. The proposed recursive algorithm for calculating the analytic solution consists of 4 phases.

(Phase 1) Piecewise linear cost function of each CC unit.

The function $F_i(P_i)$ of each CC unit will be a piecewise linear function:

$$F_i(P_i) = \begin{cases} F_{i1}(P_i) & \text{if} \quad P_i \in [m_{i1}, M_{i1}] \\ ... & ... \quad ... \\ F_{ik(i)}(P_i) & \text{if} \quad P_i \in [m_{ik(i)}, M_{ik(i)}] \end{cases}, \quad i = 1, ..., N$$

(Phase 2) Infimal Convolution of 2 CC units.

In the following proposition, we shall express the IC for each pair of linear functions.

Proposition 1. *Let* $f_i(x_i) = a_i + b_i x_i$, $(i = 1, 2)$ *with domains* $[m_i, M_i]$. *Let us assume that* $b_1 \leq b_2$. *It is verified that:*

$$(f_1 \bigodot f_2)(\xi) := \begin{cases} f_1(\xi - m_2) + f_2(m_2) & \text{if} \quad \xi \in [m_1 + m_2, M_1 + m_2] \\ f_1(M_1) + f_2(\xi - M_1) & \text{if} \quad \xi \in [M_1 + m_2, M_1 + M_2] \end{cases}$$

We now need to perform all the possible combinations of pairs of linear functions between the 2 CC units, F_1, F_2, and then calculate the minimum of them all:

$$F_1 \bigodot F_2 = \min_{(i,j)} (F_{1i} \bigodot F_{2j}); \quad i = 1, ..., k(1); \quad j = 1, ..., k(2)$$

obtaining a piecewise linear function. This result will form the basis for the subsequent generalization to the case of N functions.

(Phase 3) Infimal Convolution of N CC units.

Bearing in mind the associative nature of the IC operation, the equivalent of N CC units may now be calculated by means of a recursive process, carrying out N operations of IC. We consider the next recurrence:

$$F_1 \bigodot F_2 \bigodot \cdots \bigodot F_N = (F_1 \bigodot F_2 \bigodot \cdots \bigodot F_{N-1}) \bigodot F_N$$

(Phase 4) Optimal solution of each CC unit.

Besides the minimum value of the total cost, the IC of the N CC units yields, for any P, the vector where said minimum value is reached. We shall now determine the distribution, for any P, of what each of the N CC units has to produce (the optimal solution of each CC unit). The ED problem of CC units is thus fully solved.

4 Example

Finally, the proposed method is applied to a test ED problem ([1] and [2]) and our solution is compared with three stochastic optimization techniques: GA, EP and PS. As already stated, these heuristic methods only provide an approximate solution for the non-convex optimization problem. The costs obtained using GA, EP and PS solutions in Table I are higher as they are based on erroneous assumptions. It should be noted that our algorithm presents a much higher convergence speed than Complete Enumeration, as, in each phase, and after calculating the minimum, we shall only consider a very small fraction of the entire problem.

Table I: Comparison of the optimal solution.

	CC 1 (MW)	CC 2 (MW)	Demand (MW)	Cost ($/h)
GA	560	240	800	31888
EP	528.75	271.25	800	31544
PS	510	290	800	31460
AS	265	535	800	29871.2

5 Conclusions

In this paper we have presented a new technique, based on the calculation of the Infimal Convolution, for solving the ED problem of CC units. The technique consists in a recursive algorithm for calculating the global analytic solution. That is, we do not obtain the solution for only one value of demand, but solve a family of problems, varying P_D to obtain the solution for any value. This distinguishes our method from traditional heuristic methods. Furthermore, we have analytically obtained the solution for a test case that may serve as a comparison for subsequent studies using approximate methods.

References

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