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An economic dispatch algorithm of combined cycle units

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This paper presents a method to solve the economic dispatch (ED) problem for thermal unit systems involving combined cycle (CC) units. The ED problem finds the optimal generation of each unit in order to minimize the total generation cost while satisfying the total demand and generating-capacity constraints. A CC unit presents multiple configurations or states, each state having its own unique cost curve. Therefore, in performing ED, we need to be able to shift between these cost curves. Moreover, the cost curve is not convex for some of these states. Hence, ED becomes a non-convex optimization problem, which is difficult to solve by conventional methods. In this paper we present a new technique, developed to find the global solution, that is based on the calculation of the infimal convolution. The paper includes the results for a case test and we compare our solution with other techniques.

Keywords: non-convex optimization; infimal convolution; economic dispatch; combined cycle

2010 AMS Subject Classifications: 90C26; 65K05

1. Introduction

In today's environment of deregulated power markets, combined cycle (CC) units represent the majority of new generating plants worldwide. In the particular case of Spain, CC gas technology predominates in the energy mix, representing 26% at the end of 2010 [12]. Compared with single-cycle thermal units, CC units present numerous advantages such as higher efficiency, lower capital investment, more flexible operation, and lower environmental impact. The use of this technology has, in turn, brought new challenges to the economic dispatch (ED) problem.

The ED problem is defined as that of finding an optimal distribution of system load to the generators in order to minimize the total generation cost while satisfying the total demand and generating-capacity constraints. Cost curves of conventional thermal units can be modelled as convex functions and, traditionally, the cost function of each generator is approximated by a single quadratic function [4]. In this paper, we analyse a more complex problem and present a method to solve the ED problem for thermal unit systems involving CC units.

A CC unit consists of one or more combustion turbines (CTs), each with a heat recovery steam generator (HRSG). The steam produced by each HRSG is used to drive steam turbines (STs). The different combinations of CTs and STs in a CC unit produce multiple configurations or states. Each state has its own unique cost curve. Therefore, in performing ED, we need to be able to shift between these cost curves. However, there is another, more serious problem: the cost curve is not convex for some of these states (see [3] for a detailed explanation). Hence, ED becomes a

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non-convex optimization problem, which is difficult or even impossible to solve by conventional methods [15]. As a result, ED of CC units must use special techniques. Let us see a summary of the most important of these.

Based on the evolutionary programming (EP) technique, Yang *et al.* [16] developed an algorithm capable of determining the near global optimal solution. Bjelogrlic [3] decomposes the problem into several subproblems to then apply dynamic programming (DP) and mixed integer linear programming (MILP). Gao *et al.* [5] propose to use complete enumeration (CE), merit order loading, genetic algorithm (GA), and a hybrid technique (the lambda iteration method is applied for conventional units and CE for CC units). Gao and Sheble [6] implement three evolutionary algorithms: GA, EP, and particle swarm (PS), comparing these with the classic MILP. The three former techniques (GA, EP, and PS) involve a stochastic searching mechanism, and it should be noted that these heuristic methods do not always guarantee obtaining the globally optimal solution: they only provide an approximate solution for the non-convex optimization problem. The solution of ED problems with non-convex fuel cost functions using stochastic methods is presented in [13] for another classic problem: the valve-point effect.

In [14], a combined economic emission dispatch problem with line flow constraints is considered applying evolutionary computation methods such as GA, micro GA, and EP. Lu and Shahidehpour [10] solved a more general problem considering the unit commitment problem, applying DP and Lagrangian relaxation. However, there is a drawback to assuming that each configuration or state has a convex function model. Lu and Shahidehpour [11] consider network constraints in a security-constrained unit commitment (SCUC) and uses the same technique. Kavasseri and Nag [8] explore the algebra-based Gröbner basis technique to find the solution. MILP is also used by Liu *et al.* [9]. Recently [1] and [2] solve the SCUC problem using DP under a dual programming scheme.

In this paper we present a new technique for solving the ED problem of CC units. The technique, developed to find the global solution, is based on the calculation of the infimal convolution (IC). We believe that our work provides two very important novel contributions to the ED problem with CC units. The first is that our method calculates the global analytical solution, as opposed to heuristic methods (present in the literature), which only calculate a suboptimal solution that approximates the global solution. The second (and perhaps fundamental) contribution is that it solves not only a specific problem for a particular power demand, but also the family of problems that arise when considering all admissible power demands. We further believe that the proposed algorithm can be applied to other problems with similar characteristics that fall outside the field of ED.

The proposed recursive algorithm for calculating the analytic solution (AS) consists of four phases: calculation of the piecewise linear cost function of each CC unit, calculation of the IC of two piecewise linear functions, generalization to N units, and calculation of the optimal solution of each unit. Finally, the proposed method is applied to a test ED problem and our solution is compared with three stochastic optimization techniques: GA, EP, and PS.

The paper is organized as follows. In Section 2 we set out the mathematical modelling of CC units. Section 3 presents the optimization algorithm for the corresponding ED problem. The results obtained in a case test are then presented in Section 4 and our solution is compared with various stochastic techniques. Finally, the conclusions reached in this study are discussed in Section 5.

2. Mathematical modelling of combined cycle units

The literature [1,2,7] provides several alternatives to model CCs in electricity markets:

- *Aggregated model*. This model represents a CC unit by means of an aggregated one that is treated as a regular thermal unit. This is a very simplistic model since it ignores all the different configurations and technical constraints of the CC unit.

State	Composition
1	1 CT
2	2 CT
3	1 CT + 1 ST
4	2 CT + 1 ST

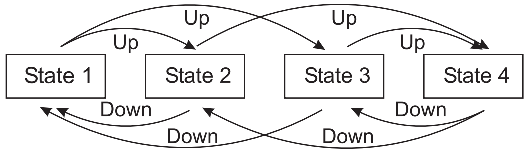

Figure 1. The states of a CC unit and the state transition diagram.

- *Pseudo unit model*. This model represents a CC unit with one or more pseudo units that comprise a single CT and its associated portion of the ST capacity. All pseudo units are required to have the same characteristics. As a consequence, this model has difficulties to represent these characteristics precisely under different operating modes.
- *Configuration-based model*. This model represents a CC unit as multiple mutually exclusive configurations or combinations of CTs and STs. Hence, each configuration has its own cost function. The feasible transitions between configurations are determined by a pre-established state transition diagram. This model presents a degree of flexibility that allows a better representation of technical parameters and bid data from each CC unit.
- *Physical unit model*. This method models the physical components of a CC unit and each CT and ST is considered as an individual resource that may submit its own startup cost, minimum up/down time, etc. From a scheduling point of view, this is not an ideal choice due to the complexity of handling dependency among components.

The last two models are the most accurate ones (see Liu *et al.* [9] for a comparison between these two models). In general, the physical unit model is more suitable for power flow and network security analysis. However, the configuration-based model is more suitable for bid/offer processing and dispatch scheduling.

This paper focuses on the ED problem that a generation company with CC units faces when preparing its offers for the *day-ahead* market. We hence consider the configuration-based model. Assuming a CC unit consists of two CTs and one ST, all configurations (or states) are shown in Figure 1. The state space for a CC unit is composed of the state space of distinct configurations and must be set up according to the individual configurations as well as the relationship between the configurations. The state transition diagram for the above CC unit can be also seen in Figure 1. It can be seen, for example, that transitions between configurations 2 and 3 are not allowed.

Each state has its own cost curve; for some states, this curve is not convex. This situation occurs just as the HRSG is ramped up. Prior to HRSG startup, only the CT is generating with a specified cost per hour. Subsequently, after HRSG startup, the fuel input remains almost constant, although the MW output of the (now) two generation units has increased due to the power produced by the ST. In the case presented in Figure 1, states 1 and 2 will be represented by conventional (convex) cost curves, but the incremental cost curves of states 3 and 4 will not increase monotonically with generation.

The most widely used model to represent the non-convexity of cost curves is a piecewise linear cost function [5,6,14]. This is the most flexible model and allows a greater approximation to reality. Sometimes, the piecewise linear cost function is approximated by more complex functions. For instance, a 4th-order polynomial function is used in [6], while a 10th-order polynomial is employed in [8]. However, some authors also simplify the problem and consider that the cost curves of all states are convex, like, for example, [16] or [10]. In the present paper, we shall use piecewise linear cost functions to represent the states of a CC unit.

The four piecewise linear cost curves, $\{G^j(P)\}_{j=1}^4$, of the CC unit considered in the example of Section 4 are shown in Figure 2. States 1 and 2 are essentially thermal unit states. Accordingly,

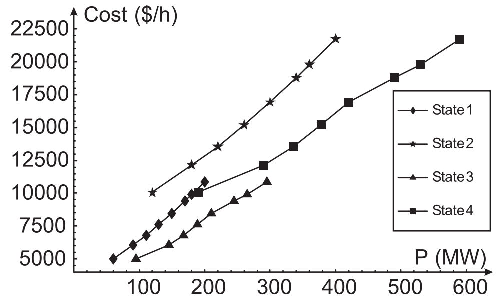

Figure 2. CC cost functions as piecewise linear functions.

the cost curves are monotonously increasing and convex. However, the curves corresponding to states 3 and 4 are monotonously increasing, though no longer convex, as they are CC unit states.

3. Algorithm of optimization

The classic ED problem can be described as an optimization (minimization) problem:

$$\min \sum_{i=1}^{N} F_i(P_i) \quad \text{subject to: } \sum_{i=1}^{N} P_i = P_D; \quad P_{i\min} \le P_i \le P_{i\max}, \quad \forall\, i = 1, \ldots, N,$$

where $F_i(P_i)$ is the fuel cost function of the ith unit, P_i the power generated by the ith unit, P_D the system load demand, $P_{i\min}$ and $P_{i\max}$ the minimum and maximum power outputs of the ith unit, and N the number of units.

To solve this problem, we have designed an algorithm based on the mathematical concept of IC, the definition and properties of which we summarize below.

Definition 1 *Let $f, g : \mathbb{R} \longrightarrow \bar{\mathbb{R}}$ be two functions of $\mathbb{R}$ in $\bar{\mathbb{R}} := \mathbb{R} \cup \{+\infty, -\infty\}$. We denote as the infimal convolution (IC) of f and g the operation defined below*:

$$(f \odot g)(x) := \inf_{y \in \mathbb{R}} \{f(x) + g(y - x)\}.$$

It is well-known that $(\mathcal{F}(\mathbb{R}, \bar{\mathbb{R}}), \odot)$ is a commutative semigroup. Furthermore, for every finite set $E \subset \mathbb{N}$, it is verified that

$$\left(\bigodot_{i \in E} f_i \right)(K) = \inf_{\sum_{i \in E} x_i = K} \sum_{i \in E} f_i(x_i).$$

When the functions are considered constrained to a domain $\mathrm{Dom}(f_i) = [m_i, M_i]$, the above definition continues to be perfectly valid redefining $f_i(x) = +\infty$ if $x \notin \mathrm{Dom}(f_i)$. In this case, the definition may be expressed as follows. Let us denote

$$(f_1 \odot f_2)(\xi) := \min_{\substack{x_1 + x_2 = \xi \\ m_i \le x_i \le M_i}} (f_1(x_1) + f_2(x_2)) = \min_{\substack{m_1 \le x \le M_1 \\ m_2 \le \xi - x \le M_2}} ((f_1(x) + f_2(\xi - x)).$$

This is the abstract functional operation that constructs the cost function of the equivalent thermal power plant to a set of units with cost functions F_i. When conventional thermal units are considered, modelled as quadratic (convex) functions, we have: $F_i(P_i) = \alpha_i + \beta_i P_i + \gamma_i P_i^2,\ i = 1, \ldots, N$. However, the problem differs when CC units are considered. As already mentioned, the cost functions may not be convex in this case.

We next show the basic phases that our algorithm comprises.

Phase 1: Piecewise linear cost function of each CC unit.

Let us consider N CC units with a configuration-based model, M being the states of each CC unit. The function $F_i(P_i)$ of each CC unit will be defined as the minimum of the M piecewise linear functions, one for each state:

$$F_i(P_i) \equiv \min\{G_i^j(P_i)\}_{j=1}^M, \quad i = 1, \ldots, N.$$

Hence, for each $F_i(P_i)$, this will in turn be a piecewise linear function:

$$F_i(P_i) = \begin{cases} F_{i1}(P_i) & \text{if } P_i \in [m_{i1}, M_{i1}] \\ \vdots & \vdots \\ F_{ik(i)}(P_i) & \text{if } P_i \in [m_{ik(i)}, M_{ik(i)}] \end{cases}, \quad i = 1, \ldots, N,$$

where $k(i)$ is the number of pieces in which each function F_i is defined (see Figure 3).

Phase 2: Infimal convolution of two CC units.

Having thus modelled the cost functions of the N CC units, we shall now construct the equivalent unit to them all. The construction is based on the IC concept presented previously. As the cost functions $F_i(x)$ are piecewise linear, we shall redefine each one as:

$$F_{ij}(x) := \begin{cases} F_{ij}(x) & \text{if } x \in [m_{ij}, M_{ij}] \\ \infty & \text{if } x \notin [m_{ij}, M_{ij}] \end{cases}, \quad i = 1, \ldots, N,\ j = 1, \ldots, k(i)$$

and we have that:

$$F_i(x) = \min_{j \in \{1, \ldots k(i)\}} F_{ij}(x).$$

Once redefined in this way, we proceed in this phase to construct the IC of only two CC units, generalizing to N units in the following phase. The calculation of the IC of two piecewise linear functions requires a combinatorial exploration that is reflected in the following theorem.

Theorem 1 *Let* $f(x) := \min_{i \in A}(f_i(x))$ *and* $g(x) := \min_{i \in B}(g_i(x))$, *then*:

$$f \odot g = \min_{(i,j) \in A \times B} (f_i \odot g_j).$$

This theorem justifies the construction of the equivalent unit to two CC units as the minimum function of all the possible ICs of pairs of linear functions. In the following proposition, we shall express the IC for each pair of linear functions.

Proposition 1 *Let* $f_i(x_i) = a_i + b_i x_i,\ (i = 1, 2)$ *with domains* $[m_i, M_i]$. *Let us assume that* $b_1 \leq b_2$. *It is verified that*:

$$(f_1 \odot f_2)(\xi) := \begin{cases} f_1(\xi - m_2) + f_2(m_2) & \text{if } \xi \in [m_1 + m_2, M_1 + m_2], \\ f_1(M_1) + f_2(\xi - M_1) & \text{if } \xi \in [M_1 + m_2, M_1 + M_2]. \end{cases}$$

We now need to perform all the possible combinations of pairs of linear functions between the two CC units, F_1, F_2, and then calculate the minimum of them all:

$$F_1 \odot F_2 = \min_{(i,j)} (F_{1i} \odot F_{2j}), \quad i = 1, \ldots, k(1),\ j = 1, \ldots, k(2),$$

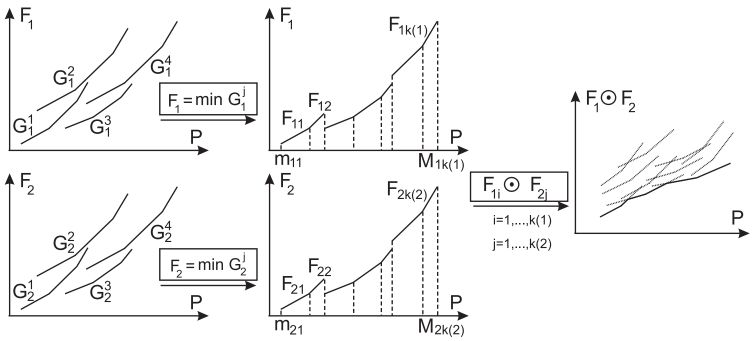

Figure 3. Illustration of the algorithm: phases 1 and 2.

obtaining the piecewise linear function outlined in Figure 3. This result will form the basis for the subsequent generalization to the case of N functions.

Phase 3: Infimal convolution of N CC units.

Bearing in mind the associative nature of the IC operation, the equivalent of N CC units may now be calculated by means of a recursive process, carrying out N operations of IC. We consider the next recurrence:

$$\begin{aligned} F_1 \odot F_2 \odot \cdots \odot F_N &= (F_1 \odot F_2 \odot \cdots \odot F_{N-1}) \odot F_N \\ &= \{H_i(P)\} = \{a_i + b_i \cdot P\}, \quad P \in [l_i, u_i], \quad i = 1, \ldots, Z. \end{aligned}$$

That is, once we have obtained the IC of the first two units, we calculate the IC of the obtained result $F_1 \odot F_2$ with the third F_3 and so on successively. The analytic expression of the IC of the N CC units yields the total cost of the optimal solution for any P.

Phase 4: Optimal solution of each CC unit.

Besides the minimum value of the total cost, the IC of the N CC units yields, for any P, the vector where said minimum value is reached; i.e. the state $G^j(P)$ in which each unit is to be found and the fuel cost function corresponding to said state:

$$I_i = \{r_i + s_i \cdot x\}_{i=1}^{N}, \quad x \in [c_i, d_i], \quad i = 1, \ldots, N.$$

We shall now determine the distribution, for any P, of what each of the N CC units has to produce (the optimal solution of each CC unit). The procedure is the following.

First, given a certain P, we take the interval $[l_i, u_i]$, $i = 1, \ldots, Z$, corresponding to the IC of the N units for which $P \in [l_i, u_i]$. We then order the I_i in increasing order of their slopes $\{s_i\}_{i=1}^{N}$. Finally, the distribution is carried out with the units with a lower s_i assuming all the available power in their operating interval, d_i, until meeting all the demand, P.

The ED problem of CC units is thus fully solved. It should be noted that our algorithm presents a much higher convergence speed than CE, as, in each phase, and after calculating the minimum, we shall only consider a very small fraction of the entire problem.

4. Example

Based on the above results, we are now ready to present an example. For this purpose, we implemented the aforementioned algorithm in Mathematica®. We shall now consider the case test presented in [5,6].

Table 1. States of the CC units.

State 1		State 2		State 3		State 4	
MW	$/h	MW	$/h	MW	$/h	MW	$/h
60	5026	120	10,051	95	5026	190	10,051
90	6084	180	12,167	145	6084	290	12,167
110	6771	220	13,542	168	6771	335	13,542
130	7602	260	15,203	189	7602	378	15,203
150	8469	300	16,939	210	8469	420	16,939
170	9390	340	18,780	245	9390	490	18,780
180	9903	360	19,806	265	9903	530	19,806
200	10,876	400	21,752	295	10,876	590	21,752

By considering this example, we shall be able to compare our solution with three evolutionary algorithms: GA, EP, and PS. As already stated, these heuristic methods only provide an approximate solution for the non-convex optimization problem. The test system consists of two identical CC units ($N = 2$). The fuels' cost functions of the two units are obtained from the data in Table 1. Each of the four states ($M = 4$) comprises seven straight lines. The corresponding graphs have already been presented in Figure 2.

As opposed to the other solutions mentioned above, our algorithm provides the AS for all values of demand. The total cost ($a_i + b_i \cdot P$) for each interval ($i = 1, \dots, Z$) of demand is listed in Table 2. This table also shows the state in which each unit operates and the fuel cost function corresponding to that state. For example, 3^2 means: state 3, fuel cost function 2.

In [6] the test system is run with a demand of 800 MW. The solutions obtained using heuristic methods GA, EP and PS, are shown in Table 3, together with the AS obtained using our method. As stated in phase 4 of the description of the algorithm, we need to only consider the appropriate interval i ($i = 1, \dots, Z$) in order to find the AS. In this case, $i = 27$:

795–855	3^6	4^7	3924.5	32.43

The optimal cost is given by:

$$a + b \cdot P = 3924.5 + 32.433 \cdot 800 = 29871.2\$.$$

Moreover, we know that unit 1 is in state 3, fuel cost function 6:

$$I_1 = r_1 + s_1 \cdot x = 3105.75 + 25.65 \cdot x, \quad x \in [c_1, d_1] = [245, 265]$$

and unit 2 in state 4, fuel cost function 7:

$$I_2 = r_2 + s_2 \cdot x = 2616.33 + 32.433 \cdot x, \quad x \in [c_2, d_2] = [530, 590],$$

and, given that $s_1 < s_2$, the distribution between the two units consists in unit 1 producing $d_1 = 265$ MW while 2 produces $800 - d_1 = 535$ MW.

The costs obtained using GA, EP and PS solutions in Table 3 are higher as they are based on the (erroneous) assumption that both units operate in state 4. As can be seen, the cost of the AS solution is lower due to allowing unit 1 to operate in state 3.

To complete our study, we shall now conduct a second test imposing precisely this condition: fix both units in state 4. The AS obtained in this case is exactly the same as that obtained using PS, which evidences that this heuristic method is not only superior to the other two (GA and EP), but also that it is able to provide the AS in this case.

Finally, note that there is no reason to compare running time between our method and the others mentioned here, as our running time corresponds to the entire family of problems resulting from varying P_D, whereas the other methods only solve one P.

Table 2. Optimal total cost and state of each CC unit.

Demand	Unit		Total cost	
l_i–u_i	1	2	a_i	b_i
120–150	1^1	1^1	5820	35.26
150–155	1^1	1^2	5957.5	34.35
155–190	1^1	3^1	6883.8	20.44
190–290	3^1	3^1	6168.4	20.44
290–313	3^1	3^2	2979.9	31.43
313–336	3^2	3^2	2979.9	31.43
336–357	3^2	3^3	246	39.57
357–371.8	3^3	3^3	246	39.57
371.8–390	3^1	3^5	5175.4	26.31
390–410	3^1	3^6	5434.5	25.65
410–433	3^2	3^6	3062.7	31.43
433–463	3^2	3^7	2630.3	32.43
463–484	3^3	3^7	−674.57	39.57
484–492.7	3^4	3^7	−1504.2	41.28
492.7–510	3^5	3^6	5872.7	26.31
510–530	3^6	3^6	6211.5	25.65
530–560	3^6	3^7	2616.3	32.43
560–590	3^7	3^7	2616.3	32.43
590–600	3^6	4^2	5111.6	30.55
600–630	3^7	4^2	3985.	32.43
630–651.7	3^7	4^3	82.41	38.62
651.7–675	3^1	4^6	8540.2	25.65
675–698	3^2	4^6	4635.5	31.43
698–758	3^2	4^7	3938.5	32.43
758–775	3^3	4^7	−1472.1	39.57
775–795	3^6	4^6	9317.2	25.65
795–855	3^6	4^7	3924.5	32.43
855–885	3^7	4^7	3924.5	32.43
885–925	4^2	4^7	5293.1	32.43
925–968	4^3	4^7	−436.8	38.62
968–985.5	4^4	4^7	−3055.6	41.33
985.5–1020	4^5	4^6	11760.	26.3
1020–1060	4^6	4^6	12423.	25.65
1060–1120	4^6	4^7	5232.6	32.43
1120–1180	4^7	4^7	5232.6	32.43

Table 3. Comparison of the optimal solution.

	CC 1 (MW)	CC 2 (MW)	Demand (MW)	Cost ($/h)
GA	560	240	800	31,888
EP	528.75	271.25	800	31,544
PS	510	290	800	31,460
AS	265	535	800	29,871.2

5. Conclusions

In this paper we have presented a new technique, based on the calculation of the IC, for solving the ED problem of CC units. The technique consists in a recursive algorithm for calculating the global AS. That is, we do not obtain the solution for only one value of demand, but solve a family of problems, varying P_D to obtain the solution for any value. This distinguishes our method from traditional heuristic methods. Furthermore, we have analytically obtained the solution for a test case that may serve as a comparison for subsequent studies using approximate methods.

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