Forecasting Electricity Prices in an Optimization Hydrothermal Problem

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Abstract. This paper presents an economic dispatch algorithm in a hydrothermal system within the framework of a competitive and deregulated electricity market. The optimization problem of one firm is described, whose objective function can be defined as its profit maximization. Since next-day price forecasting is an aspect crucial, this paper proposes an efficient yet highly accurate next-day price new forecasting method using a functional time series approach trying to exploit the daily seasonal structure of the series of prices. For the optimization problem, an optimal control technique is applied and Pontryagin's theorem is employed.

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INTRODUCTION

During the last decade, the electricity supply industry is moving from a centralized operational approach to a competitive one. This is the case for Spanish utilities since January 1st, 1998. In this paper the new short-term problems that are faced by a generation company in a deregulated electricity market are addressed and an optimization algorithm is proposed. Our model of the spot market explicitly represents the price of electricity as an *uncertain exogenous* variable. We represent generation units with high level of detail and our model distinguishes *individual generation units* and considers *inter-temporal constraints* such as hydro reserves. The methodology presented could be applied to any deregulated system based on bids. However, the Spanish regulation [1] has been used as reference market model.

Since next-day price forecasting is an aspect crucial of the problem, Since next-day price forecasting is an aspect crucial, this paper proposes an efficient yet highly accurate next-day price new forecasting method. Price forecasting techniques in power systems are relatively recent procedures. In [2] the model is based on the probability density functions of forecast prices. In [3] dynamic regression and transfer function models are proposed. In [4] electricity prices models are reviewed and a novel classification is proposed.

Trying to exploit the seasonal daily structure of the time series of prices, in this work we model the series using a functional approach which considers hourly prices as observations of daily functions of prices. Therefore, within this approach, the series of prices consists of a sequence of real-valued functions defined on the interval $[0,24]$. We model this functional time series with a linear autoregressive functional model (functional input – functional output) which formulates the relationships between each daily function of prices and the functions of previous days.

So, this paper generalizes two previous papers of the authors. In [5] our model of the spot market explicitly represents the price of electricity as a known exogenous variable, and in [6] the volatility of the spot market price of electricity is represented by a *stochastic model*. We search for past spot market sessions that can be considered similar to the session that the company is about to face using *clustering techniques*.

STATEMENT OF THE HYDROTHERMAL PROBLEM

In this section the optimization problem of one company is described. Let us assume that our hydrothermal system accounts for one hydro-plant and m thermal plants (is easy to generalize to n hydro-plants as we can see in [6]). Let $\Psi_i : D_i \subseteq \mathbb{R} \longrightarrow \mathbb{R}$ $(i = 1,\ldots,m)$ be the cost functions $(Euro/h)$ of the m thermal plants. In general, the cost functions of the thermal plants are second-order polynomials

$$\Psi_i(P_i(t)) = \alpha_i + \beta_i P_i(t) + \gamma_i P_i^2(t); i = 1, ..., m$$

where $P_i(MW)$ is the power generated, and we consider the thermal plants to be constrained by *technical restrictions* of the type

$$P_{i\min} \leq P_i(t) \leq P_{i\max}; i = 1, ..., m, \forall t \in [0,T]$$

$[0,T]$ being the optimization interval. In prior studies [7], it was proven that the problem of optimization of the fuel cost of a hydrothermal system with m thermal plants may be reduced to the study of a hydrothermal system made up of one single thermal plant, called the *thermal equivalent*. We shall denote as the equivalent minimizer of $\{\Psi_i\}_1^m$, the function $\Psi : D_1 + \cdots + D_m \to \mathbb{R}$ defined by

$$\Psi(P(t)) = \min \sum_{i=1}^{m} \Psi_i(P_i(t))$$

with $P(t)$ the power generated by said thermal equivalent. From the perspective of a generation company, and within the framework of the new electricity market, *transmission losses* are not relevant, and will not be considered.

Let $H(t, z(t), \dot{z}(t))$ be the function of *effective hydraulic contribution*, i.e. the power contributed to the system at the instant t by the hydro-plant, and we consider $H(t, z(t), \dot{z}(t))$ to be bounded by technical restrictions

$$H_{\min} \leq H(t, z(t), \dot{z}(t)) \leq H_{\max}, \forall t \in [0,T]$$

$z(t)$ being the volume that is discharged up to the instant t by the plant, and $\dot{z}(t)$ the rate of water discharge of the plant at the instant t. If we assume that b is the volume of water that must be discharged during the entire optimization interval $[0,T]$, the following boundary conditions will have to be fulfilled: $z(0) = 0, z(T) = b$.

In our problem the *objective function* is given by revenue minus cost during the optimization interval

$$F(P,z) = \int_0^T [p(t)(P(t) + H(t, z(t), \dot{z}(t))) - \Psi(P(t))]\, dt$$

Revenue is obtained by multiplying the total production of the company by the clearing price $p(t)$ in each hour t. The cost is given by Ψ, the cost function of the thermal equivalent, where $P(t)$ is the power generated by said plant. With this statement, our objective functional in continuous time form is

$$\max_{P,z} F(P,z) = \max_{P,z} \int_0^T L(t, P(t), z(t), \dot{z}(t)) dt$$

with $L(t, P(t), z(t), \dot{z}(t)) = p(t)(P(t) + H(t, z(t), \dot{z}(t))) - \Psi(P(t))$, on the set

$$\Omega = \left\{ z \in \widehat{C}^1[0,T] \middle| \begin{array}{c} z(0) = 0, z(T) = b \\ H_{\min} \leq H(t, z(t), \dot{z}(t)) \leq H_{\max}, \forall t \in [0,T] \end{array} \right\}$$

where $\widehat{C}^1$ is the set of piecewise C^1 functions.

OPTIMAL SOLUTION

The problem is formulated in the framework of Optimal Control Theory. Let us term the *coordination function* of $z \in \Omega$ the function in $[0,T]$, defined as follows

$$\mathbb{Y}_z(t) = L_{\dot{z}}(t, P(t), z(t), \dot{z}(t)) \cdot \exp\left[-\int_0^t \frac{H_z(s, z(s), \dot{z}(s))}{H_{\dot{z}}(s, z(s), \dot{z}(s))} ds \right]$$

We present the problem considering the *state variables* to be $z(t)$ and $P(t)$ and the *control variables* $u_1(t) = H(t, z(t), \dot{z}(t))$ and $u_2(t) = \dot{P}(t)$. The *optimal control problem* is thus:

$$\max_{u_1(t), u_2(t)} \int_0^T L(t, P(t), u_1(t)) dt \quad \text{with} \quad \left\{ \begin{array}{l} \dot{z} = f(t, z, u_1); \dot{P} = u_2 \\ z(0) = 0, \;\; z(T) = b \\ u_1(t) \in \{x \mid H_{\min} \leq x \leq H_{\max}\} \end{array} \right.$$

We shall use Pontryagin's Minimum Principle (PMP) [8] as the basis for proving this theorem.

Theorem 1 (Theorem of Coordination). *If* $(z^*,P^*) \in (\widehat{C}^1, C^1)$ *is a solution of our problem, then* $\exists K \in \mathbb{R}^+$ *such that:*

$$\mathbb{Y}_{z^*}(t) \;\; is \;\; \begin{cases} \leq K & if \;\; H(t,z^*(t),\dot{z}^*(t)) = H_{\min} \\ = K & if \;\; H_{\min} < H(t,z^*(t),\dot{z}^*(t)) < H_{\max} \\ \geq K & if \;\; H(t,z^*(t),\dot{z}^*(t)) = H_{\max} \end{cases}$$
$$and \qquad \dot{\Psi}(P^*(t)) = p(t)$$

This theorem is the basis for elaborating the optimization algorithm that leads to determination of the optimal solution of the hydrothermal system. The problem is thus easily broken down into the two sub-problems: Thermal and Hydro. To obtain the optimum operating conditions of the hydro-plant, we shall use the *coordination equation*

$$\mathbb{Y}_z(t) = K, \forall t \in [0,T]$$

The problem will consist in finding for each K the function z_K that satisfies $z_K(0) = 0$ and the conditions of Theorem 1, and from among these functions, the one that gives rise to an admissible function $(z_K(T) = b)$. From the computational point of view, the construction of z_K can be performed using the same procedure as in the shooting method, with the use of a discretized version of the coordination equation. The exception is that at the instant when the values obtained for z and $\dot{z}$ do not obey the constraints, we force the solution z_K to belong to the boundary until the moment when the conditions of leaving the domain (established in Theorem 1) are fulfilled.

To calculate the optimum power $P(t)$ of the thermal plant, we solve the equation

$$p(t) = \dot{\Psi}(P(t)), \forall t \in [0,T]$$

The distribution among the thermal plants is immediate by means of the definition of the thermal equivalent, imposing the corresponding constraints for each one of the power plants.

NEXT-DAY PRICE FORECASTING USING FUNCTIONAL LINEAR MODELS

A functional linear regression model [9] is an extension of the multivariate linear regression model to the case of infinite-dimensional or functional data. The data are a sample of pairs of random functions $(X(t),Y(t))$, with X the predictor and Y the response functions. The extension of the linear regression model to functional data is then:

$$E(Y(t)|X) = \mu(t) + \int X(s)\beta(s,t)ds$$

with a parameter function β and a mean response function $\mu(t)$.

The above functional model can also be extended to the case of several predictors as follows:

$$E(Y(t)|\mathbf{X}) = \mu(t) + \sum_{j=1}^{m} \int X_j(s)\beta_j(s,t)ds$$

where the data are now a sample of vectors of random functions $(X_1(t),...,X_m(t),Y(t))$, with $X_1,...,X_m$ the predictors and Y the response functions.

Given a functional time series, i.e. a series of regular functions $\{Y_\tau\}$ with $Y_\tau : \Omega \to \mathbb{R}$, $\tau \in \mathbb{N}$, the above functional framework can be used for formulating an autoregressive functional linear model for the conditional functional mean $E(Y_\tau(t)|\mathbf{X}_\tau)$ at time τ given the vector of observed functions $\mathbf{X}_\tau = (Y_{\tau-1},...,Y_{\tau-m})$.

Electricity prices show a strong daily seasonality structure that can be modeled through daily functions defined on the interval $[0,24]$ hours. Consequently, the series of prices can be seen as a sequence of daily functions $\{\pi_\tau\}$ where $\pi_\tau : [0,24] \to \mathbb{R}$ is the function of day τ in such a way that prices observed that day are observations $\pi_\tau(t)$, $t \in [0,24]$ of the function π_τ.

In this work we apply a functional autoregressive approach to the modeling and prediction of the functional time series of prices comparing several parametric [9] and non-parametric [10] tecniques. However, this series also shows a strong weekly seasonality. Trying to account also for this weekly structure, we finally postulate a mixed model which combines this functional autoregressive daily model with a functional ANOVA model which incorporates the specific effect of each week day. For the case of the linear model, the resulting mixed model has the following general form:

$$\hat{\pi}_\tau = \mu + \mathbf{d}^T \alpha + \sum_{j=1}^{m} \left\langle \pi_{\tau-j}, \beta_j(s,\cdot) \right\rangle$$

or equivalently:

$$\hat{\pi}_\tau(t) = \mu(t) + \mathbf{d}^T \alpha(t) + \sum_{j=1}^{m} \int \pi_{\tau-j}(s) \beta(s,t) ds$$

where $\mathbf{d} = (d_1,...,d_7)^T$ with $d_k = 1$ if the day is the k^{th} day of the week and $d_l = 0$ for $l \neq k$, and $\alpha(t) = (\alpha_1(t),...,\alpha_7(t))^T$ with $\alpha_k(t)$ is the specific mean effect of each day of the week, with $s,t \in [0,24]$.

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REFERENCES

1. OMEL, S.A. "Electricity market activity rules". http://www.omel.es/es/pdfs/EMRules.pdf
2. A. J. Conejo, F. J. Nogales and J. M. Arroyo, Price-taker bidding strategy under price uncertainty, *IEEE Trans. Power Sys.* **17**, 1081–1088(2002).
3. F. J. Nogales, J. Contreras, A. J. Conejo and R. Espínola, Forecasting Next-Day Electricity Prices by Time Series Models, *IEEE Trans. Power Sys.* **17**, 342–348 (2002).
4. A. Mateo, A. Muñoz and J. García, Modeling and forecasting electricity prices with input/output hidden Markov models, *IEEE Trans. Power Sys.* **20**, 13–24 (2005).
5. Bayon, L., Grau, J.M., Ruiz, M.M. and Suarez, P.M., An Environmentally Constrained Economic Dispatch: CFBC Boilers in the Day-ahead Market, *Proceedings of CMMSE 2006*, Madrid, 2006, pp. 90–99.
6. Bayon, L., Grau, J.M., Ruiz, M.M. and Suarez, P.M., An Optimization Problem in Deregulated Electricity Markets solved with the Nonsmooth Maximum Principle, *Proceedings of CMMSE 2007*, Chicago, 2007, pp. 70–80.
7. Bayon, L., Grau, J.M., Ruiz, M.M. and Suarez, P.M., New developments on equivalent thermal in hydrothermal optimization: an algorithm of approximation, *J. Comput. Appl. Math.*, **175**, 63-75 (2005).
8. Clarke, F.H., *Optimization and nonsmooth analysis*, John Wiley & Sons, New York, 1983.
9. Ramsay, J.O. and Silverman, B.W., *Functional data analysis*, Springer Verlag, New York, 1997.
10. Ferraty, F. and View, P., *Nonparametric Functional Data Analysis*, Springer, New York, 2006.