A Optimization technique of Hydrothermal Systems using Calculus of Variations

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1. Introduction

This paper studies the optimization of systems with renewable energy. We have developed a theory that is extraordinarily simpler than previous ones which resolves the problem of minimization of a functional

$$F(z) = \int_0^T L(t, z(t), z'(t))dt$$

within the set KC^1 (piecewise C^1) functions that satisfy: $z(0) = 0$, $z(T) = b$ and the constraints

$$z'(t) \geq 0 \text{ and } H(t, z(t), z'(t)) \leq P_d(t),\ \forall t \in [0, T]$$

We have established a necessary condition for the stationary functions. The method allows for elaboration of the optimization algorithm which provides us with the optimal solution of the hydrothermal system. Finally, we present a example, employing the algorithm realized with the "Mathematica" package.

Key Words: Optimization, Hydrothermal System, Calculus of Variations, Hydro Energy, Cost Functional.

2. Description of the problem

The problem [1-2] consists in minimizing the cost of fuel needed to satisfy a certain power demand during the optimization interval $[0, T]$

$$Cost = \int_0^T \Psi(P(t))dt$$

Moreover, the following equilibrium equation of active power will have to be fulfilled

$$P(t) + H(t, z(t), z'(t)) = P_d(t),\ \forall t \in [0, T]$$

If we assume that b is the volume of water that must be discharged during the entire optimization interval, we have the following boundary conditions: $z(0) = 0$, $z(T) = b$. We have developed a theory that is extraordinarily simpler than previous ones [3] which resolves the problem of minimization of a functional of the type $F(z)$ within the set KC^1 functions that satisfy $z(0) = 0$, $z(T) = b$ and the constraints

$$z'(t) \geq 0 \wedge H(t, z(t), z'(t)) \leq P_d(t), \forall t \in [0, T] \quad (1)$$

In particular, we have established a necessary condition for the stationary functions of the functional. If we did not have the restrictions (1) we could use the shooting method to resolve the problem. In this case, we would use the integral form of the Euler's equation

$$-L_{z'}(t, z(t), z'(t)) + \int_0^t L_z(s, z(s), z'(s))ds =$$
$$= -L_{z'}(0, z(0), z'(0)) = K > 0,\ \forall t \in [0, T]$$

Varying the initial condition of the derivative $z'(0)$ (initial flow rate), we would search for the extremal that fulfils the second boundary condition $z(T) = b$ (final volume). However, we cannot use this method in our case, as due to the restrictions, the extremals may not admit bilateral variations.

Firstly, we introduce the concept of *weak influence of the volume*, essential when studying superior boundary arcs. The problem will consist in finding for each K a function q_K which satisfies the conditions of the *Main Coordination Theorem* and among these functions, the one which generates an admissible function. The method allows for elaboration of the optimization algorithm which provides us with the optimal regime of functioning of the entire hydrothermal system.

Finally, we present a example, employing the algorithm realized with the "*Mathematica*" package to this end. The program resolves the optimization problem and was then applied to a hydrothermal system.

3. Conclusions

One of the main contributions of this paper is that it resolves the optimization of hydrothermal systems without being restricted to particular cases. What is more, we have obtained a very simple method that enables us to find an optimal solution in the presence of inequality constraints, and which requires very little computational effort.

References

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