NEW DEVELOPMENTS ON EQUIVALENT THERMAL IN HYDROTHERMAL OPTIMIZATION. AN ALGORITHM OF APROXIMATION

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1. Introduction

This work is embedded in the line of research entitled "Optimization of hydrothermal systems". In a previous paper [1] we considered the possibility of substituting a problem with m thermal plants and n hydroplants ($\mathrm{H_n}$-$\mathrm{T_m}$) by an equivalent problem ($\mathrm{H_n}$-$\mathrm{T_1}$) with a single thermal power station: the equivalent thermal plant. In said paper we calculated the equivalent minimizer in the case where the cost functions are second-order polynomials. We proved that the equivalent minimizer is a second-order polynomial with piece-wise constant coefficients; moreover, it belongs to the class C^1.

In this paper we shall present two fundamental contributions: first, new theoretical results relative to the equivalent thermal plant and, second, an algorithm for the approximate calculus for a general model. We assume throughout the paper the following definitions.

Let $F_i : D_i \subseteq \mathbb{R} \longrightarrow \mathbb{R}$ $(i = 1, \ldots, m)$ be the cost functions of the thermal power stations.

We assume that $\forall \xi \in D = D_1 + \cdots + D_m \subseteq \mathbb{R}$, $\exists(\xi_1, \ldots, \xi_m) \in \prod_{i=1}^{m} D_i$, the unique minimum of $\sum_{i=1}^{m} F_i(x_i)$ with the condition $\sum_{i=1}^{m} x_i = \xi$.

Definition 1.1. Let us call the i-th distribution function, the function $\Psi_i : D_1 + \cdots + D_m \longrightarrow D_i$ defined by $\Psi_i(\xi) = \xi_i$, $\forall i = 1, \ldots, m$.

Definition 1.2. We will denote as the equivalent minimizer of $\{F_i\}_1^m$, the function $\Psi : D_1 + \cdots + D_m \longrightarrow \mathbb{R}$ defined by

$$\Psi(\xi) = \min \sum_{i=1}^{m} F_i(x_i)$$

with the constraint $\sum_{i=1}^{m} x_i = \xi$.

2. New Theoretical Developments

In this paper we continue the theoretical studies of the equivalent thermal plant. First we prove, under certain assumptions, the existence and uniqueness of the equivalent minimizer Ψ.

Theorem 2.1. *Let* $\{F_i\}_{i=1}^m \subset C^1[0,\infty)$ *be a set of functions such that* F_i' *is strictly increasing* $(i = 1, ..., m)$*, with* $F_i'(0) \leq F_{i+1}'(0)$*, and let the function* $F : [0,\infty)^m \longrightarrow R$ *be* $F(x_1, \dots, x_m) := \sum_{i=1}^m F_i(x_i)$*. Let* $C_a := \{(x_1, \dots, x_m) \in \mathbb{R}^m \mid x_i \geq 0 \wedge \sum_{i=1}^m x_i = a\}$.

Then, there exists a unique set $\{\Psi_i\}_{i=1}^m$ *such that:*

(1) $(\Psi_1(a), \dots, \Psi_m(a))$ *is the minimum of* F *over* C_a, $\forall a \geq 0$.

(2) *It holds that*

$$(\Psi_1(a), \dots, \Psi_m(a)) \in \mathring{C}_a \iff a > (\sum_{i=1}^m F_i'^{-1} \circ F_m')(0)$$
$$\iff \left(\sum_{i=1}^m F_i'^{-1} \circ F_m'\right)^{-1}(a) > 0$$

being

$$\Psi_k(a) = \left(\sum_{i=1}^m F_i'^{-1} \circ F_k'\right)^{-1}(a)$$

(3) $(\Psi_1(a), \dots, \Psi_m(a)) \notin \mathring{C}_a \Longrightarrow$ *for certain* $i \in \{1, ..., m-1\}$

$$\Psi_i(a) = \Psi_{i+1}(a) = ... = \Psi_m(a) = 0$$

In the previous theorem we also obtain the distribution functions Ψ_k. Now we define the equivalent thermal plant piece-wisely, taking into account the restriction of power positivity.

Theorem 2.2. *Let* $\{F_i\}_{i=1}^m$, F*, and* C_a *be defined as in Theorem 2.1. Then there exists* $\{\delta_k\}_{k=1}^{m+1} \subset \overline{\mathbb{R}}$ *(with* $\delta_{m+1} = \infty$*) and* $\{\Psi_k\}_{k=1}^m \subset C[0,\infty)$ *such that for every* $a > 0$*, the minimum of* F *over* C_a *attains at* $(\Psi_1(a), \dots, \Psi_m(a))$*, being*

$$\delta_k = \sum_{i=1}^k (F_i'^{-1} \circ F_k')(0) \leq \sum_{i=1}^{k+1} (F_i'^{-1} \circ F_{k+1}')(0) = \delta_{k+1}$$

$$\Psi_k(a) = \begin{cases} \left(\sum_{i=1}^j F_i'^{-1} \circ F_k'\right)^{-1}(a) & \text{if } \delta_k \leq \delta_j \leq a < \delta_{j+1} \\ 0 & \text{if } \quad a \leq \delta_k \end{cases}$$

Also, we shall prove that, for a general model, the equivalent thermal plant belongs to the class C^1.

Theorem 2.3. *Let* $\{F_i\}_{i=1}^m \subset C^1[0,\infty)$ *be a set of functions defined as in Theorem* 2.1. *Then the function*

$$\Psi(a) = \sum_{k=1}^{m} F_k(\Psi_k(a)) = \min_{v\in C_a} F(v)$$

belongs to the class C^1 *and* $\Psi'(0) = F_1'(0)$.

To conclude this section, we analyse the situation that arises when the thermal plants are constrained to restrictions of the type

$$C_a := \{(x_1,\dots,x_m)\in\mathbb{R}^m | P_{\min} \le x_i \le P_{\max} \wedge \sum_{i=1}^{m} x_i = a\}$$

3. An Algorithm of Aproximation

We have developed a new algorithm for the approximate calculus of the thermal equivalent of m thermal power plants whose cost functional is general (non-quadratic). The outline is the following:

i) We linearly approximate the derivative of the cost function of each thermal plant, $F_i'(x)$, $i = 1,...,m$ in the power generation interval of each plant. This approximation may be done as finely as one wishes by simply increasing the number of splines in said interval. The integration of these functions leads us to the piece-wise defined functions $\widetilde{\Psi}_i(x)$, $i = 1,...,m$ that approximate the cost function of each thermal plant considered

$$\widetilde{\Psi}_i(x) = \begin{cases} \widetilde{\alpha}_{ik} + \widetilde{\beta}_{ik}x + \widetilde{\gamma}_{ik}x^2 \text{ if } \delta_{ik} \le x < \delta_{ik+1};\ k=1,...,l-1 \\ \widetilde{\alpha}_{il} + \widetilde{\beta}_{il}x + \widetilde{\gamma}_{il}x^2 \text{ if } x \ge \delta_{il} \end{cases}$$

ii) We next demonstrate that each function $\widetilde{\Psi}_i(x)$ can be considered as the minimizing equivalent of l fictitious thermal plants, whose cost functions, denoted by $\{F_{i1}(x), F_{i2}(x), \cdots, F_{il}(x)\}$, are second-order polynomials

$$F_{ik}(x) = \alpha_{ik} + \beta_{ik}x + \gamma_{ik}x^2;\ k=1,...,l$$

The aforementioned coefficients, deduced from those obtained in [1], are given by

$$\beta_{ik} = 2\widetilde{\gamma}_{ik}\delta_{ik} + \widetilde{\beta}_{ik}$$

$$\gamma_{i1} = \widetilde{\gamma}_{i1};\ \gamma_{ik} = \frac{\widetilde{\gamma}_{ik}}{1-\widetilde{\gamma}_{ik}\left(\sum_{j=1}^{k-1}\frac{1}{\gamma_{ij}}\right)};\ k=2,...,l$$

$$\sum_{j=1}^{l} \alpha_{ij} = \widetilde{\alpha}_{ik} - \frac{\widetilde{\beta}_{ik}^2}{4\widetilde{\gamma}_{ik}} - \sum_{j=1}^{k} \frac{\beta_{ij}^2}{4\gamma_{ik}}$$

where $(\delta_{ik}, \delta_{ik+1})$ is the domain of $\widetilde{\Psi}_i(x)$.

iii) Finally, we construct the equivalent minimizer of all the functions obtained

$$\{F_{ij}\}_{\substack{i=1,\ldots,m \\ j=1,\ldots,l}}$$

We finally show, using an example, that the developed algorithm offers very good approximate results in comparison with prior methods, such as for instance [2].

References

1. L. Bayón, J. M. Grau and P. Suárez, A new formulation of the equivalent thermal in optimization of hydrothermal systems, *Math. Probl. Eng.* (2002).

2. L. Bayón, J. M. Grau and P. Suárez,, A New Algorithm for the Optimization of a Simple Hydrothermal Problem, *Proceedings CMMSE 2002*, Vol. I, pp. 61-70, (2002).