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Plenary Lectures
We consider the family of geometric Lorenz attractors. It is well known that these attractors are chaotic and support a unique SRB measure. Here we show that the attractors are statistically stable: the SRB measure depends continuously on the dynamical system, when we consider the weak* topology in the space of probability measures.
On $C^1$-generic conservative dynamics with positive metric entropy

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We show that a $C^1$-generic volume preserving dynamical system with positive metric entropy is ergodic and nonuniformly hyperbolic.
Resonance and fractal geometry

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The phenomenon of resonance will be dealt with from the viewpoint of dynamical systems depending on parameters and their bifurcations. Resonance phenomena are associated to open subsets in the parameter space, while their complement corresponds to quasi-periodicity and chaos. The latter phenomena occur for parameter values in fractal sets of positive measure. We describe a universal phenomenon that plays an important role in modelling. This paper gives a summary of the background theory, veined by examples.
A walk into the "New Methods of Celestial Mechanics"

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Writing in 1925 about the 3 volumes of Henri Poincaré’s New Methods of Celestial Mechanics (1892, 1893, 1899), Paul Appell said: "It is probable that, during the next millenium, this book will be the mine from which more modest researchers will extract their material". And indeed, a great part of the mathematical theory of Dynamical Systems originates from the works of Poincaré and in particular from the New Methods: exponents, invariant manifolds, homoclinic and heteroclinic solutions, analytic non-integrability and divergence of the perturbation series, exponentially small splitting of separatrices, variational equations and integral invariants, recurrence theorem, surfaces of section and return maps, these notions form the backbone of the general theory; a general theory which in turn, coupled with the development of powerful computers, has renewed the classical field of Celestial Mechanics. The talk will concentrate on one of the main themes of the New Methods, namely the Planar Circular Restricted Three Body Problem, the (un)stability of which was the motivation of the treatise.
Geometric methods for instability in high dimensions

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We consider a Hamiltonian system which is a perturbation of an integrable one written in action-angle variables (with any number of degrees of freedom) and several pendula and subject it to a periodic perturbation. This is a multidimensional version of Poincare’s fundamental problem of dynamics.

We find explicit conditions which show that the system has orbits whose actions perform rather arbitrary excursions in a domain of big size which is independent of the size of the perturbation.

The main new phenomena when increasing the number of degrees of freedom is resonances of high multiplicity, but we note that they happen in sets of large co-dimension, so that they can be contoured. More precisely, we show that given any path in action space which avoids resonances of order 2 or higher there is a trajectory that follows it.
We describe $C^2$– open sets of iterated function systems on arbitrary compact manifolds admitting fully supported ergodic measures all whose Lyapunov exponents vanish. We also exploit the consequences for partially hyperbolic maps.
Explicit bifurcation diagrams for planar differential equations

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The key points for knowing the bifurcation diagrams for families of planar differential equations are the knowledge of the global behavior of the separatrices of their critical points and the control of the number of limit cycles that these equations can have.

In this talk we present a method for studying the global behavior of the separatrices based on the construction of suitable piecewise algebraic curves without contact for the flow of the differential equation. These curves are obtained using local and global analytic information of the separatrices of the finite and infinite critical points of the vector field. The study of the number of limit cycles is done by applying the Bendixson-Dulac Theorem with suitable rational Dulac functions.

We will consider several one and two parameter families of planar polynomial systems and we will determine their bifurcation diagrams. In particular we will obtain explicit algebraic curves that give upper and lower bounds of the actual bifurcation curves. For instance, for the well-known Bogdanov-Takens system, we obtain that, for small values of \( n \), the bifurcation curve corresponding to the saddle loop is given by

\[
b = \frac{5}{7} n^{1/2} + \frac{72}{2401} n - \frac{20024}{45294865} n^{3/2} - \frac{35296165}{11108339166925} n^2 + O(n^{5/2}).
\]

A similar method can also be applied to determine the shape of the traveling waves appearing in the Fisher-Kolmogorov partial differential equation. Recall that these particular solutions correspond to heteroclinic connections in an associated planar differential equation.

This talk is based on the works [1, 2, 3].
References


This talk is about piecewise smooth dynamical systems, whose phase space is partitioned into different regions, each associated with a different functional form of vector field. These systems possess unique dynamics, such as grazing, period adding and sliding. Examples occur in many areas including mechanics, robotics, electrical engineering, biology, economics and oceanography. Despite their ubiquity, the theory of piecewise smooth dynamical systems is still in its infancy, mainly due to a lack of dimension reduction techniques. In this talk, I will outline the current state of the theory and highlight many of the outstanding challenges and problems posed by such systems.
Critical points for surfaces diffeomorphisms, abundance of periodic orbits and the structural stability conjecture

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We will discuss a notion of critical set for surfaces diffeomorphisms and using them we will tack some simple question related to the $C^r$– stability conjecture and the problem about density of periodic orbits for $C^r$– generic diffeomorphisms.
Domains of Practical Stability near $L_{4,5}$ in the 3D Restricted Three-Body Problem

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It is well-known that the triangular relative equilibrium solutions (r.e.s.) of the 3D RTBP are linearly stable in a range of the mass parameter $\mu \in (0, \mu_1)$. Nonlinear stability is found also if $\mu$ is different from the exceptional values $\mu_2, \mu_3$, for a set of almost full measure on a small vicinity of the r.e.s. [2]. Furthermore, as shown in [1], normal forms give rise to Nekhorosev-like estimates of diffusion: the possible escape is extremely slow, producing a practical stability.

It is natural to formulate the question: up to which distance escape is slow, and why crossing some quasi-boundaries it becomes relatively fast. Some preliminary hints about the answer to the question can be found in [3] and [4].

The goal of this presentation is to describe some recent progress [5] which clearly shows the role that several codimension-one manifolds play in the problem. Some of these manifolds could be expected: They are the stable and unstable manifolds of the centre manifold around the collinear libration point $L_3$.

But other codimension-one manifolds have been found to play a key role, specially for orbits which reach large values of $z$. At least for small values of $\mu$ these manifolds are the stable and unstable manifolds of the centre manifolds associated to some families of periodic orbits of elliptic-hyperbolic type.

The study is done by considering simultaneously all the levels of the Jacobi constant. The methodology, results and some indications about not too small values of $\mu$ will be reported.
References


The Many Facets of Chaos

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Chaos is a concept with many facets or aspects. It has several definitions that emphasize different aspects of chaos. No definition is complete. My talk will illustrate how focusing on different aspects of chaos leads us in different directions and results in a fuller understanding of chaos.
Communications
The index of vector field and Poincaré residue

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In 1887 H.Poincaré introduced the concept of topological index for vector fields with isolated singularities given on 2-dimensional manifolds and the notion of differential residue 1-form attached to any rational differential 2-form in $\mathbb{C}^2$ with simple poles along a smooth complex curve.

The purpose of the talk is to show that the De Rham complex (also firstly considered by H.Poincaré) and its invariants naturally connects the both notions in the most widely context. Our approach is based on a new algebraic concept of the homological index for vector fields on complex analytic spaces originated by X. Gómez-Mont [3] as well as on the theory of residues of meromorphic differential forms logarithmic along hypersurface or complete intersections with arbitrary singularities developed by the author in the past few years [1, 2]. In the talk we discuss several elementary methods for computing the topological index of complex vector fields on Cohen-Macaulay curves, normal surfaces and complete intersections with singularities.

References


Communications

Secants of trajectories in dimension three

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Let $\xi$ be a real analytic vector field defined in a neighborhood of the origin of $\mathbb{R}^3$ and assume that the origin is an equilibrium point of $\xi$. Consider an orbit $\gamma$ of $\xi$ such that the origin is the only $\omega$-limit point of $\gamma$. The secants of $\gamma$ are the vectors $\gamma(t)/||\gamma(t)|| \in S^2$. We say that $\gamma$ has tangent at the origin if the limit of secants

$$\lim_{t \to +\infty} \frac{\gamma(t)}{||\gamma(t)||}$$

exists. In case we have no tangent, it is a natural question to ask for a description of the set of secants accumulation

$$Sec(\gamma) = \bigcap_{s} \{ \gamma(t)/||\gamma(t)||; t \geq s \} \subset S^2.$$

This problem can be considered as an infinitesimal version of the classical Poincaré-Bendixson Theorem [2, 4].

In this talk we give a description of the set of accumulation of secants for generic absolutely isolated singularities.

References


In this talk we study the number of limit cycles of the following family of cubic systems

\[
\begin{align*}
\dot{x} & = A(1-x-y) - Bxy^2, \\
\dot{y} & = -y(1-x-y) + Bxy^2,
\end{align*}
\]  

(1)

where $A$ and $B$ are positive real parameters.

The previous system was introduced in [2] and it models the star formation histories in giant spiral galaxies.

The main result of the talk is the following theorem.

**Theorem.** Consider system (1) in the first quadrant with $A > 0, B > 0$. It has a periodic orbit if and only if $B < (1 - 2A)/A^2$. Moreover in this case it is unique, stable and non-algebraic.

To prove our result we develop a new criterion on non-existence of periodic orbits and we extend a well-known criterion on uniqueness of limit cycles due to Kuang and Freedman, see [3]. Both results allow to reduce the problem to the control of the sign of certain functions that are treated by algebraic tools.

Finally, the non-algebraicity of the existing limit cycle is proved by applying a method introduced in [1].

**References**


Reducibility of steady-state bifurcations in coupled cell systems

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There are rapidly growing interests in the research of network dynamics since many collective phenomena in science and modern technology are modeled by large networks such as gene transcriptional networks, networks of neurons, ecological food webs, disease transmission networks, electronic circuits, and so on. In order to study network dynamics, M. Golubitsky, I. Stewart and their collaborators [1, 2, 3] introduced a theory for coupled cell system. A coupled cell system (CCS) is a network of dynamical systems or a coupled system of ODEs. In their formulation, a CCS is associated with a coupled cell network (CCN) whose architecture is a directed graph that indicates the coupling structure of the system.

A regular network [4] is a CCN where each node has the same differential equation and one kind of coupling. For regular networks, cells can be identified up to an equivalence relation on cells and the identification of cells determines a new network called the quotient network. Obviously, a CCN may have many quotient networks. In this work I consider steady-state bifurcations in CCSs associated with regular networks. By comparing bifurcations in CCSs associated with a CCN to bifurcations in CCSs associated with quotient networks of the CCN, quotient systems, define reducibility of bifurcations in CCSs: a bifurcation of equilibrium in CCS is reducible if any bifurcation branch can be lifted from a bifurcation branch in some of its quotient systems. I will show the reducibility of steady-state bifurcations in CCSs associated with \( n \)-cell bi-directional rings.
References


Cycles and suspended blenders in generic unfoldings of nilpotent singularities

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Singularities of a vector field are simplest elements from which interesting dynamics may emerge. For instance, it is proved that any generic nilpotent singularity of codimension four in $\mathbb{R}^4$ unfolds a bifurcation hypersurface of bifocal homoclinic orbits, that is, homoclinic orbits to equilibrium points with two pairs of complex eigenvalues. All return map defined over a transversal section to this homoclinic orbit is a diffeomorphism in $\mathbb{R}^3$ and thus, susceptible to exhibit heterodimensional cycles. We will approach the study of the existence of these cycles showing how suspended blenders could appear in the generic unfoldings of these nilpotent singularities. This talk is based on the paper [1].

References

Over recent years, a great deal of experimental studies and modeling simulations have been directed toward the identification of various dynamical and structural invariants to serve as key signatures uniting often diverse nonlinear systems into a single class. One such class of low order dissipative systems has been identified to possess one common, easily recognizable pattern involving spiral structures in a biparametric phase space. Despite the overwhelming number of studies reporting the occurrence of spiral structures, there is still little known about the fine construction details and underlying bifurcation scenarios for these patterns. In this talk we study the genesis of the spiral structures in several low order systems and reveal the generality of underlying global bifurcations. We will start with the Rössler model and demonstrate that such parametric patterns are the key feature of systems with homoclinic connections involving saddle-foci meeting a single Shilnikov condition [2]. Besides, we show that the organizing center for spiral structures in the Rössler model with the Shilnikov saddle-focus is related to the change of the topology of the attractor transitioning between the spiral and screw-like types [2, 3]. Other group of spiral structures is made of models with the Lorenz-like attractor. For thorough explorations of the dynamics of Lorenz-like models we have proposed [1] the algorithmically easy, though powerful toolkit based on the symbolic description of a single trajectory – an unstable separatrix of the saddle singularity of the model. A new computational technique based on the symbolic kneading invariant description for examining dynamical chaos and parametric chaos in systems with Lorenz-like attractors is proposed and tested. This technique uncovers the stunning complexity and universality of spiral structures in the iconic Lorenz equation.
References


On the confinement of Saturn’s F-ring: A planar restricted 5-body model

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The confinement of planetary narrow rings is understood in terms of the shepherd theory, which proposes the existence of two moons orbiting around the ring. The shepherd theory involves an angular momentum transfer mechanism between the shepherd moons and the ring particles, self-gravity, viscous damping due to inter-particle collisions, and assumes the existence of lower-order resonance at the ring boundaries to effectively confine the ring. Saturn’s F ring is a fascinating example of a narrow eccentric ring displaying a rich dynamical structure: besides its non-zero eccentricity and sharp-edges, it has multiple components entangled in a complicated way which shows a variety of short-time features. Two small moons orbiting close to its boundaries, Prometheus and Pandora, influence importantly many of the dynamical features observed (e.g. streamers and channels). Yet, the shepherd theory does not apply to this ring since there are no mean-motion resonances that confine the ring. The confinement of Saturn’s F-ring thus remains unexplained.

In this talk, we shall describe numerical results on a model for the confinement and structure of Saturn’s F ring. The model is a planar restricted 5-body model which includes Saturn’s flattening. The dynamics of an ensemble of non-interacting ring particles is followed during $2.4 \times 10^6$ revolutions of innermost moon Prometheus. The ring particles “escape” whenever they approach any shepherd moon within 50 km, which corresponds to a physical collision with the moon. The remaining “trapped” ring particles are filtered with respect to the
net variations of their main frequencies, i.e., we consider only those whose variance is less than a given threshold. Comparisons with the observations will be provided.
Non-Autonomous linear cellular automata

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Non autonomous linear cellular automata are systems of linear difference equations with many variables and many symmetries. They can be seen as network dynamical systems defined on Cayley graphs as well as convolution equations on discrete groups. We explore the difference Galois theory of these equations and their decoupling by means of discrete Fourier transforms. We relate the structure of the difference Galois group with the structure of the discrete group underlying the Cayley graph. In the abelian case, we give quadrature formulas for the general solution. We give some dynamical interpretation of these formulas in the autonomous case. We use infinite dimensional Picard-Vessiot theory and parameterized Picard-Vessiot theory to deal with the case on infinitely many variables, getting some theoretical results.
Symmetric dynamics in models of antigenic variation

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In this talk I will discuss how the methods of equivariant bifurcation theory can be used to analyse the dynamics of pathogens and their interactions with a host immune system [1, 2]. Isotypic decomposition of the phase space and the equivariant Hopf bifurcation are used to study symmetry-breaking bifurcation of the fully symmetric steady state in a model of antigenic variation in malaria. I will illustrate how one can perform a comprehensive classification of different periodic solutions in terms of their symmetries. Effects of immune delay on symmetric dynamics are also investigated, and numerical simulations of the full system are performed to illustrate different types of dynamical behaviour. The results of this analysis are quite generic and can be used to study symmetric properties of within-host immune dynamics of many infectious diseases.

References


Communications

On nonautonomous discrete systems

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A nonautonomous system (see [4]) is a pair \((X, f_{1,\infty})\), where \(X\) is a topological space and \(f_{1,\infty} = (f_n)\) is a sequence of continuous maps \(f_n : X \to X\). For any \(x \in X\), its orbit is given by the nonautonomous difference equation

\[
\begin{aligned}
x_{n+1} &= f_n(x_n), \\
x_0 &= x.
\end{aligned}
\]

This work is devoted to study the asymptotic behavior of the orbits of the nonautonomous system when the sequence \(f_{1,\infty}\) converges uniformly to a continuous map \(f\). In addition, we study what dynamical properties of \(f_{1,\infty}\) can be obtained from that of \(f\).

We show that the limit points of trajectories of the nonautonomous systems are contained in the non–wandering set of the limit map \(f\) when it has the shadowing property, namely, let \(\delta > 0\). A sequence \(x_n\) is a \(\delta\)–pseudo orbit of \(f\) if \(d(x_{n+1}, f(x_n)) < \delta\) for \(n \geq 1\). Given \(\varepsilon > 0\), we say that \(\text{Orb}(x, f)\) \(\varepsilon\)–shadows \(x_n\) if \(d(x_n, f^n(x)) < \varepsilon\) for \(n \geq 1\). The map \(f\) has the shadowing property if for any \(\varepsilon > 0\) there is \(\delta > 0\) such that any \(\delta\)–pseudo orbit is \(\varepsilon\)–shadowed by an orbit of \(f\). In addition, in the case of \(X = [0, 1]\) such limits points are in \(\omega\)–limit sets of \(f\) and \(f_{1,\infty}\) exhibits chaos in the sense of Li and Yorke if and only if \(f\) also is Li–Yorke chaotic, provided all the maps are surjective.

References


Using evolutionary algorithms to optimize Flower Constellations

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Nowadays Satellite Constellations are used for several purposes such as global navigation, telecommunication, Earth observation, military services, etc. There is an easy way to define a satellite constellation using the so-called 2D Lattice Flower Constellations (LFCs)[1], that completely describes the distribution of satellites in space.

In this work 2D LFCs are used to solve global positioning problems. Like GPS system, our problem needs four visible satellites at each moment to determine the user’s position. These four satellites must be geometrically well distributed. For this purpose we will use the Geometric Dilution of Precision (GDOP)[2] which is a quantity between 1 and 99 that measures how good the constellation is. The lower the GDOP, the better the constellation that we have.

Our metric for this problem, is the maximum value of the GDOP obtained for 100 ground stations uniformly distributed on the Earth surface, during the propagation time, which is the time that the constellation needs to return to its original position.

Therefore, we are dealing with an optimization problem in which the function to be minimized is the GDOP. To solve this problem we use evolutive algorithms[3], such as Genetic Algorithm or Particle Swarm Optimization.

About the results, firstly given a number of satellites, it is possible to determine the best configuration of a 2D LFC that minimizes the GDOP; secondly, thanks to the 2D LFCs, it is possible to obtain optimal constellations with elliptical orbits. Other interesting results are currently being obtained with this new Flower Constellations.
References


Communications

Melnikov approach for the splitting of two-dimensional heteroclinic surfaces in the Hopf-zero singularity

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The so-called Hopf-zero singularity consists in a vector field in $\mathbb{R}^3$ having the origin as a critical point, with eigenvalues of the linear part $0$ and $\pm i\alpha^*, \alpha^* \neq 0$.

If one considers conservative (i.e. one-parameter) unfoldings of such singularity, one can see that the truncation of the normal form at any order possesses two saddle-focus critical points with a one- and a two-dimensional heteroclinic connection. The same happens for non-conservative (i.e. two-parameter) unfoldings when the parameters lie in a certain curve (see for instance [1]).

However, when one considers the whole vector field, one expects these heteroclinic connections to be destroyed. This fact can lead to the birth of a homoclinic connection to one of the critical points, producing thus a Šil’nikov bifurcation. For the case of $C^\infty$ unfoldings, this has been proved before (see [2]), but for analytic unfoldings it is still an open problem.

In a previous work, see [3], the authors gave a complete study of the exponentially small splitting of the one-dimensional heteroclinic connection for generic analytic unfoldings. Moreover, they provide an asymptotic formula for the measure of this splitting, which is exponentially small with respect to one of the perturbation parameters.

In this talk, we will present the Melnikov approach to the splitting of the two-dimensional heteroclinic connection in the conservative case. We provide an asymptotic formula which, again, is exponentially small. We will also characterize the unfoldings where this approach is valid and give some ideas about the more sophisticated techniques needed to deal with the generic case.

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References


Global Dynamics for Symmetric Planar Maps

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We consider sufficient conditions to determine the global dynamics for equivariant maps of the plane with a unique fixed point which is also hyperbolic. When the map is equivariant under the action of a compact Lie group, it is possible to describe the local dynamics and – from this – also the global dynamics. In particular, if the group contains a reflection, there is a line invariant by the map. This allows us to use results based on the theory of free homeomorphisms to describe the global dynamical behaviour. In the absence of reflections, we use equivariant examples to show that global dynamics may not follow from local dynamics near the unique fixed point. This talk is based on the papers [1, 2].

References


A spectral theory of linear operators on Gelfand triplets and its applications to infinite dimensional dynamical systems

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The dynamics of systems of large populations of coupled oscillators have been of great interest because collective synchronization phenomena are observed in a variety of areas. The Kuramoto model is often used to investigate such phenomena, which is a system of differential equations of the form

\[
\frac{d\theta_k}{dt} = \omega_k + K \frac{1}{N} \sum_{j=1}^{N} f(\theta_j - \theta_k), \quad k = 1, \ldots, N.
\]  

(1)

In this talk, an infinite dimensional Kuramoto model is considered, and the Kuramoto’s conjecture on a bifurcation diagram of the system, which is open since 1985, will be proved.

It is well known that the spectrum (eigenvalues) of a linear operator determines a local dynamics of a system of differential equations. Unfortunately, the infinite dimensional Kuramoto model has the continuous spectrum on the imaginary axis, so that the usual spectral theory does not say anything about the dynamics. To handle such continuous spectra, a new spectral theory of linear operators based on Gelfand triplets is developed. Basic notions in the usual spectral theory, such as eigenspaces, algebraic multiplicities, point/continuous/residual spectra, Riesz projections are extended to those defined on a Gelfand triplet. They prove to have the same properties as those of the usual spectral theory.

The results are applied to the Kuramoto model to prove the Kuramoto’s conjecture. A center manifold theorem will be given with the aid of the Gelfand triplet.
and the generalized spectrum. Even if there exists the continuous spectrum on the imaginary axis, it is proved that there exists a finite dimensional center manifold on a certain space of distributions. This determines a bifurcation diagram of the Kuramoto model.

References


Symmetry-Breaking Bifurcations in Rings of Delay-Coupled Lasers

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We consider a problem in rings of delay-coupled lasers modelled using the Lang-Kobayashi rate equations. We classify the symmetry of bifurcating branches of solutions from steady-state and Hopf bifurcations that occur in 3-laser systems. This involves finding isotropy subgroups of the symmetry group of the system, and then using the Equivariant Branching Lemma and the Equivariant Hopf Bifurcation Theorem. We then utilize this result to find the bifurcating branches of solutions in DDE-Biftool. Symmetry often causes eigenvalues to have multiplicity, and in some cases, this could lead DDE-Biftool to incorrectly predict the bifurcation points. We address this issue by developing a method of finding steady-state and Hopf bifurcation points which can be used for the general case of n-laser systems with unidirectional, bidirectional, and all-to-all coupling. This method also allows us to classify a bifurcation into either symmetry-breaking bifurcation or just a regular bifurcation.
Knotted trajectories and invariant tori in fluid mechanics

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The study of the trajectories described by a small volume of fluid constitutes the basic ingredient of the Lagrangian approach to fluid mechanics. In this talk we will consider some questions related to the properties of said trajectories.

More precisely, we will discuss the existence of knotted and linked trajectories in stationary fluids, a question that goes back to the works of V.I. Arnold [1] and K. Moffatt [3] in the 1960s. These fluids are assumed to be incompressible, so that the vector field that defines the trajectories satisfies the Euler equation. We will review a theorem that ensures the existence of trajectories of any knot or link type [2]: Given any locally finite link $L$ in $\mathbb{R}^3$, we can deform it by a $C^\infty$ diffeomorphism $\Phi$ of $\mathbb{R}^3$, arbitrarily close to the identity in any $C^k$ norm, so that $\Phi(L)$ is a set of periodic trajectories of a real-analytic stationary solution to the Euler equation.

Time permitting, we will also sketch some recent results on knotted invariant tori (the so-called vortex tubes, already considered by Lord Kelvin in the XIX century) and discuss extensions of Arnold’s structure theorem.

References


Periodic orbits and invariant cones in piecewise linear systems

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Continuous piecewise linear (PWL) systems are being widely studied at the present time due to their ability to model faithfully, among other things, some mechanical devices, electronic circuits, control systems, social, financial and biological problems [4]. Furthermore, they are able to explain, in an easy way, the appearance of periodic orbits due to the change of the stability of an equilibrium point. In fact, the study of periodic orbits usually begins by analyzing the stability of the equilibrium points of the system which are in the separation boundary of the linearity zones. This analysis is simple for planar systems, but the problem becomes compounded in the three-dimensional case. For instance, in [2] it was proven that the continuous matching of two stable linear systems can be unstable. This instability is closely related to the appearance of an invariant cone for the three-dimensional system, in such a way that the absence of invariant cones guarantees the stability, when the coupled linear systems are stable [1]. Therefore, it is extremely interesting the study of the existence of invariant cones in three-dimensional continuous homogeneous PWL systems. In [1, 3] the authors perform a very complete study of the existence of invariant cones for this class of systems under the observability hypothesis. In this talk, we will focus on the analysis of the existence of invariant cones under the assumption of non-observability. After that, we will find a homogeneous non-observable system having an invariant cone foliated by periodic orbits and we will perform a perturbation which makes it observable and non-homogeneous. In this observable systems we will be able to prove the existence of periodic orbits that emanate from those of the continuum of periodic orbits covering the invariant cone.
References


Communications

Some new results on Darboux integrable differential systems

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We deal with differential systems $X$ of the form $\dot{x} = P(x, y), \dot{y} = Q(x, y)$ of degree $d$ having a Darboux first integral $H$ and an inverse integrating factor $V$. Our first main result compares a natural extension of the degree of $V$ with $d + 1$.

**Theorem 1.** Let $\Pi_1 = \prod_{i=1}^{p} f_i^{\lambda_i}, \Pi_2 = \tilde{g} / \prod_{i=1}^{p} f_i^{n_i}$. Let $\delta(\prod g_i^{\alpha_i}) = \sum \alpha_i \deg g_i$.

(a) $\delta(V) < d + 1$ if and only if $\delta(\Pi_2) > 0$.

(b) $\delta(V) = d + 1$ if and only if either $\delta(\Pi_2) < 0$ and $\Pi_1$ is not constant, or $\delta(\Pi_2) = 0$.

(c) $\delta(V) > d + 1$ if and only if $\delta(\Pi_2) < 0$ and $\Pi_1$ is constant.

Moreover in all cases we have an expression of the characteristic polynomial in terms of some inverse integrating factor.

**Corollary.** The infinity is degenerate if and only if $\delta(\Pi_1) = 0$ and either $\delta(\Pi_2) < 0$ and $\Pi_1$ is not constant, or $\delta(\Pi_2) = 0$.

The remarkable values and remarkable curves of rational first integrals were first introduced by Poincaré and afterwards studied by several authors, see [1, 3, 2]. It has been shown in the literature that the remarkable curves play an important role in the phase portrait as they are strongly related to the separatrices.

In our work we first define remarkable values and remarkable curves of Darboux first integrals and afterwards we state the following result.

**Theorem 2.** Suppose that system $X$ has a Darboux first integral $H$ which is not rational. Then $V$ is a polynomial if and only if $H$ has no critical remarkable values.
References


Power series solutions of non-linear $q$-difference equations and the Newton polygon method

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The Newton polygon construction for finding power series solutions of equations and its generalization by Puiseux and Fine [1] has been successfully used countless times both in the algebraic and in the differential contexts. We extend its use to a large class of functional equations which includes differences, $q$-differences and differential equations.

In this talk we use this method to study the properties of the generalized power series solutions of non-linear $q$–difference equations. Namely, we give a new proof of the $q$-analogue of Maillet’s theorem. Zhang’s proof of this result in [4] adapting Malgrange’s [2] is only for convergent equations whereas our result includes non-convergent $q$-Gevrey equations. We give explicit bounds for the $q$-Gevrey order of a formal power series solution in terms of that of the original equation.

The Newton polygon method allows us as well to show that the set of exponents of any generalized power series solution of a formal $q$–difference equation is finitely generated as a semigroup. This mirrors the results in [3] for differential equations. In the specific case of order and degree one, we give a bound for the rational rank of the set of exponents.
References


We consider the center problem at Hopf singular points of analytic systems in $\mathbb{R}^3$, that is, to decide when around the origin of system

$$
\begin{align*}
\dot{x} &= -y + F_1(x, y, z), \\
\dot{y} &= x + F_2(x, y, z), \\
\dot{z} &= \lambda z + F_3(x, y, z),
\end{align*}
$$

(1)

with $\lambda \in \mathbb{R}\setminus\{0\}$ and $F_i$ representing nonlinear terms there is a center on the two-dimensional center manifold $W^c$. This problem has a classical solution via the Lyapunov Center Theorem which is given in terms of the existence of a local analytic first integral which can be taken to have the form $H(x, y, z) = x^2 + y^2 + \cdots$. We give a new solution of the problem in terms of the existence of an analytic inverse Jacobi multiplier $V(x, y, z) = z + \cdots$. We recall that an inverse Jacobi multiplier $V$ for (1) is a $C^1$ scalar function satisfying the divergence-free condition $\text{div}(X/V) \equiv 0$ where $X$ is the associated vector field to system (1). See [1] for a modern reference. When studying this problem, we needed to discuss the relation between $V$ and $W^c$. In particular, we see that when (1) has a center then $W^c \subset V^{-1}(0)$ for any local $C^\infty$ inverse Jacobi multiplier of (1). On the contrary, assuming the origin is a saddle-focus of (1), we prove the following: (i) There is a local $C^\infty$ inverse Jacobi multiplier of (1) of the form $V(x, y, z) = z(x^2 + y^2)^k + \cdots$ for some integer $k \geq 2$; (ii) Any pair of linearly independent $C^\infty$ and non-flat at the origin inverse Jacobi multipliers of (1) have the same Taylor series at the origin; (iii) Given a local $C^\infty$ and non-flat at the origin inverse Jacobi multiplier $V$ of (1) there is exactly one $C^\infty$ center manifold $W^c$ satisfying $W^c \subset V^{-1}(0)$. 

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With the former properties (most of them proved in [2]) we can study multiple Hopf bifurcations by perturbing analytically (1) with a saddle-focus singularity at the origin. We prove the following: perturb analytically system (1) of the form

\[
\begin{align*}
\dot{x} &= -y + G_1(x, y, z; \varepsilon), \\
\dot{y} &= x + G_2(x, y, z; \varepsilon), \\
\dot{z} &= \lambda z + G_3(x, y, z; \varepsilon),
\end{align*}
\]  

(2)

with \( \varepsilon \in \mathbb{R}^p, \; 0 < \|\varepsilon\| << 1, \; G_i(x, y, z; 0) \equiv F_i(x, y, z), \; G_i(0, 0, 0; \varepsilon) = 0 \) and \( D_jG_i(0, 0, 0; \varepsilon) = 0 \). Then, the maximum number of limit cycles that can bifurcate from the saddle-focus at the origin in the perturbed system (2) with \( \|\varepsilon\| \) sufficiently small is \( k - 1 \), where \( V(x, y, z) = z(x^2 + y^2)^k + \cdots \) is the expression of any \( C^\infty \) and non-flat at the origin inverse Jacobi multiplier of the unperturbed system (1).

References


Existence of global connections in three-dimensional piecewise linear systems

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The proof of the existence of a global connection in differential systems is generally a difficult task, even in the case of continuous piecewise linear systems. This existence usually forces a complex dynamical behavior in a neighborhood of such connections. Hence, there are many works about their existence. However, in many of them, authors require numerical arguments to show the existence.

In the present work we give analytical proofs of the existence of global connections in a one–parameter family of autonomous three–dimensional piecewise linear continuous systems with two zones,

\[ \dot{x} = y, \quad \dot{y} = z, \quad \dot{z} = 1 - y - c|x|, \ \text{with} \ c > 0, \quad (1) \]

that can be considered a piecewise version of the well–known Michelson system [4]. These systems are reversible and have two saddle-focus equilibria, which are mapped onto each other by the reversibility.

Concretely, we prove the existence of a pair of direct homoclinic orbits [1] and a reversible T–point heteroclinic cycle [2]. We have used the Poincare half–maps in each zones of linearity to obtain the conditions that characterize these global connections. Moreover, we have verified that the proofs of their existence are similar [3]. That is, both proofs have a common part where the existence of solution of the closing-equations is considered and some specific inequalities that must be satisfied. The computations developed in these proofs are specific to the system but the techniques can be extended to others piecewise linear systems.

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References


Noise can reduce disorder in chaotic dynamics

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We evoke the idea of representation of the chaotic attractor by the set of unstable periodic orbits [1] and disclose a novel noise-induced ordering phenomenon [2]. For long unstable periodic orbits forming the strange attractor the weights (or natural measure) is generally highly inhomogeneous over the set, either diminishing or enhancing the contribution of these orbits into system dynamics. We show analytically and numerically a weak noise to reduce this inhomogeneity and, additionally to obvious perturbing impact, make a regularizing influence on the chaotic dynamics. This universal effect is rooted into the nature of deterministic chaos.

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References


Discrete equations of regulatorika and nonlinear phenomena in biology

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As usual, quantitative analysis of functioning regulatory mechanisms (regulatorika) of living systems on different organization levels leads to studying characteristic solutions of the nonlinear discrete equations of regulatorika. General analysis of these equations using the methods for quantitative analysis, studying dynamics of Lyapunov number, Kolmogorov entropy, Lämery diagrams shows varied behavior of solutions in the basin of positive attractor: stable stationary solutions, Poincare type limit cycles, dynamic chaos and collapsed solutions with transition into attraction field of trivial attractor (“black hole” effect) [1, 2]. Construction of appropriate parametric portrait has allowed to research conditions for qualitative alteration of the solutions when changing the parameters values.

Analysis of parametric portrait for discrete equations of regulatorika has shown presence of small regions with normal behavior of solutions (rwindows) in the field of the dynamic chaos. This area borders upon areas of Poincare type limit cycles and “black hole”.

Applications of considered nonlinear discrete equations of regulatorika for the quantitative studying origin and developments mechanisms of HIV/AIDS [3] on cellular communities level of immune system, viral hepatitis type B on molecular-genetic level of hepatocyte and hormonal regulation of programmed cell death (apoptosis) on cellular level of the thyroid gland have allowed to define some regularities of these diseases course and to consider questions of possible ways of treatment.
References


On the ergodicity of hyperbolic SRB measures

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We consider diffeomorphisms preserving hyperbolic SRB measure $\mu$ satisfying the dimension of the unstable manifold is constant almost everywhere. In this context, under a modified accessibility property, we show the ergodicity of $\mu$. As a corollary we obtain the following.

**Theorem** [2] Let $f : M \to M$ be a $C^{1+\alpha}$ partially hyperbolic diffeomorphism of a compact smooth Riemannian manifold $M$ preserving a hyperbolic smooth measure $\mu$ and satisfying the essential accessibility property. If the dimension of unstable manifold is constant $\mu$-almost everywhere, then $\mu$ is ergodic.

This provides partially an affirmative answer to the question by K. Burns, D. Dolgopyat and Ya. Pesin [1].

**References**


Dense orbits of flows and homeomorphisms — misunderstandings and new results

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Existence of a dense orbit (topological transitivity) or a stronger property of every orbit being dense (topological minimality) belong to the core questions of topological dynamics. In this talk I discuss these notions for continuous as well as discrete time systems in a general setting without assuming compactness of the underlying space.

Besides well known results, there are common and frequent misunderstandings related to the mentioned two properties. We discuss these in detail in the joint paper with L’ubomír Snoha, see [1]. In the talk I briefly mention several new results from the paper, particularly the ones relating density of orbits of flows and corresponding t-maps, and density of full orbits versus density of forward or backward semi-orbits.

References

Analytic Iterative Functional Equations

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The author introduced the notion of prompters in the theory of iterative functional equations, and have succeeded in obtaining formal series solutions to the iterative functional equations containing a linear term called prompter([1], [2]). For example, we can give formal solutions to the equations $f(f(x)) = x + e^{-x}$ and $f(f(x)) + f(x) = 2x + \sin x$.

In this talk, we try to extend the prompter method to the prompter-free cases. For example, for the equation $f(f(x)) = \sin x,$ we have an approximate solution (up to degree 7)

$$f_a(x) = \sin x + \frac{1}{12} \sin^3 x + \frac{3}{80} \sin^5 x - \frac{1}{96} \sin^5 x \cos x + \frac{5}{224} \sin^7 x$$

and its error estimate is given as

$$\sup_{t \in \mathbb{R}} |f_a(f_a(x)) - \sin x| \approx 0.008.$$  

Moreover, we determine the continuous one-parameter iterative group $\{f^t(x); t \in \mathbb{R}\}$ of $f(x)$ satisfying $f^t(f^s(x)) = f^{t+s}(x)$, $t, s \in \mathbb{R}$ and $f^1(x) = f(x)$ in case of $f(x)$ is prompter-free.

References


Beyond Poincaré: a non-deterministic kind of chaos

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The success story of 20th century dynamical theory predominantly concerns systems that are (i) smooth, and (ii) deterministic, and many of its key notions can be traced back to the work of Henri Poincaré, such as stability, bifurcations, and chaos. But many systems throughout nature are not smooth, and when (i) is relaxed, it can lead to a breakdown of (ii). The transition from stick to slip between frictional surfaces is nonsmooth [1], as is electrical switching [3], neuron firing [5], the triggered activation of hormone production in biological cells, and many others. In nonsmooth dynamical systems we find all of the richness of smooth systems, plus many surprises that have emerged in recent years. I will present chaotic dynamics of a fundamentally different character to deterministic chaos. Solutions evolve along infinitely repeating, yet unpredictable, trajectories, driven by the evolution of a well-posed dynamical system through an isolated point where the evolution is multi-valued [2, 4]. Solutions are determined up to a set of possible values that may form attractors, undergo bifurcations, and even form invariant sets that imply the existence of chaos. I will present the notion of non-deterministic chaos, and show how it arises in nonsmooth systems in the context of discontinuity-induced bifurcations and explosions. Finally I will describe its implications for simulation and applications.

References


Nonlinear Fourier Series Expansion Based on Chaos

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The aim of this paper is to present a nonlinear Fourier series expansion for the time series analysis of the logistic chaos: [1] and [2]. The author has considered that fractal curves describing the Weierstrass function are derived from exact chaos solutions of chaos maps: [3] and [4], and has proposed an algorithm without the accumulation of round-off errors in the iteration for chaotic time series: [5].

Firstly, we introduce the following Weierstrass function $x(t)$ from the logistic map $X_{n+1} = 2X_n^2 - 1$ and the exact chaos solution $X_n = \cos(C2^n)$;

$$x(t) = \cos(p^n t), \quad (1)$$

where $p$ is a positive integer, and for example, time series $x_i = \cos(3^5 t_i), (i = 1, 2, ..., N)$ are chaotic as illustrated in Fig. 1. Next, we propose a nonlinear Fourier series expansion;

$$g(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \{a_n \cos(p^n t) + b_n \sin(p^n t)\}, \quad (2)$$

which has orthogonal functions with coefficients $\{a_0, a_n, a_n\}, (n = 1, 2, ...).$ Thus, the time series $X_n = \cos(C2^n)$ are expanded by (2) as the discrete $g(t_i)$ in Fig. 2 with an error function $\varepsilon = \sqrt{\sum_{i=1}^{N} (y_i - g(t_i))^2}/N$ under the correction function $y_i = X_i - a_i, a \equiv (X_N - X_0)/N$ for $X_n$ to have a $2\pi$ period at $y_0 = y_{200} = 0.0$ for the
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Figure 1: Time series of $x_i = \cos(3^5 t_i)$.

Figure 2: The nonlinear time series expansion $\varepsilon = 5.39 \times 10^{-16}$.

Figure 3: The power spectrum of (3).

discrete expansion $g(t_i)$. The data $y_i$ and $g(t_i)$ are shown in Fig. 2, where $g(t)$ is given as

$$g(t) = \frac{a_0}{2} + \{a_1 \cos(89t) + a_2 \cos(89^2 t) + \ldots + a_{100} \cos(89^{100} t) \}
+ \{b_1 \sin(89t) + b_2 \sin(89^2 t) + \ldots + b_{100} \sin(89^{100} t) \}, \quad (3)$$

here the algorithm: [5] is applied to the numerical calculation of each term in (3). Then, the power spectrum of $g(t_i)$ is obtained as illustrated in Fig. 3. That is, the logistic chaos $X_n$ is not a $1/f$ fluctuation, and it is shown that any fluctuation can be generated from (3) by setting coefficients $a_n$ and $b_n$. 

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References


Amplitude and phase dynamics in systems of coupled oscillators with distributed-delay coupling

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In this talk I will discuss the effects of coupling with distributed delay on the dynamics in a system of coupled Stuart-Landau oscillators. Conditions for amplitude death will be analyzed in terms of strength and phase of the coupling, as well as the mean time delay and the width of the delay distribution for uniform and gamma distributions. I will present analytical results as well as numerical computation of the eigenvalues of the corresponding characteristic equations. These results indicate that larger widths of delay distribution increase the regions of amplitude death in the parameter space. In the case of a uniformly distributed delay kernel, for sufficiently large width of the delay distribution it is possible to achieve amplitude death for an arbitrary value of the average time delay, provided that the coupling strength has a value in the appropriate range. For a gamma distribution of delay, amplitude death is also possible for an arbitrary value of the average time delay, provided that it exceeds a certain value as determined by the coupling phase and the power law of the distribution [1]. The coupling phase has a destabilizing effect and reduces the regions of amplitude death. The case of non-identical oscillators will also be considered with an emphasis on different kinds of phase dynamics and synchronization.

References

Heteroclinic networks arise persistently in differential equations with symmetry. We discuss interesting geometrical behaviour near some types of heteroclinic networks: switching, cycling and essential asymptotic stability. We also address bifurcation from networks under forced symmetry breaking and the situations where heteroclinic networks arise.

This is joint work, some of it covered in [1, 2, 3, 4].

References


An extrapolation method to study quasi-periodic motions: application to the SRTBP

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Frequency analysis was introduced by Laskar to study secular motions of the planets in the Solar system [1]. A significant refinement of Laskar’s methods, based in the simultaneous improvement of the frequencies and the amplitudes of the signal, was given later in [2]. On another front, a methodology to compute rotation numbers of invariant curves (and more general objects) has been introduced recently (see [3] and [4]). The idea is to extrapolate the rotation number (and related quantities) from suitable averages of the iterates of an orbit.

The goal of this talk is to present a methodology to compute the frequencies of a given quasi-periodic orbit. The construction is a generalization of the mentioned averaging-extrapolation approach to study rotation numbers. We plan to describe informally this construction, that allows us to compute with high precision the components of the frequency vector. The main advantage over other high precision methods is that we do not require to compute nor to refine the amplitudes of the signal.

As an illustration, we will consider quasi-periodic motions close to the point \( L_5 \) in the spatial restricted three-body problem and also include perturbations from another bodies.

References


Aubry-Mather sets for a bouncing ball problem

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We consider the model describing the vertical motion of a ball falling with constant acceleration on a wall and elastically reflected. The wall is supposed to move in the vertical direction according to a given periodic function $f$. We apply the Aubry-Mather theory to the generating function in order to prove the existence of bounded motions with sufficiently large rotation number. Moreover, we give an example in which bounded and unbounded motions coexist.

References


Homo-heteroclinic solutions to infinity in the Restricted Three Body Problem

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In 1980 J. Llibre and C. Simó [1] proved the existence of oscillatory motions for the restricted planar three body problem, that is of orbits which leave every bounded region but which return infinitely often to some fixed bounded region. To prove their existence, they related them to the symbolic dynamics associated with a transverse homoclinic point. In their work they had to assume that the ratio between the masses of the two primaries was exponentially small with respect to the angular momentum. In the present work, we generalize their work proving the existence of oscillatory motions for any value of the mass ratio.

The Hamiltonian for the RPCTBP in polar coordinates is

\[ H(r, \alpha, y, G, t) = \frac{y^2}{2} + \frac{G^2}{2r^2} - \frac{1}{r} - \mu \tilde{U}(r, \alpha - t) \]  

(1)

where \( \tilde{U} \) is the Newtonian potential

\[ \tilde{U}(r, \phi) = \frac{1 - \mu}{(r^2 - 2\mu r \cos \phi + \mu^2)^{1/2}} + \frac{\mu}{(r^2 + 2(1 - \mu)r \cos \phi + (1 - \mu^2)^{1/2}} \]  

(2)

When \( \mu = 0 \), there exists a family of homoclinic orbits to \( r = \infty \), which is foliated by periodic orbits. These periodic orbits are parameterized by

\[ \mathcal{P}_{G_0} = \{(r, \alpha, y, G) : r = \infty, y = 0, \alpha \in \mathbb{T}, G = G_0\}, \]
where $G_0$ is the angular momentum. All the associated homoclinic orbits belong to the zero level of energy and can be parameterized by the angular momentum and its initial condition in the angular variable. We denote these orbits as

\[ z_h(u, \alpha_0; G_0) = (r_h(u; G_0), \alpha_0 + \alpha_h(u; G_0), y_h(u; G_0), G_0). \]  

(3)

It follows that, for any $G_0$, \((u, \alpha_0) \mapsto z_h(u, \alpha_0; G_0)\) is a parametrization of both the stable and unstable manifolds of the periodic orbit $P_{G_0}$.

When $\mu \neq 0$, the periodic orbits $P_{G_0}$ are preserved. We prove that for any $\mu$, the invariant manifolds of the periodic orbits at infinity split. We compute a formula for the splitting size, which is exponentially small in the angular momentum $G_0$. When $\mu$ is small, this formula is well approximated by the Melnikov function associated to the problem (already computed in [1, 2]).

References


Dissipative pendulum, coupled pendula, and sine-Gordon: When is the attractor homeomorphic to the circle?

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Consider an attractor of a dissipative non-autonomous system with one angle coordinate. Some examples are the pendulum, coupled pendula or the sine-Gordon equation. In [1] and [2] we gave conditions under which this attractor is homeomorphic to the circle and showed that they are optimal. Now we show that similar results holds for systems periodic coupled pendula. Moreover we shall see that the same happens for a sine-Gordon type equation

\[ u_{tt} + cu_t - u_{xx} + F(x, u, t) = 0, \]

\[ c > 0, \] periodic in \( x \) and assuming that this period is small.

References


Three Dimensional Chaos Maps and Fractals

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The logistic map describing biological population growth has been discussed on the rich spectrum of dynamical behavior as chaos: [1]. On the other hand, fractals have been proposed for the geometric representation of many irregular shapes and patterns in nature: [2]. However, it has been pointed out that the physics of fractals is a subject waiting to be born: [3], since the Mandelbrot fractals have been defined by complex numbers.

In this paper, we begin by introducing a generalized logistic function \( P(t) = \{a/(b + e^{-ct})\} + d \) with real constants \( \{a, b, c, d\} \), and derive a generalized logistic map;

\[
x_{n+1} = Ax_n(1 - x_n) + B, \tag{1}
\]

where \( A \) and \( B \) depend on the constants, and the constant \( B = 0 \) gives the well-known logistic map. Especially, we obtain \( x_{n+1} = 4x_n(1 - x_n) \) under \((A, B) = (4.0, 0.0)\) in (1) and an exact chaos solution \( x_n = \sin^2(C2^n) \) with a real coefficient \( C \): [4] and [5]. Then, we can construct a 2–D chaos map as follows: Starting from two chaos solutions, for example, \( x_n = \cos(C2^n) \) and \( y_n = \sin(C2^n) \) to the kernel map \( x_{n+1} = x_n^2 - y_n^2 \) and \( y_{n+1} = 2x_ny_n \), we obtain a generalized Mandelbrot map;

\[
x_{n+1} = a_1(x_n^2 - y_n^2) + b_1, \tag{2}
\]

\[
y_{n+1} = a_2x_ny_n + b_2, \tag{3}
\]

where \( \{a_1, a_2, b_1, b_2\} \) are real constants. Similarly, we can compose 3–D chaos maps, for example, from \( x_n = \cos(C2^n), y_n = \sin(C2^n) \) and \( z_n = \sin^2(C2^n) \);

\[
x_{n+1} = a_1(x_n^2 - z_n) + b_1, \tag{4}
\]
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\[ y_{n+1} = a_2 x_n y_n + b_2, \quad (5) \]
\[ z_{n+1} = a_3 x_n^2 z_n + b_3 \quad (6) \]

with real constants \( \{a_1, a_2, a_3, b_1, b_2, b_3\} \). Then, the following set;

\[ M_1 = \{a_1, a_2, a_3 \in \mathbb{R} \mid \lim_{n \to \infty} x_n, y_n, z_n < \infty, b_1 = x_0, b_2 = y_0, b_3 = z_0 \} \quad (7) \]

where \( \{x_0, y_0, z_0\} \) are initial values, gives a \( 3 - D \) fractal set of \((a_1, a_2, a_3) = (1.0, 2.0, 4.0)\) with physical analog as shown in Fig. 4.(a) and (b).

![Figure 4: A 3 – D fractal set in the x0 – y0 – z0 space illustrated by POV-Ray.](image)

(a) The whole  
(b) The part

References


Monotone and concave skew-product semiflows: a general description of the dynamics

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The dynamics of monotone and concave skew-product semiflows is analyzed. Due to its theoretical and practical interest, this question has been extensively studied in the literature: previous works of Krasnoselskii, Selgrade, Aronsson, Mellander, Johnson, Hirsch, Smith, Aiello, Freedman, Wu, Peng, Chueshov, as well as Novo, Núñez, Obaya and Sanz, among others, present interesting examples of this type of analysis. The main goal is to describe the long-term behavior of the semiorbits starting above a semicontinuous subequilibrium and the number and stability properties of the minimal sets. Several possibilities arise depending on the existence or absence of minimal sets strongly above the subequilibrium and the coexistence or not of bounded and unbounded semiorbits.

The results extend and unify previously known properties, showing scenarios which are impossible in the autonomous or periodic cases. The works [3] and [4] are an extension to the concave case of the analysis made in [1] and [2] in the sublinear case, the differences being quite significative.

References


We study the interior and the boundary crisis bifurcations from the viewpoint of the graph-based topological computation developed in [1]. We give a new formulation of these bifurcations in terms of a change of the attractor-repeller decompositions of the dynamics [2]. We prove that the attractor before the crisis changes the size by creating a chain connecting orbit to the repeller at the moment of the interior crisis. And at the moment of the boundary crisis, we also prove that the attractor before the crisis disappears by creating a chain connecting orbit to the repeller. As an illustration, we discuss these crisis bifurcations in the Ikeda map [3, 4] and the Leslie population model [5].

References


We present a numerical method for the computation and continuation of families of homoclinic and heteroclinic connections of periodic orbits of a given system Hamiltonian system with $n$ degrees of freedom. As an application, we compute families of homoclinic and heteroclinic connections of families of the Lyapunov periodic orbits associated with the collinear equilibrium points, $L_1$, $L_2$ and $L_3$ of the planar restricted three-body problem (RTBP).

Some comments are done concerning the applicability of the results on libration point missions and transport in the Solar System.
Inverse problems in the theory of foliations

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Is any embedded 3-sphere a leaf of a foliation of $\mathbb{R}^n$? Can any link be realized as the set of limit cycles of a non-vanishing Morse-Smale vector field in $\mathbb{R}^3$? Which are the conditions in order that a submanifold can be realized as a leaf of a codimension $m$ foliation of $\mathbb{R}^n$?

These problems can be solved using the theory of (weakly and strongly) integrable embeddings recently introduced by Gilbert Hector and the speaker [2]. In particular, I will discuss the following results:

- The 3-sphere is a leaf of a foliation of $\mathbb{R}^n$ if and only if $n \geq 7$. This solves a problem by Vogt [3]. In general, the submanifolds of $\mathbb{R}^n$ which are proper leaves of foliations can be classified using certain topological invariants.

- No knot in $\mathbb{R}^3$ is the zero-set of a submersion $\Phi : \mathbb{R}^3 \rightarrow \mathbb{R}^2$, but it can be realized as a compact component of $\Phi^{-1}(0)$ (this problem goes back to [1]). Moreover, any locally finite link can be realized as the set of limit cycles (hyperbolic and asymptotically stable) of a non-vanishing Morse-Smale vector field in $\mathbb{R}^3$.

The theory of integrable embeddings is a very rich framework where many classical tools of differential and algebraic topology play a prominent role: the Phillips-Gromov h-principle, the Hirsch-Smale theory of immersions, complete intersections and obstruction theory.
References


An algorithm for constructing all the planar quasi-homogeneous differential systems of a given degree

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The planar differential system \( \dot{x} = P(x, y), \quad \dot{y} = Q(x, y) \), with \( P, Q \in \mathbb{C}[x, y] \) is called quasi-homogeneous if there exist \( s_1, s_2, d \in \mathbb{N} \) such that for an arbitrary \( \alpha \in \mathbb{R}^+ \), one has that \( P(\alpha^{s_1}x, \alpha^{s_2}y) = \alpha^{s_1-1+d}P(x, y) \) and \( Q(\alpha^{s_1}x, \alpha^{s_2}y) = \alpha^{s_2-1+d}Q(x, y) \). The quasi-homogeneous systems have important properties (for example, all of them are integrable) and they had been studied from many different points of view (integrability, centers, normal forms, limit cycles). Some recent contributions about these systems can be seen, for example, in [1], [2] and [4]. But so far there was not an algorithm for constructing all the quasi-homogeneous polynomial differential systems of a given degree. In this contribution we give the key ideas to provide such an algorithm that we are developing in [3].

References


Simultaneous bifurcation of three limit cycles in piecewise linear continuous differential systems with symmetry

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The analogous case to the fold-Hopf bifurcation for differentiable systems is considered for three-dimensional symmetric piecewise linear differential systems.

In a recent paper [1], authors considered continuous piecewise linear systems with symmetry under strong assumptions on the spectra of the external linear parts involved, to study the possible occurrence of a fold-Hopf like bifurcation. Here, the restrictive assumptions of the quoted paper are suppressed and general linear parts are considered not only for the external zones, but also for the central one, maintaining the eigenvalue structure leading to a fold-Hopf bifurcation.

The simultaneous birth of three limit cycles in three-dimensional piecewise linear continuous differential systems with symmetry is proved, providing amplitudes and periods for the bifurcating limit cycles. As in the differentiable case, for the critical value of the bifurcation parameter the critical equilibrium has a zero eigenvalue and a pair of purely imaginary eigenvalues.

Finally, the theoretical results are applied to a generalized version of Chua’s circuit, showing that the fold-Hopf bifurcation takes place for a certain range of parameters.
References

Index of $\mathbb{R}^3$—orientation reversing homeomorphisms

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Let $U \subset \mathbb{R}^3$ be an open set and $f : U \to f(U) \subset \mathbb{R}^3$ be an orientation reversing homeomorphism. Let $p \in U$ be a fixed point. The main goal of this talk is to show that the fixed point index $i(f, p) \leq 1$ whenever $\{p\}$ is an isolated invariant set. As a consequence we have that there are not minimal orientation reversing $\mathbb{R}^3$–homeomorphisms.
Periodic solutions and chaotic dynamics in 3D equations with applications to Lotka-Volterra systems

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We discuss a geometric configuration for a class of homeomorphisms in 3D producing the existence of infinitely many periodic points as well as a complex dynamics due to the presence of a topological horseshoe. We also show that such a class of homeomorphisms appear in the classical Lotka-Volterra system.
Numerical study of unstable periodic orbits of some chaotic dynamical systems

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An infinite number of unstable periodic orbits are embedded in a chaotic dynamical system. However, it is usually difficult to identify unstable periodic orbits even numerically, because they cannot be identified by the forward time integration or iteration of the system. In this talk thousands of unstable periodic orbits embedded in some chaotic systems including ODE and PDE systems exhibiting chaotic behaviors are detected numerically. Time averaged values along unstable periodic orbits are investigated in relation to those of chaotic orbits. In addition, manifold structures of unstable periodic orbits are identified from Lyapunov vectors, and they are used to study the appearance of tangencies, the sequence of periodic windows, attractor merging crisis and so on.

References


Decomposing Orbits in the Poincaré Section

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The Poincaré Map [1] is a well known tool developed by Henri Poincaré (1854-1912) for visualizing the orbits of a continuous dynamic system. The Map is the intersection of a lower dimensional subspace with the orbits of a continuous dynamical system. The Poincaré Map preserves many properties of periodic and quasi-periodic orbits of the original system and can be interpreted as a discrete dynamical system that is one dimension less than the original system.

While the Poincaré Map maps consecutive puncture points on to the Poincaré section and is used to visually understand the system one often does not always explore the map itself and the information that it contains. In [2] we determined the topology of quasi-periodic orbits by decomposing the orbit into two 1D functions and analyzing the resulting periodicity. In presence of stochasticity these functions contain resonance periods and indicate unique topological structures known as “islands” which contain fixed points. Further, the ratio of the dominate period from each function form a rational approximation to the irrational periodicity.

Some orbits in the Poincaré section can contain small scale topological structures often referred to as secondary islands. These structures resides on the boundary of the basic (or primary) islands and are akin to signal superposition and the encoding of high frequency information onto the basic frequency. Just as signals can be decomposed so too can orbits. In this work we present the decomposition of such orbits via their fixed points. The results of which allows for the construction of a profile of the orbits and their stability. We apply the results our technique to analyze the evolving topology and stability of toroidal magnetic fields that surround a burning plasma in magnetic confinement fusion.
In addition, we utilize the results of the decomposition of the orbits to construct a contiguous representation on the Poincaré section using a limited number of puncture points. This technique greatly reduces the computational requirements and the influence of numerical instabilities. Such contiguous representations provides greater insight into the structure of the magnetic field compared to a traditional Poincaré plot base solely on puncture points.

References


The geometry of safety for partially controllable chaotic systems

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The aim of the Partial Control [4, 5] is to avoid escapes of some region of the phase space, in a nonlinear system in which some noise has also been added. Typically in this kind of systems we need a control higher or equal to the amount of noise added, using other control techniques, but using the partial control technique it is possible to avoid the escapes even if the control is smaller than the noise. In this talk I will report about further progress in this controlling chaos technique. In particular I will describe the Sculpting Algorithm which we have developed in order to build safe sets, which are sets in phase space where using a control smaller than noise makes it possible the chaotic dynamical system to be controllable. We have found another set, that we call asymptotic safe set, where trajectories go after the partial control strategy is applied. All these ideas are applied to two paradigmatic examples, the Hénon map and a Duffing oscillator showing the geometry of these sets in phase space.

References


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Minimal sets in monotone and concave nonautonomous recurrent differential equations

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The dynamical behaviour of solutions of nonautonomous differential equations with a certain time recurrence, such as almost periodicity, can be studied by means of a skew-product semiflow formulation. We present an in-depth study of the minimal sets in the skew-product semiflows induced by recurrent nonautonomous two-dimensional systems of differential equations given by cooperative and concave vector fields. We deal with ordinary, finite-delay and reaction-diffusion systems.

Under the assumption of the existence of a semicontinuous subequilibrium (or, roughly speaking, of a semicontinuous lower-solution) and of a minimal set situated strongly above it, we describe the behavior of the bounded semi-orbits, as well as the possible shape of the set of all the minimal sets. First of all, the omega-limit set of any semi-orbit starting strongly above the semiequilibrium is seen to be a minimal set given by a copy of the base, i.e., by the invariant graph of a continuous map. So that these omega-limit sets inherit the topological and dynamical structures of the base flow, and hence reproduce the temporal variation of the vector field. Then, a complete description of the set of all the minimal sets situated strongly above the subequilibrium is provided.
References


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Let $X$ be a germ of real analytic vector field at the origin $0$ of $\mathbb{R}^n$. Existence of local center manifolds at the origin is a very useful and well known result. They are just defined to be tangent to the eigenspace associated to all the eigenvalues with zero real part of the linear part of $X$ at $0$. They are not unique, nor infinitely differentiable but they all share the same Taylor expansion, giving rise to a unique formal center manifold at the origin.

Less known or treated are results concerning invariant manifolds inside center manifolds or subcenter manifolds. One important result in this direction is due to Bonckaert and Dumortier [1] asserting the following: if $n = 3$ and the linear part has eigenvalues $\{0, +bi, -bi\}$ with $b \neq 0$, then there exists a formal non-singular invariant curve $\hat{\Gamma}$ tangent to the eigenvalue 0 and a $C^\infty$ invariant smooth curve $\Gamma$ having $\hat{\Gamma}$ as Taylor expansion at the origin.

Our communication presents a generalization of this result, with a quite different proof. In fact we prove the following statement: Assume that $X$ has a formal invariant curve $\hat{\Gamma}$ at $0 \in \mathbb{R}^n$ (singular or not). Then there exists a solution $\Gamma$ of $X$ accumulating to the origin and having the formal curve $\hat{\Gamma}$ as asymptotic expansion.

References

Noise-induced phenomena in one-dimensional maps

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Interaction between deterministic chaos and stochastic randomness has been an important problem in nonlinear dynamical systems studies. Noise-induced phenomena are understood as drastic change of natural invariant densities by adding external noise to a deterministic dynamical systems, resulting qualitative transition of observed nonlinear phenomena. Stochastic resonance, noise-induced synchronization, and noise-induced chaos are typical examples in this scheme. The simplest mathematical model for problem is one-dimensional map stochastically perturbed by noise.

In this presentation, we discuss typical behavior of noised dynamical systems based on numerically observed noise-induced phenomena in logistic map, Belousov-Zhabotinsky map and modified Lasota-Mackey map. Our observation indicates that (i) both noise-induced chaos and noise-induced order may coexist, and that (ii) asymptotical periodicity of densities varies according to noise amplitude. An application to time-series analysis of rotating fluid is also exhibited.

References


Suppose that $f_{a,b} : \mathbb{R}^3 \to \mathbb{R}^3$ is a two-parametric family of three dimensional diffeomorphisms, having a dissipative saddle fixed point $p = p(a,b)$ for all $a, b \in \mathbb{R}$. If the eigenvalues of $p$ are real and the unstable invariant manifold of $p$ is two-dimensional, the return map $f_{a,b}^n$ associated to a homoclinic bifurcation of codimension two (called generalized homoclinic tangency, see [1]) can be approximated for $n$ large, after an $n$-dependent change of variables and parameters, by a two-dimensional non-invertible quadratic two-dimensional family $g_{a,b}$, independent of $n$. A study of this family shows that it has a very rich dynamics including the existence of several types of attractors: periodic orbits, invariant curves, strange attractors with one or two positive Lyapunov exponents (see [2, 3]). For $a = -2$ and $b = -4$ it is possible to prove that $g_{a,b}$ has a two-dimensional attractor with a absolutely continuous invariant measure and that it is conjugate to a non-invertible piecewise linear map (bidimensional tent map). In order to have a qualitative description of the dynamics of the family $g_{a,b}$, we define a family of piecewise linear and continuous maps, that we call expanding Baker maps. In this work we study the dynamics of this family. In spite of its simplicity, the numerical results show that it can capture some of the more relevant dynamics of the quadratic family, specially that related to the existence of 2D strange attractors.

References


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Analogs of Theorems of Maizel And Pliss And Their Application in Shadowing Theory

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Consider a system of linear differential equations that has a bounded solution for every bounded inhomogeneity. Maizel and Pliss’ theorems establish a connection between this property and some hyperbolicity properties of the system considered. Maizel’s theorem [1] deals with systems on the half-axis, and Pliss’ theorem [2] deals with systems on the whole axis. There are lots of papers concerning invertibility and Fredholm properties of the operators corresponding to systems of linear differential equations.

During last years, discrete analogues of these results have been widely used in the shadowing theory of diffeomorphisms of manifolds. It was shown that the Lipschitz and Holder shadowing [3, 4] and Lipschitz inverse shadowing [5] properties are equivalent to structural stability. Recently, a generalization of discrete analogues of both theorems to a new class of spaces has been proved. The generalization allows one to prove that some new sort of limit shadowing property is equivalent to structural stability.

References


A new Chebyshev family with applications to Abel equations

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We prove that a family of functions defined through some definite integrals forms an extended complete Chebyshev system. The key point of our proof consists of reducing the study of certain Wronskians to the Gram determinants of a suitable set of new functions. Our result is then applied to give upper bounds for the number of isolated periodic solutions of some perturbed Abel equations. This talk is based on the paper [1].

References

Limit cycles in discontinuous planar piecewise linear systems

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The analysis of discontinuous planar piecewise linear systems with a discontinuity line typically involves twelve parameters, becoming a formidable task. However, if we only consider orbits with no points in the sliding set then topologically conjugated canonical forms with only seven parameters can be obtained.

In the proposed canonical forms the existence and characterization of the sliding set is determined by only one parameter. Moreover, they allow us to obtain a necessary condition for the existence of invariant closed curves, facilitating a systematic analysis of the possible dynamical behavior to be found in planar discontinuous piecewise linear systems. As another byproduct of the obtained canonical form, we find that some discontinuous systems are topologically conjugated to continuous ones.

If we concentrate our attention on the case where both dynamics are of focus type, then a reduced canonical with only five parameters is achieved. We study then the possible existence of limit cycles with no points in common with the sliding set. For the case without equilibria in both open half-planes, we determine some parametric regions with no limit cycles, with one limit cycle, or with two limit cycles, describing the qualitatively different phase portraits and the bifurcations connecting them.
References


Reduced order modeling of bifurcation problems

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Bifurcation diagrams are of paramount scientific interest. The increasing necessity to account for nonlinearity in industrial devices is making unavoidable the need to analyze bifurcation problems also in industrial environments. Unfortunately, for realistic physical systems modeled by partial differential equations, current computational tools may require non-affordable CPU time and memory to construct bifurcation diagrams involving complex time dependent attractors. This is because the numerical solver must be run for a large number of values of the bifurcation parameter, in a sufficiently large time-span (which is quite large near bifurcation points), to discard transient dynamics. Thus, increasing the computational efficiency of this process is strongly needed to take advantage of the body of knowledge that has been developed along the last decades in connection with nonlinear dynamics and bifurcations.

Particular bifurcations can be analyzed projecting on the center manifold to derive the normal form equations, but these must be constructed for each bifurcation type and each particular set of equations, which penalizes flexibility. If the system is dissipative and the spatial domain is bounded, the large time behavior of the system is contained in a finite dimensional inertial manifold, which is low dimensional in many cases of interest. Thus, these systems are amenable to using reduced order modeling methods, which first identify the relevant low dimensional attractors and then project the governing equations into the manifold, to obtain a small system of ordinary differential equations.

The main object of the talk will be to provide some recent developments, including some basic ideas to constructing efficient/robust reduced order models for bifurcation problems. The ideas will be illustrated with the complex Ginzburg-Landau equation.
A bifurcation leading to limit cycles in discontinuous planar piecewise linear systems without sliding

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A family of planar discontinuous piecewise linear systems with two linearity zones is considered. By using some changes of variables and parameters, see [1], a Liénard canonical form is obtained. This canonical form has seven parameters. In the particular case analyzed with focus-focus dynamic and without sliding, a reduced canonical form with only four parameters is obtained. Under certain hypotheses the existence of a limit cycle bifurcation is assured. Analytic expressions for the amplitude, period and characteristic multiplier of the bifurcating limit cycle are provided. To illustrate the appearance of this bifurcation in real world applications, a Wien bridge oscillator without symmetry is analyzed using the provided theoretical results.

References

We consider 2D diffeomorphisms having a homoclinic figure-eight to a dissipative saddle. The bifurcation diagram was given in [1], as a function of the distances between the branches of stable and unstable manifolds. In this work, we study the rich dynamics that they exhibit under a periodic forcing. These systems are natural simplified models of phenomena with forcing and dissipation.

Under generic perturbations the manifolds can split and undulate. This gives rise to different transversal homoclinic points and to a large set of bifurcations. We will present the main tools to study the bifurcation diagram (topological methods, quadratic and cubic tangencies, return maps, cascades of sinks,...) to detect the different kinds of attractors that exist (and can coexist) in the system. We refer to [2] for a similar approach.

It should be emphasized that we look for global phenomena, that is, we study the dynamics in a fundamental domain which captures all the non-trivial facts. The analysis of the bifurcation global problem is illustrated and complemented by the numerical study of a return model which, despite being simple, has a “universal” character. All the phenomena predicted by the theoretical analysis are seen to be realized in the model. Even more, the analysis of the return model provides not only a qualitative description but also quantitative data of the dynamical phenomena taking place.

The results that we shall present have been recently submitted to publication [3]. Directions for future work will be outlined.
References


Computer assisted techniques for the verification of the Chebyshev property of Abelian integrals

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This talk deals with a criterion that appears in [2], which provides an easy sufficient condition in order for a collection of Abelian integrals $I_0, I_1, \ldots, I_{n-1}$ to have the Chebyshev property (i.e., any function in its linear span has at most $n - 1$ zeros counted with multiplicities). The condition involves the functions in the integrand of the Abelian integrals and, in the polynomial setting, can be checked in a purely algebraic way by computing resultants and applying Sturm’s Theorem. Our goal in this talk is to show its applicability in the non-algebraic setting. To this end we tackle a conjecture formulated by Dumortier and Roussarie [1] and we are able to prove it for $q \leq 2$. The proof is a combination of theoretical results, analysis of asymptotic behaviour of Wronskians and rigorous validation using computer assisted techniques based on interval arithmetic [3]

References


On a Mixed-Type Delay Differential Equation

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We deal with the existence of solutions of the following problem:

\[ \dot{x}(t) = h(t, x(t), \hat{x}_t) \]  

(1)

where \( h \) is defined on an open subset of \( R \times R \times C_1; \quad C_1 = C([-r, \rho], R), \quad r, \rho \geq 0, \hat{x}_t(\theta) = x(t + \theta) \) for \( \theta \in [-r, \rho] \). The equation (1) is called a mixed-type delay differential equation, that is with delay and advance. Such equation are used in modeling and computing of anticipatory systems in certains fields of natural and artificial systems.
Delayed bifurcations in decision making by stochastic gene regulatory networks

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Induction of a specific transcriptional program by external signaling inputs is a crucial aspect of intracellular network functioning. The theoretical concept of attractors representing particular genetic programs is reasonably adapted to experimental observations of genome-wide expression profiles or phenotypes. Attractors can be associated either with developmental outcomes such as differentiation into specific types of cells, or maintenance of cell functioning such as proliferation or apoptosis.

We consider a simple model of the genetic switch and study theoretically the influence of signaling speed on decision making to define the cellular fate[1]. We show that the decision leading to the certain cell fate in structurally symmetric circuit will depend on the speed at which the system is forced to go through the decision point. The mechanism of the effect can be understood from previous studies in dynamical bifurcations and parameter sweeping experiments in physical systems (see [2, 3] and refs therein). As in the case of a generic bistable potential, the speed with which the system crosses the critical region strongly influences the sensitivity to the transient asymmetry. This effect may have a high impact in the design of efficient synthetic logical devices and contribute to the design and application of therapies that target cellular decision making, for example, in stem cell therapy and cancer.

Finally, we generalize the concept to high-dimensional phase space [4]. We demonstrate that high-dimensional network clustering capacity is dependent on the level of intrinsic noise and the speed at which external signals operate on the transcriptional landscape. Due to the effects of fluctuations in conjunction with external signal characteristics, the paths taken in high-dimensional phase space before cell fate commitment change the probability of attractor selection.
References


The cyclicity of two classes of quadratic integrable systems under quadratic perturbations

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In this talk I present two results on the bifurcation of limit cycles in planar quadratic integrable systems under quadratic perturbations[3, 4]. By Zoladek’s classification [5], the quadratic integrable system with at least one center are divided into four cases: $Q^H_3$, $Q^R_3$, $Q^{LV}_3$ and $Q_4$, called Hamiltonian case, reversible case, Lotka-Volterra case and codimension 4 case respectively.

The first result is concerned with the bifurcation of limit cycles in general quadratic perturbations of quadratic codimension-four centers $Q_4$. Gavrilov and Iliev set an upper bound of eight for the number of limit cycles produced from the period annuli around the center. Based on Gavrilov-Iliev’s proof, we prove that the perturbed system has at most five limit cycles which emerge from the period annuli around the center. We also show that there exists a perturbed system with three limit cycles produced by the period annuli of $Q_4$.

In the second part of the talk, we consider quadratic perturbations of system

\[ \dot{x} = y + 8(b + 1)xy, \quad \dot{y} = -x - 2(3b + 1)x^2 + 6(b + 1)y^2. \]

If $b \in (-\infty, -3] \cup [-1, +\infty)$, the bifurcation of limit cycles have been studied in several papers, see [2] and reference therein. The purpose of the present talk is to investigate the bifurcation of limit cycles for the case $b \in (-3, -1)$. It is shown that, the quadratic perturbed system of (1) has at most two limit cycles in the finite phase plane. The bound is exact.

Recently, S. L. Gavrilov and I.D. Iliev[1] proposed a program for finding the cyclicity of period annuli of quadratic centers of genus one. The contents of the talk can be viewed as contributions of this program.
References


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Posters
Gursey instantons with periodic feedback in phase space

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Gursey Model which has been assumed in 1956 as a possible basis for a unitary description of elementary particles is only possible four dimensional conformally invariant nonlinear pure spinor model [1]. Recently, the behaviors of Gursey instantons in phase space were investigated depending on coupling constant [2].

In this presentation, we shall investigate the behaviors of Gursey instantons in phase space with periodic feedback to provide the better understanding quantum dynamics of spinor type instantons in vacuum.

References


Influence of dynamic external conditions on learning and memory in models of cellular control

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Processes of cellular control display remarkable signatures of primitive intelligence regulating learning and memory observed in cellular behaviour [1, 2]. A process of receiving, interpretation and adjustment to environmental conditions may be a result of cellular multilayer perceptron which has learned how to process the inputs via the evolutionary development [3].

However, the process of evolutionary learning occurred in highly dynamic or even stochastic external conditions. To investigate the influence of such external conditions on processes of learning of cellular control and adaptation we have considered a mathematical model of a simple cell-signalling pathway which gives a specified response to a pulse of an extracellular ligand. To model gene expression we have used a system first order ODEs written with appropriate Hill functions. We have shown that dynamics of external conditions can have a profound effect on learning and execution of a perceptron function. It is still an open question whether this effect has been used by cells in the process of evolutionary adaptation.

References


Random cycles in random Hopf bifurcations

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We study Hopf-Andronov bifurcations in a class of random differential equations (RDEs)

\[ x' = f_\lambda(x, \xi_t) \]

as the parameter \( \lambda \in \mathbb{R} \) is varied. Here \( x \) will belong to the plane and \( \xi_t \) will be a realization of some noise. We assume that the RDEs without noise terms, unfold a supercritical Hopf-Andronov bifurcation at \( \lambda = 0 \) then we observe the bifurcation that occurs when they are subjected to bounded noise. It is well known that attracting cycles appear in the unfolding of deterministic supercritical Hopf bifurcations. We discuss the occurrence of attracting random cycles and show that attracting random cycles appear for the RDEs with small bounded noise. Random cycles are defined in analogy with random fixed points. They are closed curves that are invariant for the skew product system and thus have a time dependent position in the state space depending on the noise realization.

References


Bifurcations of localized convective patterns under parametric disorder

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The effect of localization of states under frozen parametric disorder is well known as Anderson localization [1] and thoroughly studied for the Schrödinger equation and linear dissipation-free wave equations [2]. Some similar phenomena can occur in strongly dissipative/active systems, as the thermal convection ones [3]. Specifically, these dissipative systems, where pattern selection can occur, are governed by a kind of Kuramoto-Sivashinsky equation,

\[
\frac{\partial \theta(x,t)}{\partial t} = \frac{\partial}{\partial x} \left[ -u \theta(x,t) - \frac{\partial^3 \theta(x,t)}{\partial x^3} - q(x) \frac{\partial \theta(x,t)}{\partial x} + \left( \frac{\partial \theta(x,t)}{\partial x} \right)^3 \right],
\]

and frozen disorder in \( q(x) \) can lead to the excitation of spatially-localized patterns. The term \( u \theta \) describes an imposed transport along the system: for convective systems \( u \) is the strength of an imposed through-flow. These localized patterns significantly affects transport along the system [4, 5].

Strong enough imposed transport \( u \) can both effect the localization properties and “wash-out” the localized patterns. We report two possible scenarios of this washing-out: (i) via a pitchfork bifurcation, the pattern is time-independent for any \( u \), and (ii) via succession of Hopf bifurcations and a non-local bifurcation of homoclinic loops.

The work has been financially supported by Grant of The President of Russian Federation (MK-6932.2012.1).

References


Questions of biosystem control in dynamic diseases area based on functional-differential equations of regulatorika (functioning of regulatory mechanisms) are considered [1, 2].

Qualitative analysis of model systems for the regulatorika equations of biosystem shows presence (in the parametric space) following regimes: trivial attractor, stationary state, auto-oscillations, dynamic chaos and “black hole” effect. During ”black hole“ effect there are periodical solutions failure to trivial attractor.

Results of the quantitative study of functional-differential equations show heterogeneity of dynamic chaos area: in chaos area there are ensemble of small regions with auto-oscillations [1, 3].

Biosystem control in dynamical chaos area can be realized based on the developed equations for local control, principles of ”not worsening of system state”, ”minimal pressure” and ”ecological purity” during control process.

Conditions for existence of continuous, limited and positive solutions of function-differential equations for local control (if we have initial conditions as functions on corresponding time interval or as set of discrete values) are determined.

Scenarios for possible path for moving out the system from area of the deterministic chaos into area of self-oscillations using small regions with regular oscillations (r-windows) are investigated.

Research results are used during studying genetic mechanisms of cancer, determination of characteristic features of biosystem control and methods development for biosystem control in anomalies areas.
References


Analysis of the vibrational dynamics of molecular systems with 3 degrees of freedom

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The dynamical processes that occur in a single molecule are critical in determining their reactivity, which is the ultimate goal of any Chemical Theory. Identify the vibrational modes that are coupled in an efficient manner through Fermi resonances, and how energy flows through it, is an important task in Molecular Dynamics. When the system has two degrees of freedom there are well known mathematical tools to be used, such as the Poincaré Surface of Section (SSP). However, when the system has more than two degrees of freedom the SSP is not enough to have a complete picture of the dynamics. In recent years it has successfully studied molecular systems with three degrees of freedom by the Frequency Map (FM)[1, 2]. The FM is useful to distinguish regular and chaotic areas in phase space, and can also be seen indications of diffusion on the web formed by the resonance lines in frequency space. A complementary study based on the idea of analyzing the separation of nearby orbits, is the calculation of SALI (Small Alignment Index)[3, 4]. This index study the behavior of nearby trajectories associated vectors that can be easily calculated from the resolution of Hamilton’s equations of the system (regardless of dimension). SALI value tends rapidly to zero with time when the trajectory is chaotic (in the form \( SALI(t) \approx e^{-\lambda t} \) with \( \lambda > 0 \)). In this paper we present the study of several triatomic molecules with three degrees of freedom using the FM and SALI. SALI maps correspond to two-dimensional representations of the phase space, where the asymptotic value of SALI is shown in color scale. For a fixed energy value, the regions of chaos in the 3D system are smaller than in the 2D case, because the energy must be spread over more degrees of freedom. We have also studied the representation of SALI values on the
frequency space obtained by the FM that allows a deeper analysis of the dynamics of these systems and help to find evidence of diffusion.

References


Hindmarsh and Rose [4] proposed in 1984 a model of neuronal activity described by three nonlinear ODEs: $\dot{x} = y - ax^3 + bx^2 - z + I$, $\dot{y} = c - dx^2 - y$, $\dot{z} = \epsilon[s(x - x_0) - z]$. The parameters are typically set as follow: $a = 1$, $c = 1$, $d = 5$, $s = 4$, $x_0 = -1.6$, $\epsilon = 0.01$ and we consider $I = (1 - 0.265b)/0.0691$. The remaining parameter $b$ determines the bursting or spiking behavior.

Our Periodic Orbits (POs) search method is based on the stability transformation combined with the Newton method. The first one [2] was developed to detect unstable periodic orbits in chaotic dynamical systems. Basically, it transforms the studied dynamical system into a new one where the unstable fixed points of the original system are stable but they keep their original position in space.

On this work we have selected two values of parameter $b$ [1] corresponding to chaotic-bursting behavior ($b = 3.05$ and $b = 2.69$). For these values we have studied the topological changes of the chaotic attractors obtaining the complete set of POs up to multiplicity four. For $b = 3.05$ we found 1, 1, 0, 1 POs of multiplicity one to four with a mean period of 57.8, 81.5, 185.0 respectively and for $b = 2.69$ we found 1, 1, 2 and 3 POs of multiplicity one to four with mean period of 24.8, 47.8, 70.3, 92.1 respectively.

By using the skeleton of periodic orbits we are able to determine the topological template of the chaotic attractors [3] and to "quantify" the differences among them.
References


Global study of the Bogdanov-Takens bifurcation curve

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The bifurcation curve of the homoclinic connection that appears in the Bogdanov-Takens normal form system

\[
\begin{align*}
    x' &= y, \\
    y' &= -n + by + x^2 + xy,
\end{align*}
\]

is usually restricted to a local study near the critical point. The aim of this work is to obtain a global knowledge of this curve in the parameter space.

It is well known that for small \( n > 0 \) there exists a value \( b^*(n) \) such that the system has an unique limit cycle if and only if \( b^*(n) < b < \sqrt{n} \). Moreover \( b^*(n) = \frac{5}{7} \sqrt{n} + \ldots \). In [3] the quadratic differential equation is considered in the whole space and it is proved that \( b^*(n) \) is analytic and that the previous inequalities are satisfied for all \( n > 0 \). A detailed study of the curve \( b^*(n) \) for \( n \) small enough is presented in [2]. This previous work is based on an algebraic method for the location of bifurcation curves.

In our work, see [1], we adapt this procedure to the global study of the Bogdanov-Takens bifurcation. That is, we obtain explicit curves such that \( b_l(n) < b^*(n) < b_u(n) \) for all \( n > 0 \). With this result we proved a Perko’s conjecture, see [3], that predicts the behaviour of this curve when the parameters goes to infinity. In fact, we proved that \( b^*(n) \) goes to infinity as \( \sqrt{n} - 1 \).
References


On well-posedness of a nonresonance operator differential equation in spaces of entire functions of exponential type

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Let $E$ be a Banach space, $A$ is a closed linear operator on $E$ with domain of definition $D(A)$ that may be not dense in $E$. We suppose that $A$ has a bounded inverse operator, and prove well-posedness of the differential equation $w' = Aw + f(z)$ in a space of entire functions of exponential type. The proof is based on studying properties of entire solutions of implicit differential equation $Tw' + g(z) = w(z)$. In addition, we do not use the solution of the homogeneous equation for solving inhomogeneous one unlike the classical approach, which is based on studying the semigroup theory. Moreover, using Laurent formal series the Cauchy type integral representation for the entire solution of this implicit equation is obtained. Some results related to the mentioned above equation generalize to the case of the following differential-difference equation $Tw'(z + h) + g(z) = w(z)$. In some particular cases the represented results were obtained in [1], [2]. The closed questions for differential equations in a Banach space over a non-Archimedean field were considered [3].

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