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Mathematical modelling of the combined optimization of a pumped-storage hydro-plant and a wind park

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ABSTRACT

The new Spanish regulations allow wind farms to go to the market to sell the energy generated by their facilities. In the case of over- or under-supply, other producers must reduce or increase their production to resolve the so-called deviation, thereby incurring financial losses. Faced with this situation, wind farms have several options. In this paper we consider one promising method: the combined optimization of a pumped-storage hydro-plant and a wind park. First, we are going to present the mathematical modelling of the resulting problem. Second, we shall solve the optimization problem using Optimal Control techniques and finally we shall present several examples of the combined optimization and analyse which strategy is the best one possible.

1. Introduction

Spanish energy regulations have managed to position Spain as the second country worldwide with more wind power capacity. The new regulations (RD661/2007) (BOE [1]) allow wind farms to go to market to sell the energy generated by their facilities. However, wind power has a characteristic that makes it special: the problem of the unpredictability of wind farm production. Even though there are several commercial prediction models (for instance, Casandra [2]), the high degree of difficulty in predicting lies in its proportionality to the third power of the modulus of the wind speed. In the case of over- or under-supply, other producers must reduce or increase their production to resolve the so-called deviation, thereby incurring financial losses. These financial losses lead to what are known as deviation penalties, provided for in RD661/2007. Currently, the usual practice is that the cost resulting from deviation is fixed at 25%–30% of the clearing price on the spot market. Faced with this situation, wind farms have a number of options: they can try to offer on the intraday spot markets to re-buy or sell energy; they can pay the penalties; or they can try to store wind power energy in some way. Diverse methods have also been proposed for storing this energy: compressed air storage, batteries for electric cars, hydrogen and, what we analyse in this paper, pumped water.

In this paper we consider the combined optimization of a pumped-storage hydro-plant and a wind farm and analyse which strategy is the best one possible. The paper is organized as follows. In Section 2 we set out the corresponding mathematical problem to optimize the functioning of a pumped-storage hydro-plant and solve it using Optimal Control techniques. In a previous paper [3], the authors considered a fixed-head model for the pumped-storage hydro-plant. Section 3 summarizes the optimization algorithm presented in [3] for optimizing the functioning of a pumped-storage hydro-plant. We then go on to analyse the following problem: Is it in the interest of wind farms to go to market? To answer this question, Section 4 addresses the mathematical study of two extreme situations of the combined hydro-eolic plant. The results obtained in several examples are likewise presented in Section 4. Finally, the conclusions reached in this study are discussed in Section 5.

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2. Mathematical optimization of a pumped-storage hydro-plant

A basic physically-based relationship between the active power generated by a hydro-plant, P (in MW), the rate of water discharge, z' (in m^3/s), and the effective head, h (in m), is given by

$$P = \frac{z'h}{G} \tag{1}$$

where G is the efficiency (in m^4/h MW) (see [4,5]). For a large capacity reservoir, it is practical to assume that the effective head is constant over the optimization interval. Here the fixed-head hydro-plant model is defined and P is represented by the linear equation: $P(z'(t)) = Az'(t)$, where A represents the efficiency and diverse parameters related to the geometry of the station (see [4] for further details). Pumped-storage is a well-known type of hydro-plant option used for load balancing. At times of low electrical demand, electric power is used to pump water into the upper reservoir. During periods of high electrical demand, water is released back into the lower reservoir through a turbine, thereby generating electricity. Taking into account the conversion losses of the pumping process and evaporation losses, a maximum of 70% or 85% of the electrical energy used to pump the water into the elevated reservoir can be regained. We must therefore introduce the efficiency, η, in the model. Thus, when pumped-storage plants are considered, the function P is defined piecewise as:

$$P(z') := \begin{cases} A \cdot z' & \text{if } z' \geq 0 \\ \eta \cdot A \cdot z' & \text{if } z' < 0. \end{cases} \tag{2}$$

If we assume that b is the volume of water that must be discharged over the entire optimization interval $[0, T]$, the following boundary conditions will have to be fulfilled:

$$z(0) = 0, \qquad z(T) = b. \tag{3}$$

Besides the previous statement, we consider $z'(t)$ to be bounded by technical constraints

$$q_{\min} \leq z'(t) \leq q_{\max}, \quad \forall t \in [0, T]. \tag{4}$$

In this section we focus on the new short-term problem faced by a generation company in a deregulated electricity market when preparing its offers for the day-ahead market. Our model of the spot market explicitly represents the price of electricity as a known exogenous variable. In our problem, the objective function is given by hydraulic profit over the optimization interval, $[0, T]$. Profit is obtained by multiplying the hydraulic production of the pumped-storage hydro-plant by the clearing price, $\pi(t)$, at each hour, t. Taking our objective functional $F(z)$ in continuous time form, the problem is:

$$\max_z F(z) = \max_z \int_0^T L(t, z(t), z'(t))dt = \max_z \int_0^T \pi(t)P(z'(t))dt \tag{5}$$

$$\text{on } \Omega = \left\{ z \in \widehat{C}^1[0, T] \mid \begin{matrix} z(0) = 0, z(T) = b \\ q_{\min} \leq z'(t) \leq q_{\max} \end{matrix}; \forall t \in [0, T] \right\}. \tag{6}$$

A standard Lagrange-type Optimal Control problem can be mathematically formulated as follows:

$$\begin{aligned} &\max_{(u,z)} \int_0^T L(t, z(t), u(t))dt = \max_{(u,z)} \int_0^T \pi(t)P(u)dt \\ &z' = u; \qquad z(0) = 0, \qquad z(T) = b; \qquad u_{\min} \leq u(t) \leq u_{\max}. \end{aligned} \tag{7}$$

3. Optimization algorithm of a pumped-storage hydro-plant

For the Optimal Control problem (7), we define the Hamiltonian in normal form:

$$H(t, z, u, \lambda) := L(t, z, u) + \lambda u = \pi(t)P(u) + \lambda u. \tag{8}$$

The resulting Hamiltonian, H, is linear in the control variable, u, and results in an optimal singular/bang–bang control policy.

It is well known [6,7] that when the Hamiltonian is linear in u, the optimality condition (maximize H w.r.t. u) leads to:

$$u^*(t) = \begin{cases} u_{\max} & \text{if } \Phi(x, \lambda) > 0 \\ u_{\text{sing}} & \text{if } \Phi(x, \lambda) = 0 \\ u_{\min} & \text{if } \Phi(x, \lambda) < 0 \end{cases} \tag{9}$$

and u^* is undetermined if $\Phi(x, \lambda) \equiv H_u = 0$. The function Φ is called the switching function. If $\Phi(x^*(t), \lambda(t)) = 0$ only at isolated time points, then the optimal control switches between its upper and lower bounds, which is said to be a bang–bang type control (i.e. the problem is not singular). The times when the OC switches from $u_{\max}$ to $u_{\min}$ or vice-versa are called switching times. If $\Phi(x^*(t), \lambda(t)) = 0$ for every t in some subinterval $[t', t'']$ of $[0, T]$, then the original problem is called a singular control problem and the corresponding trajectory for $[t', t'']$, a singular arc.

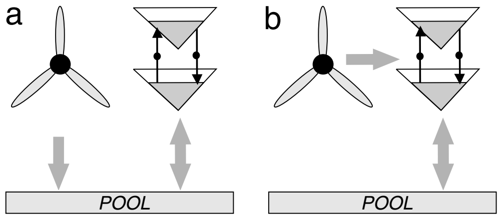

Fig. 1. Two configurations.

In general, the application of Pontryagin's Maximum Principle [6,7] is not well suited for computing singular control problems as it fails to yield a unique value for the control. In our problem, however, an added complication arises: the Hamiltonian is defined piecewise as:

$$H(t,u,\lambda) := \begin{cases} [A\cdot\pi(t)+\lambda]u & \text{if } u\geq 0 \\ [\eta\cdot A\cdot\pi(t)+\lambda]u & \text{if } u<0 \end{cases} \tag{10}$$

and the derivative of H with respect to $u(H_u)$ presents discontinuity at $u=0$, which is the point at which a sudden change in H_u is produced, as it is the border between the power generation zone (positive values of u) and the pumping zone (negative values of u). The classical gradient of H at u is defined only when H is differentiable at u. However, when non-differentiable objective functions arise in optimization problems, the generalized (or Clarke's) gradient (see [8,9]) must be considered.

On the basis of the above theoretical results, in [3] we determined the optimal solution: the bang-singular-bang segments and the boundary on which the solution is situated:

$$u^*(t) = \begin{cases} u_{\max} & \text{if } A\cdot\pi(t) > -\lambda_0 \\ u_{\text{sing}}=0 & \text{if } -\lambda_0 \in [A\cdot\pi(t), \eta\cdot A\cdot\pi(t)] \\ u_{\min} & \text{if } \eta\cdot A\cdot\pi(t) < -\lambda_0. \end{cases} \tag{11}$$

The algorithm that leads to the optimal solution comprises the following steps: (i) First, for a given λ, we have to determine the switching times: $t_1, t_2, \ldots$. These instants are calculated solving the equations

$$A\cdot\pi(t) = -\lambda; \qquad \eta\cdot A\cdot\pi(t) = -\lambda. \tag{12}$$

(ii) Second, the optimal value, λ_0, must be determined in order for:

$$z_\lambda(T) = \sum_{i=1}^{N_u} \delta_i^u \cdot q_{\max} + \sum_{i=1}^{N_l} \delta_i^l \cdot q_{\min} = b \tag{13}$$

δ_i^u and δ_i^l being the duration of the i-th bang–bang segment in the upper and lower bound respectively, N_u and N_l the number of such segments, and $z_\lambda(T)$ the final volume obtained for each λ. (iii) To calculate an approximate value of λ_0, we propose a classic iterative method (like, for example, bisection or the secant method).

The algorithm presents a series of advantages. First of all, our method needs no prior knowledge of the number and location of the bang-singular-bang arcs. Moreover, as we shall see in the next section, it shows a rapid convergence to the optimal solution and can be run in a relatively short time due to the simplicity of the operations to be performed in this method.

4. Combined optimization

With the aid of the algorithm presented in the previous section, we are now in a position to analyse the combined optimization of a pumped-storage hydro-plant and a wind farm. In this section we shall analyse whether it is in the interest of wind farms to go to market. To do so, we address two configurations (see Fig. 1):

(a) The wind farm and the pumped-storage hydro-plant work independently, each selling the energy it produces on the market.

(b) The wind farm does not sell energy on the market, but uses the generated power to pump water to the upper reservoir of the pump-plant. We shall call this pumped water b^*.

It is very important to highlight the fact that in configuration (b) we consider that the water pumped thanks to wind power, b^*, on day i is used in the hydro-plant on day $i+1$. In this way, the problem of the unpredictability of the wind is avoided completely. Accordingly, and in order for the comparison to be rigorous, the wind power production in configuration (a) is considered to be sold to the market on day i, and that of the hydro-plant on day $i+1$. We shall use superscripts to denote the day under consideration.

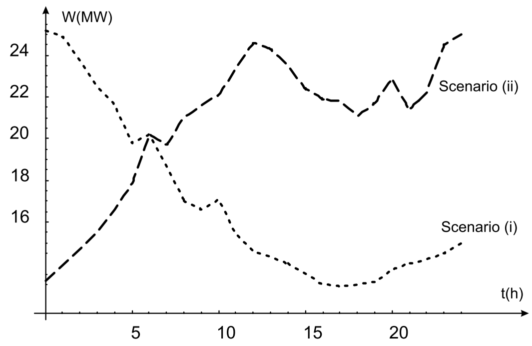

Fig. 2. Two wind power production scenarios.

In configuration (a), the total profit over the optimization interval $[0, T]$ is revenue minus cost. Revenue is obtained by multiplying the hydraulic production, $P(t)$, and the wind power production, $W(t)$, by the clearing price, $\pi(t)$, at each hour, t. The unique cost are the deviation penalties $C(t)$:

$$\int_0^T \left(\pi^{i+1}(t)P^{i+1}(t) + \pi^i(t)W^i(t) - C^i(t)\right) dt, \quad \text{with } z(T) = b. \tag{14}$$

In configuration (b), the total profit over the optimization interval, $[0, T]$, is the revenue obtained only by hydraulic production, $P(t)$:

$$\int_0^T \left(\pi^{i+1}(t)P^{i+1}(t)\right) dt, \quad \text{with } z(T) = b + b^*. \tag{15}$$

Evidently, in this configuration the water available increases by b^*. There are a number of factors that influence the final result, like for example: the efficiency of the hydro-plant, the volume of water available, deviation penalties, or wind power production. The first two factors have already been analysed in [3], so in this paper we consider the efficiency, η, and the restriction on the volume, b, as a fixed data. We shall now analyse the last two factors, which correspond to the wind farm. In order for our study to reflect a broad range of possibilities, we propose two hypothetical scenarios (see Fig. 2): (i) Low wind power production in peak hours; and (ii) High wind power production in peak hours, and then proceed to analyse the influence of deviation penalties.

4.1. Example

A program was written using the Mathematica package to apply the results obtained in this paper to an example of a hydraulic system made up of one fixed-head pumped-storage hydro-plant and a wind farm. The hydraulic model is:

$$P(t) = 0.000126821\, z'(t). \tag{16}$$

We shall also consider the technical constraints: $q_{\min} = -283866$ (m^3/h); $q_{\max} = 394, 258$ (m^3/h). We consider an efficiency $\eta = 1.2$ and a restriction on the volume $b = 1.5 \cdot 10^6$ (m^3).

In this paper, we focus on the problem that a generation company faces when preparing its offers for the day-ahead market. Thus, the classic optimization interval of $T = 24$ h was considered. The clearing price, $\pi(t)$ (euros/h MW), corresponding to one day was taken from the Spanish electricity market [10]. For the sake of simplicity, we shall also assume that the prices are equal on both days: $\pi^{i+1}(t) = \pi^i(t)$.

With the aim of not excessively complicating this study, we shall assume that the deviations, d, are a certain % of the wind power over the optimization interval. We shall also simplify the study by assuming that all the deviations go against the system. Finally, we shall consider positive errors (overproduction) to be penalized in the same way as negative errors (underproduction). We shall analyse diverse cases, from low deviations (40% on average) to high deviations (80% on average). We shall likewise consider deviation penalties of 27.5% of the clearing price on the spot market. Hence:

$$C(t) = d \cdot W(t) \cdot 0.275 \cdot \pi(t). \tag{17}$$

Table 1 shows the results obtained in Scenario (i). The hydraulic profit in configuration (a) is always the same, while the wind power profit decreases progressively with increasing deviations. There is no wind power profit in configuration (b), the only profit being hydraulic, which increases notably as the volume of water available also increases by b^*. Comparing both configurations in this Scenario (i) of low wind power production in peak hours, we see that the total profit (b) exceeds that obtained in (a) for deviations above 60%. In these cases would option (b) be more interesting.

Table 2 shows the results obtained in Scenario (ii). As can be seen, the total profit (b) in this scenario of high wind power production in peak hours exceed that obtained in (a) for deviations above 80%. The reason is that the market price is very high in peak hours and it is worth selling the wind power energy even though deviations incur many penalties.

Table 1
Comparison of profits in Scenario (i).

Config. (a)	d (%)	Wind profit	Hydr. profit	Tot. profit
	40	29743.8	26518.8	56262.7
	60	27905.7	”	54424.6
	80	26067.6	”	52586.5
Config. (b)		$b^*(10^6\ m^3)$	Hydr. profit	Tot. profit
		2.64416	55202.8	55202.8

Table 2
Comparison of profits in Scenario (ii).

Config. (a)	d (%)	Wind profit	Hydr. profit	Tot. profit
	40	38516.6	26518.8	65035.5
	60	36136.4	”	62655.2
	80	33756.1	”	60275.0
Config. (b)		$b^*(10^6\ m^3)$	Hydr. profit	Tot. profit
		3.27168	61516.7	61516.7

In conclusion, both configurations (a) and (b) seem interesting options. As we have seen, the system is highly sensitive to numerous factors and each company must carefully assess particular situations that may result in variations in the optimum configuration. Our study provides a useful, simple and efficient tool for taking such a decision.

5. Conclusions

In this paper we have presented a tool to design the optimal configuration of a wind farm combined with a pumped-storage hydro-plant. When the hydro-plant is of the fixed-head type, the optimal solution presents a very particular form: the bang-singular-bang solution. The problem studied in this paper analyses the convenience, or not, of the wind farms going to market. Our algorithm allows the optimal solution to be obtained easily and the obtained results provide real-time information to determine which configuration is preferable in each specific real situation of the electricity market.

Acknowledgment

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