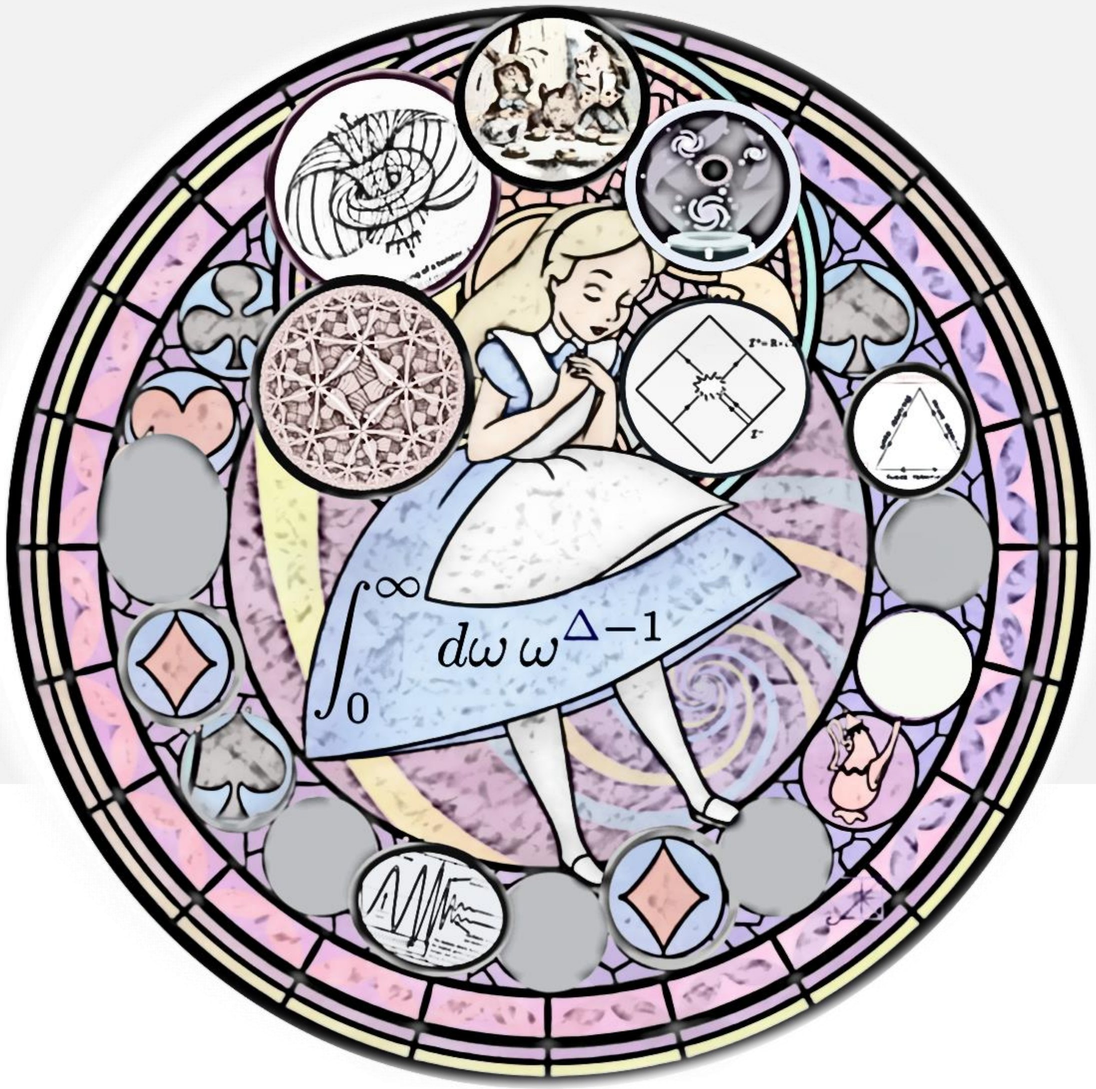



# A Carrollian perspective on Celestial holography

**Laura Donnay**
**SISSA**, Trieste

**EuroStrings** 2023, Gijón
24 - 28 April 2023

# Flat space holography

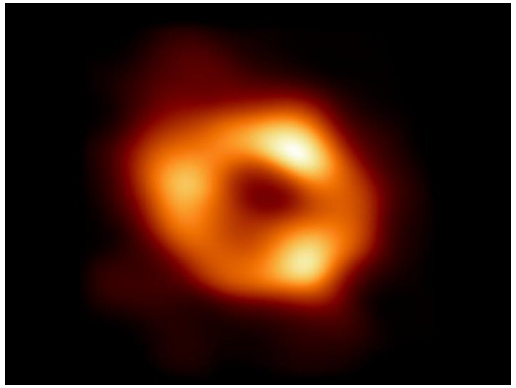
Holographic description of quantum gravity in 4d asymptotically flat spacetimes ($\Lambda = 0$)?

→ These spacetimes are relevant from collider physics … to astrophysics (< cosmological scales)

$$S_{BH} = \frac{\mathcal{A}c^3}{4G\hbar}$$

[Bekenstein][Hawking]

$\mathcal{A}$ : event horizon area



Event Horizon Telescope

↓

Holography beyond Anti-de Sitter/CFT?

$\Lambda < 0$

# Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

Early attempts:

[Susskind ‘99][Polchinski ‘99][Giddings ’99]
[de Boer, Solodukhin ‘03][Arcioni, Dappiaggi ’03 ’04]
[Dappiaggi, Moretti, Pinamonti ’06][Mann, Marolf ’06]…

…and even earlier

[Penrose ‘76][Newman ’76]

→ aimed at a reconstruction of the bulk spacetime from quantities defined only at null infinity $\mathscr{I}$

*General Relativity and Gravitation*, Vol. 7, No. 1 (1976), pp. 107–111

**Heaven and Its Properties**

EZRA T. NEWMAN

*Department of Physics, University of Pittsburgh, Pittsburgh, Pennsylvania 15213*

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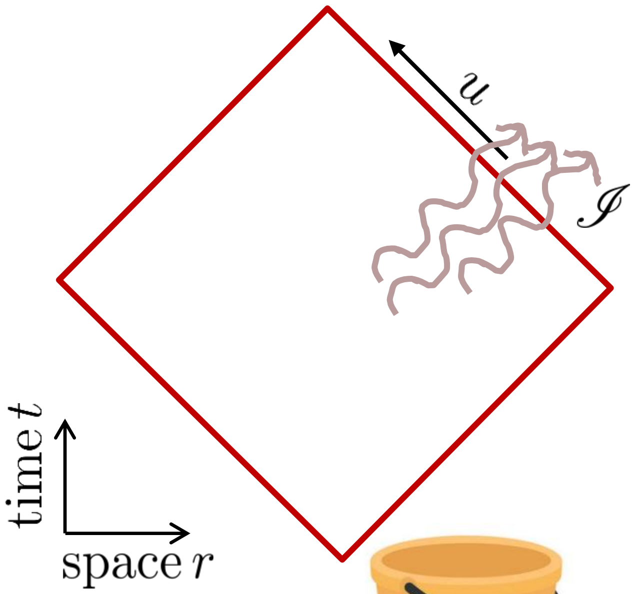
Main obstructions/difficulties:

1. The boundary is a **null** hypersurface

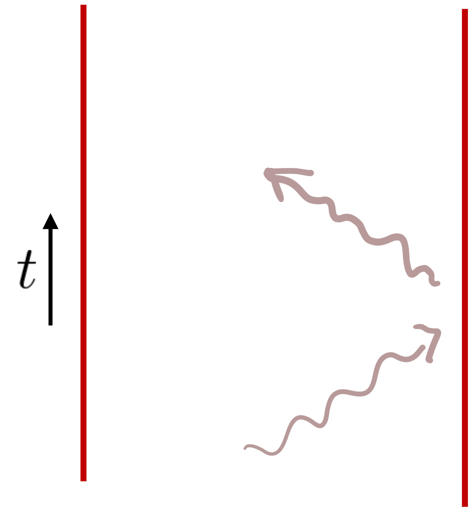
$$u = t - r$$

2. There are **fluxes** leaking out the boundary

Minkowski




AdS




Quantum gravity ‘in a **box**’

# Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

--> Road map: symmetries

What are the symmetries of asymptotically flat spacetimes?

# Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

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What are the symmetries of asymptotically flat spacetimes?

what was expected

Poincaré

what was found

Bondi-Metzner-Sachs (‘62)

# Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

--> Road map: symmetries

What are the symmetries of asymptotically flat spacetimes?

→ infinite-dimensional extension of Poincaré!

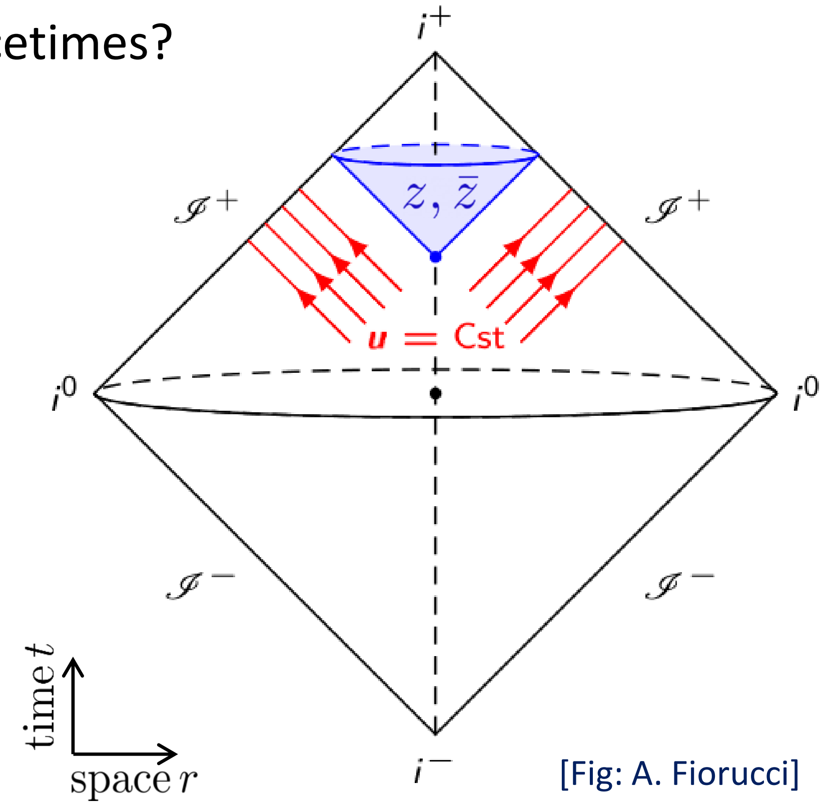
**4** Poincaré translations

↓ *Symmetry enhancement*

**∞ BMS** supertranslations



[Bondi, van der Burg, Metzner ’62] [Sachs ’62]




[Fig: A. Fiorucci]

$$\xi = \mathcal{T}(z, \bar{z})\partial_u + \cdots$$

arbitrary function
on the celestial sphere

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Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

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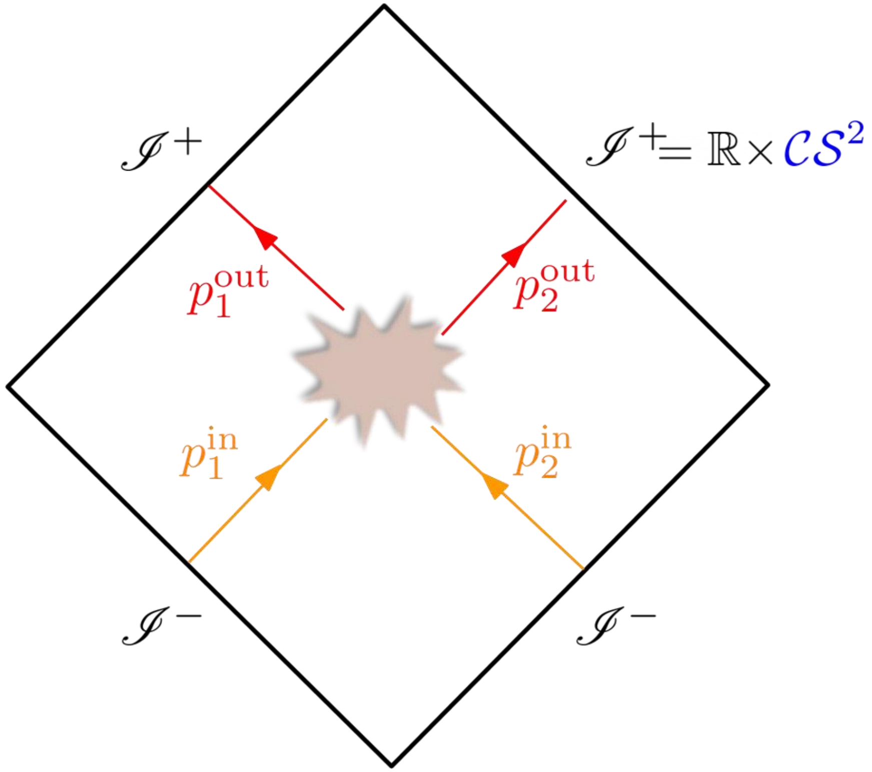
- constrain the gravitational **S-matrix**



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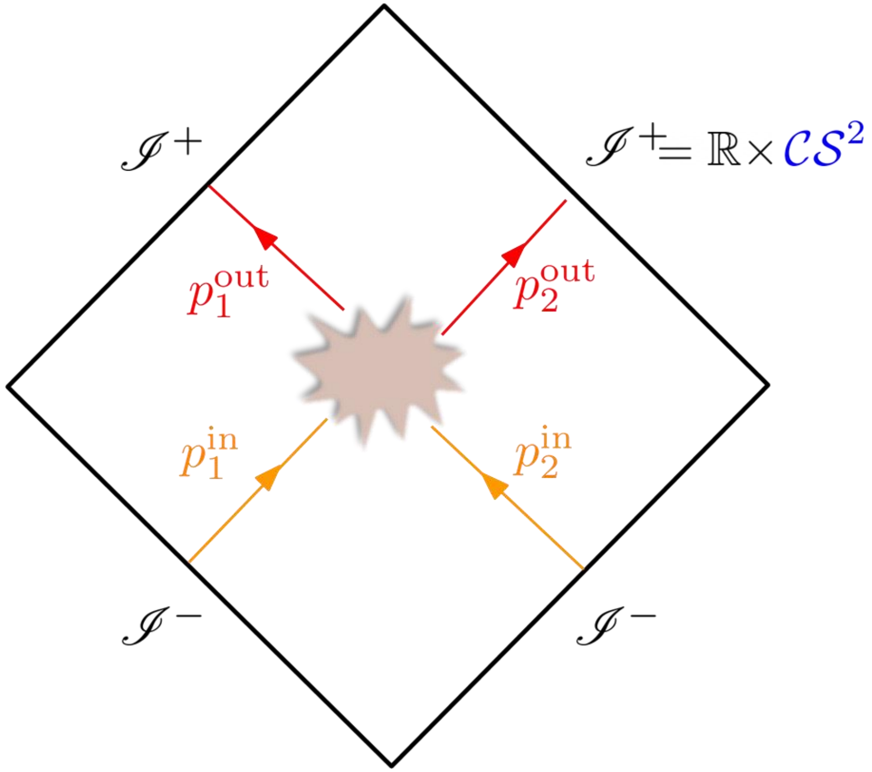

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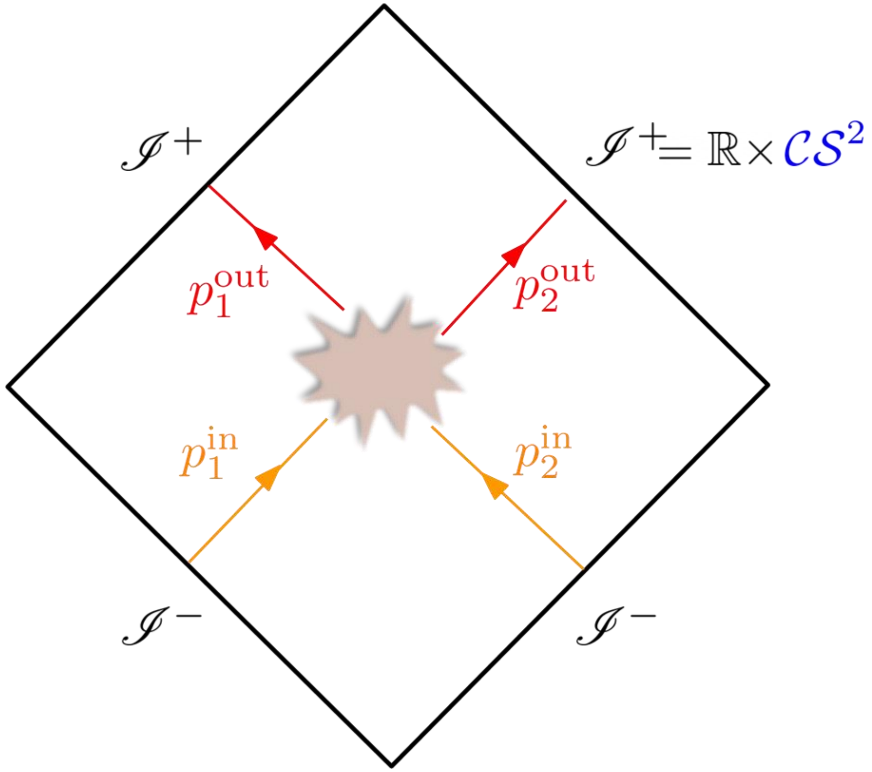
While BMS symmetries were originally disregarded, it was realized (50 years later) that they

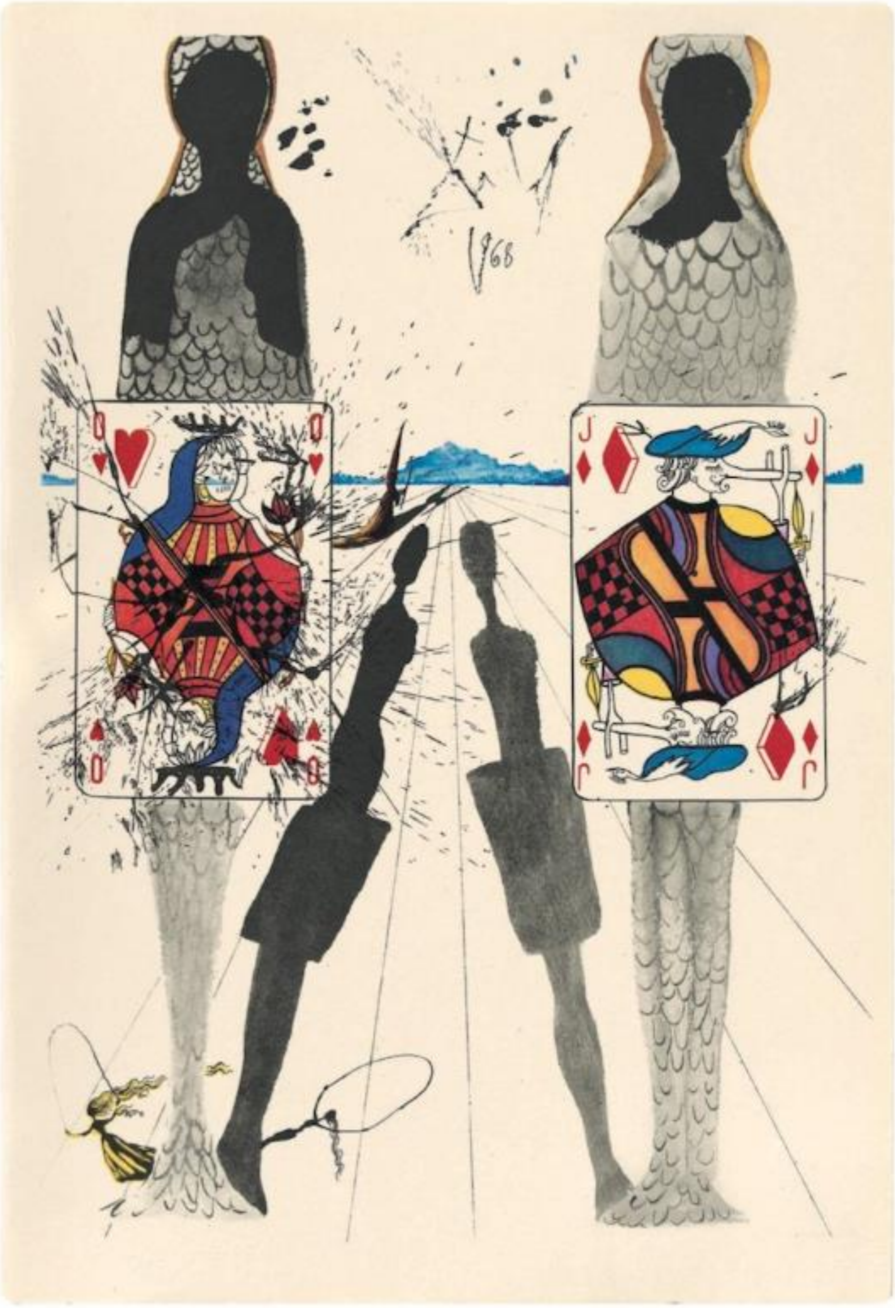
- constrain the gravitational **S-matrix**
- have associated low-energy **observables** (memory effects)
- allow further extensions, including the local **conformal** group

↓

**Celestial holography**



Salvador Dalí, illustrations for *Alice's Adventures in Wonderland*, 1969:



# Outline

1. Celestial holography
2. Carrollian holography
3. CCFT vs CCFT

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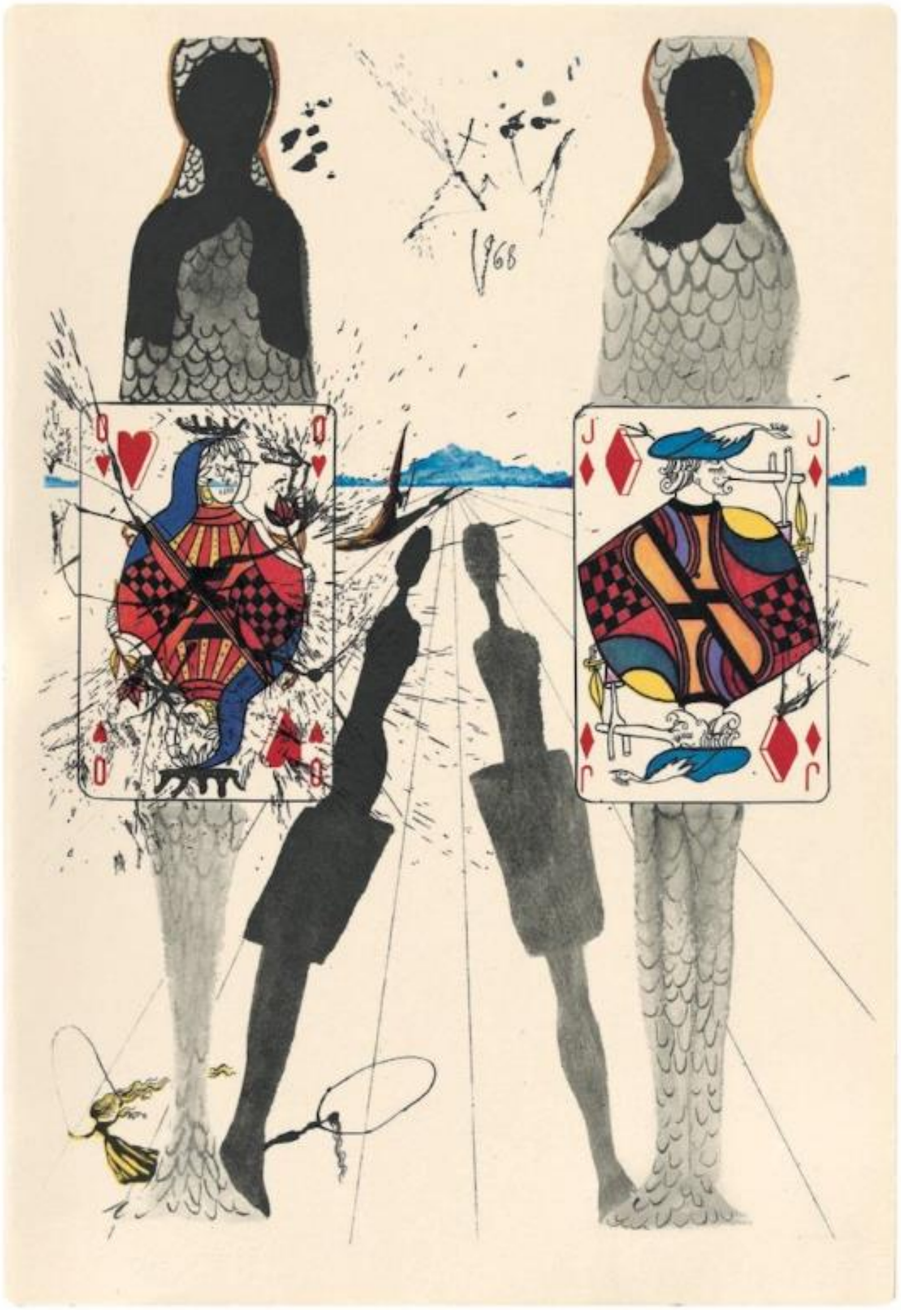

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# Celestial Holography

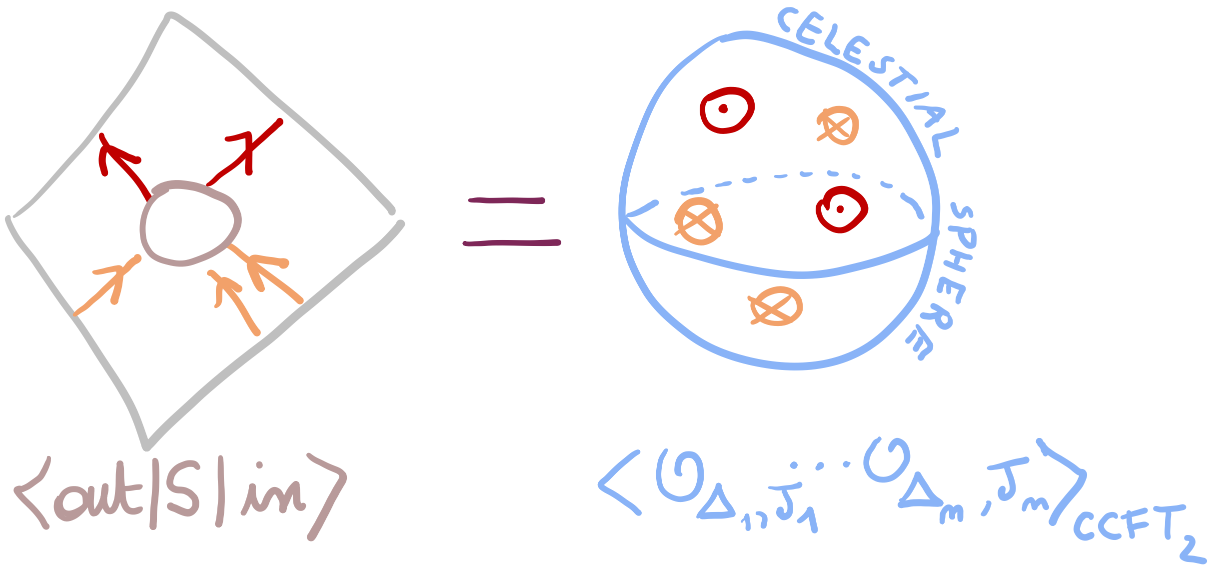
The 4d spacetime S-matrix is encoded in a 2d ‘Celestial Conformal Field Theory’

momentum of a massless particle

$p^\mu = \omega q^\mu(z, \bar{z})$

$\omega$: energy

$(z, \bar{z})$ : a point on $\mathcal{CS}^2$



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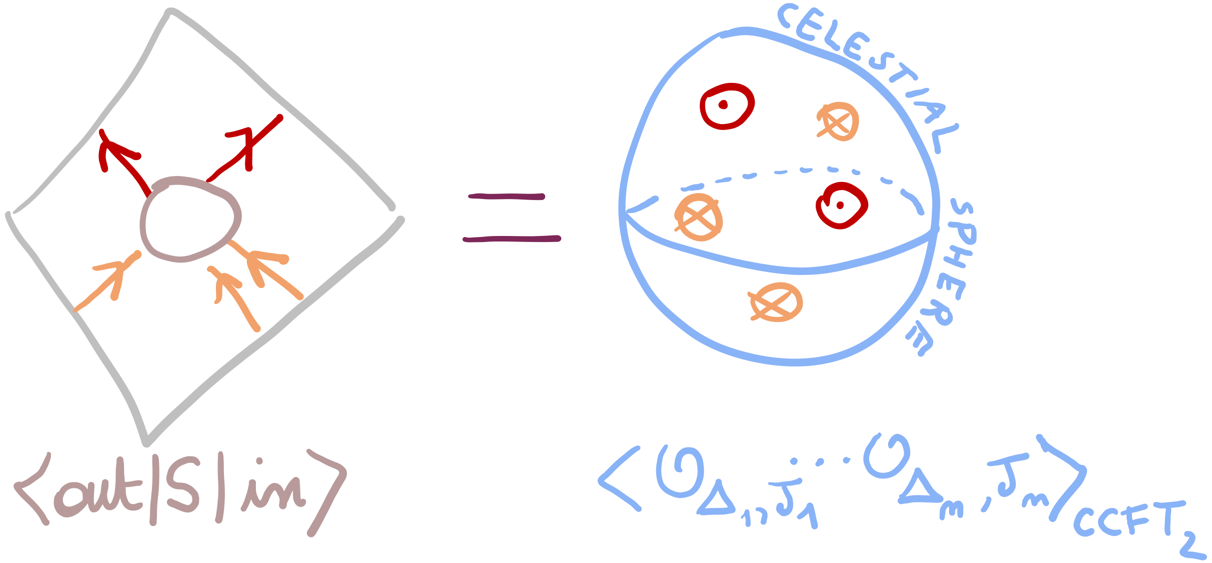


Simple idea: make conformal properties manifest

→ Plane waves are mapped to

$$\Psi^{\pm}_{\Delta}(X; z, \bar{z}) = \int_0^\infty d\omega\, \omega^{\Delta-1} e^{\pm i p\cdot X}$$

$$\Psi_{h,\bar{h}}(z,\bar{z}) \to \left(\frac{\partial z}{\partial z'}\right)^h \left(\frac{\partial \bar{z}}{\partial \bar{z}'}\right)^{\bar{h}} \Psi_{h,\bar{h}}(z,\bar{z})$$

Primary field of weight $\Delta = h + \bar{h}$

# Celestial currents

$$\mathcal{O}_{h,\bar{h}}(z,\bar{z}) \to \left(\frac{\partial z}{\partial z'}\right)^{h} \left(\frac{\partial \bar{z}}{\partial \bar{z}'}\right)^{\bar{h}} \mathcal{O}_{h,\bar{h}}(z,\bar{z})$$

The soft sector of celestial CFT is captured by 2d celestial currents.

$$(h,\bar{h}) = \frac{1}{2}(\Delta + J, \Delta - J)$$

[Kapec, Mitra, Raclariu, Strominger][Cheung, de la Fuente, Sundrum][Fotopoulos, Stieberger,Taylor]
[LD, Puhm, Strominger][Adamo, Mason, Sharma][Puhm][Guevara]

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| Asymptotic symmetry | Ward identity | Weight | 2d Celestial current |
|---|---|---|---|
| 'large gauge' $\delta A_z = D_z \epsilon$ | **Soft photon** theorem | $\Delta \to 1$ $(h,\bar{h}) = (1,0)$ | $J(z)\mathcal{O}_{h,\bar{h}}(w,\bar{w}) \sim \frac{1}{(z-w)}\mathcal{O}_{h,\bar{h}}(w,\bar{w})$ |
| supertranslations $\delta C_{zz} = D_z^2 f$ | **Soft graviton** theorem | $\Delta \to 1$ $(\frac{3}{2},\frac{1}{2})$ | $P(z,\bar{z})\mathcal{O}_{h,\bar{h}}(w,\bar{w}) \sim \frac{1}{(z-w)}\mathcal{O}_{h+\frac{1}{2},\bar{h}+\frac{1}{2}}(w,\bar{w})$ |
| superrotations $\delta C_{zz} = uD_z^3 Y^z$ | **Sub-leading** soft graviton theorem | $\Delta \to 2$ $(2,0)$ | $T(z)\mathcal{O}_{h,\bar{h}}(w,\bar{w}) \sim \frac{h}{(z-w)^2}\mathcal{O}_{h,\bar{h}}(w,\bar{w}) + \frac{\partial\mathcal{O}_{h,\bar{h}}(w,\bar{w})}{z-w}$ 2d stress tensor! |

$g_{zz} = rC_{zz} + \dots$

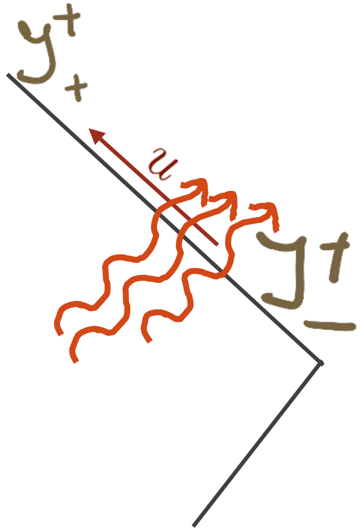
# Celestial currents

- Can be related to objects of the gravitational solution space in terms of 'BMS fluxes'

  [LD, Ruzziconi '21]

$$\int_{\mathscr{I}^+} du\partial_u\left(\cdot\right) = \left(\cdot\right)\Big|^{\mathscr{I}^+_+}_{\mathscr{I}^+_-}$$

$$\begin{aligned} ds^2 = &-du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}\,dzd\bar{z} \\ &+\frac{2M}{r}du^2 + rC_{zz}dz^2 + D^zC_{zz}dudz \\ &+\frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)dudz + c.c. + \cdots \end{aligned}$$



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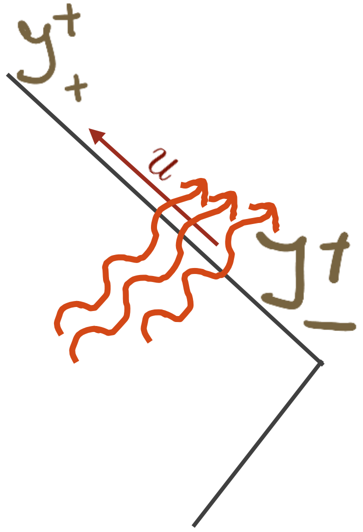
$$\int_{\mathcal{I}^+} du\partial_u\,(\cdot) = (\cdot)\Big|^{\mathcal{I}^+_+}_{\mathcal{I}^+_-}$$

$$\begin{aligned} ds^2 &= -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}\,dzd\bar{z} \\ &+\frac{2M}{r}du^2 + rC_{zz}dz^2 + D^zC_{zz}dudz \\ &+\frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)dudz + c.c. + \cdots \end{aligned}$$



- Infinite tower of currents! $\Delta \to 2, 1, 0, -1, \cdots$

  reorganized in terms of a $w_{1+\infty}$ algebra (positive helicity gravitons)

  [Guevara, Himwich, Pate, Strominger '21] [Strominger '21][Himwich, Pate, Singh '21]

  natural appearance from twistor space! [Penrose '76][Newman '76][Adamo, Mason, Sharma '21]…

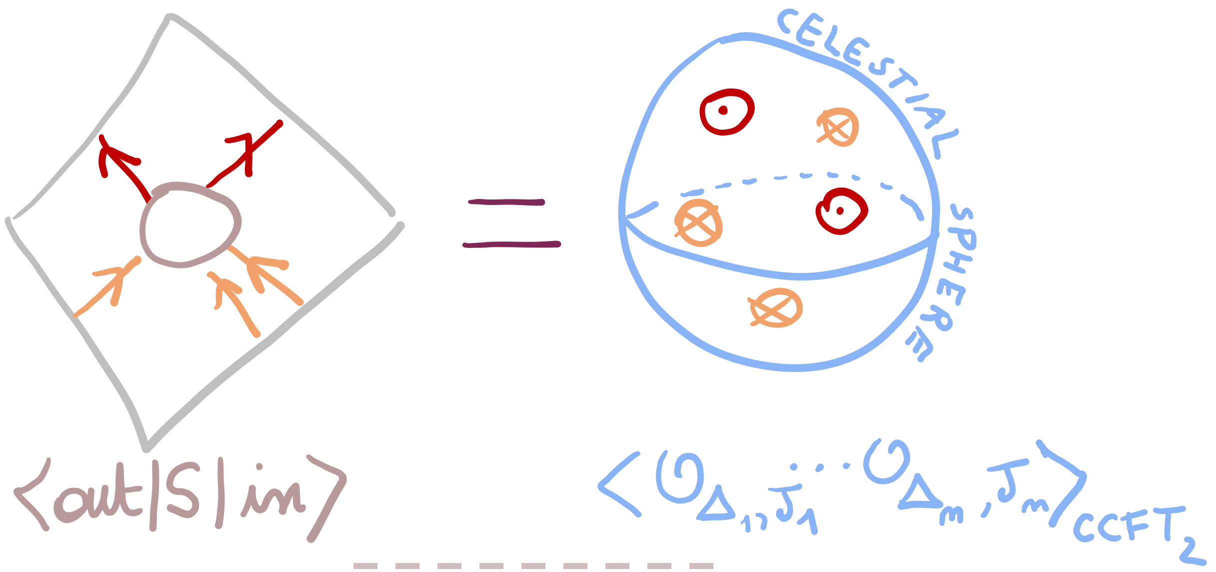
→ Powerful organizing principles for the soft sector of the S-matrix

# Summary: celestial holography

momentum of a massless particle

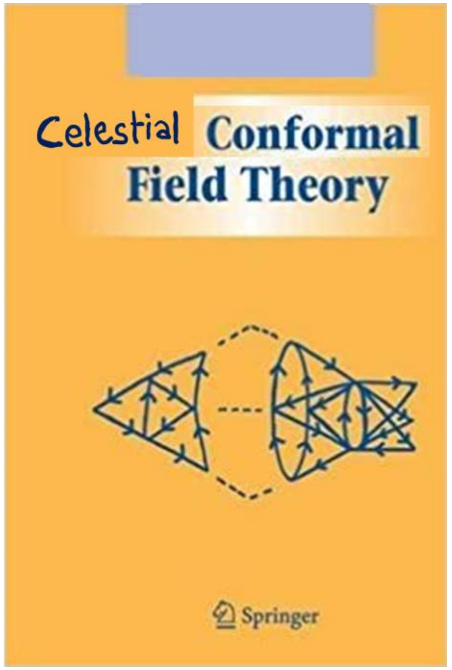
$p^\mu = \omega q^\mu(z,\bar{z})$

$(z,\bar{z})$ : a point on $\mathcal{CS}^2$




$$\int_0^\infty d\omega\, \omega^{\Delta-1}$$

$\Delta = h + \bar{h}$ : conformal dimension




?

The soft sector of scattering is captured by celestial currents $\Delta \to \mathbb{Z}$

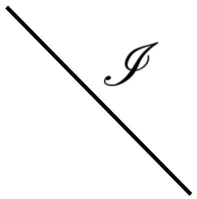
# Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes

two natural boundaries/proposals

null infinity

lighlike 3d hypersurface




4d bulk/3d holography: ‘Carroll holography’

Dual: 3d ‘BMS field theory’

[Arcioni, Dappiaggi ’03 ’04] [Dappiaggi, Moretti, Pinamonti ’06][Mann, Marolf ’06][Adamo, Casali, Skinner ‘14] [Bagchi, Basu, Kakkar, Melhra ’16] [Bagchi, Melhra, Nandi ’20] [LD, Fiorucci, Herfray, Ruzziconi ’22][Bagchi, Banerjee, Basu, Dutta ‘22][…]

Features: closer to AdS/CFT ☺

treatment of fluxes ☹

celestial sphere

Euclidean 2-sphere


4d bulk/2d holography: ‘celestial holography’

Dual: 2d ‘celestial CFT’

[de Boer, Solodukhin ’03][Pasterski, Shao, Strominger ’17] [Pasterski, Shao ’17] [Cheung, de la Fuente, Sundrum’17][…]

Features: powerful CFT techniques at hand ☺

role of translations obscured ☹

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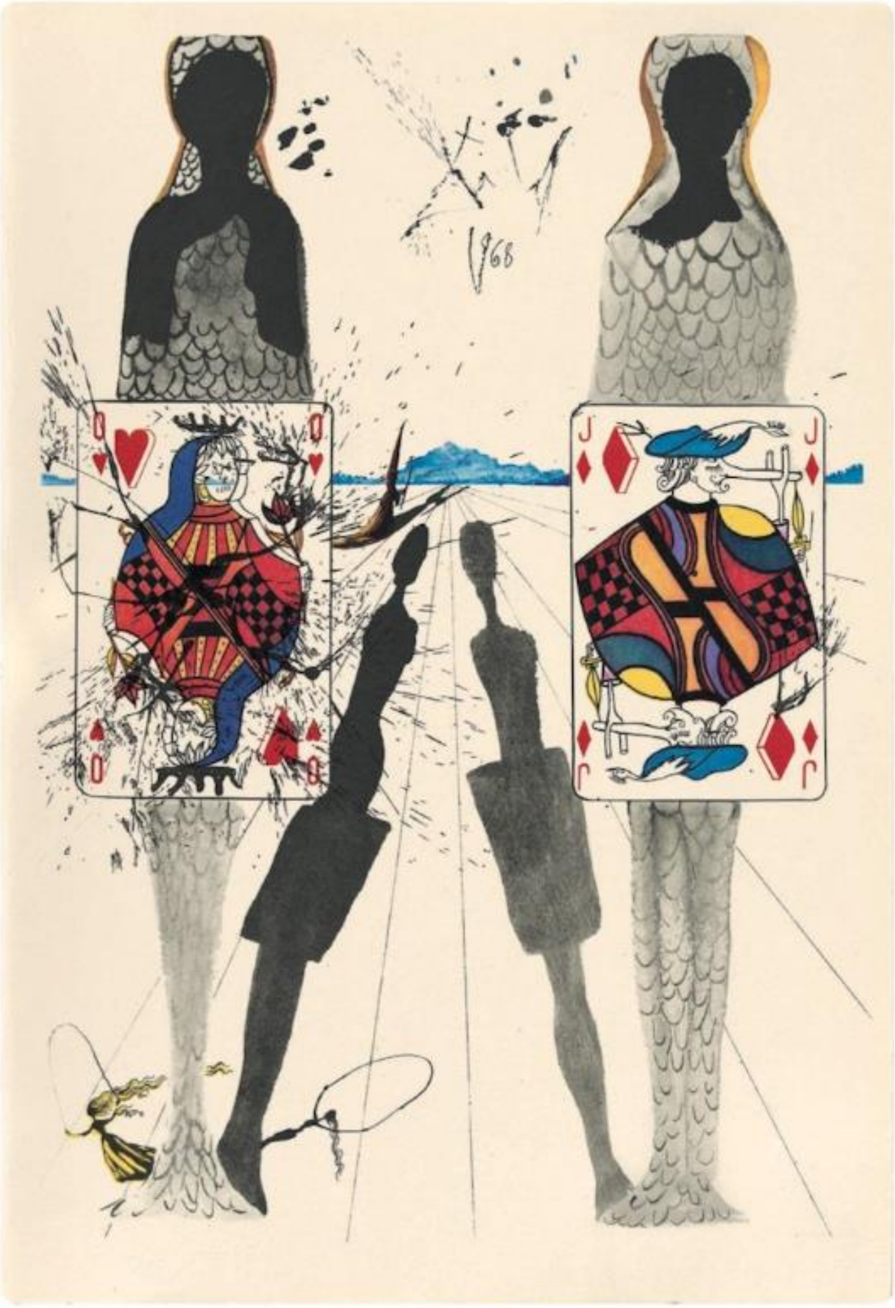

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2. Carrollian holography
3. CCFT vs CCFT

based on 2202.04702 PRL (2022) & 2212.12553
w/ Adrien **FIORUCCI,** Yannick **HERFRAY**
& Romain **RUZZICONI**

# Carrollian physics

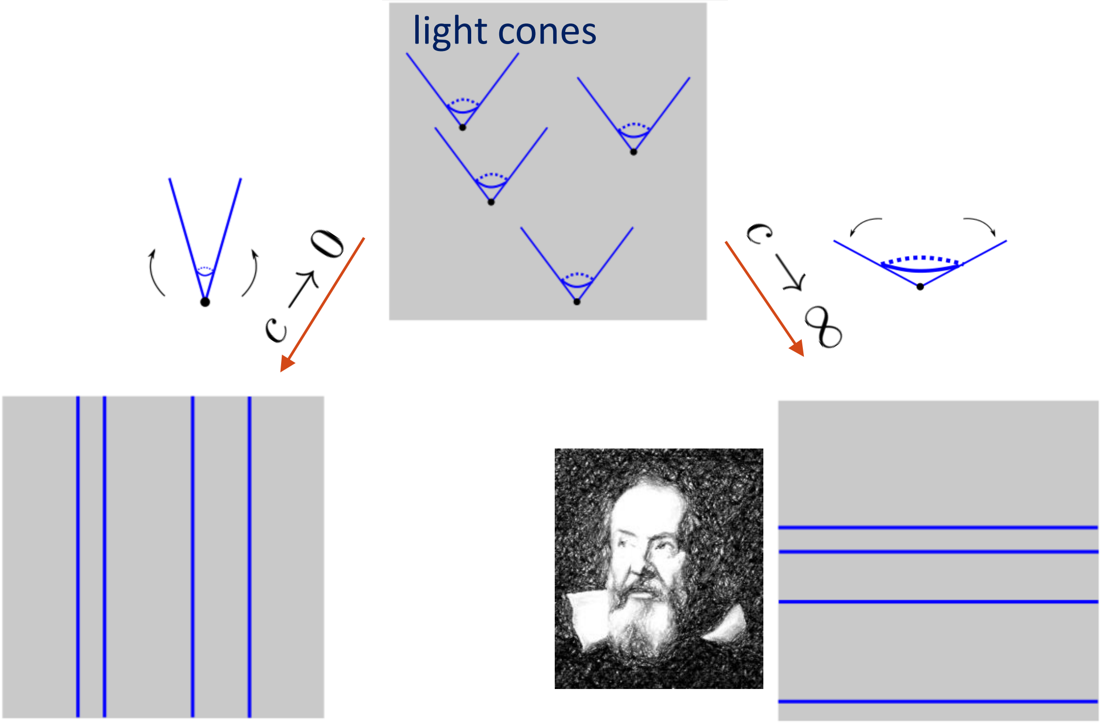
- 1965: A curiosity of Lévy-Leblond (also independently by Sen Gupta 1966)

  The $c \to \infty$ limit of the Poincaré group leads to the Galilean group.

  *But what if we take the* $c \to 0$ *limit instead?*

  → ‘**Carroll** group’

“Alice’s Adventures in Wonderland”
Lewis Carroll (1865)




Carrollian spacetime
(space is absolute)

Galilean spacetime
(time is absolute)

# Carrollian physics

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  → '**Carroll** group'


- Weird features… but (lately) found to be relevant for

  - Hamiltonian analysis of GR [Henneaux '79]

  - fluid/gravity correspondence
    [Ciambelli, Marteau, Petkou, Petropoulos, Siampos '18]
    [de Boer, Hartong, Obers, Sybesma, Vandoren '22]

  - black hole near-horizon physics [Penna'18][LD, Marteau '18]

  - cosmology [de Boer, Hartong, Obers, Sybesma, Vandoren '22]

  - …flat space holography

# BMS = conformal Carrollian symmetries

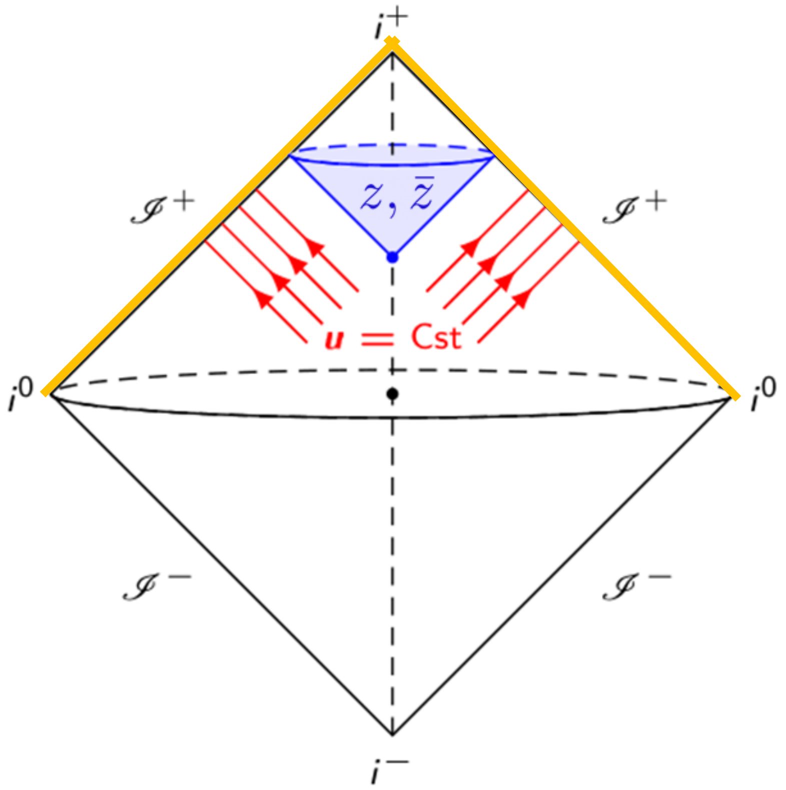
- BMS symmetries = **conformal** symmetries of a **Carrollian** structure at null infinity

  [Geroch][Penrose][Henneaux][Duval, Gibbons, Horvathy][Hartong][Ciambelli, Leigh, Marteau, Petropoulos][Bekaert, Morand][Herfray]...

$$x^a = (u, z, \bar{z})$$

$q_{ab}$ : a degenerate metric $\longrightarrow$ $q_{ab}dx^a dx^b = 0 \times du^2 + 2\gamma_{z\bar{z}}dz d\bar{z}$

a vector field satisfying $q_{ab}n^b = 0 \rightarrow n = \partial_u$



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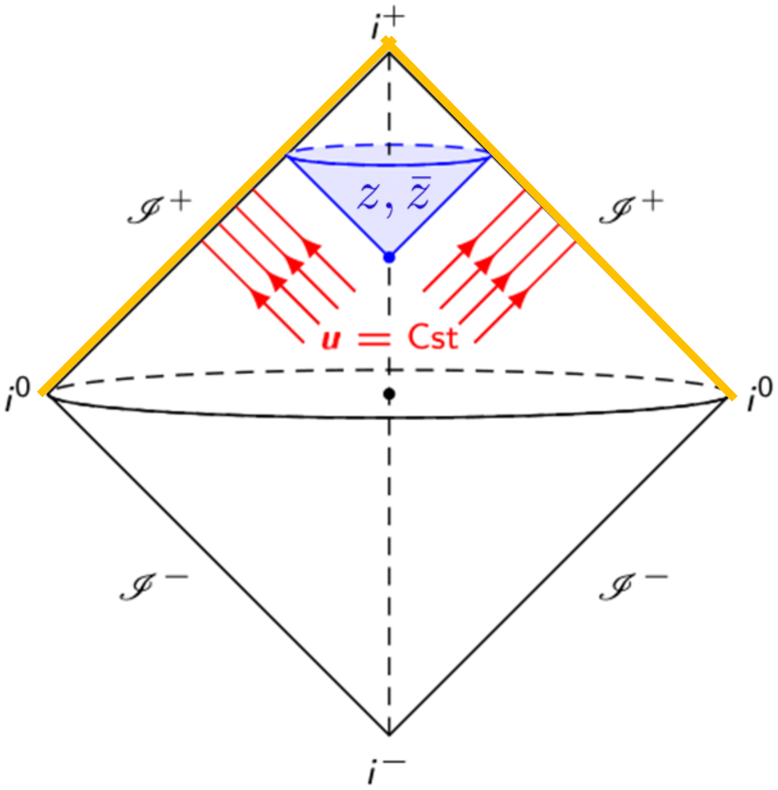



Conformal Carrollian symmetries:

$$\mathcal{L}_{\bar{\xi}} q_{ab} = 2\alpha q_{ab} \qquad \mathcal{L}_{\bar{\xi}} n^a = -\alpha n^a$$

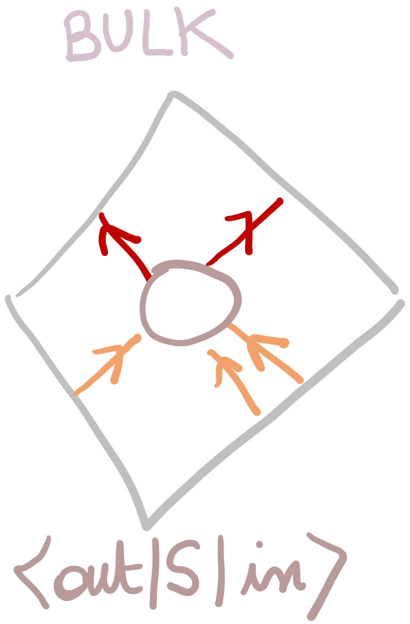
$$\alpha := \frac{1}{2}(D\mathcal{Y} + \bar{D}\bar{\mathcal{Y}})$$

$$\bar{\xi} = \left[\mathcal{T} + \frac{u}{2}(D\mathcal{Y} + \bar{D}\bar{\mathcal{Y}})\right]\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial}$$

# Towards Carrollian holography…

The S-matrix has an intrinsic holographic flavor.

Can we interpret S-matrix elements as correlation functions of a ‘**conformal Carrollian field theory**’?



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$$\langle \Phi_{k_1,\bar{k}_1}(u_1,z_1,\bar{z}_1)\ldots\Phi_{k_m,\bar{k}_m}(u_m,z_m,\bar{z}_m)\rangle_{\text{carroll FT}} \qquad \langle \text{out}|S|\text{in}\rangle$$

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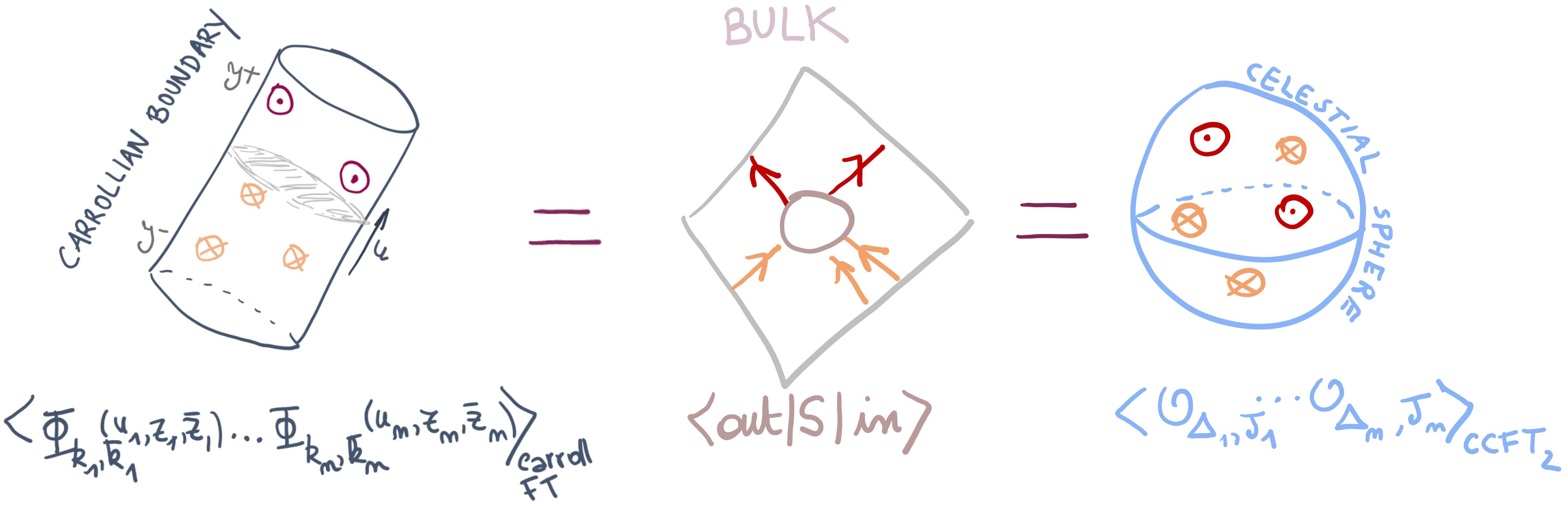


Can it give new insights for celestial CFT?

# From bulk to boundary operators (and back)

From **bulk** to **boundary** (large r expansion):

$$\Phi(X) = \int \frac{d^3p}{(2\pi)^3 2p^0} \left[a(p)e^{ip\cdot X} + a(p)^\dagger e^{-ip\cdot X}\right]$$

$$p^\mu = \omega q^\mu(\vec{w})$$

momentum of a massless particle heading towards the celestial sphere

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momentum of a massless particle heading towards the celestial sphere

Go to Bondi coordinates $X^\mu = (u, r, z, \bar{z})$ and make a large r expansion (using the stationary phase space approximation)

scalar: $$\Phi \sim \frac{1}{r}\int_0^{+\infty} d\omega \left[a(\omega, z, \bar{z})e^{-i\omega u} - a(\omega, z, \bar{z})^\dagger e^{+i\omega u}\right]$$

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spin s: $$\Phi^{(s)}_{z\ldots z}(X) \sim r^{s-1}\int_0^{+\infty} d\omega \left[a^{(s)}_+(\omega,z,\bar{z})e^{-i\omega u} - a^{(s)}_-(\omega,z,\bar{z})^\dagger e^{+i\omega u}\right]$$

(photon) $A_z \sim A^{(0)}_z(u,z,\bar{z})$

(graviton) $h_{zz} \sim rC_{zz}(u,z,\bar{z})$

# From bulk to boundary operators (and back)

From **bulk** to **boundary** (large r expansion):

spin s: $$\Phi^{(s)}_{z\dots z}(X) \sim r^{s-1} \int_0^{+\infty} d\omega \left[ a_+^{(s)}(\omega, z, \bar{z}) e^{-i\omega u} - a_-^{(s)}(\omega, z, \bar{z})^\dagger e^{+i\omega u} \right]$$

$$|||$$

$$\bar{\Phi}_{z\dots z}(u, z, \bar{z})$$

This is the boundary operator: it encodes the asymptotic behavior at null infinity. Later we will identify it with a 'Carrollian primary'.

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$$\equiv$$

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Using the usual commutation relations $[a_\alpha^{(s)}(\vec{p}), a_{\alpha'}^{(s)}(\vec{p}')^\dagger] = (2\pi)^3 2p^0 \delta^{(3)}(\vec{p} - \vec{p}')\delta_{\alpha,\alpha'}$ , one gets

$$[\bar{\Phi}_{z\dots z}(u, z, \bar{z}), \bar{\Phi}_{\bar{z}\dots\bar{z}}(u', z', \bar{z}')] = \text{sign}(u - u')\delta^{(2)}(z - z')$$

**Ex:** gravitational shear obeys the canonical relations $[C_{zz}(u, z, \bar{z}), C_{\bar{z}\bar{z}}(u', z', \bar{z}')] = \text{sign}(u - u')\delta^{(2)}(z - z')$

[Ashtekar '87]

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From **boundary** to **bulk**:

$$\Phi^{(s)}_I(X) = \int_0^{+\infty} d\omega d^2 z\ \left[\epsilon^{*\alpha}_I\ a^{(s)}_\alpha(\omega,z,\bar{z}) e^{ip\cdot X} + h.c.\right]$$

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$$a^{(s)}_+(\omega,z,\bar{z}) = \int_{-\infty}^{+\infty} d\tilde{u}\, e^{i\omega\tilde{u}} \bar{\Phi}_{z\ldots z}(\tilde{u},z,\bar{z})$$

$$\Phi^{(s)}_I(X) = \int d^2 z\, \epsilon_I^{*+}\, \partial_{\tilde{u}} \bar{\Phi}_{z\ldots z}(\tilde{u} = -q\cdot X, z, \bar{z}) + h.c.$$

Kirchhoff-d'Adhémar formula

[Penrose '80]

Allows to reconstruct the bulk field from its boundary value at $\mathscr{I}^+$

# Boundary operators as Carrollian primaries

Can we interpret **S-matrix** elements as correlation functions of a '**conformal Carrollian field theory**'?

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  The out/in boundary operators are

$$\bar{\Phi}^{\text{out}(s)}(u,z,\bar{z}) = \int_0^{+\infty} d\omega \left[ a_+^{(s)\text{out}}(\omega,z,\bar{z})e^{-i\omega u} - a_-^{(s)\text{out}}(\omega,z,\bar{z})^\dagger e^{i\omega u} \right]$$

destroys (creates) outgoing spin-s particles with positive (negative) helicity

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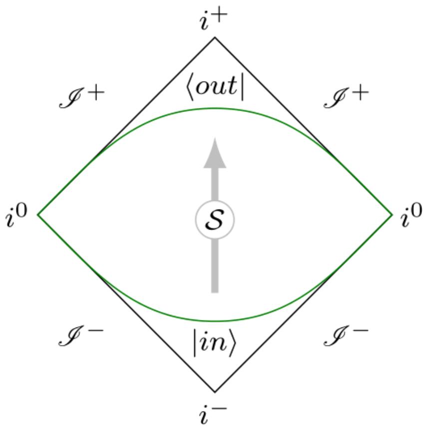

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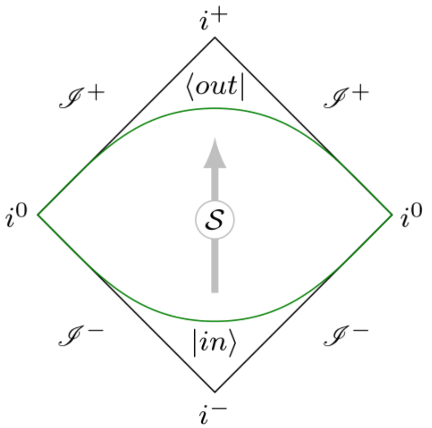


- They transform as 'conformal Carrollian primaries'

$$\delta_\xi \bar{\Phi}^{(s)}(u,z,\bar{z}) = \left[ \left(\mathcal{T} + \tfrac{u}{2}(\partial\mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}})\right)\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + k\,\partial\mathcal{Y} + \bar{k}\,\bar{\partial}\bar{\mathcal{Y}} \right] \bar{\Phi}^{(s)}(u,z,\bar{z})$$

  with weights (for outgoing) $k = \frac{1+J}{2}$ and $\bar{k} = \frac{1-J}{2}$, where $J = \pm s$

  Ex: gravitational shear $C_{zz}(u,z,\bar{z})$ is a (quasi-)Carrollian primary of weights $\left(\frac{3}{2}, -\frac{1}{2}\right)$.

  $J = +2$

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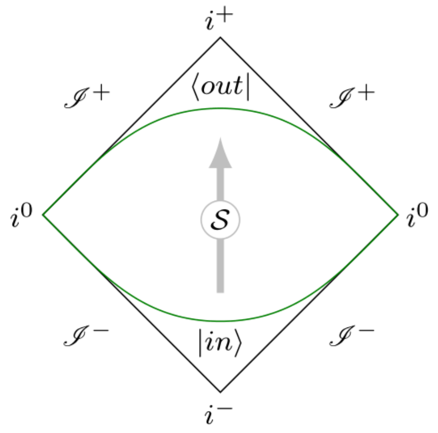



- **Goal**: S-matrix as a correlation function of conformal Carrollian primaries:

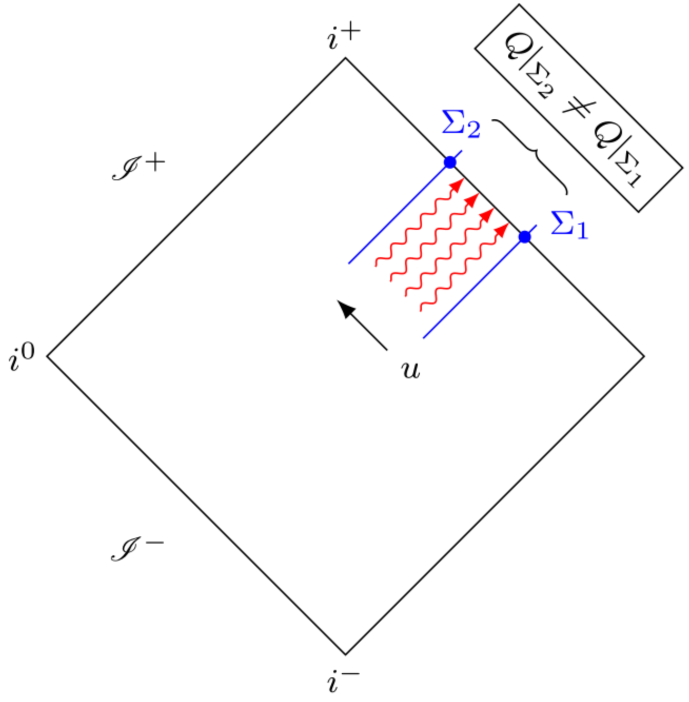
$$\langle 0|\bar{\Phi}^{(s)}(x_1)^{\text{out}}\dots\bar{\Phi}^{(s)}(x_n)^{\text{out}}\bar{\Phi}^{(s)}(x_{n+1})^{\text{in}\,\dagger}\dots\bar{\Phi}^{(s)}(x_N)^{\text{in}\,\dagger}|0\rangle = \mathcal{C}_N(u_i,z_i,\bar{z}_i)$$

**S-matrix in position basis**

# BMS charges and fluxes

- At each cut $\{u = \text{constant}\}$ of $\mathscr{I}^+$, one can construct 'surface charges' associated to BMS symmetries.

Outgoing radiation ⟶ BMS charges are *not* conserved.




$$\int_{-\infty}^{+\infty} du\, \partial_u Q_\xi = F_\xi \neq 0$$

outgoing flux

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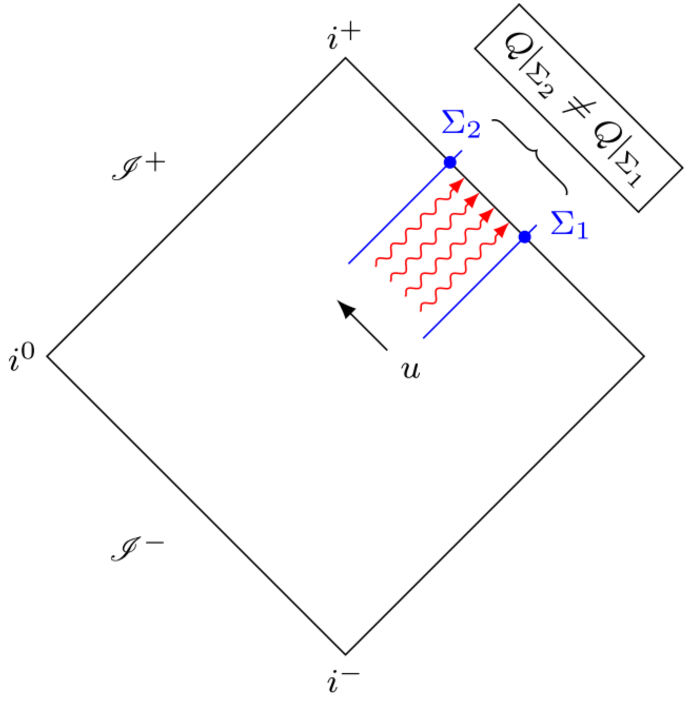
A 'good prescription' for BMS charges has emerged in recent years:

[Barnich, Troessaert '11][He, Lysov, Mitra, Strominger '14][Kapec, Lysov, Pasterski, Strominger '14][Compère, Fiorucci, Ruzziconi '19 '20][Campiglia, Peraza '20]
[LD, Ruzziconi '21][Fiorucci '21][Freidel, Pranzetti, Raclariu '21][LD, Nguyen, Ruzziconi '22]




$$Q_\xi = \frac{1}{8\pi G}\int_S d^2 z\, [2\mathcal{T}\widetilde{M} + \mathcal{Y}\bar{\widetilde{N}} + \bar{\mathcal{Y}}\widetilde{N}]\,,$$

$$\widetilde{M} = M + \frac{1}{8}(C_{zz}N^{zz} + C_{\bar{z}\bar{z}}N^{\bar{z}\bar{z}})$$

$$\widetilde{N} = N_{\bar{z}} - u\bar{\partial}M + \frac{1}{4}C_{\bar{z}\bar{z}}\bar{\partial}C^{\bar{z}\bar{z}} + \frac{3}{16}\bar{\partial}(C_{zz}C^{zz}) + \frac{u}{4}\bar{\partial}\Big[\Big(\partial^2 - \frac{1}{2}N_{zz}\Big)C^z_{\bar{z}} - \Big(\bar{\partial}^2 - \frac{1}{2}N_{\bar{z}\bar{z}}\Big)C^{\bar{z}}_z\Big]$$

$$\int_{-\infty}^{+\infty} du\, \partial_u Q_\xi = F_\xi \neq 0$$

outgoing flux

# Sourced conformal Carrollian Ward identities

- This suggests to consider external sources: Noether currents $j^a_K$ are no longer conserved:

$$\partial_a j^a_K = F_K \neq 0$$

flux term

- Noether currents associated to conformal Carrollian symmetries $\bar\xi = \left[\mathcal{T} + \frac{u}{2}(\partial\mathcal{Y} + \bar\partial\bar{\mathcal{Y}})\right]\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar\partial$

$$j^a_{\bar\xi} = \mathcal{C}^a{}_b \bar\xi^b \qquad \mathcal{C}^a{}_b = \begin{bmatrix} \mathcal{M} & \mathcal{N}_\mathcal{B} \\ \mathcal{B}^A & \mathcal{A}^A{}_B \end{bmatrix}$$

: encodes Carrollian momenta

$x^a = (u, z, \bar z)$

Carrollian stress tensor

[Ciambelli, Marteau, Petkou, Petropoulos, Siampos '18]²
[Ciambelli, Marteau '18][LD, Marteau '19]

- Global conformal Carrollian symmetries (Carrollian rotation, translations, boosts, dilatation, ~~special CT~~) impose the following constraints

$z\partial_z - \bar z\partial_{\bar z}$ $\quad \partial_a$ $\quad z\partial_u, \bar z\partial_u$ $\quad x^a\partial_a$

$$\begin{aligned} \partial_u\mathcal{M} &= F_u\,, & \mathcal{B}^A &= 0\,, \\ \partial_u\mathcal{N}_z - \tfrac{1}{2}\partial\mathcal{M} + \bar\partial\mathcal{A}^{\bar z}{}_z &= F_z\,, & 2\mathcal{A}^z{}_z + \mathcal{M} &= 0\,, \\ \partial_u\mathcal{N}_{\bar z} - \tfrac{1}{2}\bar\partial\mathcal{M} + \partial\mathcal{A}^z{}_{\bar z} &= F_{\bar z}\,, & 2\mathcal{A}^{\bar z}{}_{\bar z} + \mathcal{M} &= 0 \end{aligned}$$

[LD, Fiorucci, Herfray, Ruzziconi '22]

# Sourced conformal Carrollian Ward identities

$X \equiv \Psi^{i_1}(x_1)\dots\Psi^{i_N}(x_N)$

The sourced Ward identities $\partial_a \langle j^a_K(x) X\rangle = \sum_{k=1}^{N} \delta^{(n)}(x-x_k)\,\delta_{K^{i_k}}\langle X\rangle + \langle F_K(x) X\rangle$

of a conformal Carrollian field theory imply

$j^a_{\xi} = \mathcal{C}^a{}_b \bar{\xi}^b \qquad \mathcal{C}^a{}_b = \begin{bmatrix} \mathcal{M} & \mathcal{N}_B \\ \mathcal{B}^A & \mathcal{A}^A{}_B \end{bmatrix}$

$$\partial_u \langle \mathcal{M}\, X\rangle + \sum_i \delta^{(3)}(x-x_i)\partial_{u_i}\langle X\rangle = \langle F_u\, X\rangle$$

$$\partial_u \langle \mathcal{N}_z\, X\rangle - \frac{1}{2}\partial\langle \mathcal{M}\, X\rangle + \bar\partial\langle \mathcal{A}^{\bar z}{}_z\, X\rangle + \sum_i \left[\delta^{(3)}(x-x_i)\partial_i\langle X\rangle - \partial\left(\delta^{(3)}(x-x_i)\, k_i\,\langle X\rangle\right)\right] = \langle F_z\, X\rangle$$

$$\partial_u \langle \mathcal{N}_{\bar z}\, X\rangle - \frac{1}{2}\bar\partial\langle \mathcal{M}\, X\rangle + \partial\langle \mathcal{A}^{z}{}_{\bar z}\, X\rangle + \sum_i \left[\delta^{(3)}(x-x_i)\bar\partial_i\langle X\rangle - \bar\partial\left(\delta^{(3)}(x-x_i)\, \bar k_i\,\langle X\rangle\right)\right] = \langle F_{\bar z}\, X\rangle$$

$$\langle \mathcal{B}^A\, X\rangle = 0$$

$$\langle (\mathcal{A}^z{}_z + \frac{1}{2}\mathcal{M})X\rangle + \sum_i \delta^{(3)}(x-x_i)\, k_i\,\langle X\rangle = 0\,,$$

$$\langle (\mathcal{A}^{\bar z}{}_{\bar z} + \frac{1}{2}\mathcal{M})X\rangle + \sum_i \delta^{(3)}(x-x_i)\, \bar k_i\,\langle X\rangle = 0$$

[LD, Fiorucci, Herfray, Ruzziconi ’22]

# Duality Carrollian momenta/gravitational data

We propose

$$\langle \mathcal{M} \rangle = \frac{\widetilde{M}}{4\pi G}\,,$$
$$\langle \mathcal{N}_A \rangle = \frac{1}{8\pi G}\left(\widetilde{N}_A + u\partial_A \widetilde{M}\right)\,,$$
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[LD, Fiorucci, Herfray, Ruzziconi ’22]

$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}\,dzd\bar{z}$$
$$+\frac{2M}{r}du^2 + rC_{zz}dz^2 + D^z C_{zz}dudz$$
$$+\frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)dudz + c.c. + \cdots$$

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$$\begin{aligned} ds^2 &= -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\,\mathrm{d}z\mathrm{d}\bar{z} \\ &\quad + \frac{2M}{r}\mathrm{d}u^2 + rC_{zz}\mathrm{d}z^2 + D^z C_{zz}\mathrm{d}u\mathrm{d}z \\ &\quad + \frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)\mathrm{d}u\mathrm{d}z + c.c. + \cdots \end{aligned}$$

*cf.* AdS/CFT where the holographic stress-energy tensor is identified with some subleading order in the bulk metric expansion [Balasubramanian, Kraus '99] [Haro, Solodukhin, Skenderis '01]

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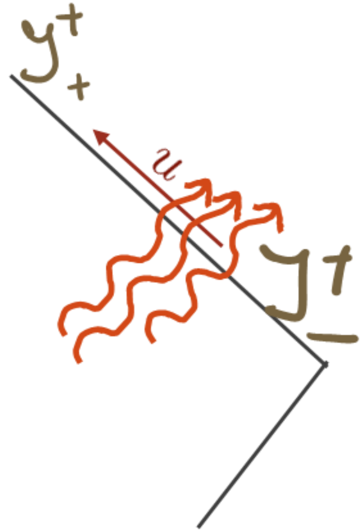
The external sources at the boundary are identified with the asymptotic shear

$$\sigma_{AB} = C_{AB}$$




Fluxes:

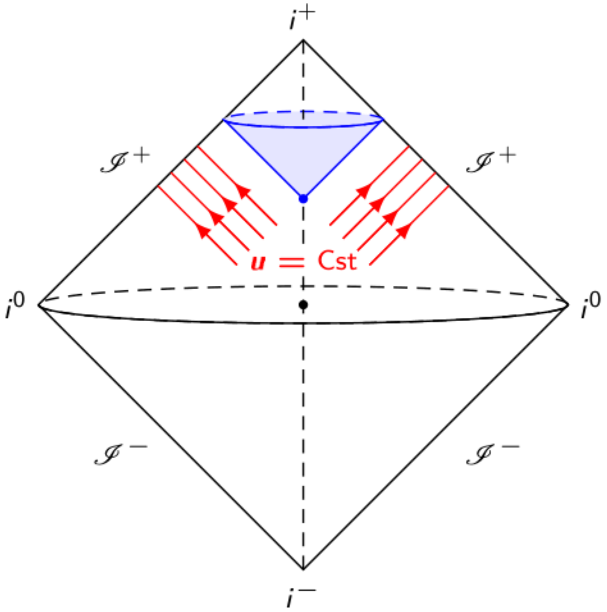
$$F_u = \frac{1}{16\pi G}\left[\partial_z^2\partial_u\sigma_{\bar{z}\bar{z}} + \frac{1}{2}\sigma_{\bar{z}\bar{z}}\partial_u^2\sigma_{zz} + \text{c.c.}\right],$$

$$F_z = \frac{1}{16\pi G}\left[-u\partial_z^3\partial_u\sigma_{\bar{z}\bar{z}} + \sigma_{zz}\partial_z\partial_u\sigma_{\bar{z}\bar{z}} - \frac{u}{2}(\partial_z\sigma_{zz}\partial_u^2\sigma_{\bar{z}\bar{z}} + \sigma_{zz}\partial_z\partial_u^2\sigma_{\bar{z}\bar{z}})\right]$$

Consistently, these expressions plugged into the sourced Ward id. of the conformal Carrollian theory reproduce the flux-balance laws (e.g. Bondi mass loss).

# Constraints for a holographic conformal Carrollian theory

## Gluing the future and the past

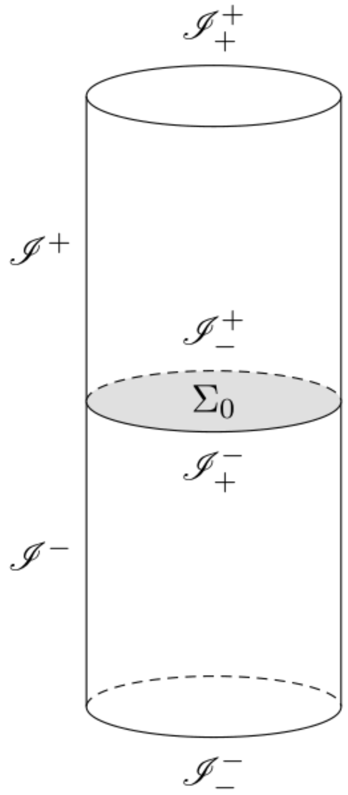



- We want to treat the conformal boundary as a whole by gluing the two pieces around spatial infinity.

$$\hat{\mathscr{I}} \equiv \mathscr{I}^- \sqcup \mathscr{I}^+$$

Separating surface
= locus where the Carrollian vector $n^a$ vanishes

- We get only one smooth automorphism of $\hat{\mathscr{I}}$.
Consistent with antipodal matching of [Strominger '13].



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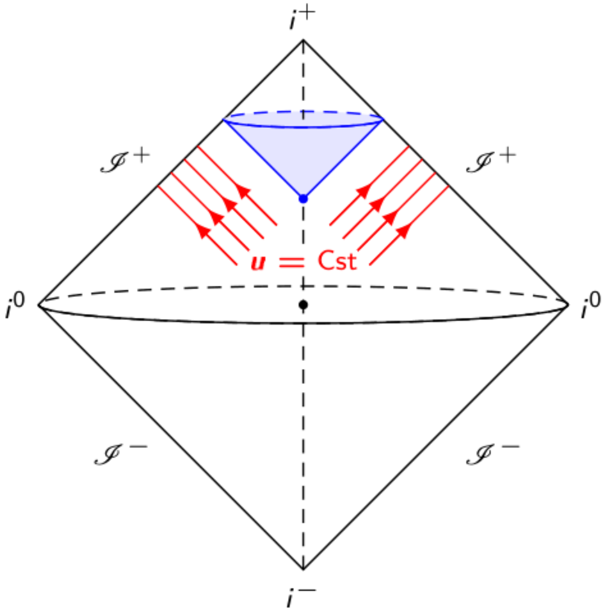


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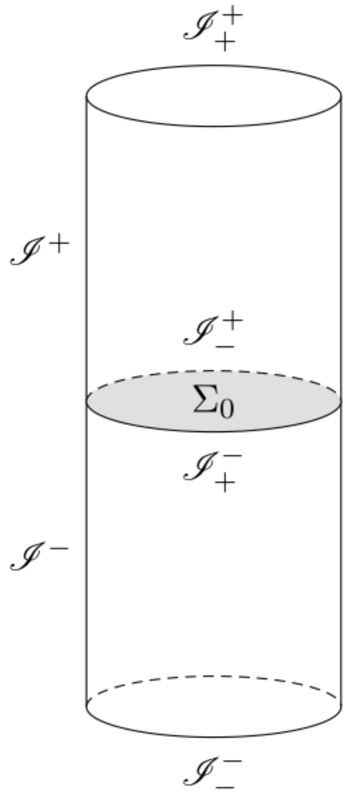


## Ward id. for massless scattering

Assuming that the Noether current vanishes at $\mathscr{I}^-_-$ and $\mathscr{I}^+_+$ :

$$\delta_{\bar{\xi}} \langle X^{\sigma}_N \rangle = 0$$

Invariance of the correlators under conformal Carroll symmetries

# Conformal Carroll invariant low-point correlators

$$\delta_{\bar{\xi}} \langle X_N \rangle = 0$$

$$\langle X_2 \rangle = \langle \Phi_{(k_1,\bar{k}_1)}(u_1, z_1, \bar{z}_1), \Phi_{(k_2,\bar{k}_2)}(u_2, z_2, \bar{z}_2) \rangle$$

[Bagchi, Mandal ‘09][Bagchi, Gary, Zodinmawia ‘17][Chen, Liu, Zheng ‘21][Bagchi, Banerjee, Basu, Dutta ‘22]

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Carrollian translations and boosts $\longrightarrow$ $\langle X_2 \rangle = f(z_{12}, \bar{z}_{12}) + g(u_{12})\delta^{(2)}(z_{12})$

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- Time-**independent** branch

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Carrollian rotation and dilatation → $\langle X_2 \rangle^f = \dfrac{c_1\, \delta_{k_1,k_2} \delta_{\bar{k}_1,\bar{k}_2}}{(z_1 - z_2)^{k_1+k_2} (\bar{z}_1 - \bar{z}_2)^{\bar{k}_1+\bar{k}_2}}$

$z\partial_z - \bar{z}\partial_{\bar{z}}$ $\quad x^a\partial_a$

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  Carrollian rotation and dilatation $\rightarrow$ $\langle X_2 \rangle^f = \dfrac{c_1\, \delta_{k_1,k_2} \delta_{\bar{k}_1,\bar{k}_2}}{(z_1 - z_2)^{k_1+k_2} (\bar{z}_1 - \bar{z}_2)^{\bar{k}_1+\bar{k}_2}}$

  $z\partial_z - \bar{z}\partial_{\bar{z}}$ $\quad x^a\partial_a$

- Time-**dependent** branch

# Conformal Carroll invariant low-point correlators

$$\delta_{\bar\xi} \langle X_N \rangle = 0$$

$$\langle X_2 \rangle = \langle \Phi_{(k_1,\bar k_1)}(u_1, z_1, \bar z_1), \Phi_{(k_2,\bar k_2)}(u_2, z_2, \bar z_2) \rangle$$

[Bagchi, Mandal '09][Bagchi, Gary, Zodinmawia '17][Chen, Liu, Zheng '21][Bagchi, Banerjee, Basu, Dutta '22]

Carrollian translations ($\partial_a$) and boosts ($z\partial_u, \bar z\partial_u$) → $\langle X_2 \rangle = f(z_{12}, \bar z_{12}) + g(u_{12})\delta^{(2)}(z_{12})$

$z_{12} = z_1 - z_2$
$u_{12} = u_1 - u_2$

- Time-**independent** branch

Carrollian rotation ($z\partial_z - \bar z\partial_{\bar z}$) and dilatation ($x^a\partial_a$) →

$$\langle X_2 \rangle^f = \frac{c_1\, \delta_{k_1,k_2}\delta_{\bar k_1,\bar k_2}}{(z_1 - z_2)^{k_1+k_2}(\bar z_1 - \bar z_2)^{\bar k_1+\bar k_2}}$$

- Time-**dependent** branch

Carrollian rotation and dilatation →

$$\langle X_2 \rangle^g = \frac{c_2}{(u_1 - u_2)^{k_{12}^+ - 2}}\delta^{(2)}(z_{12})\delta_{k_{12}^-,0}$$

$$k_{12}^\pm \equiv \sum_{i=1,2}(k_i \pm \bar k_i)$$

$$\langle X_2 \rangle^g_{(n)} = c_3 \frac{d^n}{du_1^n}\text{sign}(u_1 - u_2)\, \delta^{(2)}(z_{12})\, \delta_{k_{12}^-,0}\, \delta_{k_{12}^+,2+n}$$

$n \in \mathbb{N}$

# Conformal Carroll invariant low-point correlators

$$\delta_{\bar{\xi}} \langle X_N \rangle = 0$$

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[Bagchi, Mandal '09][Bagchi, Gary, Zodinmawia '17][Chen, Liu, Zheng '21][Bagchi, Banerjee, Basu, Dutta '22]

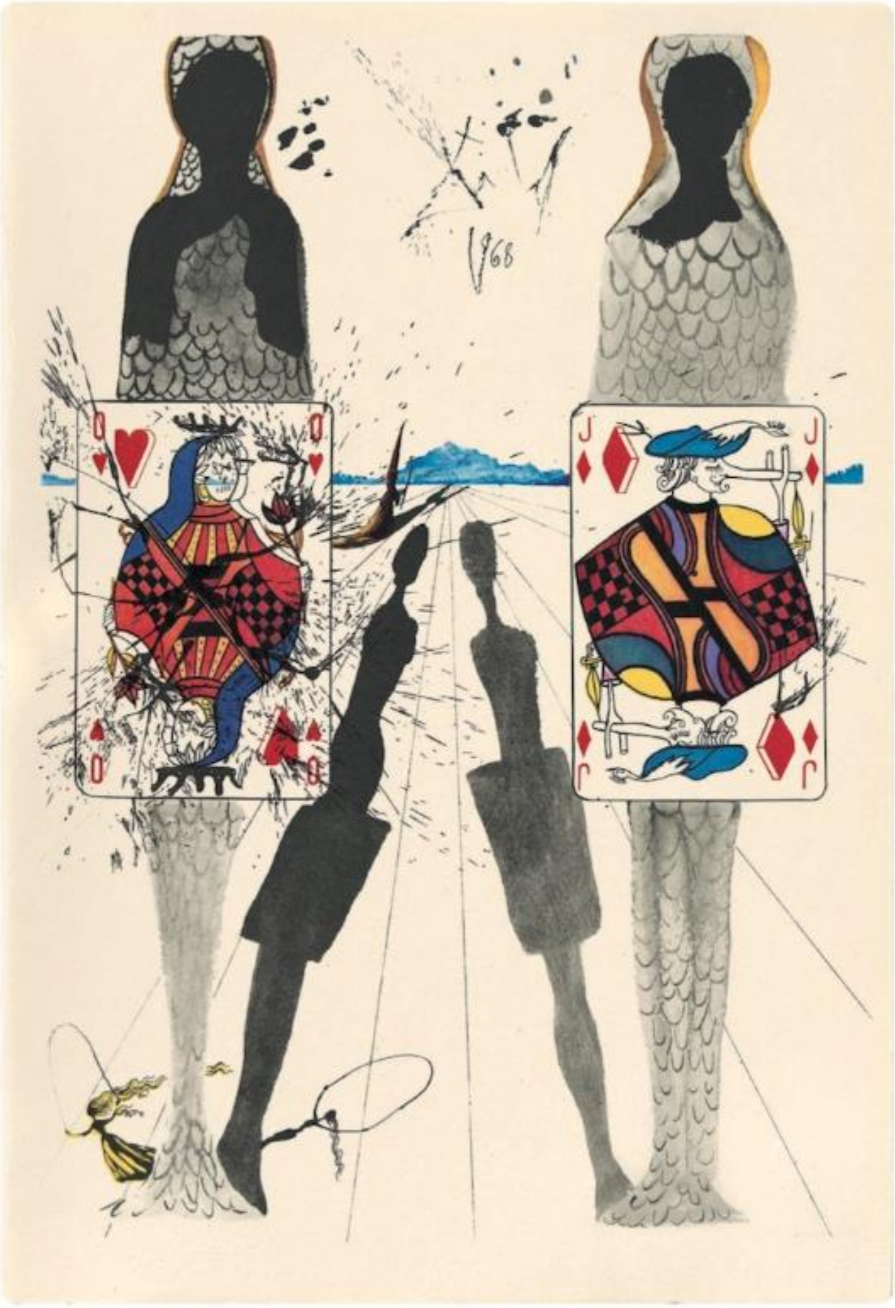
- Conclusion: the time-**dependent** branch gives the Carrollian 2-point function

$$\langle X_2 \rangle = \left[ \frac{1}{\beta} - \left( \gamma + \ln|u-v| + \frac{i\pi}{2} \mathrm{sign}(u-v) \right) \right] \delta^{(2)}(z_1 - z_2) \delta_{k_{12}^+,0} \delta_{k_{12}^-,0}$$

- Three-point correlators were computed as well using the embedding space formalism.

[Salzer '23]

Salvador Dalí, illustrations for *Alice's Adventures in Wonderland*, 1969:



# Outline

1. Celestial holography
2. Carrollian holography
3. CCFT vs CCFT

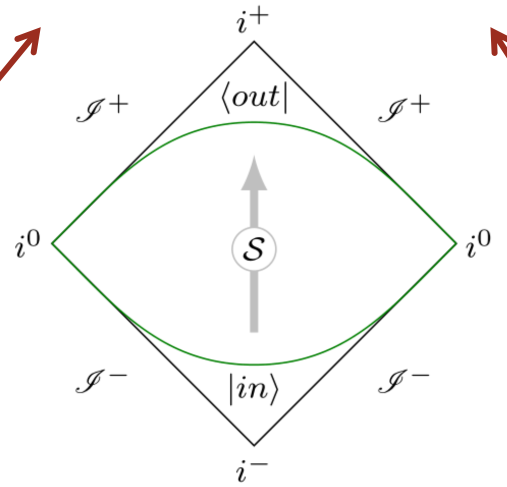
# From Carrollian to celestial

**Momentum basis**

$$\mathcal{A}_N = \langle \text{out}|\mathcal{S}|\text{in}\rangle_{\text{momentum}}$$

Mellin

$$\int_0^{+\infty} d\omega\, \omega^{\Delta-1}$$




Fourier

$$\int_0^{+\infty} d\omega\, e^{-i\omega u}$$

**Celestial basis**

$$\mathcal{M}_N = \langle \text{out}|\mathcal{S}|\text{in}\rangle_{\text{boost}}$$

**Position space basis**

$$\mathcal{C}_N = \langle \text{out}|\mathcal{S}|\text{in}\rangle_{\text{position}}$$

# From Carrollian to celestial

**Momentum basis**

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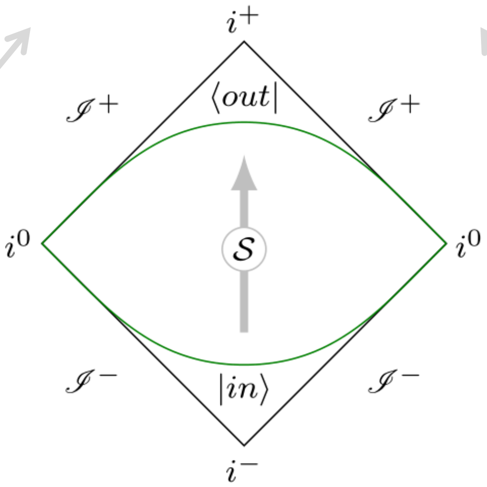


Fourier

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**Celestial basis**

$$\mathcal{M}_N = \langle \text{out}|\mathcal{S}|\text{in}\rangle_{\text{boost}}$$

Mellin + Fourier

$$\int_{-\infty}^{+\infty} du\, u^{-\Delta}$$

**Position space basis**

$$\mathcal{C}_N = \langle \text{out}|\mathcal{S}|\text{in}\rangle_{\text{position}}$$

see also ‘extrapolate dictionary’
[Pasterski, Puhm, Trevisani ’21]

# Relationship with celestial Ward identities

- The map between conformal Carrollian and celestial operators is

[LD, Fiorucci, Herfray, Ruzziconi '22]

$$\mathcal{O}^{\text{out}}_{(\Delta_i,J_i)}(z_i,\bar{z}_i) = \lim_{\epsilon\to 0^+}\int_{-\infty}^{+\infty}\frac{du_i}{(u_i+i\epsilon)^{\Delta_i}}\,\sigma^{\text{out}}_{(k_i,\bar{k}_i)}(u_i,z_i,\bar{z}_i),$$

$$\mathcal{O}^{\text{in}}_{(\Delta_j,J_j)}(z_j,\bar{z}_j) = \lim_{\epsilon\to 0^+}\int_{-\infty}^{+\infty}\frac{dv_j}{(v_j-i\epsilon)^{\Delta_j}}\,\sigma^{\text{in}}_{(k_j,\bar{k}_j)}(v_j,z_j,\bar{z}_j)$$

$$k=\frac{1}{2}(1\pm J)\,,\qquad \bar{k}=\frac{1}{2}(1\mp J)$$

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$$k=\frac{1}{2}(1\pm J)\,,\qquad \bar k=\frac{1}{2}(1\mp J)$$

- Conformal Carrollian Ward identities can reproduce the ones for celestial CFT:

$$\left\langle P(z,\bar z)\prod_{i=1}^{N}\mathcal{O}_{\Delta_i,J_i}(z_i,\bar z_i)\right\rangle+\sum_{q=1}^{N}\frac{1}{z-z_q}\left\langle\dots\mathcal{O}_{\Delta_q+1,J_q}(z_q,\bar z_q)\dots\right\rangle=0$$

$$\left\langle T(z)\prod_{i=1}^{N}\mathcal{O}_{\Delta_i,J_i}(z_i,\bar z_i)\right\rangle+\sum_{q=1}^{N}\left[\frac{\partial_q}{z-z_q}+\frac{h_q}{(z-z_q)^2}\right]\left\langle\prod_{i=1}^{N}\mathcal{O}_{\Delta_i,J_i}(z_i,\bar z_i)\right\rangle=0$$

leading & subleading soft graviton theorem

[He, Lysov, Mitra, Strominger '15][Kapec, Mitra, Raclariu, Strominger '17]
[LD, Puhm, Strominger '18][Fan, Fotopoulos, Taylor '19]

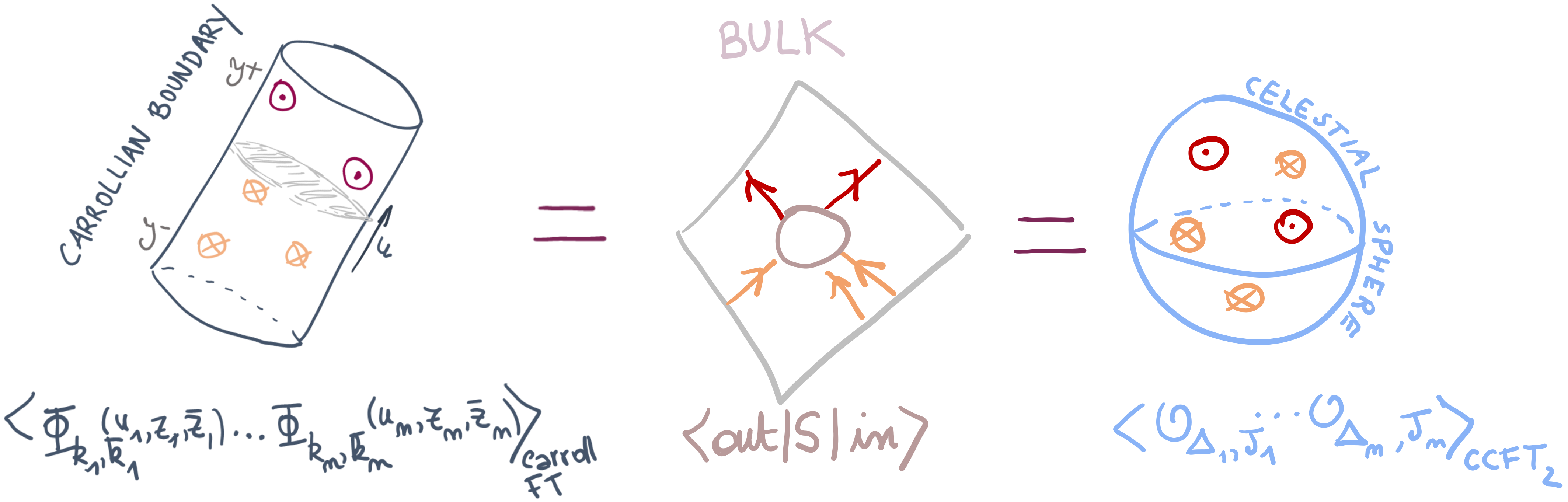
## Summary and outlook

# Summary and outlook

Celestial CFT living on the celestial sphere

↕ ⟷ quantum gravity in flat spacetime

Conformal Carrollian field theory living at null infinity



# Summary and outlook

Celestial **CFT** living on the celestial sphere

↕

Conformal Carrollian field theory living at null infinity

⟷ quantum gravity in flat spacetime

What is a **CCFT**?

→ **B**eyond kinematics? **T**op-down constructions?

# Summary and outlook

Celestial CFT living on the celestial sphere
↕
Conformal Carrollian field theory living at null infinity

⟷ quantum gravity in flat spacetime

What is a CCFT?
→ Beyond kinematics? Top-down constructions?

full tower of currents
link with AdS/CFT, dS/CFT
building representations
log corrections
bootstrapping CCFT
higher dimensions
massive particles
relationship to string theory
adding black holes
…

# Summary and outlook

Celestial **CFT** living on the celestial sphere

↕

**C**onformal **C**arrollian field theory living at null infinity

⟷ quantum gravity in flat spacetime


amplitudes

gravitational waves observation

conformal field theory

twistor theory

asymptotic symmetries

quantum field theory

mathematical GR

fluid/gravity

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**Thank you!**