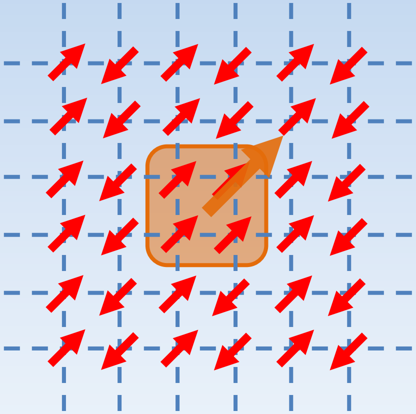
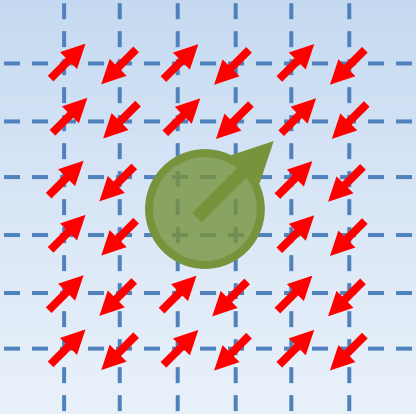
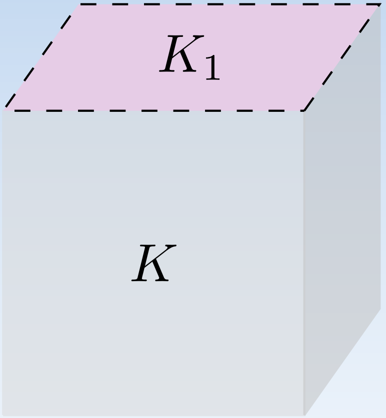
Defects and the renormalisation group







Márk Mezei (Oxford)

Eurostrings 2023
25/04/2023

Outline

Introduction

RG monotonicity

Examples

Conclusions and the future

Thank you

Gabriel Cuomo

Zohar Komargodski

Avia Raviv-Moshe

Ofer Aharony

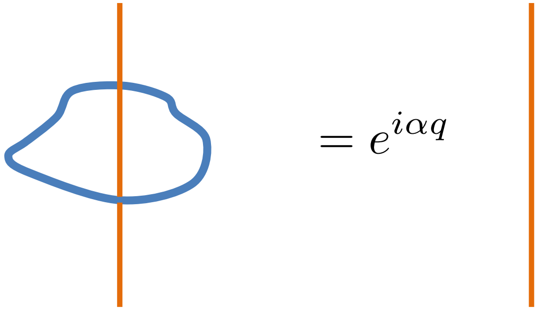
Yifan Wang

Four directions

Currently four major directions in the study of defects

- Topological defects and generalised symmetries: defects as symmetry generators
  [Gaiotto, Kapustin, Seiberg, Willett; Chang, Lin, Shao, Wang, Yin; …]

E.g. 4d Maxwell



$$= e^{i\alpha q}$$

$$\mathrm{WL}_q = e^{iq\int_\gamma A}$$

$$\mathrm{GW}_\alpha = e^{\frac{2i\alpha}{e^2}\int_\Sigma *F}$$

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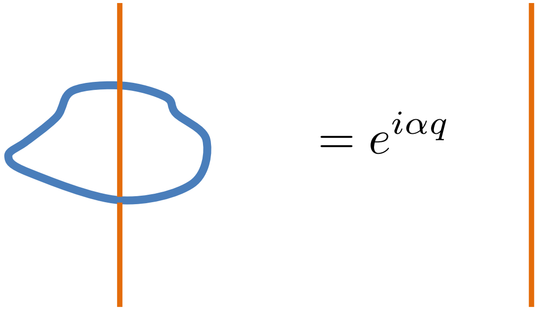

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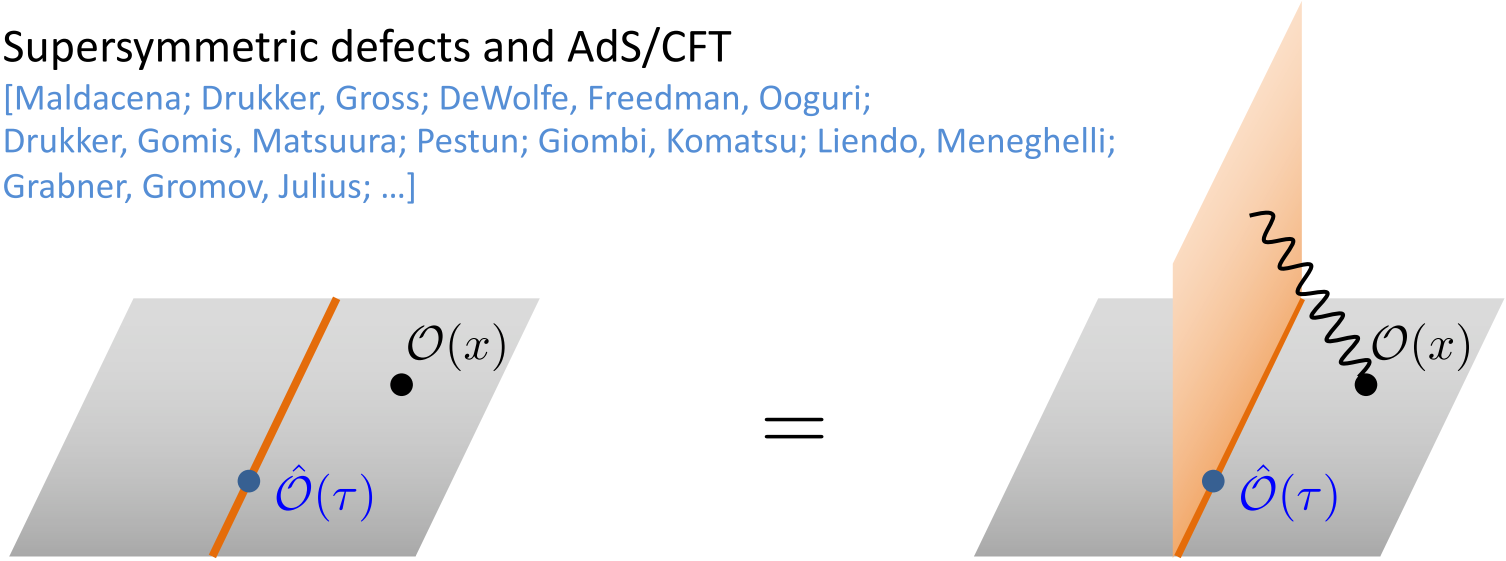
- Supersymmetric defects and AdS/CFT
  [Maldacena; Drukker, Gross; DeWolfe, Freedman, Ooguri;
  Drukker, Gomis, Matsuura; Pestun; Giombi, Komatsu; Liendo, Meneghelli;
  Grabner, Gromov, Julius; …]



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  Boundaries of SPTs often have gapless modes, often survive closing of the bulk gap
  [Scaffidi, Parker, Vasseur; Verresen, Thorngren, Jones, Pollmann; Verresen; …]

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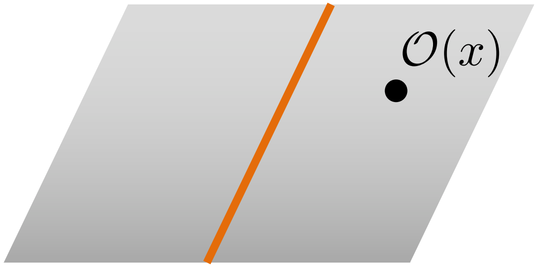
  Can have RG even when bulk is critical



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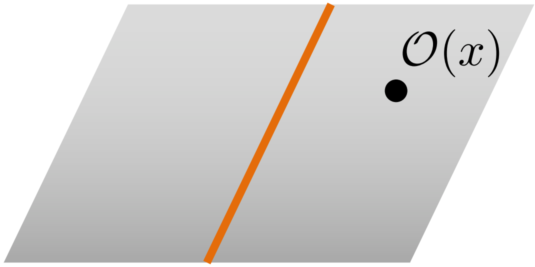


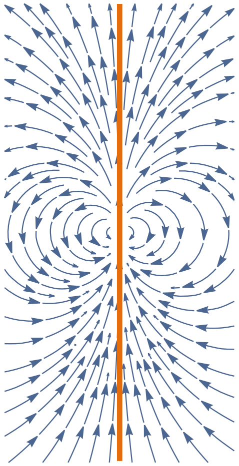
Fixed point: $$\langle \mathcal{O}(x) \rangle = \frac{a}{|x_\perp|^\Delta}$$

RG flow: $$\langle \mathcal{O}(x) \rangle = \frac{a(\mu|x_\perp|)}{|x_\perp|^\Delta}$$

$$a(\mu|x_\perp|) = \begin{cases} a_{\mathrm{UV}} + \dots & |x_\perp| \ll 1/\mu \\ a_{\mathrm{IR}} + \dots & |x_\perp| \gg 1/\mu \end{cases}$$

Physical setup

Symmetries and RG



- **Defect line** preserves $SL(2,\mathbb{R}) \times SO(D-1) \subset SO(D+1,1)$

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- Fixed points preserve maximal symmetry (also for higher d)

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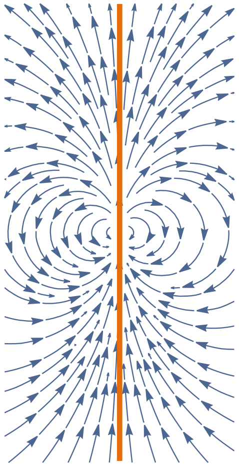

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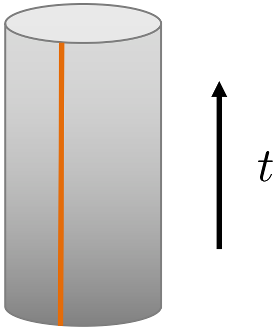

- Impurity or heavy probe: changes Hilbert space

  Applications: order parameter for confinement;
  physical boundaries (SP criticality, …);
  impurities, dislocations, …

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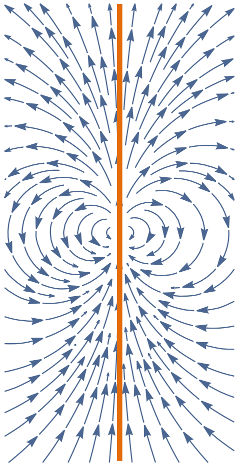

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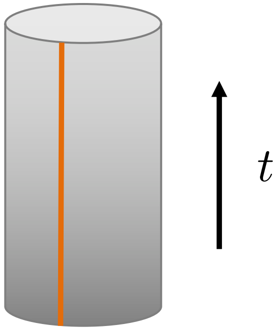
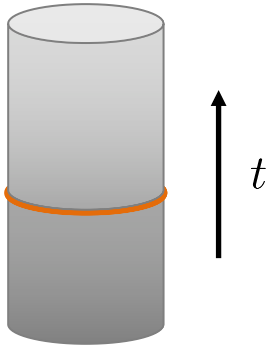


- On time slice: operator acting on Hilbert space

  Applications: area vs perimeter law of WLs;
  charges of generalised symmetries



Methods

- Perturbation theory: defect can be (strongly) interacting even when bulk is free
  [Allais, Sachdev; Cuomo, Komargodski, Mezei; Cuomo, Komargodski, Mezei, Raviv-Moshe; …]
- Large N: vector, melonic, matrix
  [Metlitski; Cuomo, Komargodski, Mezei; Popov, Wang; Krishnan, Metlitski; Drukker, Gross; …]
- Semiclassics at large charge
  [Cuomo, Komargodski, Mezei, Raviv-Moshe; Rodriguez-Gomez; Rodriguez-Gomez, Russo; …]
- Bootstrap
  [Gliozzi, Liendo, Meineri, Rago; Billo, Goncalves, Lauria, Meineri; Lauria, Liendo, van Rees, Zhao; Behan, Di Pietro, Lauria, van Rees; Collier, Mazanc, Wang; Padayasi, Krishnan, Metlitski, Gruzberg, Meineri; Herzog, Shrestha; …]
- Integrability and localization
  [Pestun; Giombi, Komatsu; Liendo, Meneghelli; Grabner, Gromov, Julius; Komatsu, Wang; …]
- RG monotonicity
  - ➢ Dilaton effective action
    [Jensen, O'Bannon; Cuomo, Komargodski, Raviv-Moshe; Wang]
  - ➢ Entanglement entropy inequalities
    [Casini, Testé, Torroba; Casini, Landea, Torroba]
- (Quantum) Monte Carlo and quantum simulation
  [Assaad, Herbut; Allais; Toldin, Assaad, Wessel; Ebadi et al.]
- Combinations of the above

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In odd dimensions RG monotonicity expected from universal piece in partition function
[Jafferis; Jafferis, Klebanov, Pufu, Safdi]

- **Line** defect insertion
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$$b(\mu R) \equiv \left(1 - R\frac{d}{dR}\right) \log\langle D\rangle$$

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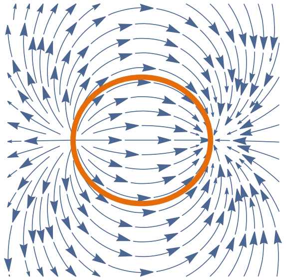
- Key step ↙ spurion

$$S = S_{\mathrm{DCFT}} + \left(\mu\, e^{\Phi(\phi)}\right)^{1-\hat{\Delta}}\int_\gamma \hat{\mathcal{O}}$$

$$\Phi \sim \Phi + \epsilon\left(\dot{\xi}_D + \xi_D\dot{\Phi}\right)$$



RG monotonicity

In even dimensions trace anomaly coefficient is RG monotone
[Zamolodchikov; Cardy; Komargodski, Schwimmer; Jensen, O'Bannon; Wang]

- Defect contribution to trace anomaly

$$\langle T^{\mu}_{\mu} \rangle = \mathcal{A}_{\text{bulk}} + \delta(\Sigma_d) \left[-(-1)^{d/2}\, b_d\, E_d + (\text{other int. terms}) + (\text{ext. terms}) \right]$$

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$$S = S_{\text{IR DCFT}} + S_{\text{dilaton}}[\Phi] + \dots$$

$$S_{\text{dilaton}}[\Phi] \supset \begin{cases} \Delta b_2 \int (\partial \Phi)^2 & d = 2 \\ \Delta b_4 \int (\partial\Phi)^2 \left[2\partial^2 \Phi - (\partial \Phi)^2 \right] & d = 4 \end{cases}$$

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Spherical 2d defects provide another avenue to monotonicity **[Sinha's talk]**

$$b(\mu R) \equiv -R\frac{d}{dR}\left(1 - \frac{1}{2}R\frac{d}{dR}\right)\log\langle D_{S^2}\rangle$$

RG monotonicity from entanglement entropy

Entanglement entropy also provides a count of dofs

- Equivalent to sphere partition function
  [Casini, Huerta, Myers]

- Inequalities indicate proof method: SSA, QNEC
  [Lieb, Ruskai; Bousso et al.; Balakrishnan et al.; Casini, Huerta; Casini, Testé, Torroba]

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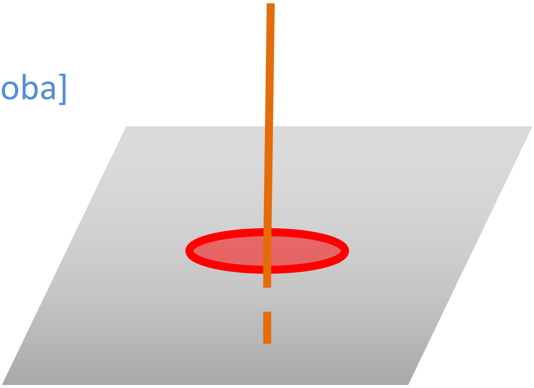
  [Affleck, Ludwig; Lewkowycz, Maldacena]

$$\Delta S = \log\langle D\rangle + \int \langle T_{\tau\tau}\rangle_D$$



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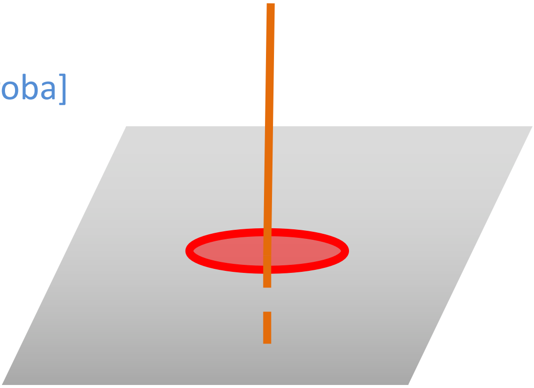

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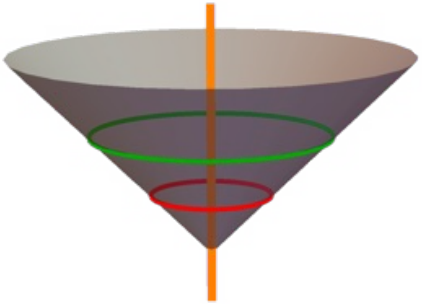
- Improved construction
  [Casini, Landea, Torroba]

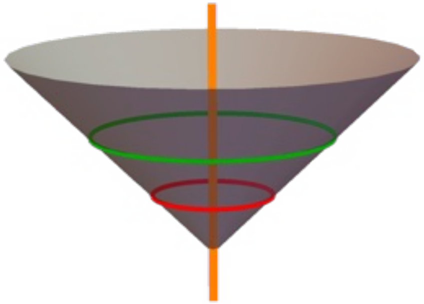


$$S_{\rm rel}\left(\rho_R|\sigma_R\right) \equiv {\rm Tr}_R\left[\left(\log\rho_R - \log\sigma_R\right)\rho_R\right]$$

$$= \Delta\langle H_\sigma\rangle - \Delta S$$

Relative entropy to be evaluated on the light cone, where CFT vacuum has Markov property

RG monotonicity from entanglement entropy



- Construction eliminates $\int \langle T_{\tau\tau} \rangle_D$ term

$$S_{\text{rel}}\left(\rho_R | \sigma_R\right) = \begin{cases} 0 + \dots & R \ll 1/\mu \\ b_{\text{UV}} - b_{\text{IR}} + \dots & R \gg 1/\mu \end{cases}$$

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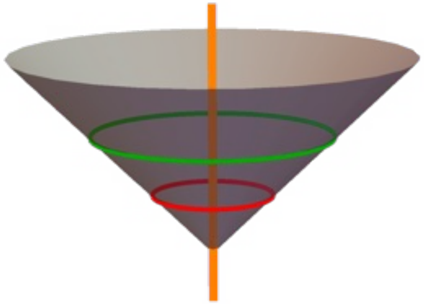

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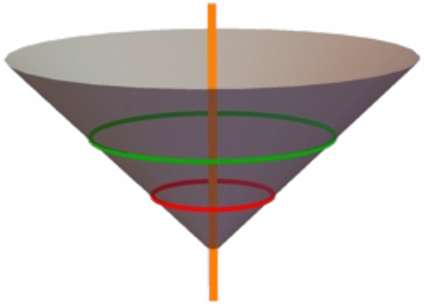

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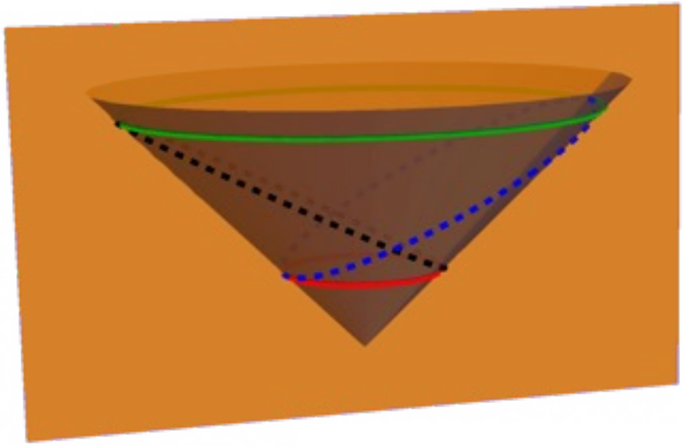
Key equation $R\, S''_{\text{rel}}(R) - (d-3)S'_{\text{rel}}(R) \geq 0$



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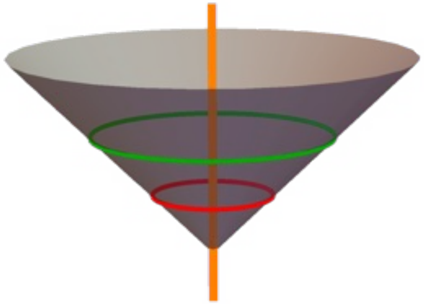

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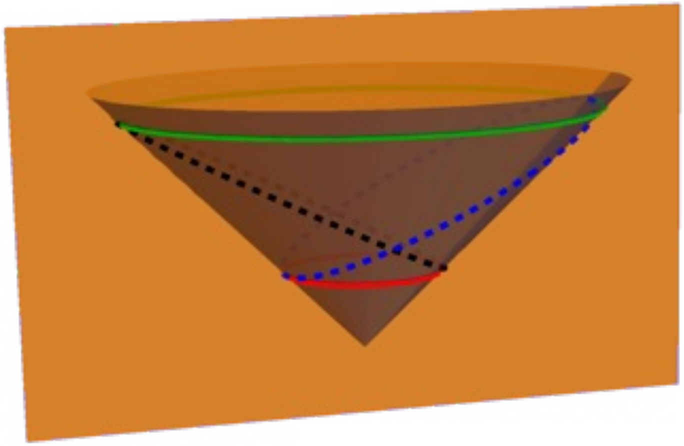

- For d=D this proves c-theorems for bulk RG flows

For d>4 would need more derivatives, but such entropy inequalities are currently not known

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External field defect

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- E.g: $\exp\left(-h\int d\tau\, \phi_1(\tau,0)\right)$ in O(N) Wilson-Fischer CFT

  DCFT saddle point at large N

  [Cuomo, Komargodski, Mezei]

$$s(x) = \frac{1}{4|x_\perp|^2}\,, \qquad b_{\mathrm{IR}} = -0.1537\,N\,, \qquad a^2_{\phi_1} = 0.5581\,N$$

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  DCFT saddle point at large N
  [Cuomo, Komargodski, Mezei]

$$s(x) = \frac{1}{4|x_\perp|^2}\,, \qquad b_{\mathrm{IR}} = -0.1537\,N\,, \qquad a^2_{\phi_1} = 0.5581\,N$$

Marginal tilt operator rotates the external field $\Delta\left(\hat{t}_{\hat{a}}\right) = 1$

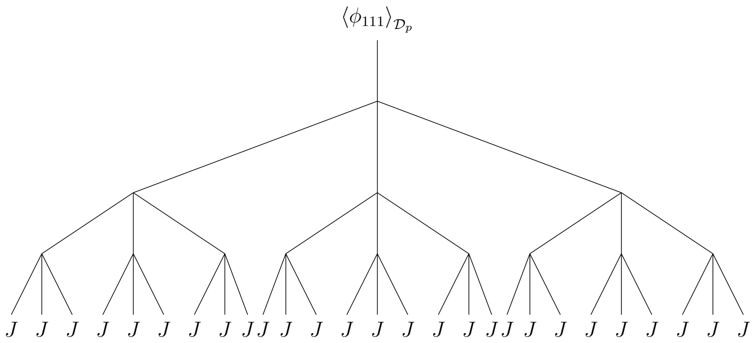
Long DRG flow: $\Delta(\hat{\phi}_1)_{\mathrm{UV}} = 0.5 \quad \rightarrow \quad \Delta(\hat{\phi}_1)_{\mathrm{IR}} = 1.542$

Consistent with ϵ - expansion, 2d Ising interface, Monte Carlo
[Allais, Sachdev; Cuomo, Komargodski, Mezei; Assaad, Herbut; Allais; Toldin, Assaad, Wessel]

External field defects

External field defects in other models

- Closed set of Schwinger-Dyson equations for one-point functions in melonic theories [Popov, Wang]



melonic tree

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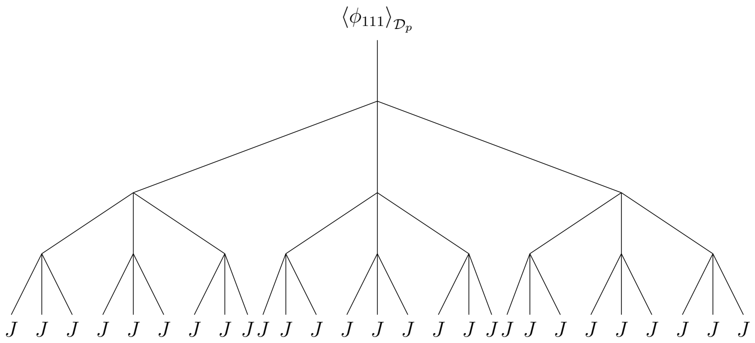


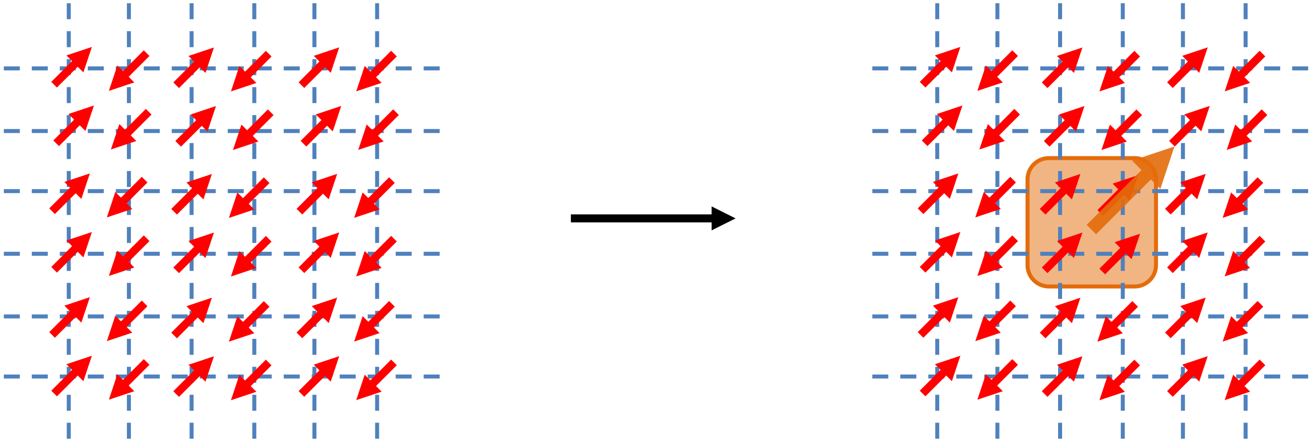
melonic tree

- In Gross-Neveu-Yukawa model $\Delta(\sigma_{\text{HS}}) < 1$, can deform trivial line $\exp\left(-h\int_\gamma \sigma_{\text{HS}}\right)$
  [Giombi, Helfenberger, Khanchandani]

Applications

External field defect

- $\exp\left(-h\int d\tau\,\phi_1(\tau,0)\right)$ models turning on magnetic field for a couple of lattice sites



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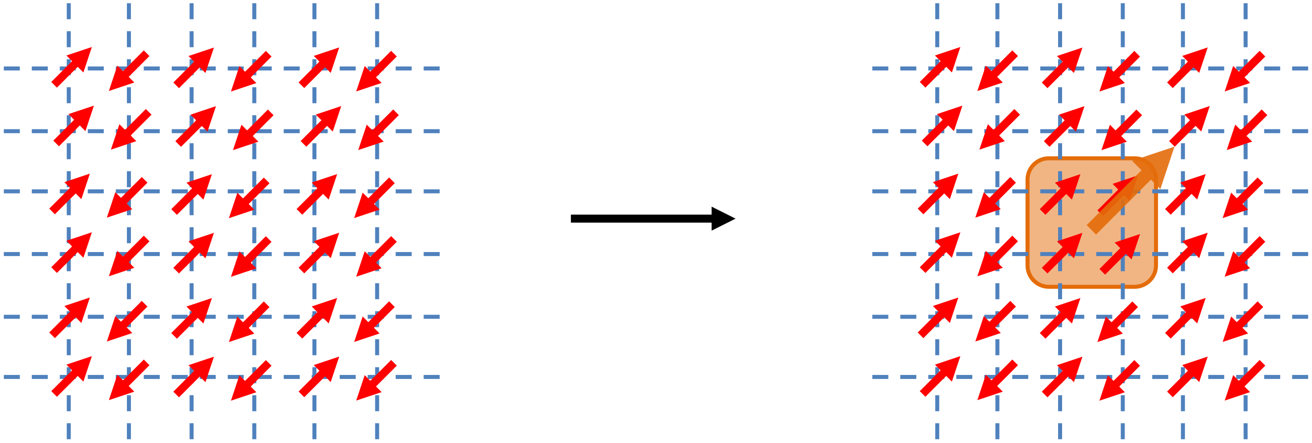
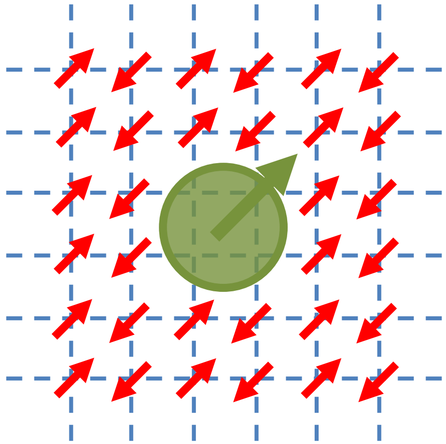


Impurity: new degrees of freedom at the defect

[Kondo; Wilson; Sengupta; Sachdev Buragohain Vojta; …]

$$\boxed{\begin{aligned} S_{\rm DQFT} =& S_{\rm CFT} + S_{\rm QM} \\ & - \gamma\int d\tau\,\phi_a S^a \end{aligned}}$$



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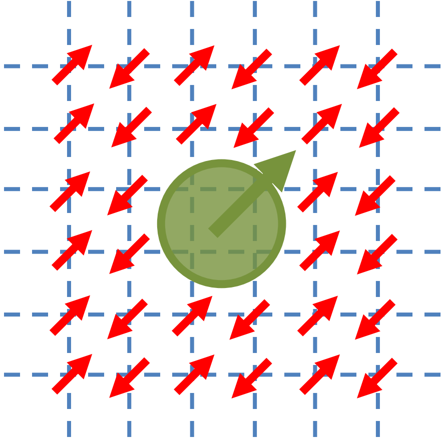

Impurity: new degrees of freedom at the defect

$$S_{\text{DQFT}} = S_{\text{CFT}} + S_{\text{QM}} - \gamma \int d\tau \, \phi_a S^a$$

- Natural setup O(3) CFT coupled to spin-1/2 impurity

- Variants: O(N) model coupled to impurity in spinor rep.
  $\phi_{\alpha\beta} = -\phi_{\beta\alpha},\ (\alpha, \beta = 1, \ldots N)$ sigma model coupled to impurity in O(N) spinor rep.
  … [Liu, Shapurian, Vishwanath, Metlitski]

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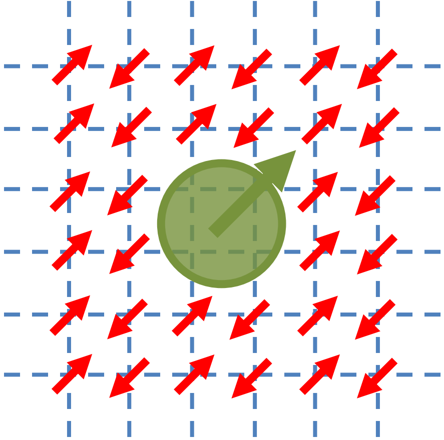

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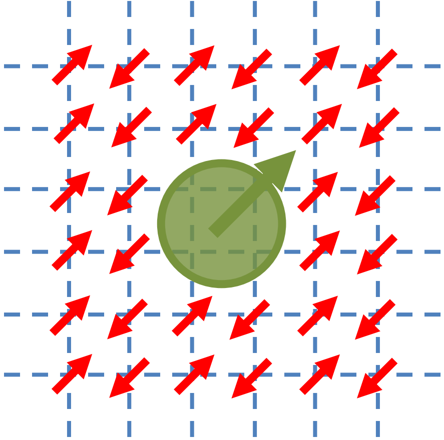
- O(3) model coupled to spin-s impurity
  [Cuomo, Komargodski, Mezei, Raviv-Moshe]

$$S_{\mathrm{QM}} = \int d\tau \, \bar{z}\dot{z} \,, \qquad \bar{z}z = 2s \,, \qquad S^a = \bar{z}\frac{\sigma^a}{2}z \,, \qquad b_{\mathrm{UV}} = 2s + 1$$

Applications



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1/s expansion: impurity spin is slow

$$Z = \int \underset{\uparrow}{d^2\hat{n}} \int D\phi_a D\chi\, \exp\left[-\underset{\uparrow}{S_{\rm ext.\ field}}(\hat{n}) - \int d\tau\, \underset{\uparrow}{\bar{\chi}\dot{\chi}} - \frac{\kappa}{\sqrt{s}} \int d\tau\, \underset{\uparrow}{\text{“}\hat{t}\chi\text{”}} + \dots\right]$$

averaging over ext. field direction

ext. field defect

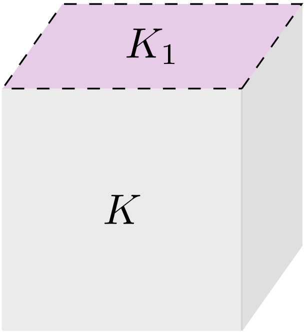
free spin s

interactions fixed by sym.

Boundary CFT

Boundary universality in the 3d O(N) CFT

[Metlitski; Toldin; Gliozzi, Liendo, Meineri, Rago; Padayasi, Krishnan, Metlitski, Gruzberg, Meineri]



$$H = -\sum_{\text{bdy layer},\langle\alpha\beta\rangle} K_1 \vec{S}_\alpha \cdot \vec{S}_\beta - \sum_{\text{bulk},\langle ij\rangle} K \vec{S}_i \cdot \vec{S}_j$$

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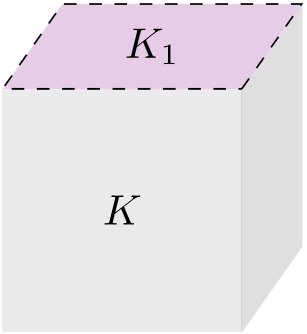

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- Phase diagram for $2 \leq N \leq N_c$

Dirichlet (*ordinary*)

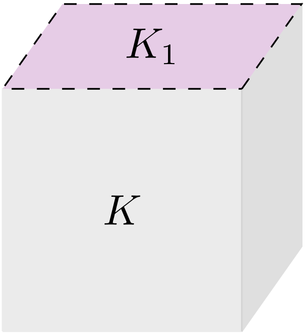
Neumann (*special*)

symmetry breaking disallowed
by CMWH mechanism (*extraordinary log*)

- Extraordinary log described by symmetry breaking BC coupled to boundary sigma model through tilt operator

Wilson lines

Novelty: existence of DCFT/BCFT was a dynamical question in O(N) CFT



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  [Aharony, Cuomo, Komargodski, Mezei, Raviv-Moshe; Shytov, Katsnelson, Levitov]

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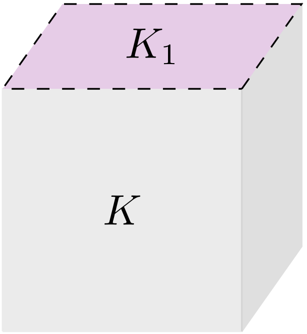

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$$\mathrm{WL} = \exp\left[i \int d\tau \, q A_\tau \right]$$

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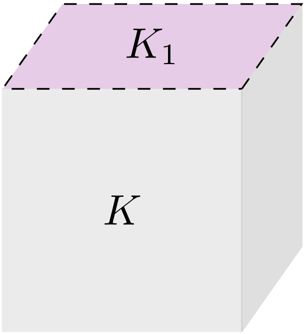

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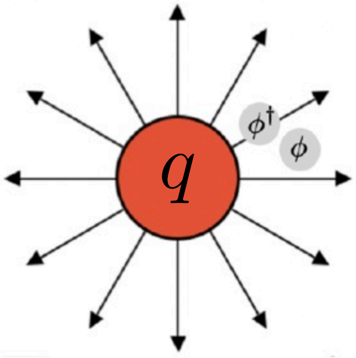
$$e^2 \to 0\,,\quad q\to\infty\,,\quad e^2 q = \text{fixed}$$

Similar double scaling limits
[Badel, Cuomo, Monin, Rattazzi; Rodriguez-Gomez; Rogriduez-Gomez, Russo; …]



$$A_0 = \frac{e^2 q}{4\pi r}$$

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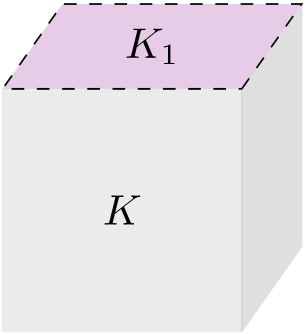

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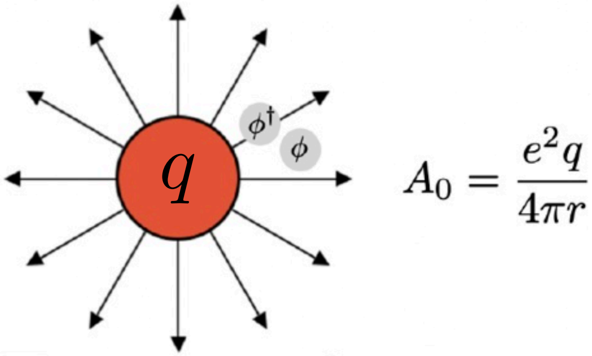

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- Large anomalous dimension for the defect operator

$$\Delta\left(\widehat{\bar\psi\psi}\right) = 1 + 2\sqrt{1-\frac{e^4q^2}{16\pi^2}}$$

$$= 3 - \frac{e^4q^2}{16\pi^2} - \frac{e^8q^4}{1024\pi^4} + \dots$$

Wilson lines

- Naïve WL action is fine tuned

$$\Delta\left(\widehat{\bar{\psi}\psi}\right) = 2 \pm \sqrt{1 - \frac{e^4 q^2}{16\pi^2}}$$

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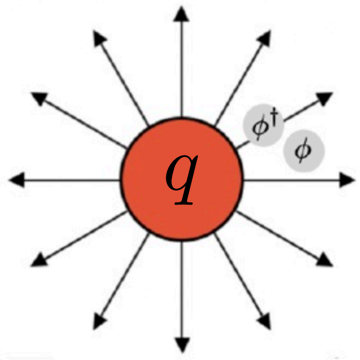

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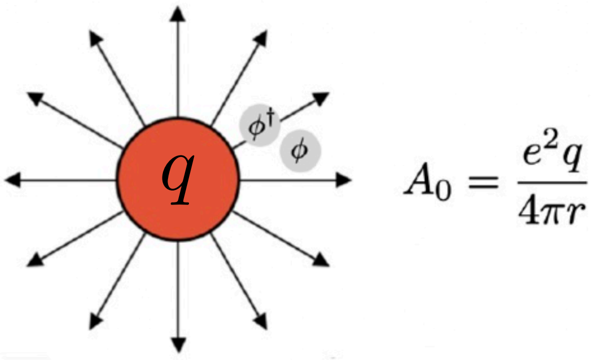
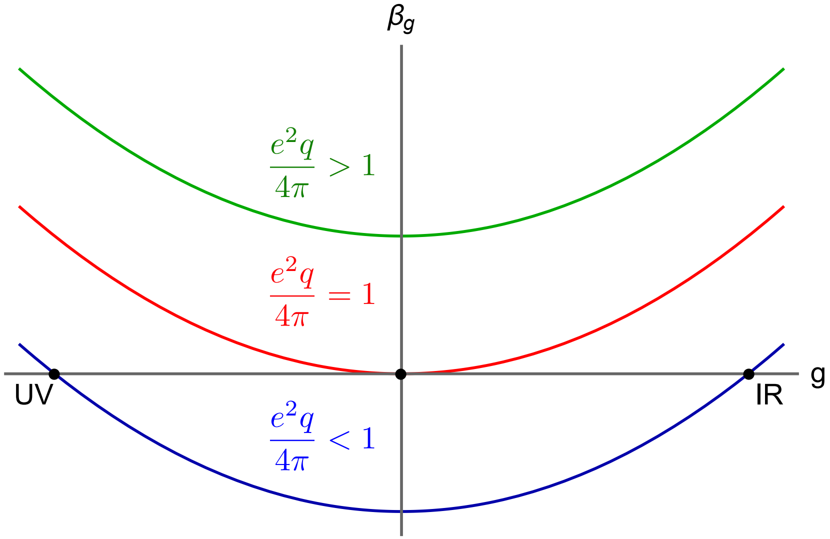


- DRG flow



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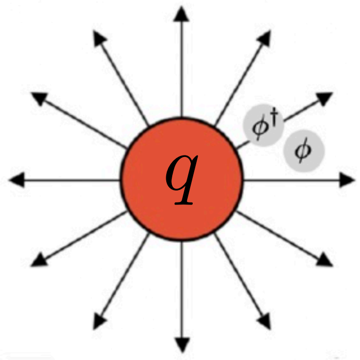

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- New DCFT $\sim$ alternative quantization

- Annihilation of fixed points: screening of WL by pair production of fermions down to

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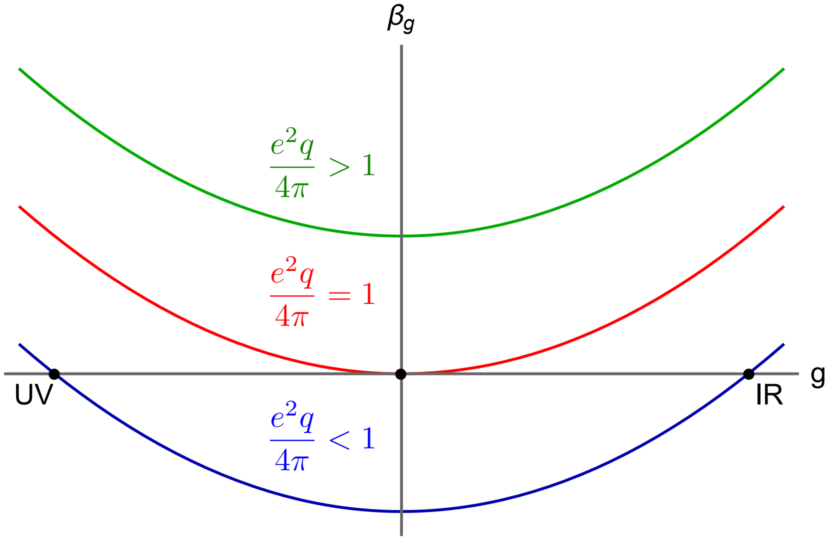

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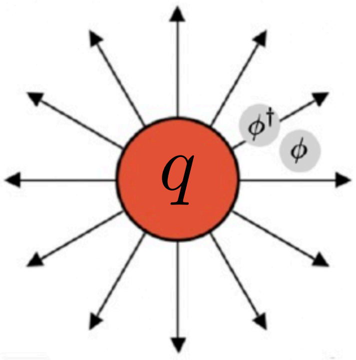
- In contrast, with bosons complete screening
  Exponentially large screening clouds



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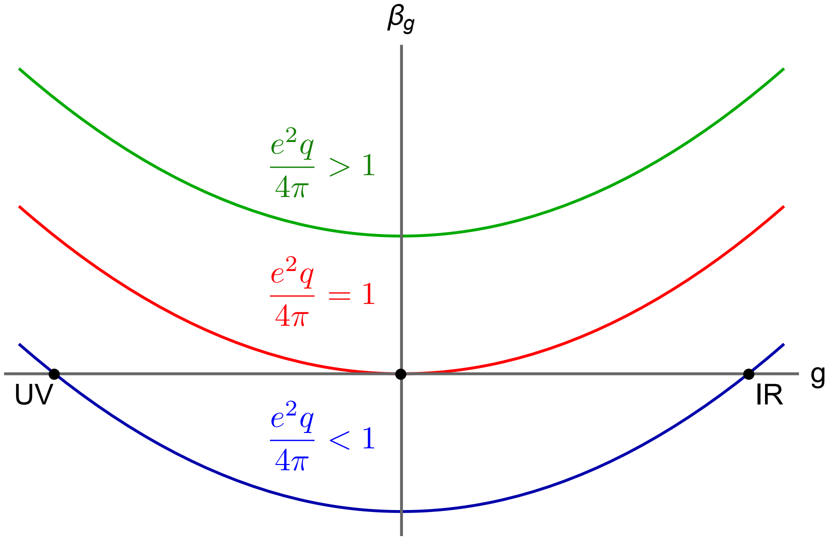


$$R_\psi \sim r_0 \exp\left(\frac{2\pi^2}{e^2}\sqrt{\frac{q - q_c}{2q_c}}\right)$$

$$R_\phi \sim r_0 \exp\left(\frac{\pi}{\sqrt{\frac{q-q_c}{2q_c}}}\right)$$

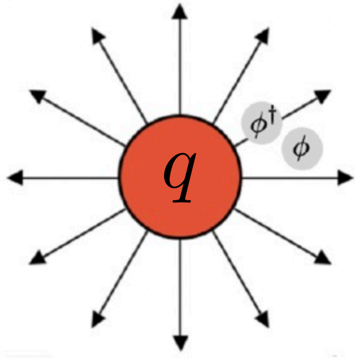
Wilson lines

Screening common feature in gauge theories

- 3d QED at large N_f

$$\Delta\left(\widehat{\bar{\psi}\psi}\right) = 1 \pm \sqrt{1 - 4E(q/N_f)^2}$$

$$\boxed{A_0 = \frac{E(q/N_f)}{r}}$$



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$U(1)_{\pm N_f}$ CS theory with N_f WLs
In UV we have just enough $1.12N_f$ distinct WLs

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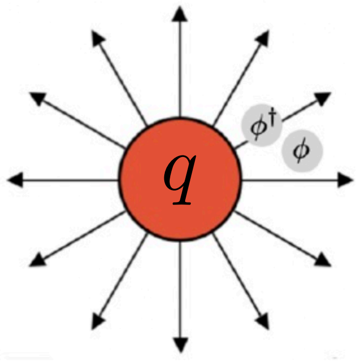

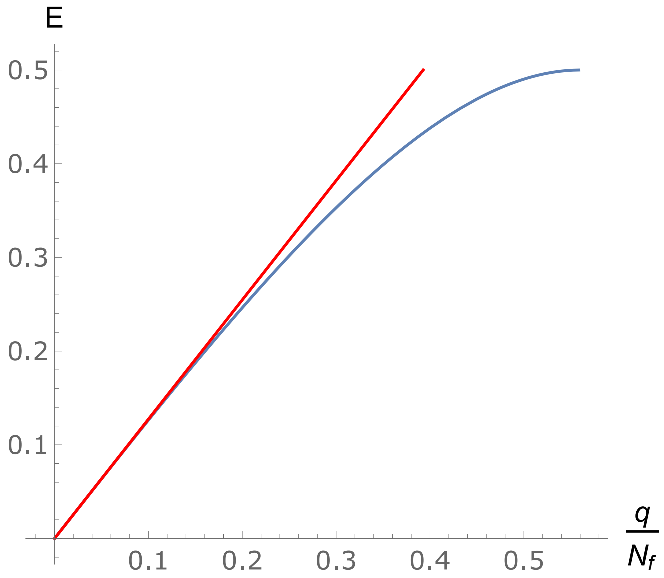

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- In graphene (mixed dimensional QED) screening experimentally observed
  Would need Z>137 to be accessible in atomic physics
  [Wang et al.; Pomeranchuk, Smorodinsky]

- Two WL DCFTs in planar CS matter theories, closed set of loop equations
  [Gabai, Sever, Zhong]

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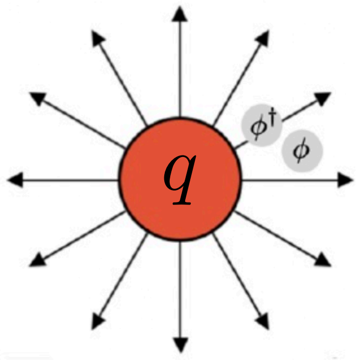
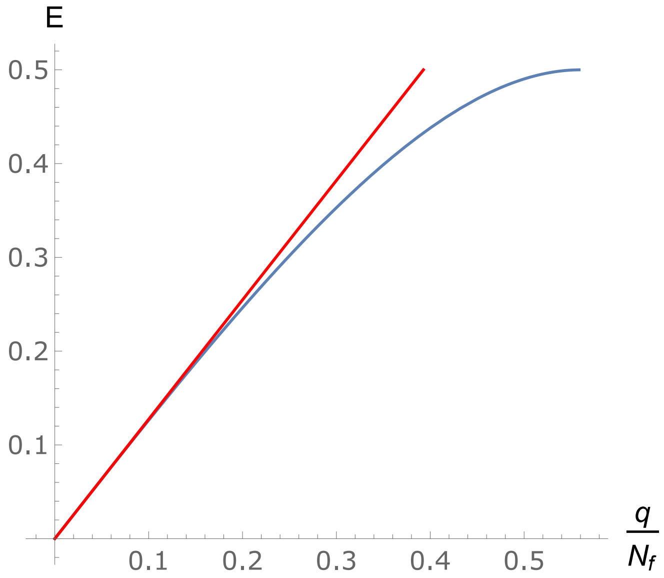
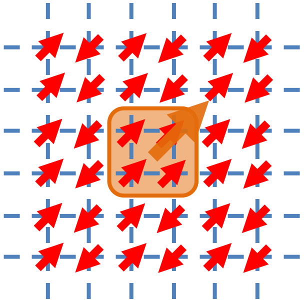


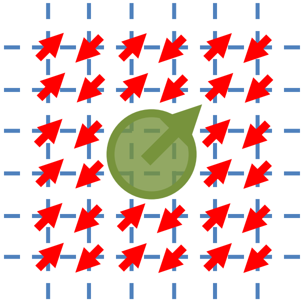


Defect dynamics

Comparison and summary



Ext. field defect
Protected by RG
monotonicity



Spin impurity
Partial protection by
one-form sym., existence
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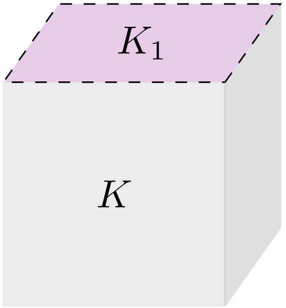
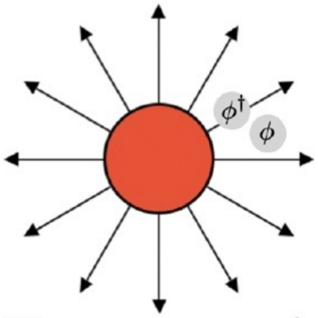


Conformal boundaries
Existence dynamical
question



Wilson line
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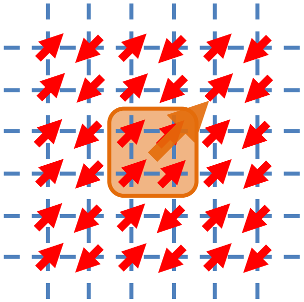

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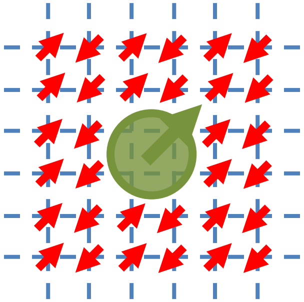

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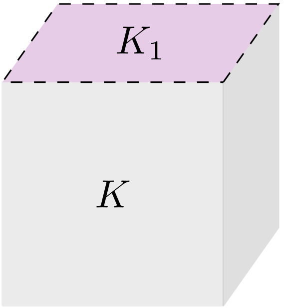

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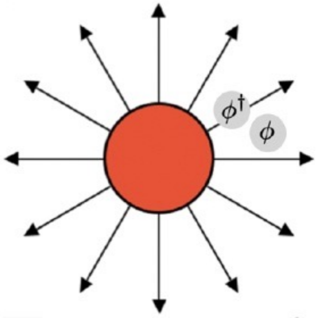
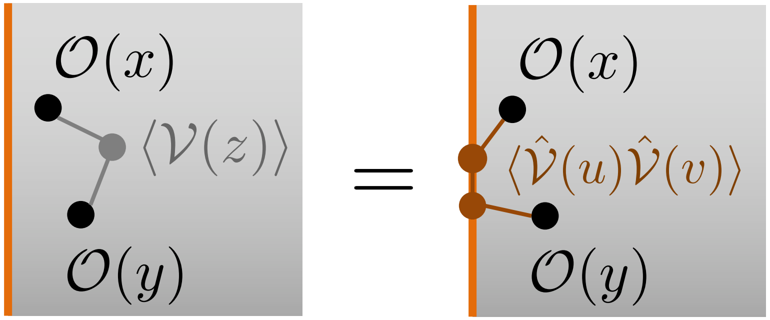


Wilson line
Screening dynamics

For dynamical questions we have perturbative expansions and numerical methods

- $\Delta(\hat{\phi})$ in Ordinary BCFT in O(N) model

	N=2	N=3	N=4
ϵ - expansion	1.19	1.153	1.125
Bootstrap	1.2342(9)	1.198(1)	1.172
Monte Carlo	1.2286(25)	1.194(3)	1.158(3)



Bootstrap

Bootstrap dream: classify all CFTs
Defect version: classify all defects for a given CFT

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  [Collier, Mazanc, Wang]

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- Fusion of defects in its infancy

  - Fusion of conformal boundary with topological defect lines in CFT$_2$
    [Collier, Mazanc, Wang]

  - Free theories and ϵ -expansion
    [Söderberg]

Bootstrap

Start with local operator bootstrap in free theories

- Codim-2 defects trivial in one real free scalar and in 4d Maxwell
  [Lauria, Liendo, van Rees, Zhao; Herzog, Shrestha]
- New BCFT even for one real free scalar from kink in bootstrap
  [Behan, Di Pietro, Lauria, van Rees]
- Rich DCFT physics in O(3) free scalar (in fractional d)
  [Cuomo, Komargodski, Mezei, Raviv-Moshe; Beccaria, Giombi, Tseytlin; Nahum]
- $SL(2,\mathbb{Z})$ orbits of BCFTs for 4d Maxwell
  [Di Pietro, Gaiotto, Lauria, Wu; Witten]

Outline

Introduction

RG monotonicity

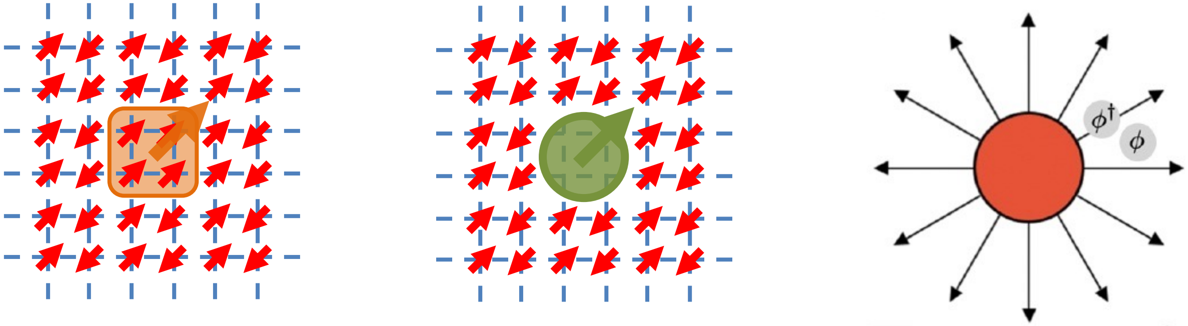
Examples

Conclusions and the future

Conclusions and the future

- Defect RG monotonicity from dilaton effective actions and quantum information

- Three routes do defects: external field, new dofs, gauging



- Bootstrap and (Quantum) Monte Carlo

- Future: interplay between DCFT, topological defects, integrability and AdS/CFT, bootstrap, **quantum simulation**