The quantum entropy of extremal black holes and the Schwarzian theory

Sameer Murthy
King's College London

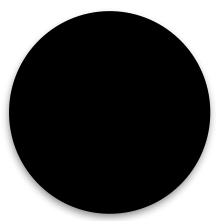
Eurostrings
Gijón, April 24, 2023

Black holes are quantum-statistical systems

Quantum entropy

$$\log \dim(\mathcal{H}_{\rm BH})(M,Q,J) \; = S_{\rm BH}^{\rm class}(M,Q,J) + \cdots$$

(Boltzmann)



(M,Q,J)

Deviations from GR!

Universal law in GR

$$S_{\rm BH}^{\rm class} \; = \; \frac{1}{4}\frac{A_{\rm H}}{\ell_{\rm P}^2} \; = \; \frac{A_{\rm H}\, c^3}{4\,\hbar\, G_N}$$

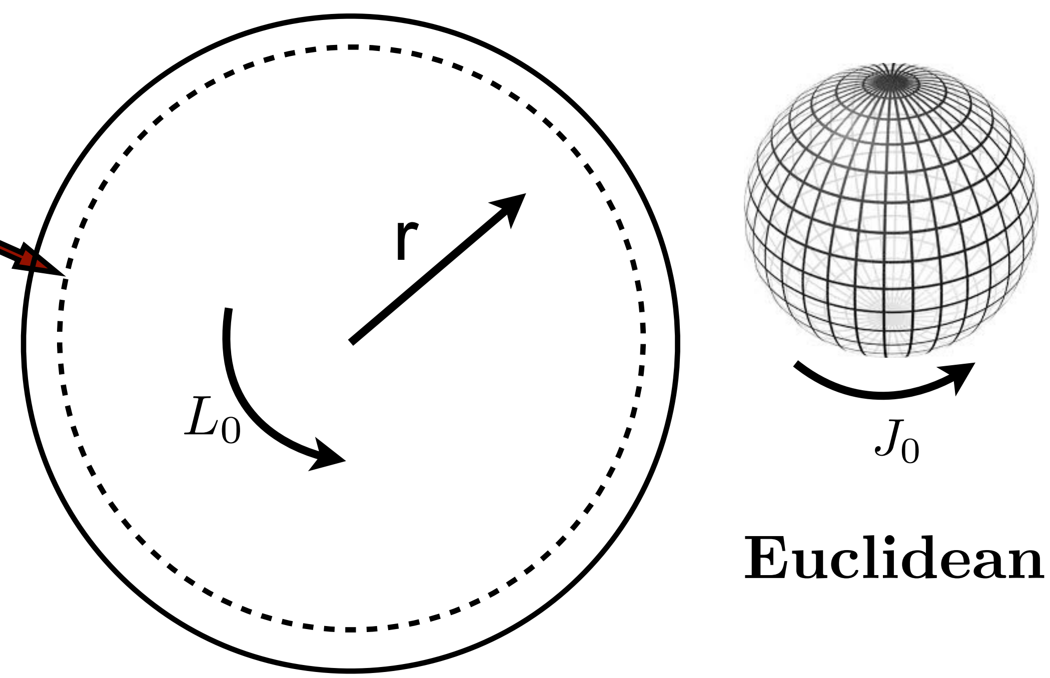
(Bekenstein-Hawking ’74)

Can we calculate $\dim(\mathcal{H}_{\mathrm{BH}})$ in gravity?

- Without positing additional discretization of horizon [cf Wheeler]
- Microcanonical: Supersymmetric BHs believed to be independent quantum systems

4d BPS BH: Near-horizon

Shape and size fixed by charges (attractor mech.)



$\mathbf{Euclidean\ AdS_2 \times S^2}$

Macroscopic Euclidean functional integral

$\mathrm{AdS}_2/\mathrm{CFT}_1$

Microscopic counting in flat space (or embed in AdS_{d+1}/CFT_d , d>1)

Can we calculate holographic observables at finite N independently in AdS and CFT?

$$Z_{\mathrm{CFT}}(N;\mu) \quad = \quad Z_{\mathrm{AdS}}(G_N;\mu)$$

$$= \mathrm{Tr}_{\mathcal{H}_{\mathrm{CFT}}}\, \mathrm{e}^{\mu_i q_i}\,(-1)^F \quad = \sum_{\alpha\in\mathrm{Saddles}} \exp(-S^{\alpha}_{\mathrm{grav}}(\mu))$$

Modular(-like) symmetry

All-orders in 1/N (localization)

- Supersymmetric indices provide a set of prototype examples: well-defined and calculable on both sides

- $\mathrm{AdS}_2/\mathrm{CFT}_1$: $\dim(\mathcal{H})$ of susy BHs (Main example today)
  [A.Dabholkar, J.Gomes, S.M. '10 -'14; L. Iliesiu, S.M., G.J. Turiaci, '22]
- $\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$: similar structure in $d = 2,3,4$

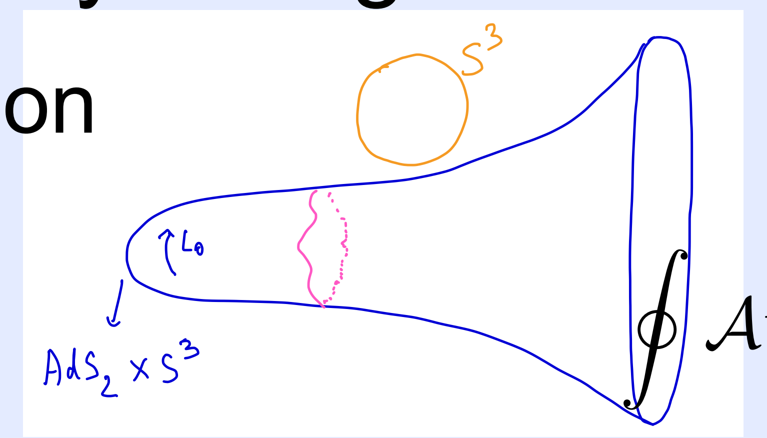
Supersymmetric index in gravity

How does susy BH contribute to the index?

$$W^{\text{index}}_{\text{grav}} = \int_{\{Q\psi_M(g_{MN})=0\}} Dg_{MN}\; \mathrm{e}^{\mu_i \int \mathcal{A}_i - S_{\text{grav}}(g_{\mu\nu})}\; Z_{\text{1-loop}}(g_{MN})$$

- Susy BHs are extremal and naively have infinite action.

- Susy BH contribution to index defined as limit $T \to 0$ of non-extremal susy configurations
  - complex deformation
  - smooth geometry
  - smooth limit



$\oint \mathcal{A}^R = \pi\mathrm{i}$

Cabo-Bizet, Cassani, Martelli, S.M. ’18 in AdS5/CFT4

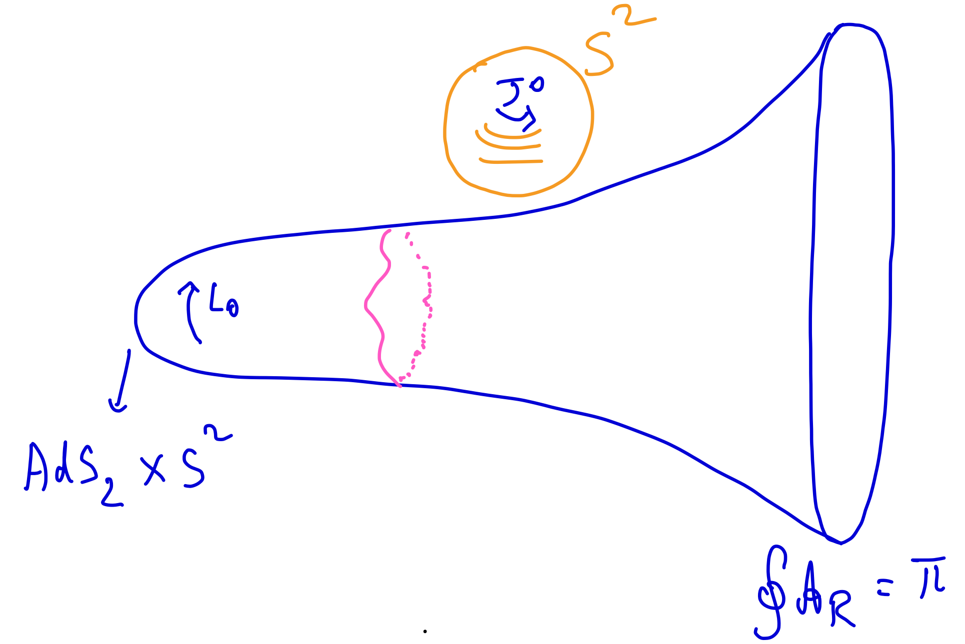
Cassani, Papini ’19
Bobev, Crichigno ’19
in AdS4,6,7, …

- Same is true for susy BHs in asymptotically flat space

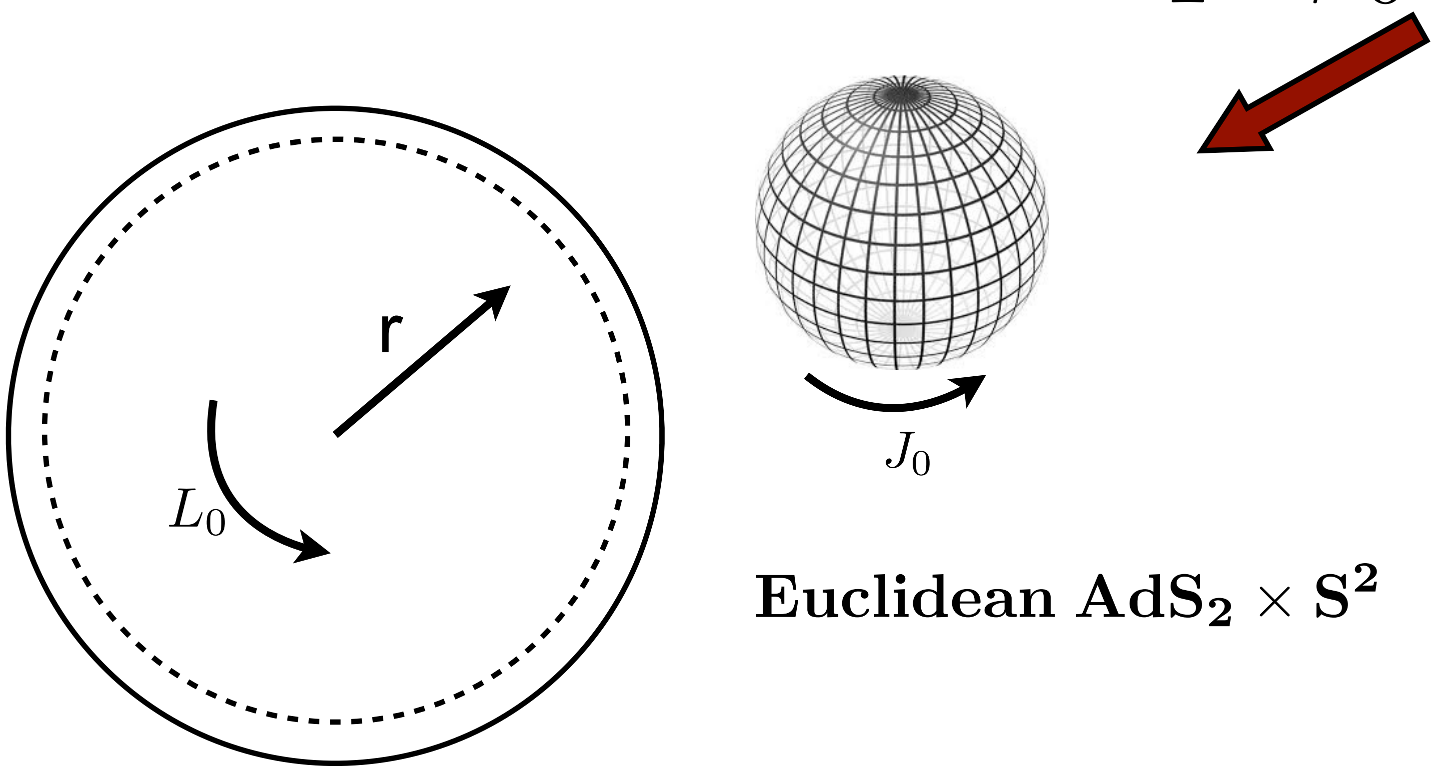
[Iliesiu, Kologlu,Turiaci ’19] [Boruch, Iliesiu, Murthy,Turiaci, in progress]

To focus on the BH, zoom in to the near-horizon near-AdS_2 region

4d BPS BH in $\mathbb{R}^3 \times S^1$



$T \to 0$



Euclidean $\mathbf{AdS_2 \times S^2}$

Supersymmetry

$$Q^2 = L_0 - J_0 \,.$$

- We start from AdS_2 limit and then consider T-deformation.

Localization in supergravity

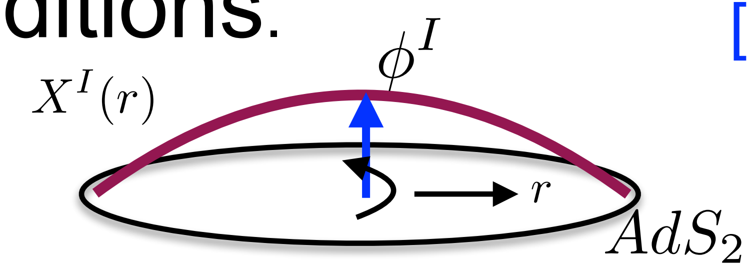
[A.Dabholkar, J.Gomes, S.M. '10 -'14; L. Iliesiu, S.M., G.J. Turiaci, '22]

1. Formalism: Construct rigid supercharge Q in off-shell supergravity (deformation of BRST structure).

   [B. de Wit, S.M., V. Reys, '18] [I. Jeon, S.M., '18]

2. All solutions of localization equations $Q\,\Psi = 0$, w/ $AdS_2 \times S^2$ boundary conditions.

   [R.Gupta, S.M. '12]



3. Evaluate full supergravity action on these solutions (include all higher derivative terms). D-terms do not contribute!

   [S.M., V.Reys, '13]

4. Compute one-loop determinant.

   [Cardoso, de Wit, Mahapatra '12, S.M., V. Reys, '15, Y. Ito, R. Gupta, I. Jeon,'15, I. Jeon, S.M. '18, L.Iliesiu, S.M., G.J. Turiaci, '22]

⅛ BPS BH in N=8 string theory

- Write N=8 string theory as an effective N=2 sugra with 15 vectors, 10 hypers, 6 spin 3/2 multiplets

- Prepotential for vector multiplets is tree-level exact

$$F(X) = -\frac{1}{2}\frac{C_{abc}X^a X^b X^c}{X^0}$$

$$W^{(1)}_{\rm grav}(N) = \int \frac{d\sigma}{\sigma^{9/2}} \exp\left(\sigma + \pi^2 N/4\sigma\right) = \widetilde{I}_{7/2}(\pi\sqrt{N})$$

- Microscopic index $W_{\rm micro}(N)$ known from D1-D5 counting. Coefficients of a known modular form.

[Maldacena, Moore, Strominger '00]

Comparison to microscopics

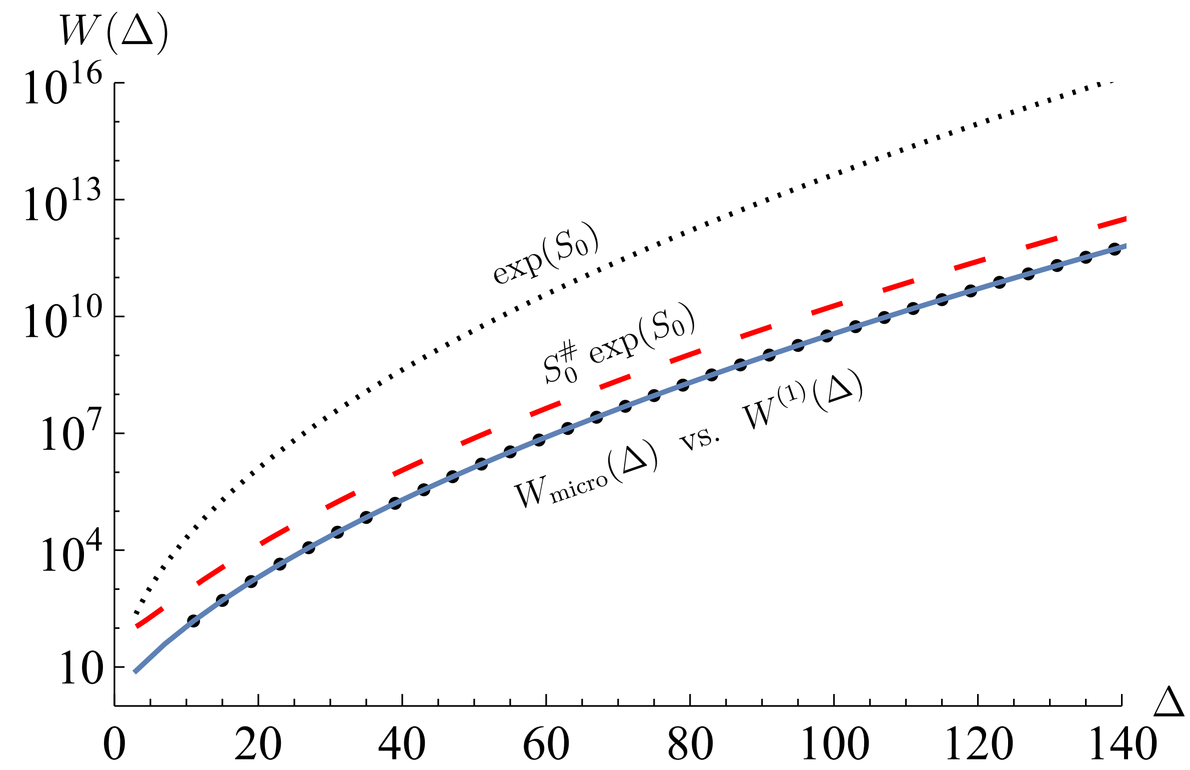
⅛ BPS BH in N=8 string theory

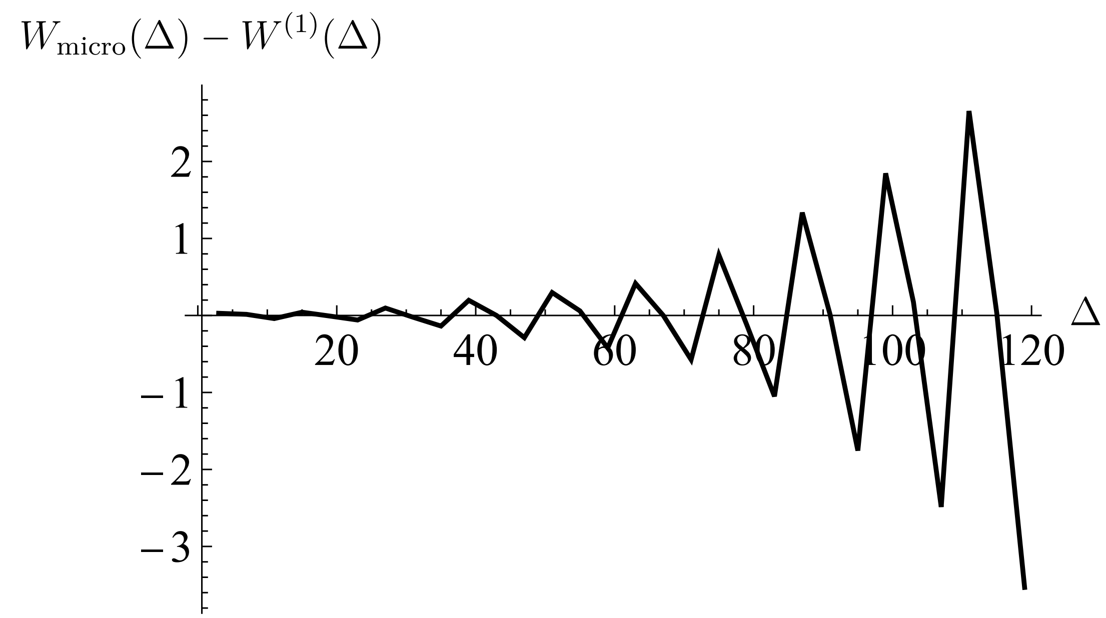
[A.Dabholkar, J.Gomes, S.M. '11]

N	$W_{\text{micro}}(N)$	$W^{\text{index}(1)}_{\text{grav}}(N)$
3	8	7.97
4	12	12.2
7	39	38.99
8	56	55.72
11	152	152.04
12	208	208.45
15	513	512.96
...	...	...
10^5	exp(295.7)	exp(295.7)

$$d_{\text{micro}}(N) = Z^{\text{index}(1)}_{\text{grav}}(N)\big(1 + \mathrm{O}(\mathrm{e}^{-\pi\sqrt{N}/2})\big)$$

Can we calculate the exponentially suppressed corrections?





[L. Iliesiu, S.M., G.J.Turiaci '22]

- Treatment of subtle effects coming from taking a BH to zero temperature. Some modes become massless.

[Sachdev '15; Almheiri, Kang '16; Maldacena, Stanford, Yang '16; Moitra, Trivedi, Vishal '18, Maldacena, Turiaci, Yang '19; Ghosh, Maxfield,Turiaci '19; Iliesiu,Turiaci, 20; Iliesiu, Kruthoff,Turiaci, Verlinde, 20; Heydeman, Iliesiu, Turiaci, Zhao, '20;…]

Modular symmetry → Analytic formula (infinite sum) for microscopic degeneracy

[Analytic number theory: G. Hardy, S. Ramanujan ('20); H. Rademacher ('38)]

$$W_{\rm micro}(N) = \sum_{c=1}^{\infty} c^{-9/2}\, K_c(N)\, \widetilde{I}_{7/2}(\pi\sqrt{N}/c)$$

$$= \widetilde{I}_{7/2}(\pi\sqrt{N})$$

→ All order pert. around AdS_2

$$+ \sum_{c>1}^{\infty} c^{-9/2}\, K_c(N)\, \widetilde{I}_{7/2}(\pi\sqrt{N}/c)$$

→ Orbifolds of AdS_2

[A.Dabholkar, J.Gomes, S.M.'14]

$$\widetilde{I}_\rho(z) = 2\pi\left(\frac{z}{4\pi}\right)^{-\rho} I_\rho(z)$$

$$= e^z + \dots$$ (I-Bessel function)

$$K_c(N) = \sum_{\substack{0<d<c \\ (c,d)=1}} e^{2\pi i \frac{N}{4}\frac{d+d^{-1}}{c}} M(c,d)$$ (Kloosterman sum)

- Expect all-order quantum fluctuations around orbifold

$$Z_c = \frac{1}{c} \; K_c(N) \; \frac{1}{c^{7/2}} \, \widetilde{I}_{7/2}(\pi\sqrt{N}/c)$$

CS theory contribution

[Dabholkar, Gomes, S.M. ’14]

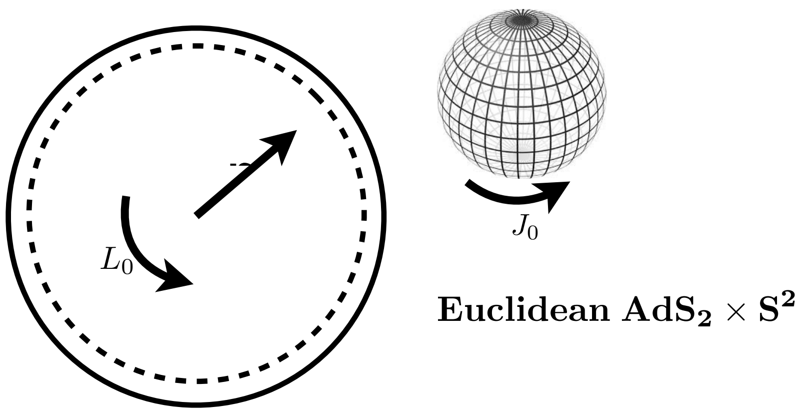
Quantum fluctuations of all bulk modes

[Iliesiu, S.M., Turiaci ’22]

Zero modes and mass gap

Subtlety in localized path integral

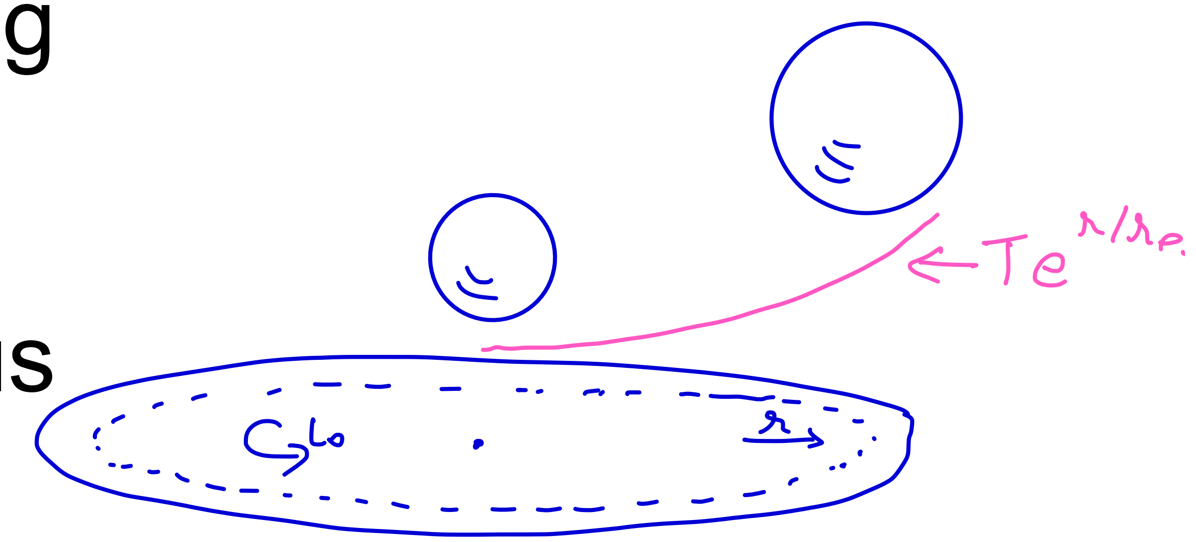
- Zero modes in AdS2 coming from "pure gauge transformations" which don't vanish at boundary. [Camporesi, Higuchi '95]



- Effect of zero mode at large charges calculated using ultra-locality [Witten] : important contribution to logarithmic corrections to Bekenstein-Hawking entropy [Sen]

- Regulate zero-modes by turning on a small (susy) temperature.

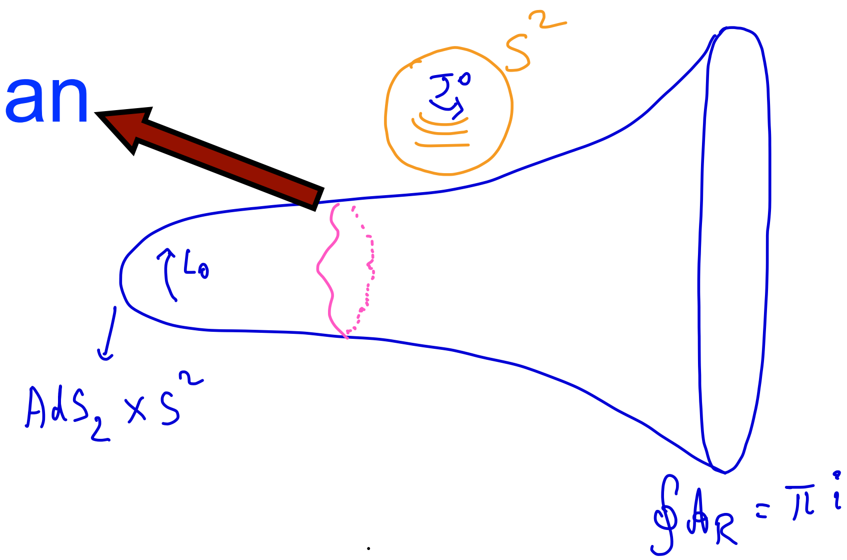
- Schwarzian mode: spontaneous vs explicit symmetry breaking



Regulate zero modes and then take $T \to 0$

[L. Iliesiu, S.M., G.J.Turiaci '22]

Super-Schwarzian modes



[…, Stanford, Witten '17; Mertens, Turiaci, Verlinde '17; Heydeman, Iliesiu, Turiaci, Zhao, '20]

Results:

- No mass-gap for generic extremal BHs
  $$Z_{\rm grav}(T,Q) = T^{\alpha}\, f(Q)$$
- For supersymmetric BHs, $\alpha = 0$, mass-gap $1/S_{\rm BH}^{3/2}$
- Volume of space of super-Schwarzian modes on orbifold $= 1/c$

From index to entropy

Black hole degeneracy = index

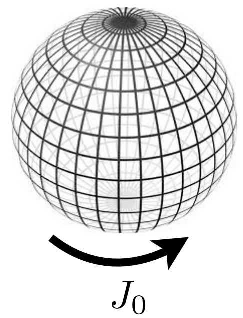
- In the microscopic theory, one actually counts the supersymmetric index $\mathrm{Tr}_{\mathcal{H}}\,(-1)^{\mathrm{F}}$. This is protected.

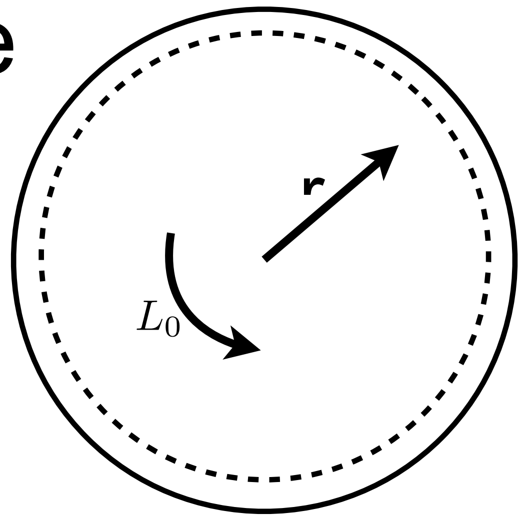
- 4d supersymmetric black holes are spherically symmetric and therefore have zero net angular momentum $J_0 = F$

- AdS_2 geometry ⇨ microcanonical ensemble

⇨ every state has F=0

⇨ $\mathrm{Tr}_{\mathcal{H}}\,(-1)^{\mathrm{F}} = \mathrm{Tr}_{\mathcal{H}}\,1$





[A. Sen ’10; A. Dabholkar, J. Gomes, S.M., A. Sen, ’10]

- This argument can be extended to Schwarzian modes. The result is that only J=0 states contribute.

[Iliesiu, Kologlu,Turiaci ’19; L. Iliesiu, S.M., G.J.Turiaci ’22]

Higher-dimensional AdS spaces: status

Structure of $\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$ in d>1

$$\mathcal{I}_N(\tau) \simeq \sum_{\frac{d}{c} \in \mathbb{Q}} \exp\left(\frac{1}{c} F^{\rm grav}_{\rm BH}(\tau - d/c)\right)$$

- Family of saddles of CFT index. CFT4 [Benini, Milan'18] [Cabo-Bizet, S.M.'19] [...] CFT3 [Benetti-Genolini, Cabo-Bizet, S.M.,'23]
- Family of gravitational saddles (orbifolds in string/M-theory) AdS3 [Maldacena, Strominger.,'98] AdS4 [Benetti-Genolini, Cabo-Bizet, S.M.,'23] AdS5 [Aharony, Benini, Mamroud, Milan,'21]
- 1/N Perturbation expansion around each saddle. Hints.
  AdS5 [Aharony, Benini, Mamroud, Milan,'21] [Mamroud '22] AdS4 [Pando Zayas, Xin,'20] [Bobev, Reys, + Charles, Hristov, Choi '21, '22]
  AdS3 [Ciceri, Jeon, S.M.,'23]
- Number theory structure. CFT2 [Dijgraaf-Maldacena-Moore-Verlinde '00][Maschot-Moore '10] CFT4 [Cabo-Bizet, S.M.'19] [Garoufalidis, S.M., Zagier, ??]

Thank you for your attention!