Aspects of Gauge-Strings Duality

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Outline

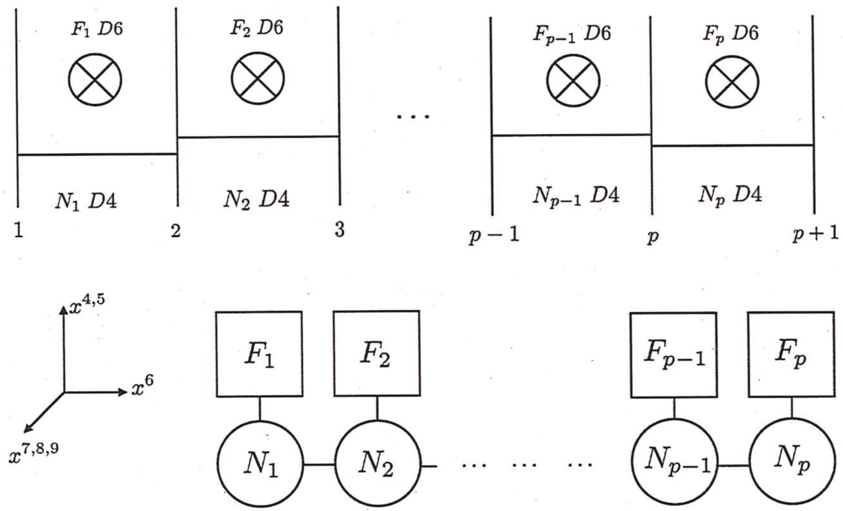
1. I will discuss work in the area of AdS/CFT. Focus on ideas and outcomes over the many technical details.
2. How holography "sees" a general structure in 8-SUSYs SCFTs. Using the free energy (central charge) as sample 'observable'. The final goal: calculate correlation functions in the SCFTs, their deformation and RG-flows.
3. Today, I will focus on four classes of examples: $N = 2$ SCFTs in four dimensions, $N = 4$ SCFTs in three dimensions, $N = 1$ SCFTs in five dimensions (both **balanced**) and $N = (1, 0)$ SCFTs in six dimensions. Comments about RG-flow away from CFTs. What I discuss today are generic lessons.
4. Works with Mohammad Akhond, Ali Fatemiabhari, Paul Merrikin, Andrea Legramandi, Leonardo Santilli, Lucas Schepers and Ricardo Stuardo. Previous work with Yolanda Lozano, Anayeli Ramirez, Niall Macpherson, Stefano Speziali, Anton Faedo and Chris Rosen.

A classification of a family of solutions of Type II or M-theory

Classify half BPS SUSY backgrounds with AdS-factor. Writing holographic duals to field theories: Conformal, SUSY, (8 SUSYs). Dimension:1,2,3,4,5,6.

These theories have (at least) $SU(2)$ R-symmetry. They can have other flavour-like symmetries.

These QFTs are usually realised on Hanany-Witten set-ups with D_p, D_{p+2} and NS5 branes.



SCFTs in diverse dimensions (8+8 SUSY). An *incomplete* picture.

- d=6:Hanany-Zaffaroni, Brunner-Karch —D6-D8-NS5— Gaiotto, Tomasielo, Apruzzi, Fazzi, Rosa, Passias, Cremonesi.
- d=5:Aharony-Hanany-Kol —D5-D7-NS5– D'Hoker, Gutperle, Uhlemann, Trivella, Karch. Legramandi, Nunez
- d=4:Gaiotto —D4-D6-NS5— Gaiotto, Maldacena; Aharony, Berkooz, Berdichevsky; Stefanski, Reid-Edwards. Nunez, Speziali, Roychowdhuri, Zacarias.
- d=3: Gaiotto-Witten —D3-D5-NS5— D'Hoker, Estes, Gutperle; Assel, Bachas, Gomis. Akhond, Legramandi, Nunez.
- d=2:(0,4) SCFT –D2-D4-D6-D8-NS5. Lozano, Macpherson, Nunez, Ramirez. Couzens, Martelli, Schaffer-Nameki, Wong.
- d=1: $N = 4$ SCQM– D0-D2-D4-D8-NS, Lozano, Nunez, Ramirez, Speziali.

Conformality, eight Poincare SUSYs. At least $SU(2)$ R-symmetry.

$$ds^2 \sim f_1 AdS_{d+1} + f_2 d\Omega_2 + f_3 d\Sigma_{7-d}(\vec{y}), \quad f_i(\vec{y}).$$

There are also NS B_2, Φ and RR fields respecting the isometries above.

Generically, what happens in all these cases is the following

We write the BPS equations. These are first order, nonlinear, coupled, partial differential equations.

Algebraic manipulations with these PDEs, allow us to write all the $f_i(\vec{y})$ in terms of a single function, that we refer to as 'potential function'.

Interestingly, the potential function satisfies a *linear* second order PDE (Laplace).

This Laplace equation needs of boundary conditions. Here is where the kinematic information about the SCFT enters, the good behaviour of the backgrounds is encoded, etc. Other boundary conditions imply other field theories.

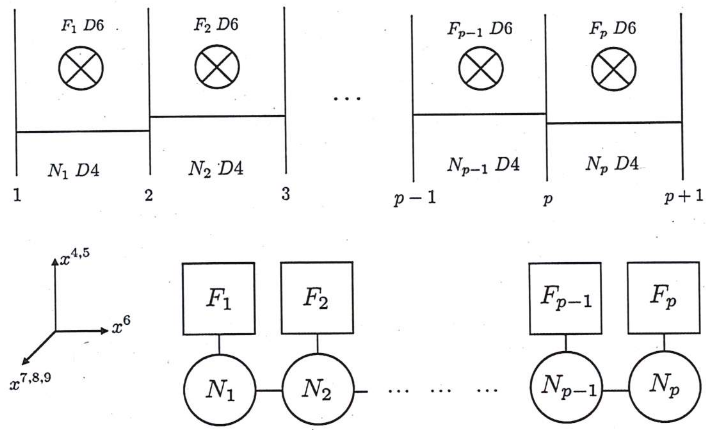
Examples: start with $N = 2$ four dimensional SCFTs. Continue with $N = 4$ SCFTs in 3d, then $N = 1$ 5d SCFTs (both balanced). Finish with 6d $N = (1, 0)$ SCFTs.

BPS → PDE for $f_i(\vec{y})$ → $f_i = f_i(V(\vec{y}), \vec{\partial} V(\vec{y}))$ → $V(\vec{y})$ → Laplace equation

QFT → Boundary condition

Consider 4d $\mathcal{N} = 2$ SCFTs.

For its historical and conceptual value, we start with these 4d SCFTs. A quiver-like description and a Hanany-Witten set-up.

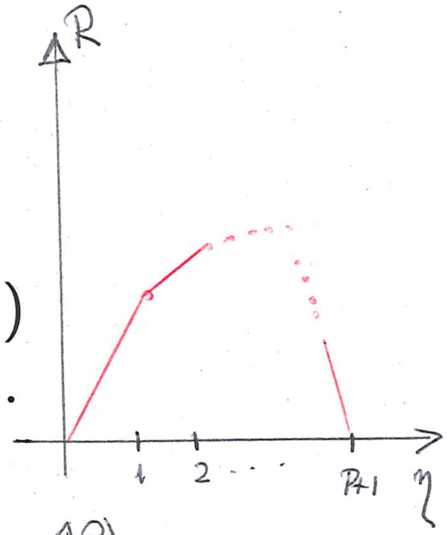


The bosonic global symmetries are $SO(2,4) \times SU(2)_R \times U(1)_r$. This will be reflected in the string dual containing $AdS_5 \times S^2 \times S^1$. The beta function of these theories is $\beta \sim (2N_c - N_f)$. This implies

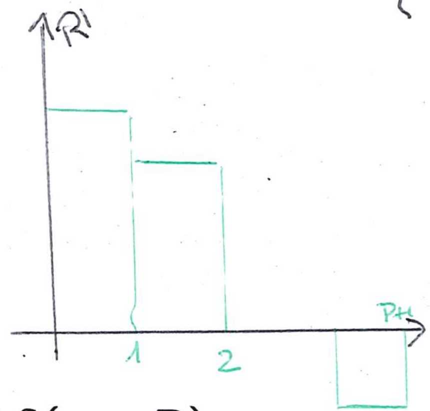
$$2N_1 - N_2 = F_1, \quad 2N_2 - N_1 - N_3 = F_2, ..., 2N_P - N_{P-1} = F_P.$$

One can define a 'Rank-function' $R(x)$, that is convex polygonal. For the quiver pictured above, we have

$$R(x) = \begin{cases} N_1 x & 0 \leq x \leq 1 \\ N_1 + (N_2 - N_1)(x-1) & 1 \leq x \leq 2. \\ \ldots. & \\ N_k + (N_{k+1} - N_k)(x-k) & k \leq x \leq (k+1) \\ N_P(P+1-x) & P \leq x \leq P+1. \end{cases}$$



$$R'(x) = \begin{cases} N_1 & 0 \leq x \leq 1 \\ (N_2 - N_1) & 1 \leq x \leq 2. \\ \ldots. & \\ (N_{k+1} - N_k) & k \leq x \leq (k+1) \\ -N_P & P-1 \leq x \leq P. \end{cases}$$



$$R'' = (2N_1 - N_2)\delta(x-1) + (2N_2 - N_1 - N_3)\delta(x-2) + .. + (2N_P - N_{P-1})\delta(x-P).$$

Lin, Lunin and Maldacena wrote (2005) the Type IIA backgrounds ($\alpha' = g_s = 1$). Gaiotto-Maldacena (2010).

$$ds_{10}^2 = 4f_1 ds_{AdS_5}^2 + f_2(d\sigma^2 + d\eta^2) + f_3 d\Omega_2(\chi,\xi) + f_4 d\beta^2.$$
$$B_2 = f_5 d\Omega_2(\chi,\xi), \quad C_1 = f_6 d\beta, \; A_3 = f_7 d\beta\wedge d\Omega_2, \; e^{2\phi} = f_8.$$

The functions $f_i(\sigma,\eta)$ can be all written in terms of a function $V(\sigma,\eta)$ and its derivatives, $f_i \sim f_i(V, \partial_\sigma V, \partial_\eta V)$. For example

$$f_1(\sigma,\eta)^2 = \frac{2\dot{V} - \ddot{V}}{V''}, \quad f_2 = 2f_1\frac{V''}{\dot{V}}, \ldots.$$

The function $V(\sigma,\eta)$ satisfies a Laplace-like equation with certain given boundary conditions to avoid nasty singularities,

$$\sigma^2\partial_\sigma^2 V + \sigma\partial_\sigma V + \sigma^2\partial_\eta^2 V = 0,$$
$$\sigma\partial_\sigma V(\sigma\to\infty,\eta)\to 0, \quad \sigma\partial_\sigma V(\sigma,\eta=0) = \sigma\partial_\sigma V(\sigma,\eta=P) = 0.$$
$$R(\eta) = \sigma\partial_\sigma V(\sigma,\eta)|_{\sigma=0}, \quad R(0) = R(P) = 0.$$

"Laplace like equation"

Boundary Conditions

Importantly, the rank function $R(\eta)$ appears as an initial condition for the Laplace problem.

Given $R(\eta)$, we write the solution for $V(\sigma,\eta)$ as a Fourier series

$$V(\sigma,\eta) = -\sum_{n=1}^{\infty} \frac{c_n}{w_n} K_0(w_n\sigma)\sin(w_n\eta), \qquad w_n = \frac{n\pi}{P}.$$

$$c_n = \frac{n\pi}{P^2}\int_{-P}^{P} R(\eta)\sin(w_n\eta)d\eta.$$

The backgrounds are trustable if the numbers P, N_k are large.

Other choices of boundary conditions are interesting.

Using this, one can calculate the Page charges in correspondence with D4,D6 and NS branes in the Hanany-Witten set-up.

$$Q_{NS5} = \frac{1}{4\pi^2}\int_{\eta,\Omega_2,\sigma\to\infty} H_3 = P,$$

$$Q_{D6} = \frac{1}{2\pi}\int_{[\eta,\beta]\sigma=0} F_2 = R'(\eta=0) - R'(\eta=P),$$

$$Q_{D4} = \frac{1}{8\pi^3}\int_{[\eta,\Omega_2,\beta]\sigma=0} F_4 - B_2\wedge F_2 = \int_0^P R(\eta)d\eta.$$

Linking numbers, Entanglement Entropy, central charge can be computed holographically and have expressions in terms of $R(\eta)$.
Roughly defined as a 'weighted version' of the volume of the internal space.

$$ds^2 = a(r,\vec{y})\left[-dt^2 + dx_d^2 + b(r)dr^2\right] + ds_{int}^2(r,\vec{y}), \quad \Phi(r,\vec{y}),$$

$$V_{int} = \int_{X_{int}} \sqrt{e^{-4\Phi}\det[g_{int}]a(r,\vec{y})^d}. \qquad H = V_{int}^2$$

$$c_{hol} \sim \frac{b(r)^{\frac{d}{2}}}{G_N}\frac{H^{\frac{2d+1}{2}}}{(H')^d}.$$

When computed for these backgrounds dual to 4d N=2 SCFTs,

$$ds_{10}^2 = 4f_1 ds_{AdS_5}^2 + f_2(d\sigma^2 + d\eta^2) + f_3 d\Omega_2(\chi,\xi) + f_4 d\beta^2, \;\; e^{2\phi} = f_8.$$

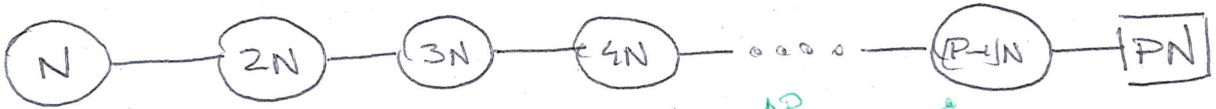
$$a = f_1(\sigma,\eta)r^2, \quad b = \frac{1}{r^4}, \;\; d = 3 \to V_{int} \to H \to$$

$$4c = \int_0^P R(\eta)^2 d\eta \sim P^3 \sum_{j,l=1}^{P-1} F_j F_l \mathrm{Re}\left[Li_4(e^{\frac{i\pi(j+l)}{P}}) - Li_4(e^{\frac{i\pi(j-l)}{P}})\right].$$

Compare with results by Shapere-Tachikawa

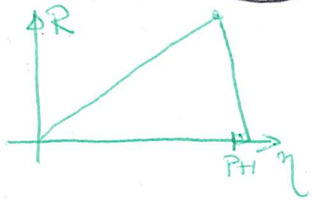
$$a = \frac{5n_v + n_h}{24}, \qquad c = \frac{2n_v + n_h}{12}.$$

Generic expressions, for generic quivers. Focus on an example.



I will use this example for comparisons (nothing special about it!)

We find



$$n_v = \sum_{k=1}^{P-1} N^2 k^2 - 1, \quad n_h = \sum_{k=1}^{P-1} N^2 k(k+1),$$

$$a = \frac{N^2 P^3}{12} - \frac{5N^2 P^2}{48} + \frac{P}{48}(N^2 - 10) + \frac{5}{24},$$

$$c = \frac{N^2 P^3}{12} - \frac{N^2 P^2}{12} - \frac{P}{6} + \frac{1}{6},$$

$$c = \frac{1}{4}\int_0^P R(\eta)^2 d\eta = \frac{N^2 P^3}{12} - \frac{N^2 P^2}{6} + \frac{PN^2}{12}.$$

Let us discuss the case of 3d *balanced* N=4 SCFTs.

N=4 SCFTs in d=3. $SO(2,3)\times SU(2)\times SU(2)$, 8 SUSYs. First discussed by D'Hoker, Estes and Gutperle (2007).

In the language of Akhond, Legramandi, C.N. (2021).

$$ds^2_{10,st} = f_1\Big[ds^2(\mathrm{AdS}_4) + f_2 d\Omega_{2,L} + f_3 d\Omega_{2,R} + f_4(d\sigma^2 + d\eta^2)\Big],$$

$$e^{-2\Phi} = f_5,\quad B_2 = f_6 \mathrm{Vol}\Omega_{2,L},\quad C_2 = f_7 \mathrm{Vol}\Omega_{2,R},\quad \tilde{C}_4 = f_8 \mathrm{Vol}(\mathrm{AdS}_4).$$

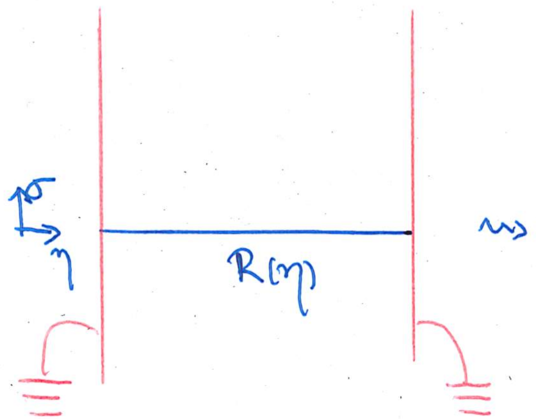
$$f_i(\sigma,\eta) = f_i(\hat{W}, \partial_\sigma \hat{W}, \partial_\eta \hat{W}, \ldots).$$

$$\partial^2_\sigma \hat{W}(\sigma,\eta) + \partial^2_\eta \hat{W}(\sigma,\eta) = \delta(\sigma)\mathcal{R}(\eta),$$ ⇝ Laplace

$$\hat{W}(\sigma,\eta=0) = 0,\qquad \hat{W}(\sigma,\eta=P) = 0,$$

$$\partial_\sigma \hat{W}(\sigma=0^+,\eta) - \partial_\sigma \hat{W}(\sigma=0^-,\eta) = -\mathcal{R}(\eta).$$

Initial and Boundary conditions



$$\hat{W}(\sigma,\eta) = \sum_{k=1}^{\infty} \frac{P}{k\pi} R_k \cdot \sin\left(\frac{k\pi}{P}\eta\right) \cdot e^{-\frac{k\pi}{P}|\sigma|}$$

Quantising the Page charges.

$$N_{NS5} = \frac{1}{4\pi^2}\int_{\Sigma_3} H_3 = P$$

$$N_{D3} = \frac{1}{(2\pi)^4}\int_{\Sigma_5} \hat{F}_5 = \mathcal{R}(\eta) - (\eta - \Delta)\mathcal{R}'(\eta),$$

$$N_{D5} = \frac{1}{4\pi^2}\int_{\hat{\Sigma}_3} F_3 = \mathcal{R}'(\eta_i) - \mathcal{R}'(\eta_f).$$

This forces us to choose

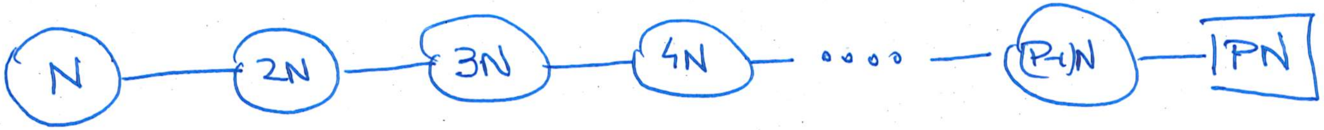
$$\mathcal{R}(\eta) = \begin{cases} N_1\eta & 0 \leq \eta \leq 1 \\ N_l + (N_{l+1} - N_l)(\eta - l) & l \leq \eta \leq l+1, \quad l := 1, ..., P-2 \\ N_{P-1}(P - \eta) & (P-1) \leq \eta \leq P. \end{cases}$$

	t	x_1	x_2	y_1	y_2	y_3	z_1	z_2	z_3	η
NS5	—	—	—	·	·	·	—	—	—	·
D5	—	—	—	—	—	—	·	·	·	·
D3	—	—	—	·	·	·	·	·	·	—

A very similar calculation for the holographic central charge

$$c_{hol} = -\frac{P^2}{16\pi^3} \sum_{l,k=1}^{P-1} F_l F_k \text{Re}\left(Li_3(e^{\frac{i\pi(k+l)}{P}}) - Li_3(e^{\frac{i\pi(k-l)}{P}}) \right).$$

One can exactly obtain the same expression using a matrix model. Coccia-Uhlemann (2021), Akhond, Legramandi, C.N, Santilli, Schepers (2022). For the particular case of the quiver



$$c = \frac{N^2 P^4}{32\pi^3} \left[2\zeta(3) - 2\text{Re Li}_3(e^{\frac{2\pi i(P-1)}{P}}) \right] \sim \frac{N^2 P^2}{8\pi} \log P.$$

There is also a nice way of phrasing Mirror symmetry in this language. Computing Wilson loops is also possible, these results are exactly checked with a matrix model.

Balanced 5d N=1 SCFTs. $SO(2,5)\times SU(2)$, 8 SUSYs.

Studied in works by D'Hoker, Gutperle, Uhlemann, Karch (2016).
In the language of Legramandi, C.N (2021).

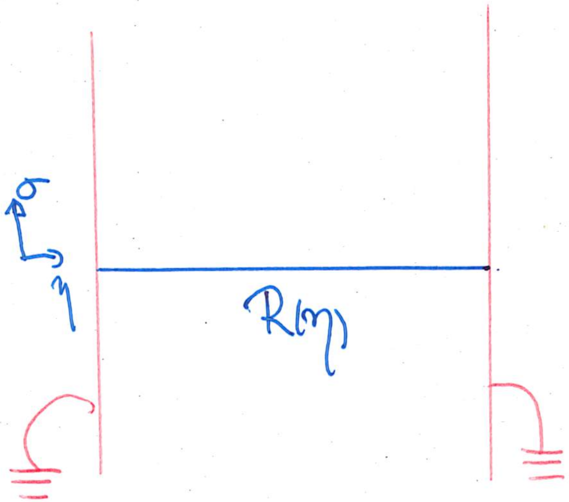
$$ds^2_{10} = f_1\Big[ds^2(\text{AdS}_6) + f_2 ds^2(S^2) + f_3(d\sigma^2 + d\eta^2)\Big],$$

$$B_2 = f_4 \text{Vol}(S^2),\quad C_2 = f_5\text{Vol}(S^2),\quad e^{-2\Phi} = f_6,\quad C_0 = f_7,$$

$$f_i(\sigma,\eta) = f_i(\hat V, \partial_\sigma \hat V, \partial_\eta \hat V, \ldots.),\quad \boxed{\partial^2_\sigma \hat V + \partial^2_\eta \hat V = \delta(\sigma)\mathcal{R}(\eta).} \rightarrow \text{Laplace}$$

$$\hat V(\sigma\to\pm\infty,\eta) = 0,\qquad \hat V(\sigma,\eta=0) = \hat V(\sigma,\eta=P) = 0.$$

$$\lim_{\epsilon\to 0}\Big(\partial_\sigma \hat V(\sigma=+\epsilon,\eta) - \partial_\sigma \hat V(\sigma=-\epsilon,\eta)\Big) = \mathcal{R}(\eta).$$



$$\hat V(\sigma,\eta) = \sum_{k=1}^{\infty} \frac{\pi k}{P} R_k \sin\left(\frac{k\pi}{P}\eta\right)\cdot e^{-\frac{k\pi}{P}|\sigma|}$$

Quantised Page charges, relates $\mathcal{R}(\eta)$ and a balanced quiver.

$$\mathcal{R}(\eta) = \begin{cases} N_1 \eta & 0 \leq \eta \leq 1 \\ N_k + (N_{k+1} - N_k)(\eta - k) & k \leq \eta \leq k+1, \quad k = 1, ..., P-2 \\ N_{P-1}(P - \eta) & (P-1) \leq \eta \leq P. \end{cases}$$

$$\begin{aligned} &Q_{NS5} = P \\ &Q_{D7}[k, k+1] = \mathcal{R}''(k) = (2N_k - N_{k+1} - N_{k-1}), \\ &Q_{D5}[k, k+1] = \mathcal{R}(\eta) - \mathcal{R}'(\eta)(\eta - k) = N_k \\ &Q_{D7,total} = \int_0^P \mathcal{R}''(\eta) d\eta, \quad Q_{D5,total} = \int_0^P \mathcal{R}\, d\eta. \end{aligned}$$

Follow similar line as described above, compute $\hat{V}$ in Fourier series, central charge, etc.

For a generic rank function, calculate $\hat{V}$ and c_{hol}.

$$\hat{V} = \frac{P^2}{2\pi^3}\sum_{s=1}^{P-1} F_s \mathrm{Re}\left(\mathrm{Li}_3(e^{-\frac{\pi(|\sigma|+i\eta-is)}{P}}) - \mathrm{Li}_3(e^{-\frac{\pi(|\sigma|+i\eta+is)}{P}})\right).$$

$$c = -\frac{P^4}{4\pi^{10}}\sum_{l=1}^{P-1}\sum_{s=1}^{P-1} F_l F_s \mathrm{Re}\left(\mathrm{Li}_5(e^{i\frac{\pi(l+s)}{P}}) - \mathrm{Li}_5(e^{i\frac{\pi(l-s)}{P}})\right).$$

Checked by a matrix model, Akhond, Legramandi, C.N, Santilli, Schepers (2022). For Wilson loops, Uhlemann (2022), Fatemiabhari, C.N (2022).

For the quiver,

$$\hat{V} = \frac{NP^3}{2\pi^3}\mathrm{Re}\left(\mathrm{Li}_3(-e^{-\frac{\pi}{P}(|\sigma|+i+i\eta)}) - \mathrm{Li}_3(-e^{-\frac{\pi}{P}(|\sigma|-i+i\eta)})\right),$$

$$c = \frac{N^2P^6}{8\pi^{10}}\left(2\zeta(5) - \mathrm{Li}_5(e^{\frac{2\pi i}{P}}) - \mathrm{Li}_5(e^{-\frac{2\pi i}{P}})\right) \sim \frac{N^2P^4}{2\pi^8}\zeta(3)$$

The case of 6d $\mathcal{N} = (1,0)$ SCFTs.

Formulation by Tomasiello+ (Apruzzi, Dibitetto, Gaiotto, Fazzi, Rota, Passias, Tizzano, Petri, Cremonesi), Bobev, Gautason.

Dual to SCFTs with $SO(2,6) \times SU(2)_R$

$$ds^2 = f_1(\eta) AdS_7 + f_2(\eta) d\Omega_2 + f_3(\eta) d\eta^2,$$
$$B_2 = f_4(\eta) d\Omega_2, \quad F_2 = f_5(\eta) d\Omega_2, \quad F_0(\eta), \quad e^{\Phi} = f_6(\eta).$$

All the functions $f_1(\eta), ..., f_6(\eta)$ are written in terms of a potential function $V(\eta)$ solving

$$V'''(\eta) = -162\pi^3 F_0.$$

$$\frac{-V(\eta)}{81\pi^2} = \begin{cases} a_1\eta + \frac{a_2}{2}\eta^2 + \frac{a_3}{6}\eta^3 & 0 \leq \eta \leq 1 \\ b_0 + b_1(\eta - 1) + \frac{b_2}{2}(\eta - 1)^2 + \frac{b_3}{6}(\eta - 1)^3 & 1 \leq \eta \leq 2 \\ c_0 + c_1(\eta - 2) + \frac{c_2}{2}(\eta - 2)^2 + \frac{c_3}{6}(\eta - 2)^3 & 2 \leq \eta \leq 3 \\ .... & i \leq \eta \leq i+1 \\ p_0 + p_1(\eta - P) + \frac{p_2}{2}(\eta - P)^2 + \frac{p_3}{6}(\eta - P)^3 & P \leq \eta \leq P+1 \end{cases}$$

We impose quantisation of Page charges and continuity of V, V', V''. The rank function is identified as $R(\eta) \sim V''(\eta)$.

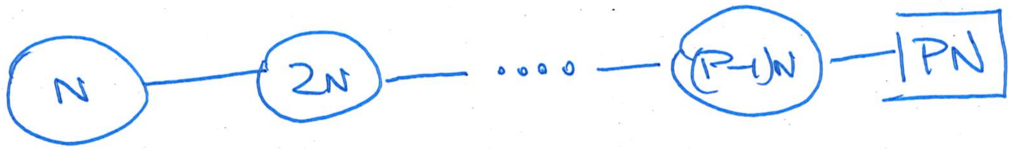
$$Q_{NS} = P, \quad Q_{D8}^{[k,k+1]} = \frac{1}{81\pi^2}\left(V'''(k) - V'''(k+1)\right),$$
$$Q_{D6}^{[k,k+1]} = \frac{1}{81\pi^2}\left(V''(\eta) - V'''(\eta)(\eta - k)\right).$$

	t	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
NS5	●	●	●	●	●	●	·	·	·	·
D6	●	●	●	●	●	●	●	·	·	·
D8	●	●	●	●	●	●	·	●	●	●

We can calculate the central charge

$$\frac{3^8\pi^6 c}{2} = \int_0^P V(\eta)V''(\eta)d\eta \sim P^5 \sum_{j,l=1}^{P-1} F_j F_l \mathrm{Re}\left[Li_6(e^{\frac{i\pi(j+l)}{P}}) - Li_6(e^{\frac{i\pi(j-l)}{P}})\right].$$

For the quiver



$$c \sim \frac{2P^5 N^2}{45\pi^2}.$$

Let me compare the results for the same quiver in dimensions: 3,4,5,6.

$$c^{(3d)} \sim \frac{P^2 N^2}{4\pi} \log P, \quad c^{(4d)} \sim \frac{P^3 N^2}{12},$$
$$c^{(5d)} \sim \frac{P^4 N^2}{2\pi^8}\zeta(3), \quad c^{6d} \sim \frac{2P^5 N^2}{45\pi^2}.$$

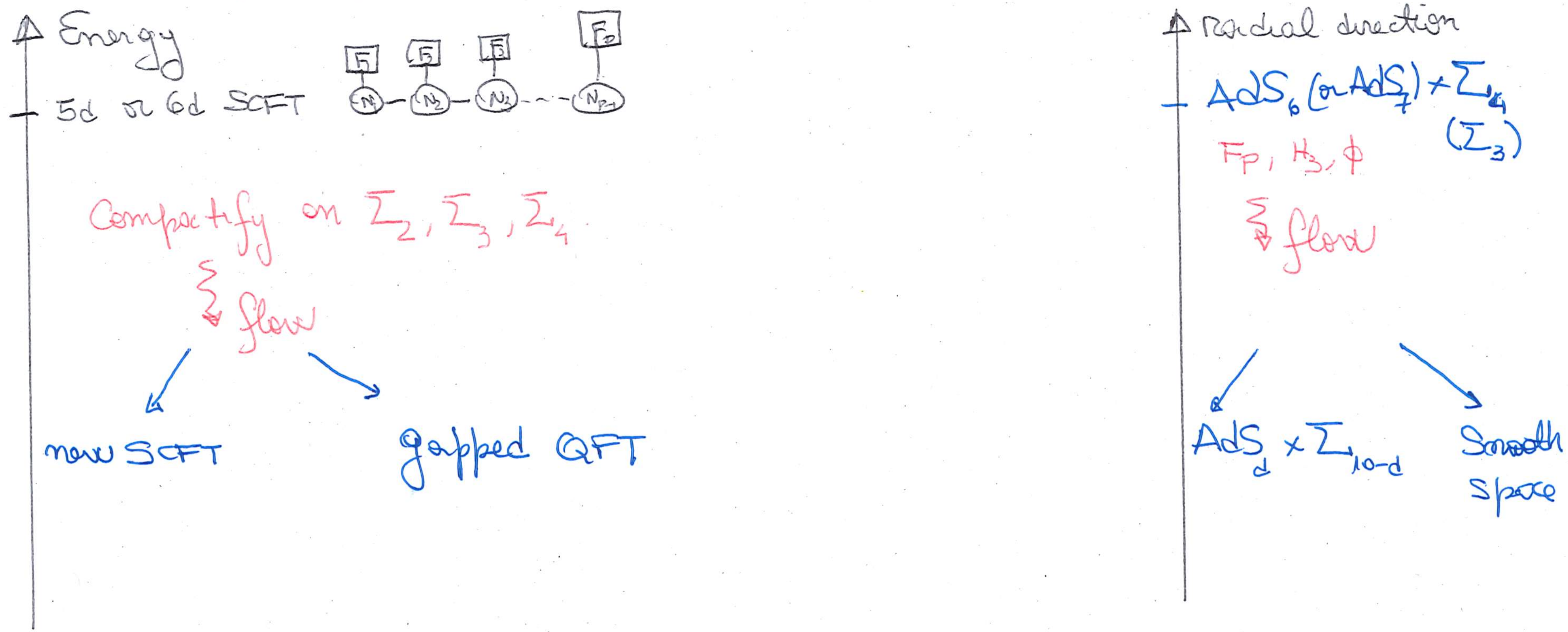
Let me leave this here and discuss some applications and predictions rather than tests!

Let us discuss some applications and predictions

Consider RG-flows away from the 5d N=1 or 6d N=(1,0) SCFTs.

We can compactify the 6d or 5d-SCFTs, deform by relevant operators, leading at low energies to a lower dimensional QFT. Under certain circumstances, these are strongly coupled CFT_d, with $d = 1, 2, 3, 4$.

This is captured by backgrounds written together with Legramandi (2021), and with Merrikin and Stuardo (2022).



For the case SCFT5 flowing to SCFT3

$$ds_{st}^2 = f_1\Big[e^{2f(r)}\overbrace{(-dt^2+dy_1^2+dy_2^2+dr^2)}^{\text{would give } AdS_4} + e^{2h(r)}\overbrace{\frac{(dz^2+dx^2)}{z^2}}^{\Sigma_2} +$$

$$f_2 ds^2(\tilde{S}^2) + f_3(d\sigma^2+d\eta^2)\Big], \quad f_i = f_i(\sigma,\eta,X(r))$$

$$B_2 = f_4 \mathrm{Vol}(\tilde{S}^2) + \frac{2}{9}\eta\frac{\cos\theta}{z^2}dx\wedge dz, \qquad C_0 = f_7,$$

$$C_2 = f_5 \mathrm{Vol}(\tilde{S}^2) + 4\partial_\sigma(\sigma V)\frac{\cos\theta}{z^2}dx\wedge dz,, \qquad e^{-2\Phi} = f_6.$$

$$F_5 = (1+*)\frac{2e^{4f(r)-2h(r)}}{3X^2}dr\wedge dt\wedge dy_1\wedge dy_2\wedge d\left(\cos\theta\sigma^2\partial_\sigma V\right),$$

$$ds^2(\tilde{S}^2) = d\theta^2 + \sin^2\theta(d\phi - A^3)^2, \quad \mathrm{Vol}(\tilde{S}^2) = \sin\theta d\theta\wedge\left(d\phi - A^3\right)$$

An infinite family of Type IIB backgrounds with AdS_4 factor, preserving four supercharges. Field theoretically, a twisted compactification of the $SCFT_5$ into three dimensions. There are BPS eqs. for $f(r), h(r), X(r)$.

For the flow between a 6d $N = (1, 0)$ SCFT to a 4d $N = 1$ SCFT, let me quote the metric (omit F_0, F_2, F_4, B_2, Φ)

$$ds_{st}^2 = X(r)^{-\frac{1}{2}} \Big[$$

$$f_1(\eta)\, e^{-\frac{4\Phi(r)}{5}} \Big(e^{2f(r)} dx_{1,3}^2 + dr^2 + e^{2h(r)} \left(d\theta_1^2 + \sinh^2(\theta_1) d\phi_1^2 \right) \Big)$$

$$+ X(r)^3 \left(f_2(\eta) d\eta^2 + \frac{f_3(\eta)}{\omega(r,\eta)} \left(d\theta_2^2 + \sin^2(\theta_2)(d\phi_2 - \cosh\theta_1 d\phi_1)^2 \right) \right) \Big]$$

$f(r), h(r), X(r), \Phi(r)$ solve BPS equations. They interpolate between AdS_7 and AdS_5. The mixing is in $\omega(r, \eta)$.

It is interesting that these solutions work for *any* quiver/rank function we choose. This will reflect in the functions $f_i(\eta)$ and $\omega(r, \eta)$.

In this way, we construct an infinite family of AdS_5 backgrounds in massive IIA preserving 4 Poincare SUSY.

Central charge for the 4d $N = 1$ SCFT or 3d $N = 2$ SCFT

If the 'mother' theory is the quiver we have been using

$$c_{3d} \sim P^4 N^2 (g-1), \qquad c_{4d} \sim P^5 N^2 (g-1).$$

That is, we get a scaling with the genus of the compactification manifold $(g-1)$ and factors of the length of the quiver reminiscent of a higher dimension.

It would be interesting to write clearly a 3d or 4d SCFT with these scalings.

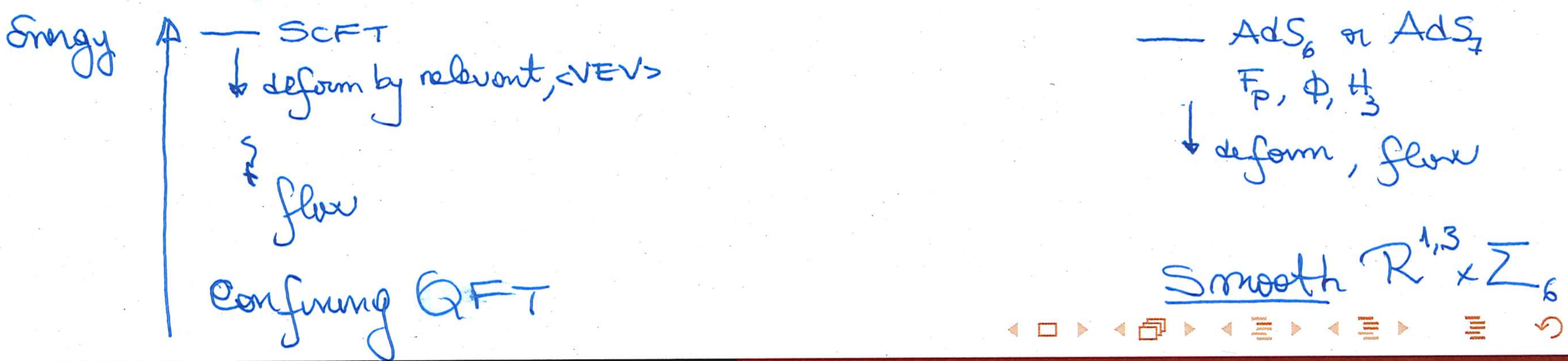
The classification of 4 SUSYs AdS_4 in IIB (Passias, Solard, Tomasiello) or AdS_5 backgrounds (Bah) in (massive) IIA is hard to deal with. Here, we construct infinite solutions easily.

For the future: the possibility of constructing SUSY compactifications that end the space 'smoothly'. This would describe the dynamics of a large number of gauge nodes, with fundamental and bifundamental matter in a confining situation.

There are various backgrounds that end smoothly, encoding the dynamics of adjoints and bi-fundamental matter undergoing confinement. These were carefully explored in the past. Some tests and many predictions!

Finding the new compactifications above would add the dynamics of fundamental fields (flavours), with $SU(N_f)$ symmetry into the mix.

One example (**not necessarily so nice**): compactify d=5 $N=1$ SCFTs family on a circle, with SUSY breaking boundary conditions



Consider SCFT5 flowing to a non-SUSY 4d theory

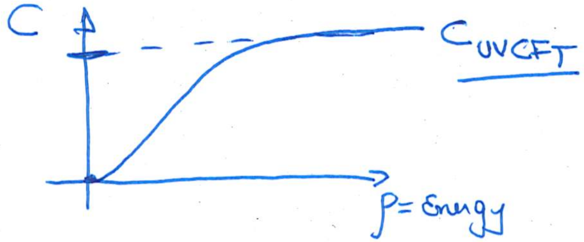
$$ds_{st}^2 = f_1\Big[e^{2\rho}\left(-dt^2 + d\vec{x}_3^2 + g(\rho)d\psi^2\right) + \frac{d\rho^2}{g(\rho)} + f_2 ds^2(S^2) + f_3(d\sigma^2 + d\eta^2)\Big],$$

$$B_2 = f_4 \text{Vol}(S^2), \quad C_2 = f_5 \text{Vol}(S^2), \quad C_0 = f_7, \quad e^{-2\Phi} = f_6,$$

$$g(\rho) = 1 - e^{5(\rho_* - \rho)}.$$

The central charge, in this case reads

$$c = \frac{\mathcal{N}_{AdS_6}}{128\pi^6} \frac{\left(1 - e^{-5(\rho-\rho_*)}\right)^2}{\left(1 - \frac{3}{8}e^{-5(\rho-\rho_*)}\right)^4}.$$



The quantity divides into a part coming from the $SCFT_5$ and one coming from 'the flow". We have the strong dynamics of flavours+colours encoded here.

Some conclusions

The features discussed here for 3d $N = 4$, 4d $\mathcal{N} = 2$, 5d $\mathcal{N} = 1$, 6d $N = (1,0)$ SCFTs repeat in the cases of SUSY CFTs in 1d, 2d.

These systems can be thought in terms of D_p-D_{p+2}-NS_5 branes.

We study RG-flows away from these SCFTs, finding new SCFTs or a mass-gap behaviour. Interesting universality: some observables 'decouple' the flow from the CFT data.

Other field theory observables can be computed and checked in the conformal cases. There are nice connection with Integrability, not discussed today.

Unify the taxonomy of these backgrounds. Study how integrability and other observables behave under RG flows. It would be nice to extend these classifications to $\mathcal{N} = 1$ cases.