Consistent truncations, Kaluza-Klein spectrometry, and beyond

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plan: Consistent truncations & Kaluza-Klein spectrometry

motivation

- compactification, Kaluza-Klein spectra

tools

- consistent truncations
- exceptional field theory

examples

- $AdS_p \times S^q$ and deformations / squashed spheres
- non-supersymmetric AdS vacua, perturbative stability
- beyond consistent truncations

beyond spectra

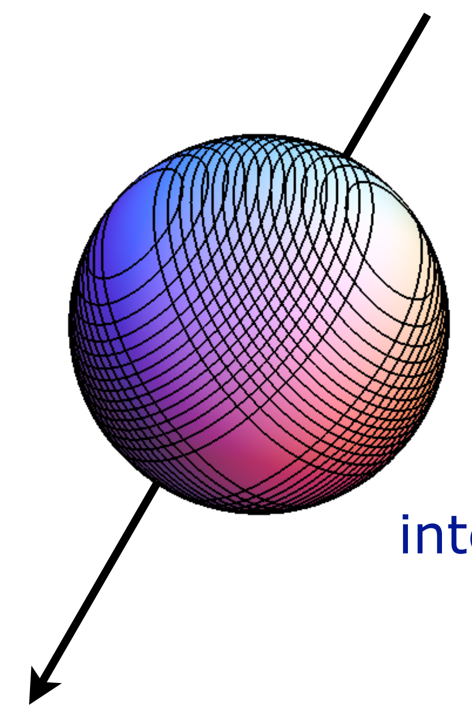
- cubic couplings / holography

based on work with Emanuel Malek, Bastien Duboeuf,

Nikolay Bobev, Camille Eloy, Michele Galli, Adolfo Guarino, Alfredo Giambrone, Olaf Hohm, Gabriel Larios, Hermann Nicolai, Brandon Robinson, Colin Sterckx, Mario Trigiante, Jesse van Muiden

motivation compactification & Kaluza-Klein spectra

higher-dimensional sugra



internal space

lower-dimensional sugra

- background $\mathscr{M}_{10} = \mathscr{M}_4 \times \mathscr{M}_6$
- expanding fields in harmonics on the internal space

  e.g. scalar field

$$\phi(x,y) = \sum_{\Sigma} \phi_{\Sigma}(x)\, \mathcal{Y}^{\Sigma}(y)$$

fluctuations ($\phi_{\Sigma}(x)$) — harmonics ($\mathcal{Y}^{\Sigma}(y)$)

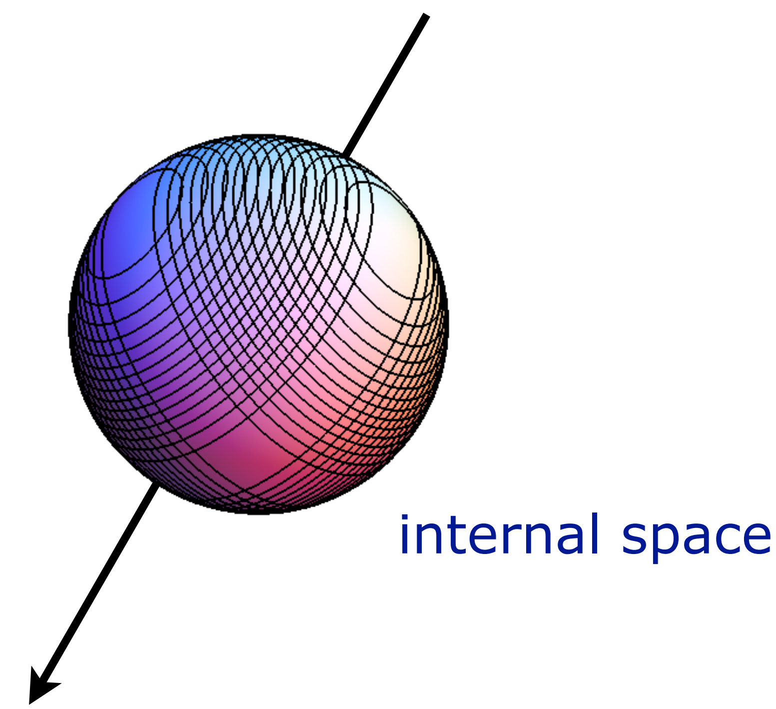
- such that the dynamics of the KK fluctuations is described by a lower-dimensional theory
- infinitely many fields (KK towers)
- mass spectrum of the KK-fluctuations
- in general: complicated problem
  - diagonalize various Laplacians on the internal manifold
  - disentangle mass eigenstates from different higher-dimensional origin
  - flux compactifications: higher-dimensional p-forms
- important: phenomenology, stability, holography, …

compactification & Kaluza-Klein spectra

$$\phi(x,y) = \sum_{\Sigma} \phi_\Sigma(x)\, \mathcal{Y}^\Sigma(y)$$

fluctuations ($\phi_\Sigma(x)$) — harmonics ($\mathcal{Y}^\Sigma(y)$)

higher-dimensional sugra



lower-dimensional sugra

- mass spectrum of the KK-fluctuations
- in general: complicated problem
- important: phenomenology, stability, holography, …

- some windows into the KK-spectrum:
  - universality in the spin-2 sector [Bachas, Estes]
    massless scalar wave equation in 10D
  - symmetry helps [Salam, Strathdee]
    symmetric spaces, large isometry groups
  - supersymmetry helps
    masses from representation theory

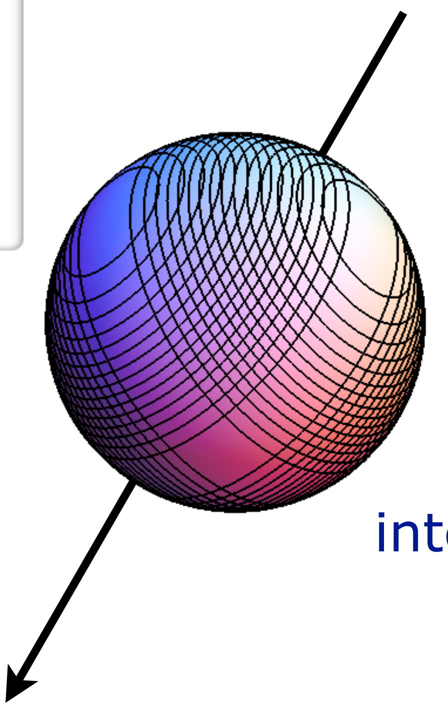
- if the background has no (super-)symmetries, we need <u>new tools</u> !

tools: consistent truncations

- definition

  - (non-linear) truncation to a finite number of KK-modes
  - described by a lower-dimensional supergravity
  - such that any solution of the lower-dimensional theory lifts to a solution of the higher-dimensional theory

higher-dimensional sugra



internal space

lower-dimensional sugra

- non-linear reduction ansatz, e.g. D=11 sugra on $AdS_4 \times S^7$

$$
\begin{aligned}
ds^2 &= \Delta^{-1}(x,y)\, g_{\mu\nu}(x)\, dx^\mu dx^\nu \\
&\quad + G_{mn}(x,y) \left(dy^m + \mathcal{K}_{[ab]}{}^m(y) A_\mu^{ab}(x) dx^\mu\right) \left(dy^n + \mathcal{K}_{[cd]}{}^n(y) A_\nu^{cd}(x) dx^\nu\right) \\
G^{mn}(x,y) &= \frac{1}{8}\, \Delta(x,y)\, \mathcal{K}_{[ab]}{}^m(y) \mathcal{K}_{[cd]}{}^n(y) \left(u^{ij}{}_{ab} + v^{ijab}\right)(x) \left(u^{ijab} + v^{ijab}\right)(x) \\
A_{kmn}(x,y) &= -\frac{\sqrt{2}i}{96}\, \Delta(x,y)\, G_{kp}(x,y)\, \mathcal{K}_{[ab]mn}(y) \mathcal{K}_{[cd]}{}^p(y) \left(u^{ij}{}_{ab} - v^{ijab}\right)(x) \left(u^{ijab} + v^{ijab}\right)(x)
\end{aligned}
$$

[de Wit, Nicolai 1987]

- used to be rare, now we have many...

> warning:
> in general, these are not effective field theories: the truncation removes modes of mass comparable to the modes kept

- still (very) useful!

tools: consistent truncations

- D=11 sugra on $AdS_4 \times S^7$ [de Wit, Nicolai 1987]
  - truncation to D=4 maximal supergravity (70 scalar fields)

gauge group SO(8)

$$\ell = 0: \quad \mathbf{35}_v + \mathbf{35}_c$$
$$\ell = 1: \quad \mathbf{112}_v + \mathbf{224}_c$$
$$\ell = 2: \quad \mathbf{1} + \mathbf{35}_s + \mathbf{300} + \mathbf{294}_v + \mathbf{840}_s$$
$$\ell = 3: \quad \mathbf{8}_v + \mathbf{224}_s + \mathbf{672}_v + \mathbf{1400}_v + \mathbf{2400}_v$$

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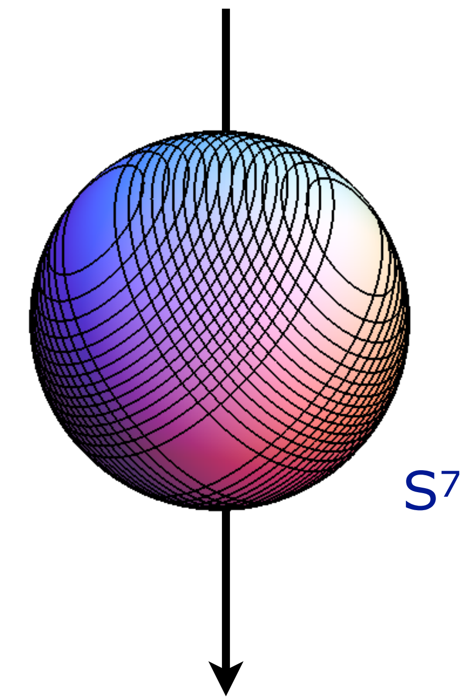
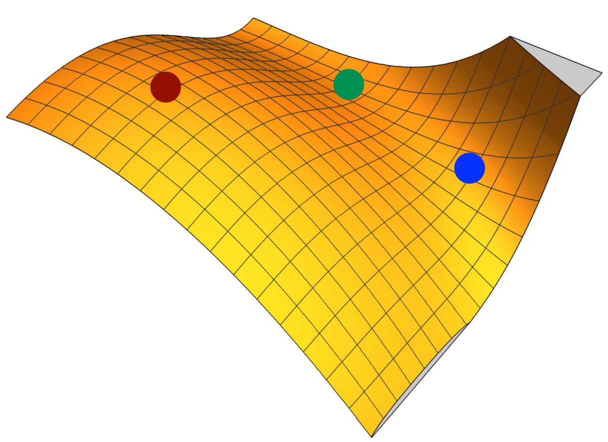
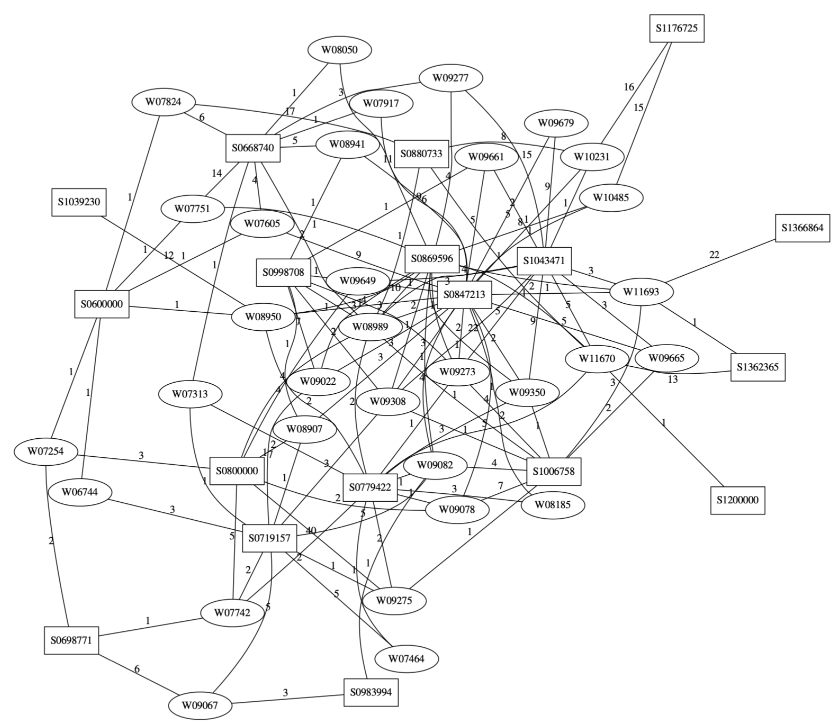
D=11 sugra



D=4 maximal sugra (70 scalars)

- every stationary point of the D=4 scalar potential lifts to a D=11 background of the form $\mathcal{M}_{11} = AdS_4 \times \mathcal{M}_7$





- around these backgrounds: compute the masses of the 70 scalars
- instabilities in almost all non-supersymmetric AdS_4 vacua !

[Comsa, Firsching, Fischbacher: "SO(8) Supergravity and the Magic of Machine Learning"]

new tools: exceptional field theory (ExFT) [Hohm, HS]

duality covariant formulation of D=11 supergravity

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\rm top} - V(\mathcal{M}, e).$$

- built after D=4 $\mathcal{N} = 8$ supergravity: 56 vectors $\mathcal{A}_\mu^M$, scalar target space E7/SU(8) [Cremmer,Julia]
- all fields live on external space-time : $\{x^\mu\}$ $\mu = 0, \ldots, 3$
  internal space : $\{y^m\}$ $m = 1, \ldots, 7$
- non-abelian gauge structure
  - (infinite-dimensional) non-abelian gauge structure: the internal diffeomorphisms (Kaluza-Klein) + tensor gauge symmetry
    ⟶ generalized diffeomorphisms [Coimbra, Strickland-Constable, Waldram]
  - non-abelian field strength

$$\mathcal{F}_{\mu\nu}{}^M = 2\partial_{[\mu} A_{\nu]}{}^M - 2\, A_{[\mu}{}^K \partial_K A_{\nu]}{}^M - \tfrac{1}{2} \left(24\, (t_\alpha)^{MK} (t^\alpha)_{NL} - \Omega^{MK} \Omega_{NL}\right)\, A_{[\mu}{}^N\, \partial_K A_{\nu]}{}^L - 12\, (t^\alpha)^{MN}\, \partial_N B_{\mu\nu\,\alpha}$$

- embedding $\partial_m \longrightarrow \partial_M$ subject to the section constraint $(t_\alpha)^{MN}\, \partial_M \otimes \partial_N = 0$
  covariant restriction from 56 down to 7 (6) coordinates

new tools: consistent truncations from ExFT

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

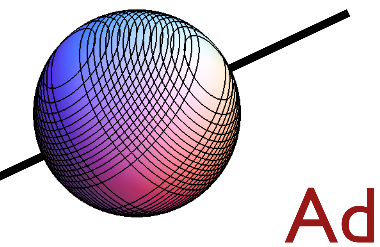
dictionary

generalized Scherk-Schwarz reduction of ExFT

$$\mathcal{M}_{MN}(x,Y) \;=\; U_M{}^K(Y)\, M_{KL}(x)\, U_N{}^L(Y)$$

$$\mathcal{A}_\mu{}^M(x,Y) \;=\; \rho^{-1}(Y)\, (U^{-1})_K{}^M(Y)\, A_\mu{}^K(x)$$

D=11 sugra



AdS$_4$ x S^7

$$ds^2 = \Delta^{-1}(x,y)\, g_{\mu\nu}(x)\, dx^\mu dx^\nu + G_{mn}(x,y) \left(dy^m + \mathcal{K}_{[ab]}{}^m(y) A_\mu^{ab}(x) dx^\mu\right) \left(dy^n + \mathcal{K}_{[cd]}{}^n(y) A_\nu^{cd}(x) dx^\nu\right)$$

$$G^{mn}(x,y) = \frac{1}{8} \Delta(x,y)\, \mathcal{K}_{[ab]}{}^m(y) \mathcal{K}_{[cd]}{}^n(y) \left(u^{ij}{}_{ab} + v^{ijab}\right)(x) \left(u^{ijab} + v^{ijab}\right)(x)$$

$$A_{kmn}(x,y) = -\frac{\sqrt{2}i}{96} \Delta(x,y)\, G_{kp}(x,y)\, \mathcal{K}_{[ab]mn}(y) \mathcal{K}_{[cd]}{}^p(y) \left(u^{ij}{}_{ab} - v^{ijab}\right)(x) \left(u^{ijab} + v^{ijab}\right)(x)$$

D=4 maximal sugra

$E_{7(7)}$ valued twist matrix $U_M{}^N(Y)$ and scale factor $\rho(Y)$

consistency equations (generalized Leibniz parallelizable)

$$\left[(U^{-1})_M{}^P (U^{-1})_N{}^L\, \partial_P U_L{}^K\right]_{\mathbf{912}} \overset{!}{=} \rho\, X_{MN}{}^K$$

embedding tensor of the D=4 gauged supergravity

powerful tool to construct consistent truncations

new tools: Kaluza-Klein spectroscopy from ExFT

[E. Malek, HS]

- consistent truncation to lowest KK-multiplet

$$\mathcal{A}_\mu{}^M(x,Y) \;=\; \rho^{-1}(Y)\,(U^{-1})_K{}^M(Y)\,A_\mu{}^K(x)$$

- extend to the higher Kaluza-Klein modes (linearized)

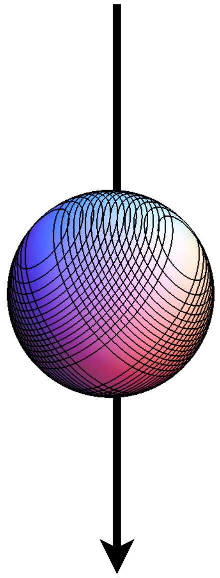
$$\mathcal{A}_\mu{}^M(x,Y) \;=\; \rho^{-1}(Y)\,(U^{-1})_K{}^M(Y)\,\sum_\Sigma A_\mu{}^{K,\Sigma}(x)\,\mathcal{Y}^\Sigma$$

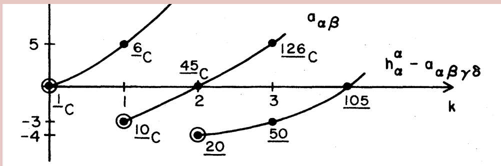
$$\mathcal{M}_{MN}(x,Y) \;=\; U_M{}^K(Y)\,U_N{}^L(Y)\Big(\delta_{KL}+\sum_\Sigma \phi^{\alpha,\Sigma}\,\mathbb{T}_{\alpha,KL}\,\mathcal{Y}^\Sigma\Big)$$

with fluctuations $A_\mu{}^{K,\Sigma}\,,\;\phi^{\alpha,\Sigma}\,,$

and the tower of scalar harmonics $\mathcal{Y}^\Sigma$

D=11 sugra





- (lowest KK multiplet) $\otimes$ (scalar harmonics) —ExFT field equations→ KK spectrum

trace of exceptional symmetry in the full spectrum ⟶ holography!

$$\mathcal{A}_\mu{}^M(x,Y) = \rho^{-1}(Y)\,(U^{-1})_K{}^M(Y) \sum_\Sigma A_\mu{}^{K,\Sigma}(x)\,\mathcal{Y}^\Sigma$$

$$\mathcal{M}_{MN}(x,Y) = U_M{}^K(Y)\,U_N{}^L(Y)\left(\delta_{KL} + \sum_\Sigma j_{KL,\Sigma}(x)\,\mathcal{Y}^\Sigma\right)$$

- plug into the ExFT action and linearize in fluctuations
- e.g. mass matrix for <u>all</u> vector fluctuations $A_\mu{}^{M,\Sigma}$

$$M_{M\Sigma,N\Omega} \propto \frac{1}{3} X^{\rm s}_{ML}{}^K X^{\rm s}_{NK}{}^L \delta^{\Sigma\Omega} + 2\left(X^{\rm s}_{MK}{}^N - X^{\rm s}_{NM}{}^K\right)\mathcal{T}_{K,\Omega\Sigma} - 6\left(\mathbb{P}^K{}_M{}^L{}_N + \mathbb{P}^M{}_K{}^L{}_N\right)\mathcal{T}_{L,\Omega\Lambda}\mathcal{T}_{K,\Lambda\Sigma} + \frac{8}{3}\mathcal{T}_{N,\Omega\Lambda}\mathcal{T}_{M,\Lambda\Sigma}$$

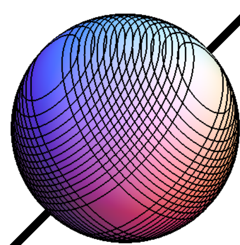
in terms of essentially four-dimensional data !

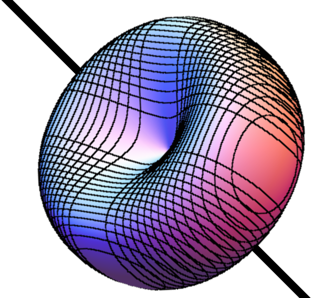
- symmetrized D=4 embedding tensor $X^{\rm s}_{MN}{}^K \equiv X_{MN}{}^K + X_{MK}{}^N$
- adjoint projector $\mathbb{P}^M{}_N{}^K{}_L = (t^\alpha)_N{}^M (t_\alpha)_L{}^K$
- representation of scalar harmonics $\mathcal{K}_M{}^m \partial_m \mathcal{Y}^\Sigma = \mathcal{T}_{M,\Sigma\Omega}\,\mathcal{Y}^\Omega$

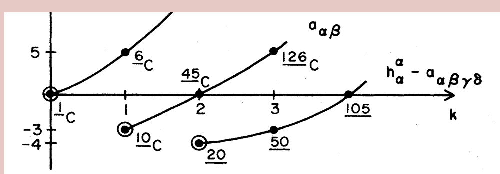
- similar for the scalar mass matrix
- similar for fermion masses [Cesàro, Varela]
- entire KK mass spectrum!

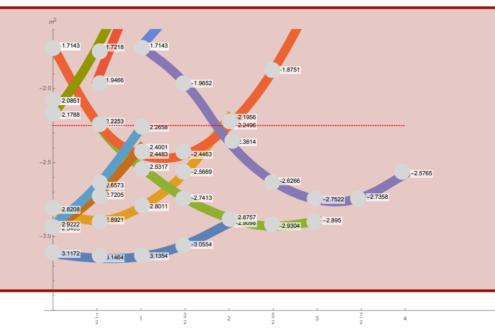
examples: AdS$_4$ x S^7 and deformations

D=11 sugra









simple and compact (re-)derivation of the supergravity spectrum on S^7

[1980's: Biran, Casher, Englert, Nicolai, Rooman, Spindel, Sezgin]

direct identification of BPS multiplet components within D=11 supergravity

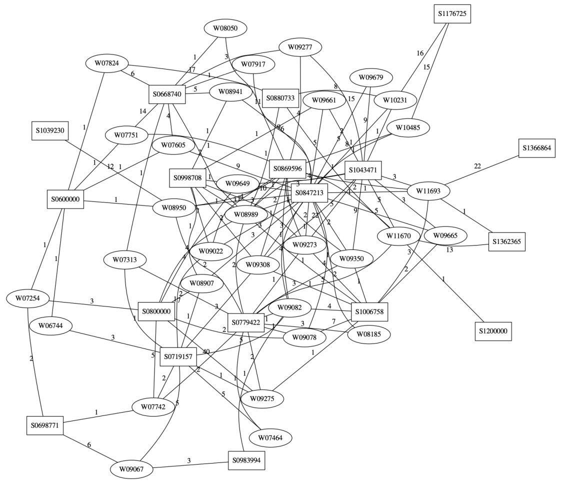
$$A_\mu{}^{K,\Sigma}\ ,\ \phi^{\alpha,\Sigma}$$

tools for non-supersymmetric vacua (where masses are not controlled by symmetry)

example: non-supersymmetric AdS$_4$ vacua SO(3) x SO(3)

- in D=4 SO(8) supergravity, the supergravity potential has been carefully scanned for AdS$_4$ vacua

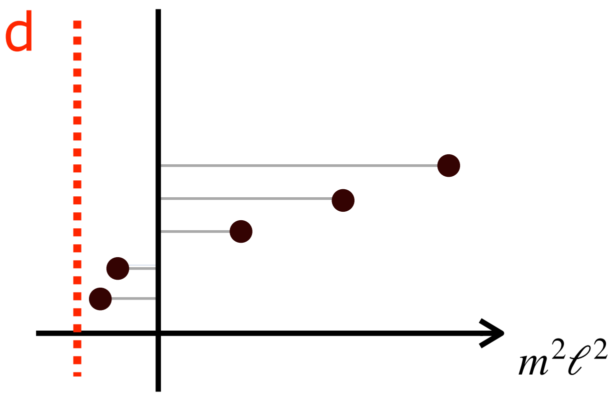
  [Comsa, Firsching, Fischbacher]



all non-supersymmetric vacua are unstable already within D=4 supergravity, i.e. have instabilities within the lowest Kaluza-Klein multiplet

- except for a distinguished SO(3) x SO(3) invariant extremal point [Warner]
  - stable within D=4 supergravity [Fischbacher, Pilch, Warner]
  - uplift to D=11 supergravity [Godazgar, Godazgar, Krüger, Nicolai, Pilch]
  - brane-jet instabilities [Bena, Pilch, Warner]

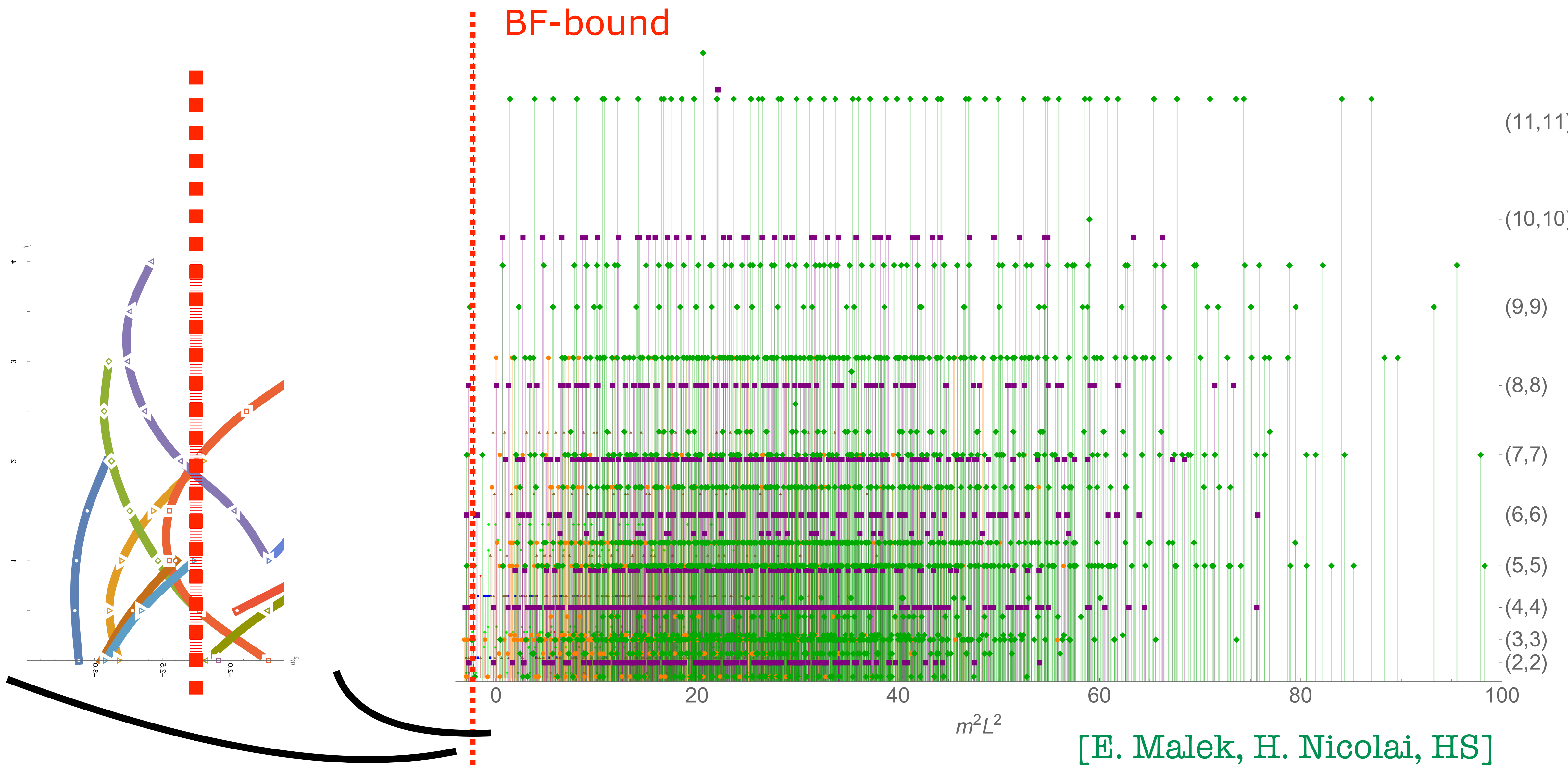
70 scalars: BF-bound



beyond ?

example: non-supersymmetric AdS_4 vacua SO(3) x SO(3)

- ExFT formulas: full scalar Kaluza-Klein spectrum up to level 6
  (~ 100.000 scalar fields), from D=4 data



[E. Malek, H. Nicolai, HS]

instabilities starting from KK level 2

$\Longrightarrow$ In D=4, SO(8) supergravity, all known non-supersymmetric vacua are perturbatively unstable!

example: non-supersymmetric AdS$_4$ vacua in ISO(7) supergravity

- massive IIA admits a consistent truncation on S^6 [Guarino, Jafferis, Varela]
  ⟶ to (dyonic) ISO(7) gauged supergravity [Dall'Agata, Inverso]
  with $\mathcal{N}=3$ AdS$_4$ vacuum

- the D=4 scalar potential carries a wealth of AdS vacua:
  ⟶ non-supersymmetric vacua, stable within D=4 supergravity

- most symmetric: $\mathcal{N}=0$ G$_2$ vacuum, deformed S^6
  ⟶ no brane-jet instabilities [Guarino, Tarrio, Varela]

- ExFT analysis yields the full KK spectrum!

$$\begin{aligned}\langle \mathcal{J}\mathcal{M}\mathcal{J}\rangle \;\propto\; & \tfrac{1}{5}\left(X_{\underline{AE}}{}^{\underline{F}}X_{\underline{BE}}{}^{\underline{F}} + X_{\underline{EA}}{}^{\underline{F}}X_{\underline{EB}}{}^{\underline{F}} + X_{\underline{EF}}{}^{\underline{A}}X_{\underline{EF}}{}^{\underline{B}} + 5\,X_{\underline{AE}}{}^{\underline{F}}X_{\underline{BF}}{}^{\underline{E}}\right)\mathcal{J}_{\underline{AD},\Sigma}\,\mathcal{J}_{\underline{BD},\Sigma}\\ &+\frac{2}{5}\left(X_{\underline{AC}}{}^{\underline{E}}X_{\underline{BD}}{}^{\underline{E}} - X_{\underline{AE}}{}^{\underline{C}}X_{\underline{BE}}{}^{\underline{D}} - X_{\underline{EA}}{}^{\underline{C}}X_{\underline{EB}}{}^{\underline{D}}\right)\mathcal{J}_{\underline{AB},\Sigma}\,\mathcal{J}_{\underline{CD},\Sigma}\\ &-\frac{4}{5}\left(X_{\underline{AC}}{}^{\underline{D}}\mathcal{T}_{\underline{B},\Omega\Sigma} + 6\,X_{\underline{AC}}{}^{\underline{B}}\mathcal{T}_{\underline{D},\Omega\Sigma}\right)\mathcal{J}_{\underline{AB},\Sigma}\,\mathcal{J}_{\underline{CD},\Omega}\\ &-\frac{4}{5}\left(X_{\underline{CA}}{}^{\underline{B}}\mathcal{T}_{\underline{C},\Omega\Sigma} + 6\,X_{\underline{BC}}{}^{\underline{A}}\mathcal{T}_{\underline{C},\Omega\Sigma}\right)\mathcal{J}_{\underline{AD},\Sigma}\,\mathcal{J}_{\underline{BD},\Omega}\\ &+12\,\mathcal{J}_{\underline{AD},\Sigma}\,\mathcal{J}_{\underline{BD},\Omega}\mathcal{T}_{\underline{A},\Omega\Lambda}\mathcal{T}_{\underline{B},\Lambda\Sigma} - \mathcal{J}_{\underline{AB},\Sigma}\,\mathcal{J}_{\underline{AB},\Omega}\mathcal{T}_{\underline{C},\Omega\Lambda}\mathcal{T}_{\underline{C},\Lambda\Sigma}\end{aligned}$$

  analytic mass formula for all scalars:

$$m^2\ell^2 = (n+2)(n+3) - \frac{3}{2}\,\mathcal{C}_{[n_1,n_2]}$$

[A. Guarino, E. Malek, HS]

KK level n, G$_2$ Casimir $\mathcal{C}_{[n_1,n_2]}$

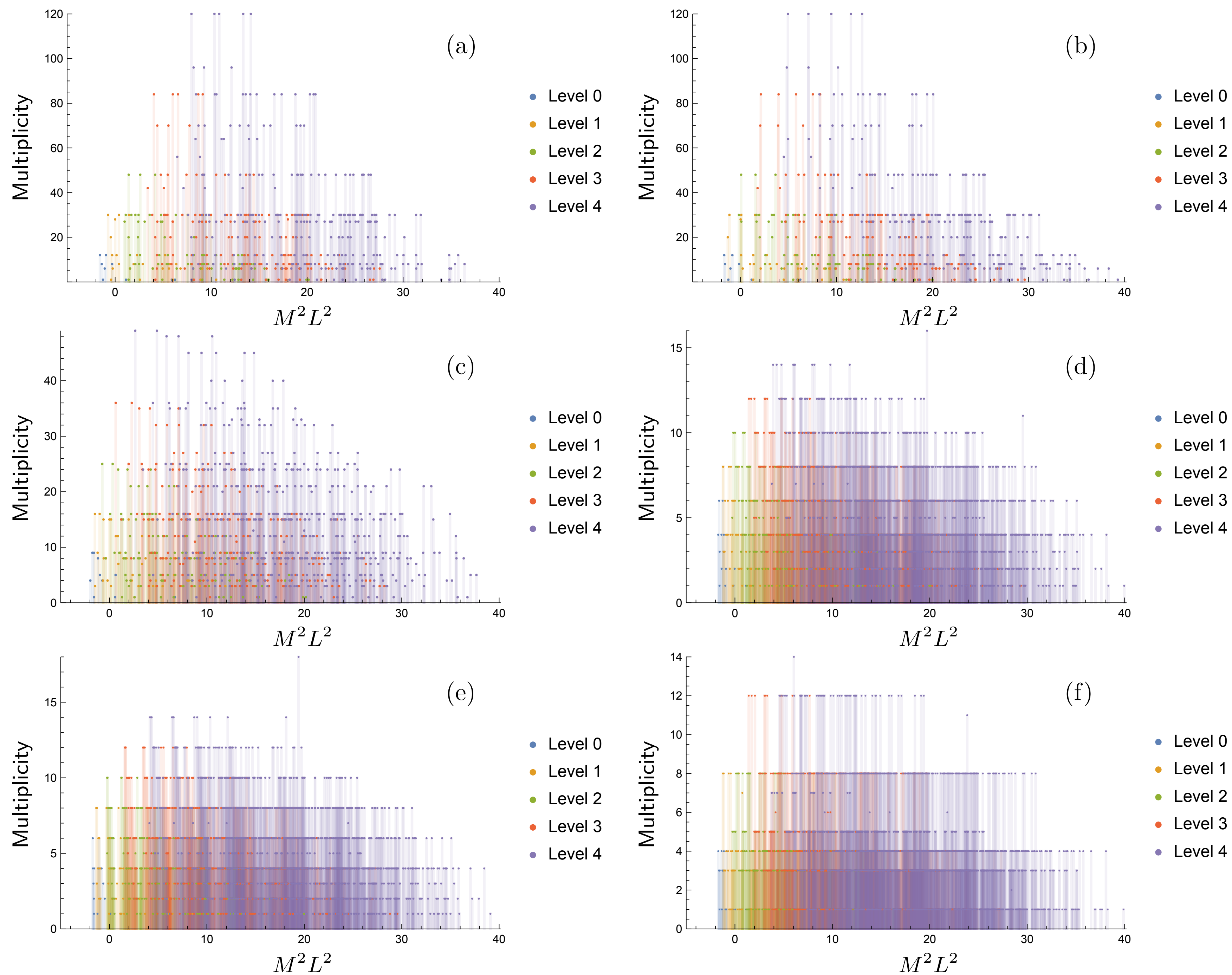
- proves stability of the KK spectrum: $m^2\ell^2 \geq m^2_{\rm BF}\,\ell^2$

- (perturbatively) stable non-supersymmetric AdS$_4$ vacuum

- bubble instabilities… [Bomans, Cassani, Dibitetto, Petri]

example: non-supersymmetric AdS₄ vacua in ISO(7) supergravity

- likewise: KK-spectra for more non-supersymmetric vacua (numerical) with remaining SU(3), SO(4), U(2), SO(3): all (perturbatively) stable!

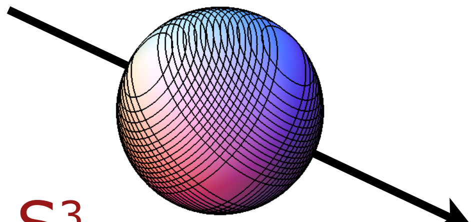


example: $AdS_3 \times S^3$ and deformations

[with C. Eloy and G. Larios]

example: $AdS_3 \times S^3$ and deformations

D=6 minimal sugra

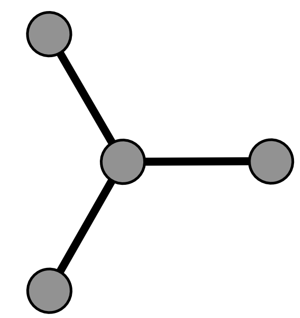


$AdS_3 \times S^3$

D=3 gauged sugra

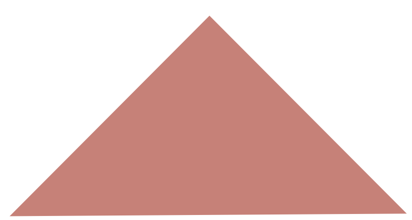
gauge group SO(4)
scalar coset SO(4,4) / (SO(4) x SO(4))
with matter: SO(4,4+n) / (SO(4) x SO(4+n))

- twist matrix $U_M{}^N \in \mathrm{SO}(4,4)$ ⟶ triality (equivalent uplifts)



$\mathcal{N} = (2,0)$ sugra

$U_M{}^N$



$\mathcal{N} = (1,1)$ sugra $U_M{}^N$ $U_M{}^N$ $\mathcal{N} = (1,1)$ sugra
different consistent truncation

- with matter: inequivalent embeddings $\mathrm{SO}(4,4) \subset \mathrm{SO}(4,4+n)$

example: AdS_3 x S^3 and deformations

- full Kaluza-Klein spectra [C. Eloy] in SU(2|1,1) supermultiplets $(\mathbf{n})^{j_1,j_2}_{\Delta_L}$

$n = 0$: $(\mathbf{3})^{(1/2,1/2)}_{1/2} \quad (\mathbf{3})^{(0,0)}_{2} + (\mathbf{3})^{(0,1)}_{1}$

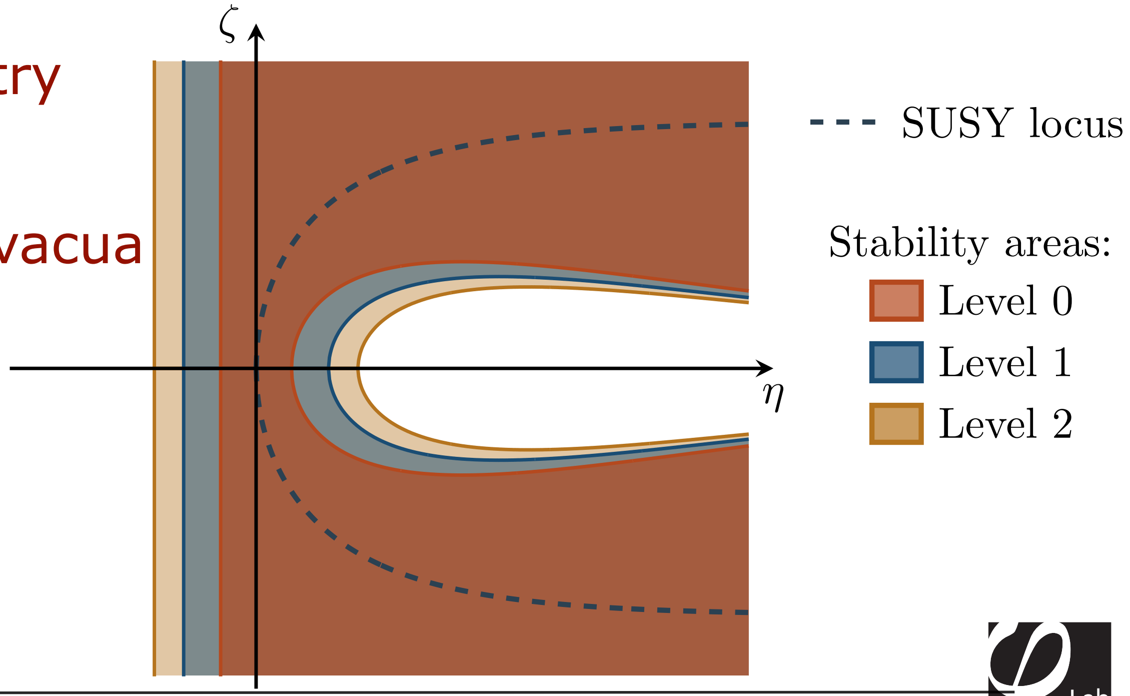
$n = 1$: $(\mathbf{4})^{(1/2,1)}_{1} + (\mathbf{4})^{(1/2,0)}_{2} \quad (\mathbf{2})^{(0,1/2)}_{5/2} + (\mathbf{2})^{(0,3/2)}_{3/2} + (\mathbf{2})^{(0,1/2)}_{1/2} + (\mathbf{4})^{(0,1/2)}_{5/2} + (\mathbf{4})^{(0,3/2)}_{3/2} + (\mathbf{4})^{(0,1/2)}_{1/2} \quad (\mathbf{2})^{(1/2,0)}_{2} + (\mathbf{2})^{(1/2,1)}_{1}$

$n = 2$: $(\mathbf{5})^{(1/2,(1)/2)}_{5/2} + (\mathbf{5})^{(1/2,(3)/2)}_{3/2} \quad (\mathbf{3})^{(0,2/2)}_{3} + (\mathbf{3})^{(0,2}_{2} + (\mathbf{3})^{(0,0)}_{2} + (\mathbf{3})^{(0,1)}_{1} + (\mathbf{5})^{(0,1)}_{3} + (\mathbf{5})^{(0,2)}_{2} + (\mathbf{5})^{(0,0)}_{2} + (\mathbf{5})^{(0,1)}_{1} \quad (\mathbf{3})^{(1/2,1/2)}_{5/2} + (\mathbf{3})^{(1/2,3/2)}_{3/2}$

$n = 3$: $(\mathbf{6})^{(1/2,1)}_{3} + (\mathbf{6})^{(1/2,2)}_{2} \quad (\mathbf{4})^{(0,3/2)}_{7/2} + (\mathbf{4})^{(0,5/2)}_{5/2} + (\mathbf{4})^{(0,1/2)}_{5/2} + (\mathbf{4})^{(0,3/2)}_{3/2} + (\mathbf{6})^{(0,3/2)}_{7/2} + (\mathbf{6})^{(0,5/2)}_{5/2} + (\mathbf{6})^{(0,1/2)}_{5/2} + (\mathbf{6})^{(0,3/2)}_{3/2} \quad (\mathbf{4})^{(1/2,1)}_{(6)/2} + (\mathbf{4})^{(1/2,2)}_{2}$

$\tilde{n} = 0$, $\tilde{n} = 1$, $\tilde{n} = 2$, $\tilde{n} = 3$

- two-parameter deformation (η, ζ), (flat directions in the scalar potential)
  - $SO(4) \longrightarrow U(1)^2$ squashed S^3
  - one-parameter curve preserving supersymmetry
  - moduli-dependent KK spectrum: stability region for non-supersymmetric AdS_3 vacua



example: squashed S^7 / beyond consistent truncations

[B. Duboeuf, E. Malek, HS]

example: squashed S^7

- $AdS_4 \times S^7_{\text{squashed}}$ Freund-Rubin solution of 11D supergravity ($\mathcal{N} = 0,1$)

$$\frac{SO(8)}{SO(7)} \sim S^7 \sim \frac{Sp(2)\times Sp(1)}{Sp(1)\times Sp(1)}$$

[Awada, Duff, Nilsson, Pope]

- deformation of the round S^7 by scalars from higher KK levels

$$\ell = 0: \quad \mathbf{35}_v + \boxed{\mathbf{35}_c}$$
$$\ell = 1: \quad \mathbf{112}_v + \mathbf{224}_c$$
$$\ell = 2: \quad \boxed{\mathbf{1}} + \mathbf{35}_s + \boxed{\mathbf{300}} + \mathbf{294}_v + \mathbf{840}_s$$
$$\ell = 3: \quad \mathbf{8}_v + \mathbf{224}_s + \mathbf{672}_v + \mathbf{1400}_v + \mathbf{2400}_v$$
$$\ell = 4: \quad \mathbf{35}_v + \boxed{\mathbf{840}_s} + \mathbf{1386}_v + \mathbf{4312}_v + \mathbf{5775}_v$$

$Sp(2)\times Sp(1)$ singlets

- preserves only $Sp(2)\times Sp(1)$ isometries
- not within D=4, SO(8) supergravity!
- space invader scenario (massless gravitino from higher KK level)

general formulas for mass operators in terms of different Laplacians on the internal space [Duff, Nilsson, Pope]

combine with spectrum of different Laplacians on the internal space [Nilsson, Pope, Yamagishi, Ekhammar, Karlsson]

Table 5
Mass operators from the Freund–Rubin ansatz

Spin	Mass operator
2^+	Δ_0
$(3/2)^{(1),(2)}$	$\not{D}_{1/2} + 7m/2$
$1^{-(1),(2)}$	$\Delta_1 + 12m^2 \pm 6m(\Delta_1 + 4m^2)^{1/2}$
1^+	Δ_2
$(1/2)^{(4),(1)}$	$\not{D}_{1/2} - 9m/2$
$(1/2)^{(3),(2)}$	$3m/2 - \not{D}_{3/2}$
$0^{+(1),(3)}$	$\Delta_0 + 44m^2 \pm 12m(\Delta_0 + 9m^2)^{1/2}$
$0^{+(2)}$	$\Delta_L - 4m^2$
$0^{-(1),(2)}$	$Q^2 + 6mQ + 8m^2$

example: squashed S^7

ExFT embedding

- consistent $\mathcal{N}=1$ truncation to $\mathrm{Sp}(2)\times\mathrm{Sp}(1)$ singlets: 4 scalars

$$
\begin{aligned}
\ell=0:&\quad \mathbf{35}_v+\boxed{\mathbf{35}_c}\\
\ell=1:&\quad \mathbf{112}_v+\mathbf{224}_c\\
\ell=2:&\quad \boxed{\mathbf{1}}+\mathbf{35}_s+\boxed{\mathbf{300}}+\mathbf{294}_v+\mathbf{840}_s\\
\ell=3:&\quad \mathbf{8}_v+\mathbf{224}_s+\mathbf{672}_v+\mathbf{1400}_v+\mathbf{2400}_v\\
\ell=4:&\quad \mathbf{35}_v+\boxed{\mathbf{840}_s}+\mathbf{1386}_v+\mathbf{4312}_v+\mathbf{5775}_v
\end{aligned}
$$

- not a truncation of D=4, SO(8) sugra
- coset space $\left(\mathrm{SL}(2)/\mathrm{SO}(2)\right)^2$

  cf. [Cassani, Koerber]

- embedding by a new twist matrix

$$
\mathcal{M}_{MN}(x,Y) \;=\; U_M{}^K(Y)\,U_N{}^L(Y)\left(\delta_{KL}+\sum_\Sigma j_{KL,\Sigma}(x)\,\mathcal{Y}^\Sigma \;+\ldots\right)
$$

$$
\boxed{U_M{}^N(Y)=\hat{U}_M{}^K(Y)\,\mathbb{S}(Y)_K{}^N}
$$

round S^7 — rotation

- generalized ~~Leibniz~~ parallelizable $\quad [\mathcal{U}_A,\mathcal{U}_B]=X_{AB}{}^C(y)\,\mathcal{U}_C$

  (no maximal truncation)

- (modified) ExFT mass formulas still apply!

example: squashed S^7

- ExFT embedding
  - consistent $\mathcal{N}=1$ truncation to $\mathrm{Sp}(2)\times\mathrm{Sp}(1)$ singlets: 4 scalars
  - embedding by a new twist matrix
  - (modified) ExFT mass formulas still apply!

- provide the complete spectrum

$\mathcal{N}=1$ multiplets $L[J]$

e.g. $\underbrace{[k,q,k]}_{\mathrm{Sp}(2)\times\mathrm{Sp}(1)\ \text{irrep}}$: $L[\tfrac{3}{2}]\otimes[0] \oplus L[\tfrac{3}{2}]\otimes[\tfrac{1}{2}] \oplus L[\tfrac{1}{2}]\otimes[\tfrac{1}{2}\otimes\tfrac{1}{2}] \oplus L[0]\otimes[\tfrac{1}{2}\otimes 1]$

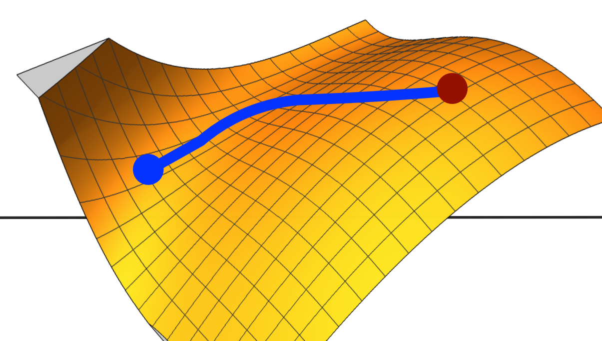
extra charge s

- universal mass formula:

$$\Delta_{J,s} = 1 + \frac{5}{3}s + \frac{1}{3}\sqrt{(3J+2s^2)^2 + 5\,\mathscr{C}_{\mathrm{Sp}(2)} + 15\,\mathscr{C}_{\mathrm{Sp}(1)}}$$

- holographic RG flow

[with M.Galli]



correlation functions

beyond spectra: cubic couplings

[B. Duboeuf, E. Malek, HS]

beyond: cubic couplings

- cubic couplings around $AdS_5 \times S^5$

$$\mathscr{L}_{\phi^3} = g_{I_1 I_2 I_3}\, \phi^{I_1} \phi^{I_2} \phi^{I_3}$$

  - information on the holographic 3-pt functions $\left\langle \mathcal{O}_{I_1} \mathcal{O}_{I_2} \mathcal{O}_{I_3} \right\rangle$
  - e.g. computed explicitly for

$$\mathcal{O}_{I_1} = \mathcal{O}^{i_1 \cdots i_n} = \mathrm{Tr}\left[X^{i_1} X^{i_2} \cdots X^{i_n} \right]$$

[Lee, Minwalla, Rangamani, Seiberg]
[Arutyunov, Frolov]

  - and matched with $\mathcal{N}=4$ SYM
  - expand/diagonalize/disentangle IIB field equations
  - gauge fixing, non-linear field redefinitions

- in ExFT variables
  - & basis of fluctuations $\mathcal{M}_{MN} = U_M{}^K(Y)\, U_N{}^L(Y) \Big(\delta_{KL} + \sum_\Sigma \phi^{\alpha,\Sigma}\, \mathbb{T}_{\alpha,KL}\, \mathcal{Y}^\Sigma \Big)$
  - tensor product structure (algebra) $\otimes$ (scalar harmonics)

$$\mathscr{L}_{\phi^3} = g_{\alpha\Sigma,\beta\Delta,\gamma\Omega}\, \phi^{\alpha,\Sigma} \phi^{\beta,\Delta} \phi^{\gamma,\Omega}$$

  - universal expressions for the cubic couplings

beyond: cubic couplings

- universal expressions for the cubic couplings

$$\mathcal{L}_{\phi^3} = \phi^{\alpha\Sigma}\phi^{\beta\Delta}\phi^{\gamma\Omega}\left\{c^{\Sigma\Delta\Omega}\,\mathbb{X}_{\alpha\beta\gamma} + c^{\Delta\Omega\Lambda}\,\mathcal{T}_A{}^{\Lambda\Sigma}\,\mathbb{X}_{A\,\alpha\beta\gamma} + c^{\Delta\Omega\Lambda}\,\mathcal{T}_A{}^{\Lambda\Theta}\mathcal{T}_B{}^{\Theta\Sigma}\,\mathbb{X}_{AB\,\alpha\beta\gamma}\right\}$$

representation matrix on harmonics

symmetric SO(6) tensors

  - with tensors

$$\mathbb{X}_{AB\,\alpha\beta\gamma} = 6\,\mathbb{T}_{\gamma\alpha\beta\,A}{}^{B} - \frac{3}{2}\,\mathbb{T}_{\gamma\,A}{}^{B}\,\eta_{\alpha\beta}$$

$$\mathbb{X}_{A\,\alpha\beta\gamma} = -X_{BC}{}^{D}\,\mathbb{T}_{[\alpha\gamma]\,D}{}^{C}\,\mathbb{T}_{\beta A}{}^{B} - X_{BC}{}^{D}\,\mathbb{T}_{\alpha\,D}{}^{B}\mathbb{T}_{\beta\gamma\,A}{}^{C} + X_{AB}{}^{C}\,\mathbb{T}_{\beta\alpha\gamma\,C}{}^{B} + \ \dots$$

$$\mathbb{X}_{\alpha\beta\gamma} = \frac{3}{5}\,X_{AB}{}^{C}X_{DE}{}^{F} \times \left\{\delta^{BE}\mathbb{T}_{\alpha\,A}{}^{B}\mathbb{T}_{\beta\gamma\,F}{}^{C} + \delta^{AD}\mathbb{T}_{\alpha\,B}{}^{E}\mathbb{T}_{\beta\gamma\,F}{}^{C} + \delta_{CF}\mathbb{T}_{\beta\,A}{}^{D}\mathbb{T}_{\alpha\gamma\,B}{}^{E} + \ \dots \ \right\}$$

products of E_6 generators

embedding tensor

- reproduce and extend the known results on $AdS_5 \times S^5$

- apply to all other vacua in the theory! e.g. $\mathcal{N}=2$ Leigh-Strassler SCFT

conclusions

- new tools for the analysis of Kaluza-Klein spectra from ExFT
- entire Kaluza-Klein spectra entirely encoded in 4-dim data:
  - embedding tensor $X_{MN}{}^{K}$ of the lower-dimensional supergravity
  - representation $(\mathcal{T}_M)_\Sigma{}^\Lambda$ of the scalar harmonics

$$M_{M\Sigma,N\Omega} \;\propto\; \frac{1}{3}\, X^{\rm s}_{ML}{}^{K}\, X^{\rm s}_{NK}{}^{L}\, \delta^{\Sigma\Omega} + 2\left(X^{\rm s}_{MK}{}^{N} - X^{\rm s}_{NM}{}^{K}\right) \mathcal{T}_{K,\Omega\Sigma}$$
$$-6\,\left(\mathbb{P}^{K}{}_{M}{}^{L}{}_{N} + \mathbb{P}^{M}{}_{K}{}^{L}{}_{N}\right) \mathcal{T}_{L,\Omega\Lambda}\mathcal{T}_{K,\Lambda\Sigma} + \frac{8}{3}\,\mathcal{T}_{N,\Omega\Lambda}\mathcal{T}_{M,\Lambda\Sigma} \qquad \text{etc.}$$

- access to vacua
  - with few or no (super-)symmetries
  - within and beyond consistent truncations
- applications to holography, stability analysis, moduli-dependence
- extension to cubic and higher couplings
- universal patterns in mass spectra & cubic couplings: holography!