QFT in AdS and the flat-space limit

Balt van Rees

Ecole Polytechnique

CPHT Centre de Physique Théorique

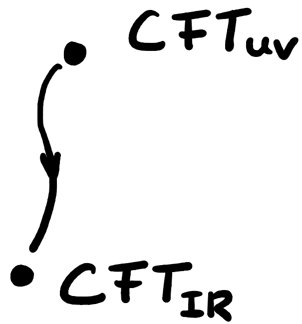
cnrs

l'X ECOLE POLYTECHNIQUE

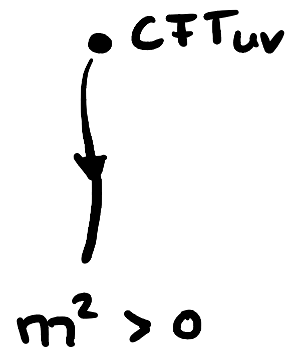
INSTITUT POLYTECHNIQUE DE PARIS

Can we bootstrap non-conformal QFTs?

two types:



gapless



gapped

this talk: progress for both using QFT in AdS

Generalities

Q: UV fixed point: CFT in AdS?

A: easy, since:

$$ds^2 = \left(\frac{R}{z}\right)^2 (d\vec{x}^2 + dz^2) \underset{\text{Weyl}}{\sim} ds^2 = d\vec{x}^2 + dz^2 \quad \text{with } z \geq 0$$

recipe:

- CFT on half space (BCFT)

  ex: $\langle O(x) \rangle = a\, z^{-\Delta}$ $\qquad \langle O(x) O(y) \rangle = f(\xi)\, z_x^{-\Delta} z_y^{-\Delta}$

- Weyl rescale to AdS

  ex: $\langle O(x) \rangle = a\, R^{-\Delta}$ $\qquad \langle O(x) O(y) \rangle = f(\underline{\xi})\, R^{-2\Delta}$

  geodesic distance

- done!

More generalities

Q : perturbed CFTs in AdS?

Note: in BCFT: $\langle \dots \exp\left[-\lambda \int_M \mathcal{O} - \hat{\lambda} \int_{\partial M} \hat{\mathcal{O}}\right] \rangle$

ex: Virasoro minimal model flows

$$\begin{tikzcd} m \arrow[d] \\ m-1 \end{tikzcd} \quad \rightsquigarrow \text{ add boundary}$$

$$\begin{tikzcd}
\cdots & (a_1, a_2)_m \arrow[l] \arrow[r] \arrow[d] & \bigoplus_\ell (a_1 + a_2 - 2\ell, 1)_m \arrow[d] & (a_1, a_2 - 1)_m \arrow[l] \arrow[d] \\
\cdots & (a_2 - 1, a_1)_{m-1} \arrow[l] \arrow[r] & \bigoplus_\ell \cdots & \cdots \arrow[l]
\end{tikzcd}$$

[Fredenhagen, Gaberdiel, Schmidt-Colinet 2009]

A : in AdS: bdy. couplings <u>fixed</u> by symmetry

ex: $\begin{tikzcd} m \arrow[d] \\ m-1 \end{tikzcd} \quad \rightsquigarrow \text{ in } AdS_2 \qquad \begin{tikzcd} (a_1, a_2)_m \arrow[d] \\ (a_2, a_1)_{m-1} \end{tikzcd}$

[w.i.p. with E. Lauria and M. Milam]

Even more generalities

AdS covariance along the flow $\longrightarrow$

boundary conformal covariance along the flow

e.g. $\langle \hat{O}(\vec{x})\, \hat{O}(0) \rangle = 1/|\vec{x}|^{2\hat{\Delta}}$

with $\hat{\Delta} = \hat{\Delta}(\mu R)$

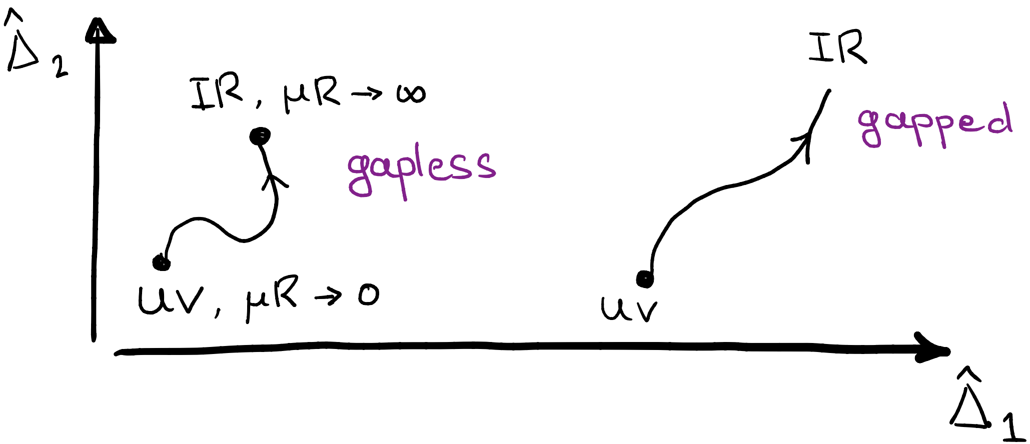
AdS radius, $Ric \sim R^{-2}$

RG scale

$\longrightarrow$ one-parameter family of solutions to conformal crossing equations.

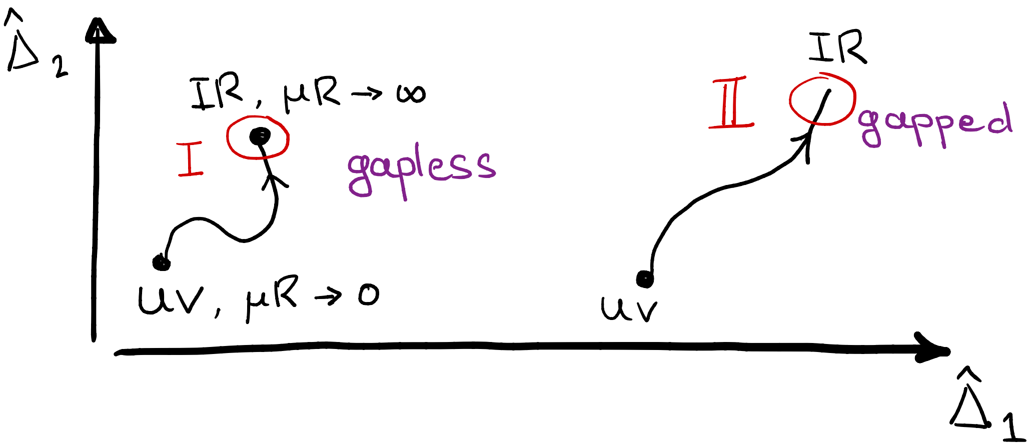
Q: can we bootstrap it?

Targets



$$m^2 R^2 = \hat{\Delta}(\hat{\Delta} - d)$$

Targets



$$m^2 R^2 = \hat{\Delta}(\hat{\Delta} - d)$$

I : vicinity of IR fixed point (in D = 2)

[w.i.p. with E. Lauria and A. Antunes]

II : large $\hat{\Delta}$ limit (flat-space limit)

[2210.15683 + w.i.p. with X. Zhao]

I : vicinity of IR fixed point

IR

gapless theory + irrelevant perturbations

free fields :

$$\int \left[\frac{1}{2} (\partial\phi)^2 + \lambda (\partial\phi)^4 + \dots \right]$$

claim : $\lambda > 0$

[Adams et al, 2006]

[Caron-Huot et al, ...]

non-trivial CFT :

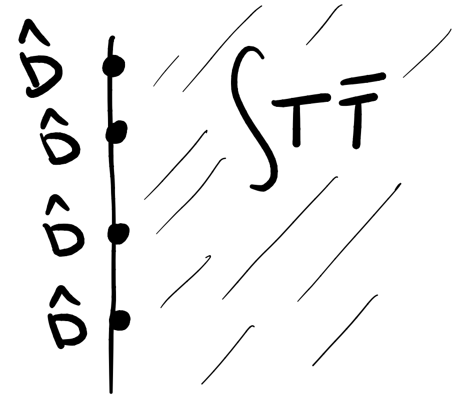
$$\left\langle \dots \exp\left[-g \int \mathcal{O} \right] \right\rangle$$

$$\Delta_{\mathcal{O}} > D$$

Q : constraints ?

$T\bar{T}$ deformation in AdS_2

$$\langle \hat{D} \ldots \hat{D} \; \exp\left[-g \int T\bar{T} + \cdots \right] \rangle$$



$$\hat{D}(\vec{x}) = \lim_{z \to 0} T_{\perp\perp}(\vec{x}, z)$$

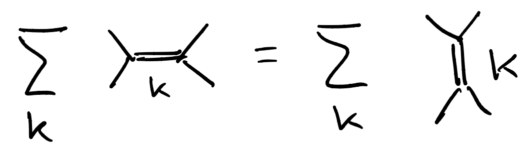
analytically:

- conf. perturbation thy. in AdS_2 background

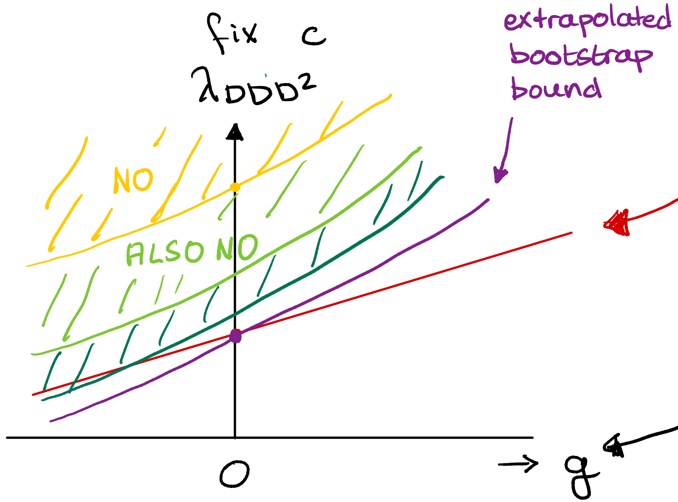
$$\Delta_D = 2 + \#g + \mathcal{O}(g^2)$$
$$\Delta_{D^2} = 4 + \#g + \mathcal{O}(g^2)$$
$$\lambda^2_{DDD} = \frac{8}{c} + \#g + \mathcal{O}(g^2)$$
$$\lambda_{DDD^2} = \frac{2}{c}(c + 22/5) + \#g + \mathcal{O}(g^2)$$

numerically:

- analyze $\langle \hat{D}\hat{D}\hat{D}\hat{D} \rangle$ with numerical conf. bootstrap



$T\bar{T}$ deformation in AdS_2



$$\lambda_{DDD^2} = \frac{2}{c}\left(c + \frac{22}{5}\right) + \# g$$

$$\left(\begin{aligned} \Delta_D &= 2 + \# g \\ \Delta_{D^2} &= 4 + \# g \\ \lambda^2_{DDD} &= \frac{8}{c} + \# g \end{aligned}\right)$$

upshot: only $g > 0$ is consistent!

remarks:

- agrees with integrable deformation ...
- ... but holds for <u>any</u> flow ending with $T\bar{T}$.
- same constraint from hydro:

  [Delacretaz, Fitzpatrick, Katz, Walters 2021]

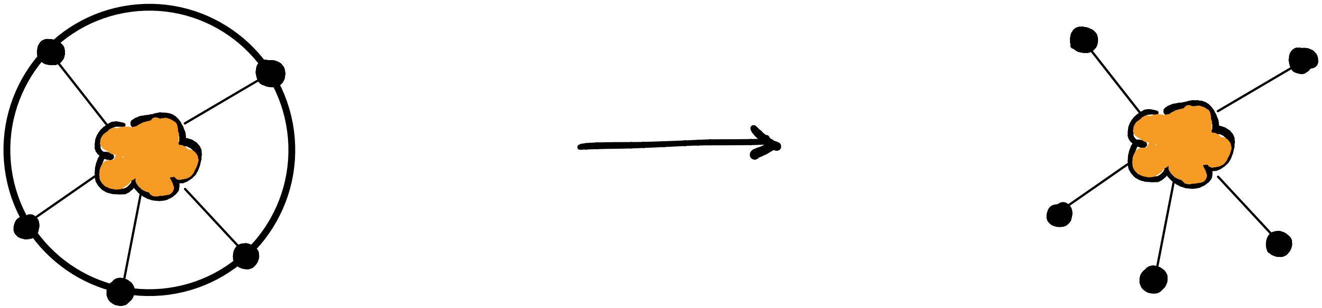
I : vicinity of IR fixed point

- $T\bar{T}$ is sign-constrained
- other constraints ? higher dimensions?
  analytic approaches ?
- bootstrap rest of the flow ?

II : large $\hat{\Delta}$ limit

II : large $\hat{\Delta}$ limit (flat-space limit)

main idea: as $\hat{\Delta} \to \infty$, conformal correlators become scattering amplitudes



$\langle \hat{O}_1(\vec{x}_1) \dots \hat{O}_5(\vec{x}_5) \rangle$

$\langle p_1 \dots | \hat{S} | \dots p_5 \rangle$

$p_i^2 = m_i^2$

II.I : why should we try?

II.II : how does this work?

II.III : what can we learn?

II.I: why should we try?

$\langle \hat{O} \dots \hat{O} \rangle$

well-understood:

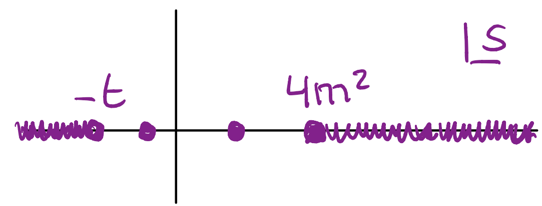
- convergent OPE
- huge domain of analyticity
- Regge bounds

$\langle \dots | S | \dots \rangle$ = disconnected $+ \, \delta(\sum p) \, T(s,t,u,\dots)$

partially understood:

sometimes, from LSZ:

- dispersion relations:



- Froissart-Martin bound

$$\sigma_{tot}(s) \lesssim \ln^2(s)$$

Can we do better?

II.II how does it work?

amplitudes from correlators

- direct formula: [Dubovsky, Gorbenko, Mirbabayi 2017]

$$\langle p_1, \ldots | S | \ldots p_K \rangle = \lim_{R\to\infty} \langle \hat{O}(\vec{x}_1) \ldots \hat{O}(\vec{x}_K) \rangle \Big|_{\text{s-matrix}}$$

- Mellin formula: [PPTvRV 2016]

$$T(p_i \cdot p_j) = \lim_{R\to\infty} \tilde{M}(\gamma_{ij} = \ldots\ )$$

where

$$\langle \hat{O}(\vec{x}_1) \ldots \hat{O}(\vec{x}_K) \rangle = \int [d\gamma]\, \tilde{M}(\gamma_{ij}) \prod_{i<j} \Gamma(\gamma_{ij}) |\vec{x}_{ij}|^{-2\gamma_{ij}}$$

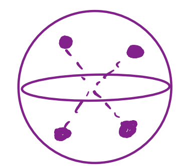
- phase shift formula [PPTvRV 2016]
- momentum space formula [Hijano 2019]
- ... [Y-Z Li 2021]

direct formula

claim: sometimes

$$\langle \underline{p}_1, \ldots | S | \ldots \underline{p}_K \rangle = \lim_{R\to\infty} Z^{K/2} \langle \hat{O}(n_1) \ldots \hat{O}(n_K) \rangle \Big|_{S\text{-matrix}}$$

$ds^2 = d\rho^2 + R^2 \sinh^2(\rho/R)\, d\Omega^2_{D-1}$



bdy S^d with coords. n^μ

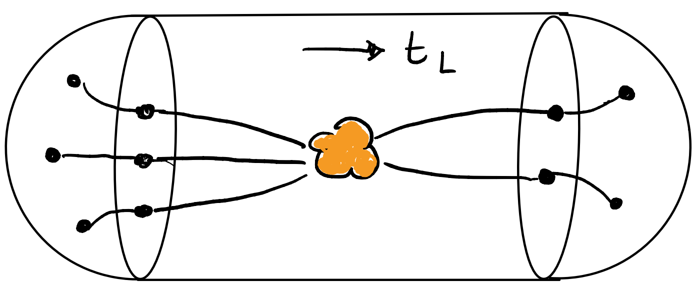
$\mu = 1..D$, $n^2 = 1$

where

in: $(n^0, \underline{n}) = -(p^0, -i\underline{p})/m$

out: $(n^0, \underline{n}) = +(p^0, -i\underline{p})/m$

pictorially:



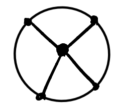
[Dubovsky, Gorbenko, Mirbabayi 2017]

[Hijano 2019]

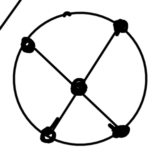
[Komatsu, Paulos, BvR, Zhao 2020]

direct formula

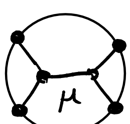
examples:

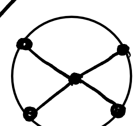
- 2-pt. fn: $\frac{Z}{x^{2\Delta}} \to$ sphere coords $\to$ S-matrix config. $\xrightarrow[R\to\infty]{} (2\pi)^3 \delta^{(D-1)}(p_1 - p_2)$
- 4-pt. contact diagram: Z^4  $\xrightarrow[R\to\infty]{} (2\pi)^D \delta^{(D)}\left(\sum_{i=1}^{4} p_i\right)$

amplitude prescription:

$\lim_{R\to\infty} \langle \hat{O} \ldots \hat{O} \rangle_c \Big/$  $\Big|_{\text{S-matrix}} \rightsquigarrow T(s,t)$

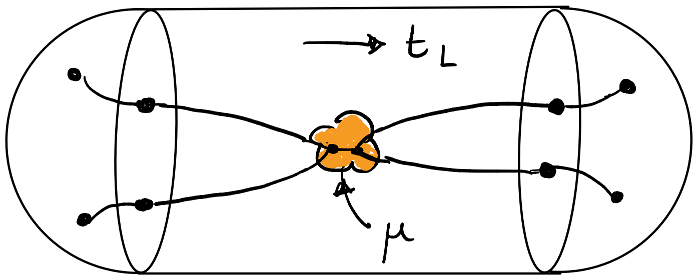
- 4-pt. exchange diagram:



/  $\xrightarrow[R\to\infty]{} \frac{-1}{s-\mu^2}$ (sometimes!)

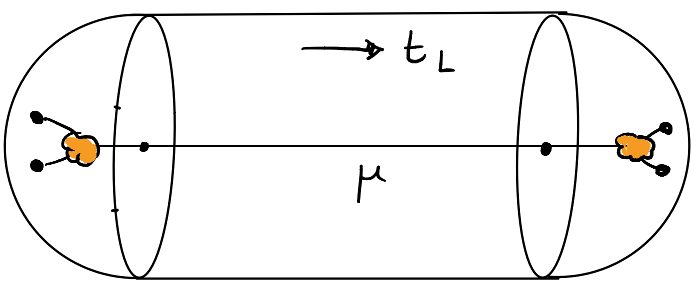
Exchange diagram

Saddle pt. approx. gives:

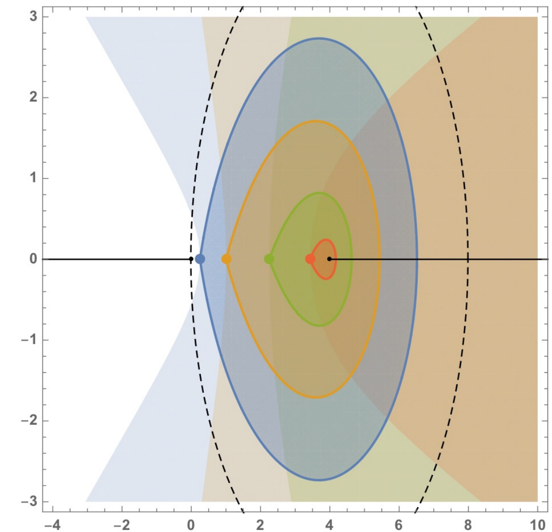


$\longrightarrow \dfrac{-1}{s-\mu^2}$

but sometimes extra contr:



$\longrightarrow \infty$

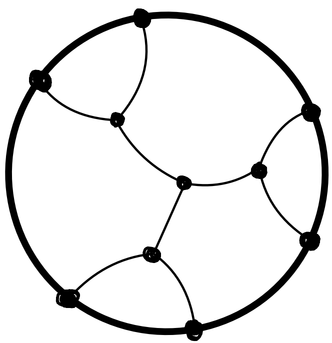


- only for $0 < \mu^2 < 4$
- only within $|s-4| < 4$

$(m^2 = 1)$

[Komatsu, BvR, Paulos, Zhao 2020]

AdS Landau diagrams

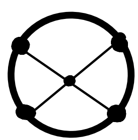


- on-shell particles propagating over large distances
- "momentum conservation" at vertices

can always be drawn,
do not always dominate.

upshot: to get amplitude: [Komatsu, BvR, Paulos, Zhao 2020]

$$\lim_{R \to \infty} \frac{\langle \hat{O}(x_1) \ldots \hat{O}(x_4) \rangle_c}{\text{(diagram)}} \Bigg|_{\text{S-matrix}} = \begin{cases} T(s,t) & \text{most times} \\ \infty & \text{sometimes} \end{cases}$$

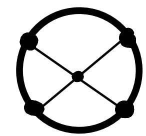


but: understandable

II.III what can we learn?

ideally:

$$T(s,t) = \lim_{\Delta_i \to \infty} \frac{\langle \mathcal{O}(x_1) \ldots \mathcal{O}(x_4) \rangle_c}{\text{(diagram)}} \Bigg|_{\text{S-matrix}}$$



to study analyticity, bounds, etc.

Q: what can we prove? can we reproduce results from 1960s, or improve them?

Q: what to assume?

QFT in AdS instead of LSZ

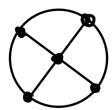
QFT in AdS instead of LSZ

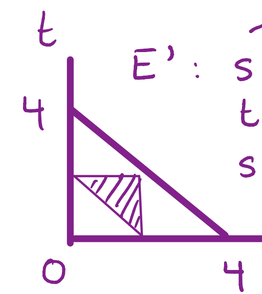
<u>claim</u>: consider conformal four-point function of scalar $\mathcal{O}_\Delta$.

suppose OPE structure : $\mathcal{O}_\Delta \times \mathcal{O}_\Delta = \mathbb{1} + \left(\begin{array}{l}\text{operators with} \\ \text{dimension} \geq \sqrt{2}\,\Delta\end{array}\right)$

and that, pointwise,

$$\lim_{\Delta \to \infty} \left. \frac{\langle \mathcal{O}_\Delta(x_1) \ldots\ldots \mathcal{O}_\Delta(x_4) \rangle}{\text{(diagram below)}} \right|_{\text{S-matrix}} = T(s,t) < \infty \quad \textbf{inside } E'.$$





E' : $s \leq 2$
$t \leq 2$
$s + t \geq 2$

Then $T(s,t)$ is <u>analytic</u>,

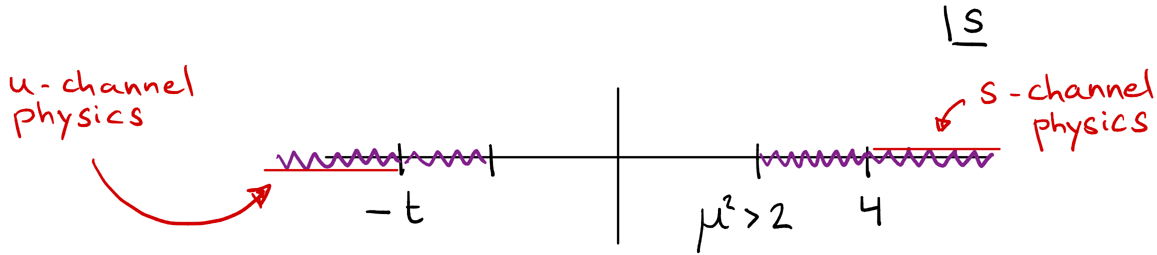
obeys the <u>Froissart bound</u>,

and consistent with <u>unitarity</u>.

[BvR, Zhao (2022)]

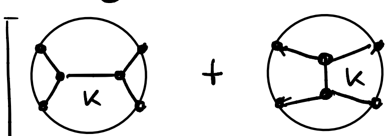
QFT in AdS instead of LSZ

analyticity: for $0 \le t < 4$ we have only cuts:



proof uses subtracted conformal dispersion relation, leading to "Polyakov-Regge" block decomposition:

$$\langle \phi\phi\phi\phi \rangle_c = \sum_{k \neq 1} a_k^2 \left[\quad + \quad \right] \rightsquigarrow$$

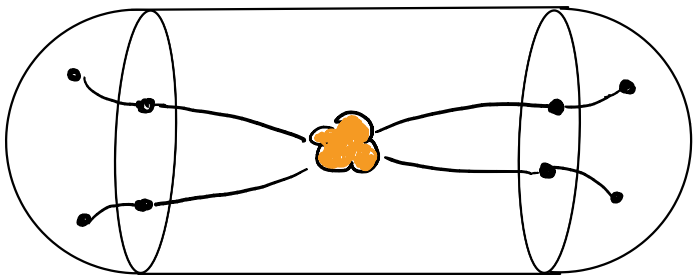


$$T(s,t) \sim \sum_{\ell} \int d\mu^2 \, a_{\mu^2,\ell} \, \Xi_{\mu^2,\ell} \left[\frac{1}{s-\mu^2} + \frac{1}{4-s-t-\mu^2} \right] + \cancel{\left[\text{Landau diagram contr.} \right]}$$

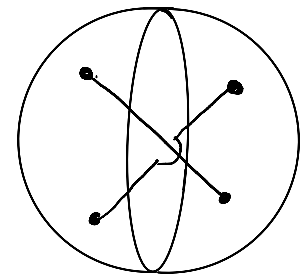
[Caron-Huot, Mazac, Rastelli, Simmons-Duffin 2020, ...]

QFT in AdS instead of LSZ

unitarity: pictorially:



$$\leqslant$$



in more detail:

$$\langle \Psi | \exp(-i\pi H) | \Psi \rangle \leq \langle \Psi | \Psi \rangle \xrightarrow[R \to \infty]{} \langle \Psi | \Psi \rangle_{\text{free}}$$

(up to Landau diagrams at least)

Since only 2-2 contributes, we get:

$$2\,\mathrm{Im}(T) \geqslant T\,T^{\dagger}$$

(postulate asymptotic completeness? cf. $\mathcal{H}_{in} = \mathcal{H}$)

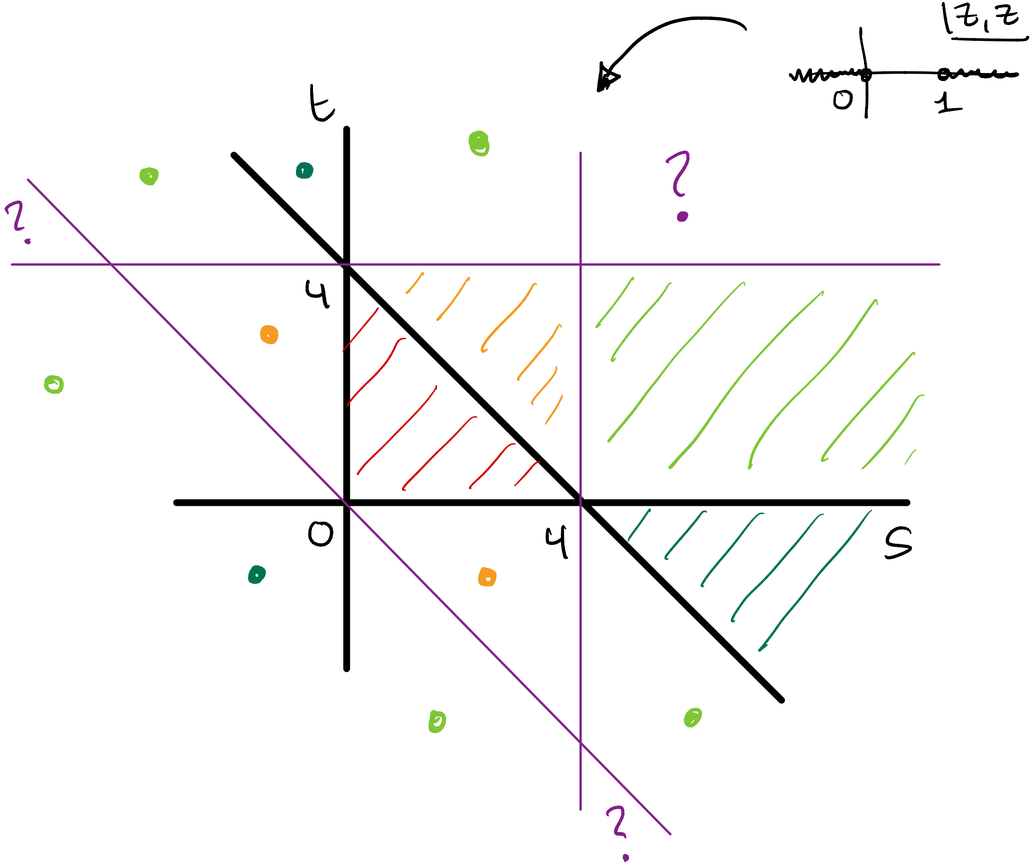
I : vicinity of the IR fixed point

II : large $\hat{\Delta}$ limit

- dot mathematical i's
- find best axioms
- $2 \to 2$: unequal particles
- $m \to n$?

Thank you!

From cross ratios to Mandelstam invariants



: Eucl. correlator
$z^* = \bar{z}$

: $z^* = \bar{z}$, but:
$z \to e^{2\pi i} z$

: Lor spacelike
$0 < z, \bar{z} < 1$

: $0 < z, \bar{z} < 1$, but:
$z \to e^{2\pi i} z$