BLACK HOLES IN

# EINSTEINIAN CUBIC GRAVITY

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Based on: -Bueno, PAC, Phys.Rev. D94 (2016) no.10, 104005

-Bueno, PAC, 1610.08019

-Bueno, PAC, Min, Visser, 1610.08519

1. **Introduction**
2. **Construction of Einsteinian cubic gravity (ECG)**
3. **Black holes in ECG**
4. **Conclusions**

# WHY HIGHER-DERIVATIVE GRAVITY (HDG)?

- Natural expectation: quantum corrections to the EH action have the form of higher-order curvature terms

$$S=\int_{\mathcal{M}} d^4x\sqrt{|g|}\left[\frac{1}{2\kappa}(-2\Lambda_0+R)+\alpha_1 R^2+\alpha_2 R_{\mu\nu}R^{\mu\nu}+\alpha_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}+\ldots\right]$$

$$\kappa=8\pi G$$

$$\alpha_i\sim\hbar$$

- In particular, string theory predicts an infinite series of such terms

e.g. Gross, Sloan

- Higher-order terms drastically change the **UV behavior** of gravity
- In particular, we expect Planck-scale modifications of **Black Holes**
- Moreover, higher-derivative gravity is **renormalizable** K.S.Stelle
- Also, HDG has many applications in other contexts such as **cosmology** or **holography** e.g. Brigante, Liu, Myers, Shenker, Yaida;

e.g. Sotiriou, Faraoni; Clifton, Ferreira; Nojiri, Odintsov

- Although HDG is renormalizable, **it propagates ghost degrees of freedom** e.g. Bueno, PAC, Min, Visser

- **Classically:** Hamiltonian unbounded from below, modes of negative energy (Ostrogradsky instability)

- **Quantum point of view:** non-normalizable or negative norm states.
  **Unitarity breaking** → **The theory is inconsistent**

- In order to work as an EFT, the ghost must be removed

- There are ghost-free theories: Lovelock, f(R), f(Lovelock)…
  Lovelock; Sotiriou, Faraoni; Bueno, PAC, Lasso, Ramírez

- **In this talk I will construct a new (perturbatively) unitary higher-order correction to the EH action and I will find black hole solutions of this new theory, unveiling fundamental differences with respect to the Schwarzschild black hole**

# PART 1: CONSTRUCTION OF EINSTEINIAN CUBIC GRAVITY (ECG)

# SPECTRUM OF HDG

We consider a L(Riemann) theory

$$S=\int d^Dx\sqrt{-g}\,\mathcal{L}(R_{\mu\nu\rho\sigma},g_{\alpha\beta})$$

- **Goal:** find the degrees of freedom propagated on the vacuum.
- Vacuum: **maximally symmetric space (m.s.s.)**

$$\boxed{\bar{R}_{\mu\nu\alpha\beta}=2\Lambda\bar{g}_{\mu[\alpha}\bar{g}_{\beta]\nu}}$$

- $\Lambda$=0 **flat**, $\Lambda$>0 **dS**, $\Lambda$<0 **AdS**
- The curvature $\Lambda$ is determined by the **Background Embedding Equation**

$$\boxed{\Lambda\frac{d\bar{\mathcal{L}}(\Lambda)}{d\Lambda}=\frac{D}{2}\bar{\mathcal{L}}(\Lambda)}$$

Bueno, PAC, Min, Visser

# SPECTRUM OF HDG

- Expand the metric on the vacuum $$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$
- Expand the field equations linearly in the perturbation
- Decomposition of the metric perturbation

$$h_{\mu\nu} = t^{(m)}_{\mu\nu} + t^{(M)}_{\mu\nu} + \frac{\bar{\nabla}_{\langle\mu}\bar{\nabla}_{\nu\rangle}h}{(m_s^2 + D\Lambda)} + \frac{1}{D}\bar{g}_{\mu\nu}h$$

- Linearized Equations:

Bueno, PAC, Min, Visser

Look at the sign!

$$-\frac{1}{2\kappa_{\rm eff}}\left[\bar{\Box} - 2\Lambda\right] t^{(m)}_{\mu\nu} = 0$$

$$+\frac{1}{2\kappa_{\rm eff}}\left[\bar{\Box} - 2\Lambda - m_g^2\right] t^{(M)}_{\mu\nu} = 0,$$

$$-\left[\frac{(D-1)(D-2)\Lambda(m_g^2 - (D-2)\Lambda)}{4\kappa_{\rm eff} m_g^2 (m_s^2 + D\Lambda)}\right]\left[\bar{\Box} - m_s^2\right] h = 0$$

**There is a massless spin-2 graviton, a ghost-like massive graviton and a scalar**

- The theories are unitary around the vacuum if the massive graviton is not present $\Longleftrightarrow$ $\boxed{m_g^2 = +\infty}$
- This condition imposes constraints on the couplings of the theory at every order in curvature
- **Example:** most general cubic gravity

$$
\begin{aligned}
S = & \frac{1}{16\pi G}\int_M d^D x\sqrt{-g}\Big\{ -2\Lambda_0 + R + L^2\left(\alpha_1 R^2 + \alpha_2 R_{ab}R^{ab} + \alpha_3 R_{abcd}R^{abcd}\right) \\
& + L^4\big(\beta_1 R_{a\ b}^{\ c\ d} R_{c\ d}^{\ e\ f} R_{e\ f}^{\ a\ b} + \beta_2 R_{ab}^{cd} R_{cd}^{ef} R_{ef}^{ab} + \beta_3 R_{abcd}R^{abc}_{\ \ \ e}R^{de} + \beta_4 R_{abcd}R^{abcd}R \\
& + \beta_5 R_{abcd}R^{ac}R^{bd} + \beta_6 R_a^b R_b^c R_c^a + \beta_7 R_{ab}R^{ab}R + \beta_8 R^3\big)\Big\}.
\end{aligned}
$$

- **No massive graviton:**

$$
\begin{aligned}
& \frac{1}{2}\alpha_2 + 2\alpha_3 + \Lambda L^2\Big( -\frac{3}{2}\beta_1 + 12\beta_2 + 2D\beta_3 + 2D(D-1)\beta_4 \\
& + \left(D - \frac{3}{2}\right)\beta_5 + \frac{3}{2}(D-1)\beta_6 + \frac{1}{2}D(D-1)\beta_7\Big) = 0.
\end{aligned}
$$

Imposing the constraint for every Lambda and D yields

$$S=\int d^Dx\sqrt{|g|}\Big\{\frac{1}{2\kappa}(R-2\Lambda_0)+\kappa^{\frac{4-D}{D-2}}(\tilde{\alpha}_1R^2+\tilde{\alpha}_2\mathcal{X}_4)+\kappa^{\frac{6-D}{D-2}}(\tilde{\beta}_1R^3+\tilde{\beta}_2\mathcal{X}_6+\tilde{\beta}_3R\mathcal{X}_4+\tilde{\beta}_4\mathcal{P}+\tilde{\beta}_5\mathcal{Y})\Big\}$$

Bueno, PAC, Min, Visser

$$\mathcal{P}\equiv 12R_{a\ b}^{\ c\ d}R_{c\ d}^{\ e\ f}R_{e\ f}^{\ a\ b}+R_{ab}^{cd}R_{cd}^{ef}R_{ef}^{ab}-12R_{abcd}R^{ac}R^{bd}+8R_a^bR_b^cR_c^a$$

$$\mathcal{Y}\equiv R_{\mu\ \nu}^{\ \alpha\ \beta}R_{\alpha\ \beta}^{\ \rho\ \sigma}R_{\rho\ \sigma}^{\ \mu\ \nu}-3R_{\mu\nu\rho\sigma}R^{\mu\rho}R^{\nu\sigma}+2R_\mu^{\ \nu}R_\nu^{\ \rho}R_\rho^{\ \mu}$$

- **(Perturbatively) ghost free cubic gravity in any dimension**
- **Linearly: Einstein+scalar**
- **We can further impose that there is no scalar:** $m_s^2=+\infty$

## This way, we get Einsteinian Cubic Gravity (ECG)

$$S=\int d^Dx\sqrt{|g|}\left\{\frac{1}{2\kappa}(R-2\Lambda_0)+\kappa^{\frac{4-D}{D-2}}\alpha\mathcal{X}_4+\kappa^{\frac{6-D}{D-2}}(\beta\mathcal{X}_6+\lambda\mathcal{P})\right\}$$

Bueno, PAC

- Lovelock+new interaction P
- Perturbatively unitary around any M.S.S. in any dimension
- Linearly equivalent to Einstein gravity with effective gravitational and cosmological constants
- Unlike Lovelock, **P is non-trivial in four dimensions!**
- **Provides a useful model to study effects of higher-order terms in four dimensions**

# PART II:

# BLACK HOLES

# BLACK HOLES IN HIGHER-DERIVATIVE GRAVITY

- Problem of finding black hole solutions in HDG: very complicated equations. For example:

$$ds^2_{N,f} = -N^2(r)f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{(D-2)}$$

Non-linear, fourth-order coupled differential equations for N(r) and f(r)

- There are special theories in which the problem can be simplified

- For example, in D>4, Lovelock+Quasitopological gravity

Boulware, Deser; Cai

Myers, Robinson; Oliva, Ray

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{|g|}\left\{\frac{12}{L^2} + R + \frac{\lambda L^2}{2}\mathcal{X}_4 + \frac{7\mu L^4}{4}\mathcal{Z}_5\right\}$$

$$\mathcal{Z}_5 = R_{a\ b}^{\ c\ d}R_{c\ d}^{\ e\ f}R_{e\ f}^{\ a\ b} + \frac{1}{56}\Big(21\,R_{abcd}R^{abcd}R - 72\,R_{abcd}R^{abc}{}_{e}R^{de} + 120\,R_{abcd}R^{ac}R^{bd} + 144\,R_a{}^bR_b{}^cR_c{}^a - 132\,R_a{}^bR_b{}^aR + 15\,R^3\Big)$$

$$ds^2 = -\left(1+\frac{r^2}{L^2}f(r)\right)N^2(r)dt^2 + \frac{dr^2}{1+\frac{r^2}{L^2}f(r)} + r^2 d\Omega^2_{(3)}$$

with N(r)=constant and

$$1 - f + \lambda f^2 + \mu f^3 = \frac{\omega^4}{r^4}$$

- However, there is no known theory in D=4 with these properties

# SOLUTIONS OF ECG IN FOUR DIMENSIONS

Bueno, PAC

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}} d^4x\sqrt{|g|}\left\{R + \alpha\mathcal{X}_4 - \lambda\mathcal{P}\right\}$$

$\boxed{[\lambda] = E^{-4}}$

$\boxed{[\alpha] = E^{-2}}$

In principle Planck-scale, but could be any other scale

- Static spherically-symmetric ansatz: $ds^2 = -N(r)f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{(2)}$
- Field equations: $\frac{\delta S[N,f]}{\delta N} = \frac{\delta S[N,f]}{\delta f} = 0$
- Solution: **N(r)=1** and f satisfies

$$-(f-1)r - \lambda\left[4f'^3 + 12\frac{f'^2}{r} - 24f(f-1)\frac{f'}{r^2} - 12ff''\left(f' - \frac{2(f-1)}{r}\right)\right] = r_0$$

$$r_0 = 2GM$$

- Problem reduced to an ordinary 2nd order diff. equation
- **Up to cubic order, ECG is unique in D=4**

# BOUNDARY CONDITIONS

1. Asymptotically flat metric $\boxed{\lim_{r\to\infty} f(r) = 1}$

Asymptotic perturbations over Schwarzschild:

$$f(r) = 1 - \frac{r_0}{r} - \lambda\left(\frac{108r_0^2}{r^6} - \frac{92r_0^3}{r^7}\right) + B\exp\left(-\frac{r^{5/2}}{15\sqrt{\lambda r_0}}\right) + \cancel{A\exp\left(\frac{r^{5/2}}{15\sqrt{\lambda r_0}}\right)}$$

**1-parameter family of asymptotically flat solutions**

2.Regular Horizon: $f(r_h) = 0$ $\kappa_g = f'(r_h)/2$ (Surface gravity)

Series expansion around the horizon:

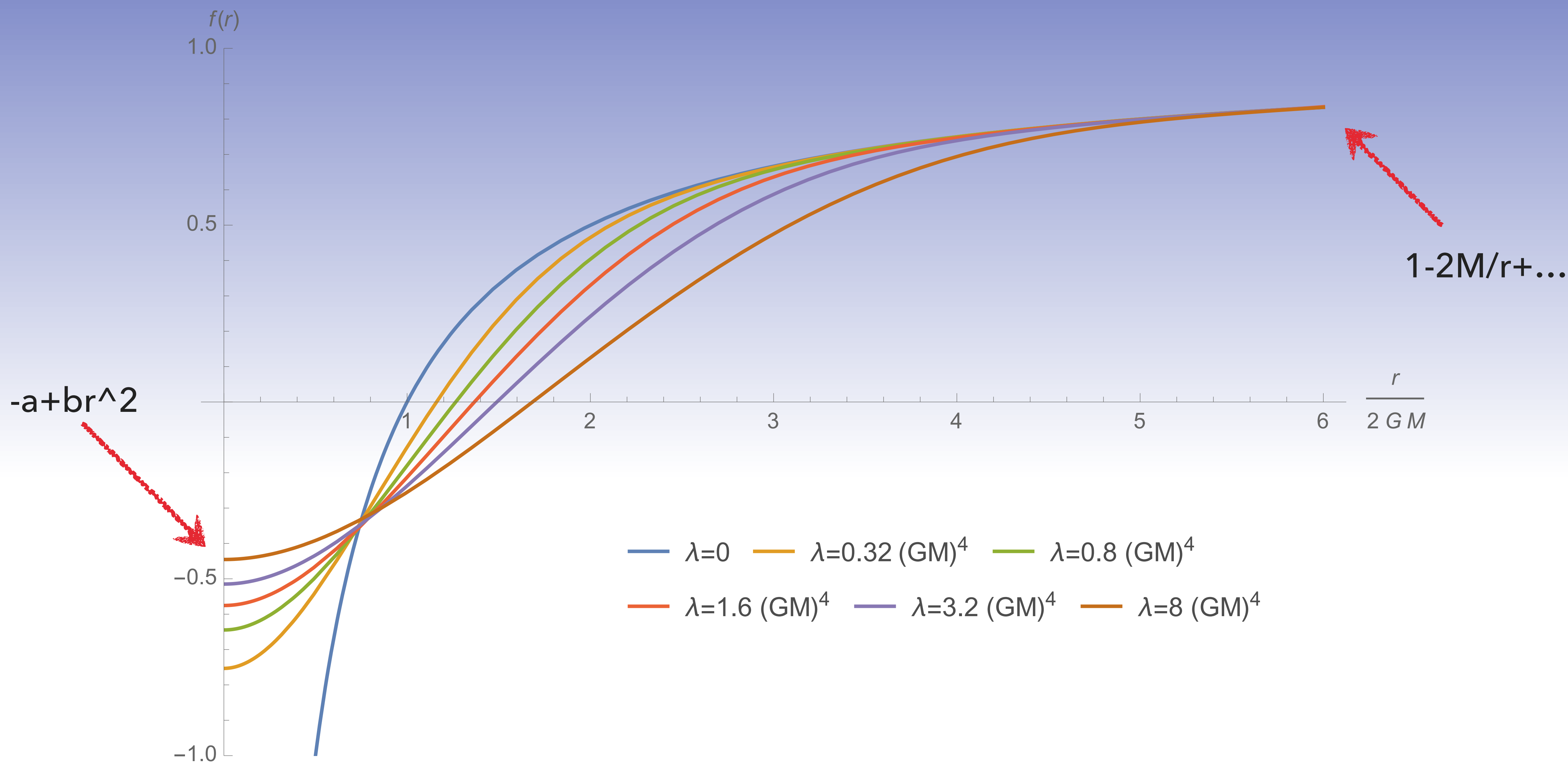
$$f(r) = 2\kappa_g(r - r_h) + \sum_{n=2}^{\infty} a_n(r - r_h)^n$$

**-rh and kg are fixed by the equations!**

-Only free parameter: f''(rh) → **1-parameter family of reg. horizon solutions**

# NUMERIC SOLUTION

Unique solution satisfying both requirements




Singularity softened: $R_{abcd}R^{abcd} = \frac{4(f(0)-1)^2}{r^4} + \mathcal{O}\left(\frac{1}{r^2}\right)$ compared to $R_{abcd}R^{abcd} = 48G^2M^2/r^6$

# HORIZON

- The horizon radius and the surface gravity are completely fixed by the mass

$$GM = \frac{r_h}{2} - \frac{8\lambda}{r_h^3}\left(1+\sqrt{1+48\lambda/r_h^4}\right)^{-3}\left(5+3\sqrt{1+48\lambda/r_h^4}\right)$$

$$\kappa_g = \frac{1}{r_h(1+\sqrt{1+48\lambda/r_h^4})}$$

Compare with Schwarzschild

$GM = \frac{r_h}{2}$ $\kappa_g = \frac{1}{2r_h}$

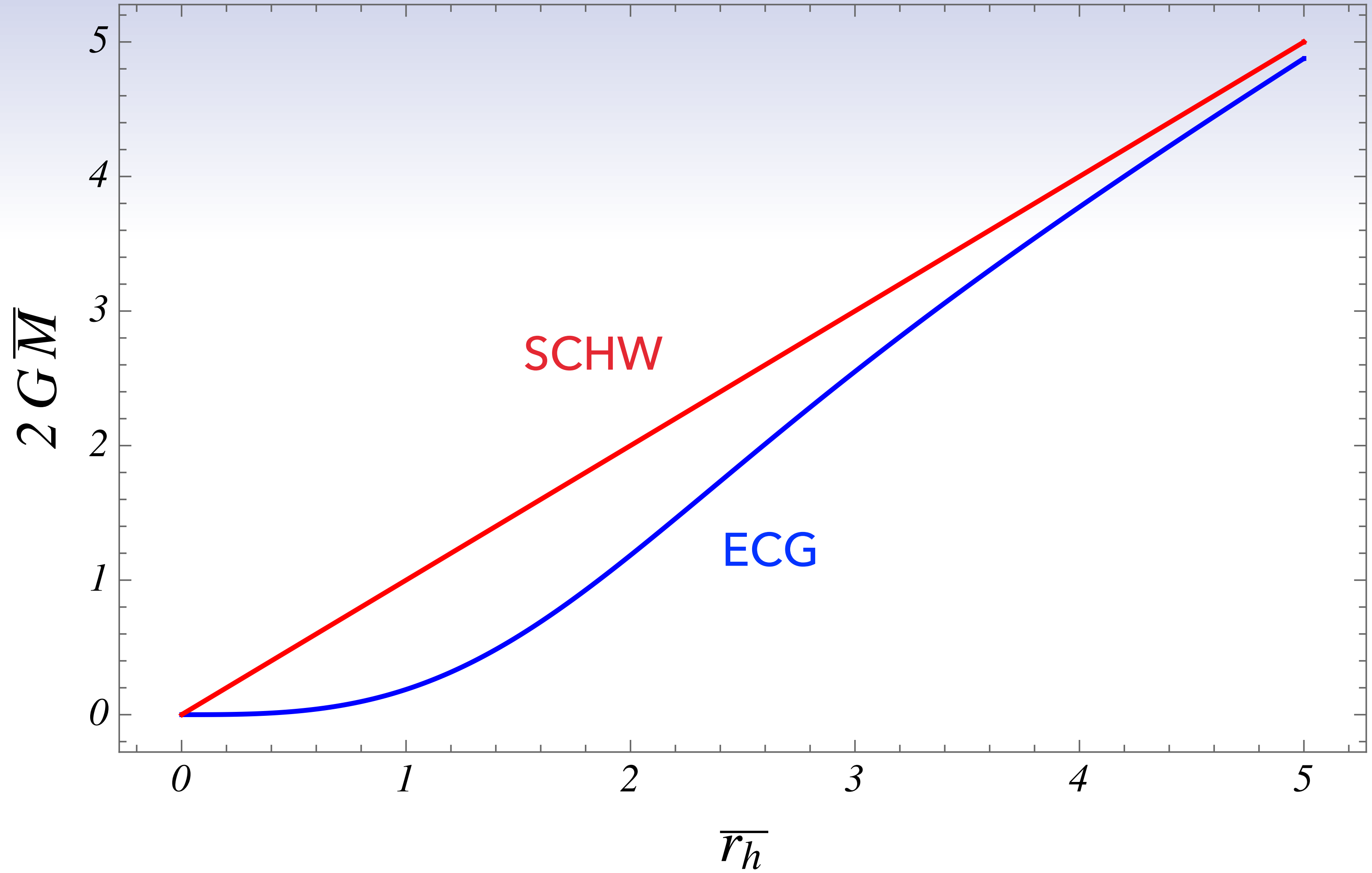
**For small masses** $GM << \lambda^{1/4}$

$$r_h \approx (6\sqrt{3\lambda}GM)^{1/3}$$

Planck scale $r_h \sim l_p$ reached

when $\boxed{M \sim \sqrt{G/\lambda}}$

(if lambda>G^2)



# ENTROPY

- Wald's Black Hole Entropy

$$S = -2\pi \int_H d^2x\sqrt{h}\frac{\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}}\epsilon_{\mu\nu}\epsilon_{\rho\sigma}$$

- Entropy of ECG Black hole

$$S = \frac{\pi r_h^2}{G}\left[1 - \frac{48\lambda}{r_h^4}\frac{3+2\sqrt{1+48\lambda/r_h^4}}{\left(1+\sqrt{1+48\lambda/r_h^4}\right)^2}\right] + \frac{2\pi}{G}\alpha$$

- Compare with Schwarzschild

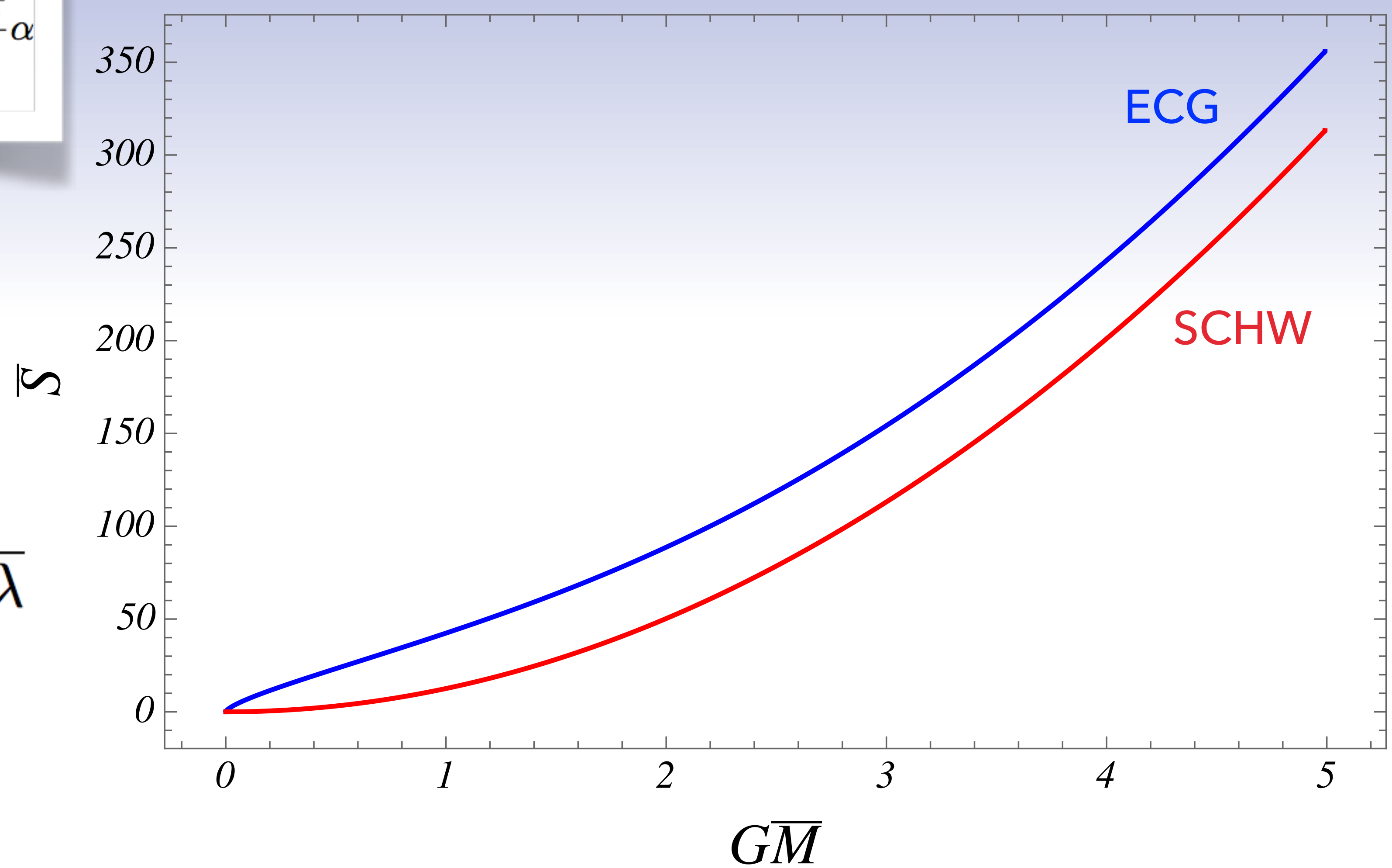
$$S_{\text{Schw.}} = \frac{\pi r_h^2}{G} = \frac{A}{4G}$$

- Positivity of entropy

S(M=0)=0 $\Longleftrightarrow$ $\alpha = \sqrt{48\lambda}$

- Second law?

Entropy vs Mass



# TEMPERATURE

- Hawking temperature is defined as $T_H = \frac{\kappa_g}{2\pi}$
- For the ECG BH $$T_H = \frac{1}{2\pi r_h(1+\sqrt{1+48\lambda/r_h^4})}$$ compare with $T_{\text{Schw}} = \frac{1}{4\pi r_h}$
- First Law of Black Hole mechanics!

$$\boxed{dM = T_H dS}$$

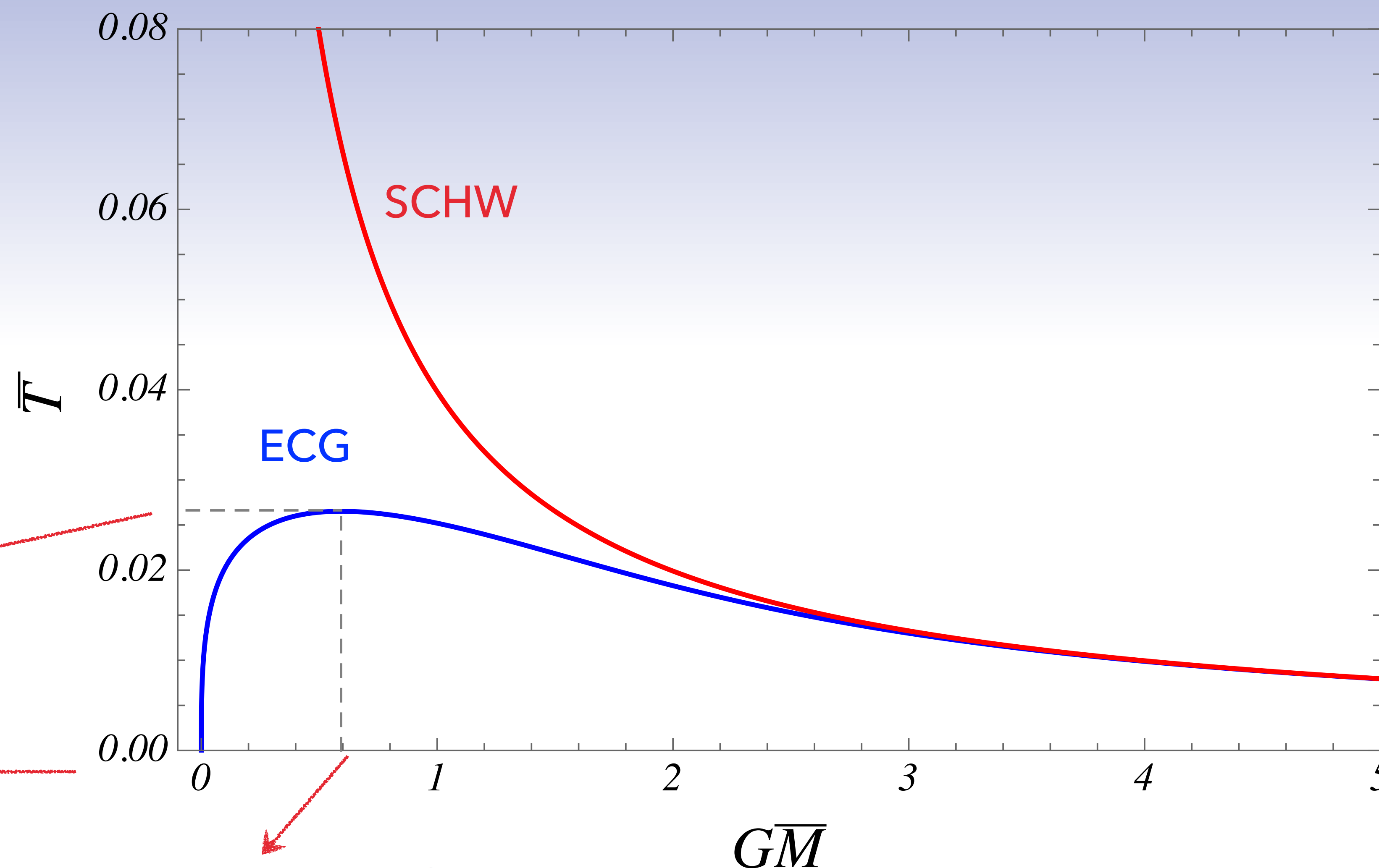
- The temperature vanishes as rh goes to 0

$$T_H \approx \frac{r_h}{2\pi\sqrt{48\lambda}}$$

Maximum temperature $\boxed{T_{\max} = \frac{1}{12\pi\lambda^{1/4}}}$

Vanishing Temperature

Temperature vs Mass



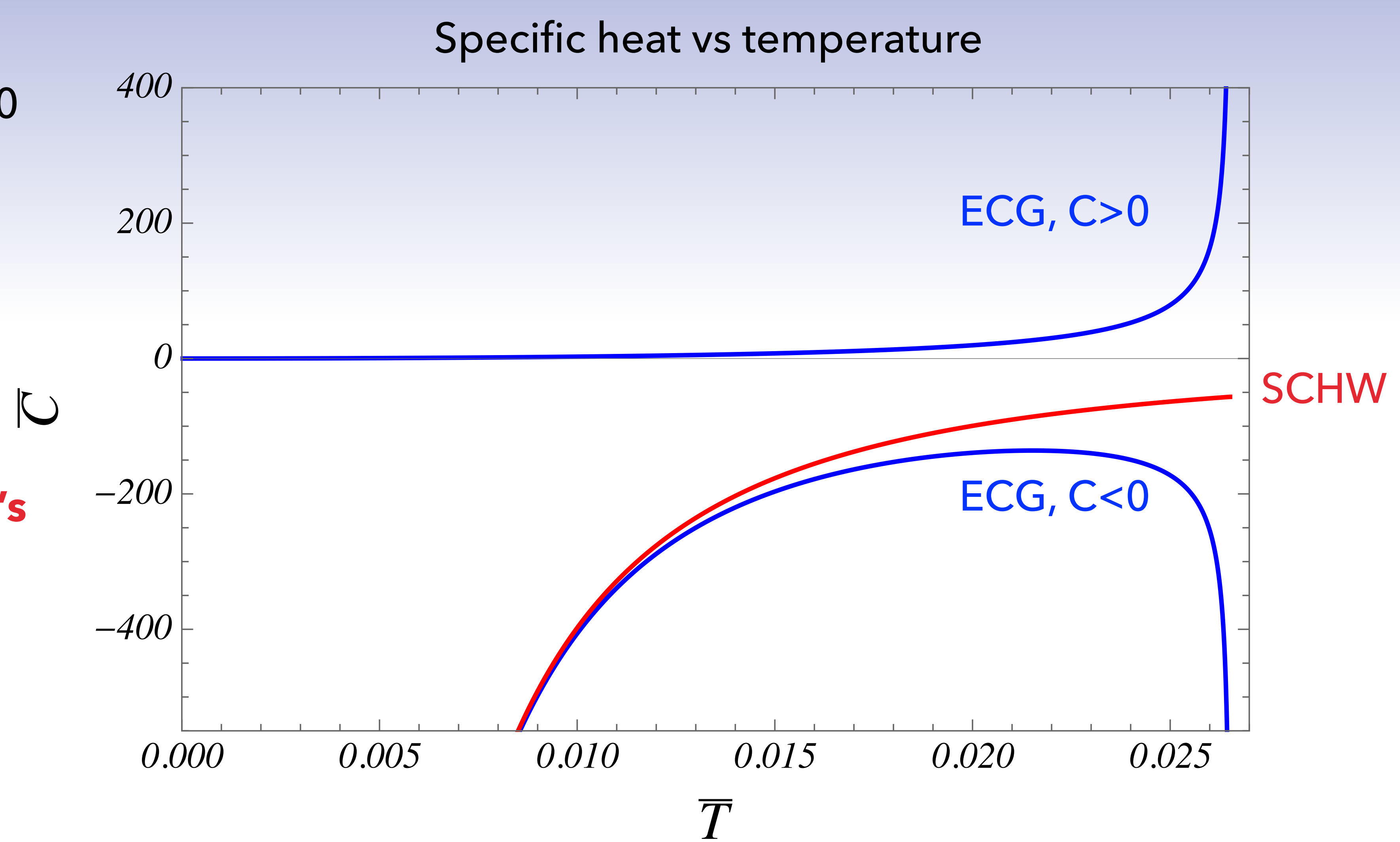

$GM_{\max} = \frac{16}{27}\lambda^{1/4}$

# SPECIFIC HEAT

- The specific heat is defined as $$C = T\left(\frac{\partial S}{\partial T}\right)_M$$
- Explicitly:

$$C = -\frac{8\pi\left(1152G^4\lambda^2 + 24G^2\lambda r_h^4\left(4\sqrt{1+\frac{48G^2\lambda}{r_h^4}}+5\right) + r_h^8\left(\sqrt{1+\frac{48G^2\lambda}{r_h^4}}+1\right)\right)}{Gr_h^2\left(\sqrt{1+\frac{48G^2\lambda}{r_h^4}}+1\right)^2\left(-48G^2\lambda + r_h^4\left(\sqrt{1+\frac{48G^2\lambda}{r_h^4}}+1\right)\right)}$$

- Schwarzschild always has C<0 but in ECG there are two phases
- C<0: macroscopic unstable BH's (they evaporate)
- **C>0: microscopic stable BH's**
- Phase transition at T=Tmax



# EVAPORATION OF BLACK HOLES

- SCHW BH's have a finite lifetime
- ECG BH's have an infinite lifetime → They never vanish completely
- Energy loss rate: $\frac{dM}{dt} = -\frac{\pi^3 r_h^2}{15} T_H^4$ **When M –>0 the temperature vanishes!**
- For small masses, $GM << \lambda^{1/4}$ , the evolution is

$$M(t) = \frac{M_0}{1 + t/t_0}$$

$$t_0 = \frac{5120\pi\lambda}{G^2 M_0}$$

**Black hole's half-life**

→ LONG-LIVED BLACK HOLE REMNANT

- Some numbers: $\lambda \sim (1GeV)^{-4}$

$$M_0 = 1GeV$$

(initial microscopic black hole mass)

→ $t_0 \sim 10^{38} \times$ Age of the universe

# CONCLUSIONS

- In this talk we have seen that there exist HDG which are perturbatively unitary around the vacuum and we have constructed a cubic theory satisfying this property (ECG)
- ECG represents a new interesting model which can be used as an EFT in order to study posible effects of quantum gravity
- We have computed the modification of the SCHW. black hole induced by the ECG term
- This black hole has several remarkable features: There is no metric divergence, the curvature singularity is softened, the temperature doesn't diverge, there are stable microscopic BH's and they leave black hole remnants.
- Observational status: no deviations from GR if $\lambda^{1/4}/GM << 1$
- For macroscopic (solar mass) BH's, very weak bound $\lambda << (10^{-10}eV)^{-4}$
- If lambda is big enough, microscopic black holes have enormous half-lifes. This could be used to explain **Dark matter as microscopic primordial black hole remnants**

- **Final remarks: ECG is most general cubic gravity whose spectrum is Einstein-like in any dimension (with dimension-independent couplings)**
- **In addition, ECG is the unique cubic theory in D=4 allowing non-trivial single-blackening factor solutions**
- **The solution allows for generalizations: asymptotically AdS/dS, different horizon topology or adding more fields** Bueno,Cano; Hennigar, Mann
- **ECG will also have applications in holography** Dey, Roy, Sarkar

# THANK YOU!