Exceptional Generalised Geometry: some applications

LPTHE
LABORATOIRE DE PHYSIQUE
THEORIQUE ET HAUTES ENERGIES

Oscar de Felice

LPTHE - Université Pierre et Marie Curie Paris

Oviedo Postgraduate Meeting

Summary

- Motivation and inspiration
  - Extended symmetry in String theory
  - Geometrical interpretation of symmetries
- Generalised Geometry
  - Extended bundles
  - Encoding Fluxes and gauge transformations
- How to use these weapons
  - Example: consistent truncations

Why studying dimensional reductions?

String theory has an intrinsic phenomenological problem: it's defined in 9+1 dimensions

One looks for solutions of the following form

$$\mathcal{M}_D \to \mathcal{M}_{D-d} \times M_d$$

External space

Internal space

Often one is interested in the low energy effective theory in *(D-d)* dimensions: (un)gauged supergravity.

The structure of the internal space determines the lower dimensional theory. Preserved susy, gauge group, spectrum…

- The problem is to construct the effective action in *(D-d)* dimensions.
  - KK compactifications are the standard approach to dimensional reductions.
    - There is an infinite number of KK modes
    - We need to “truncate” to a finite number of d.o.f.
    - We call *Truncation Ansatz* the prescription of selecting the degrees of freedom to be kept.
- “Consistency” of the ansatz means that the dependence on the internal manifold factorises out once the ansatz is inserted in the eom.
  - All solutions of the lower dimensional theory lift to solutions of the higher dimensional one.
  - Consistent reductions allow to establish a map between theories in different dimensions

Extended symmetry in String theory and M-theory

- Our goal is to construct effective actions for lower dimensional theories

- The *D-d* dimensional effective action on tori has the following global symmetry group

d	3	4	5	6	7
group	$SL(5)$	$SO(5,5)$	$E_{6(6)}$	$E_{7(7)}$	$E_{8(8)}$

- These are the U-duality groups

- They all contain *O(d,d)* as a subgroup

T-duality group

Extended symmetry in String theory and M-theory

- These symmetries can be seen from a geometrical point of view on the model of GR
  - In GR we have diffeomorphisms symmetry and all the quantities have defined transformation rules under the group of diffeomorphims *GL(d)*
- One can construct U-duality covariant formalisms
  - Double/Exceptional Field Theory [Hull, Zwiebach; Samtleben, Hohm]
  - (Exceptional) Generalised Geometry [Hitchin; Gualtieri; Hull; Pacheco, Waldram]

Generalised Geometry

[Hitchin, Gualtieri, '01]

- Gauge symmetries of the lower-dimensional theory come from the metric and *p*-form potentials of the higher dimensional supergravity.
- One needs a formalism treating diffeomorphisms and *p*-form gauge transformations in a unified fashion.

The main idea: define a generalised tangent bundle

$$E \cong TM \oplus T^*M$$

$O(d,d)$

Structure group

generalised vector $\quad V = v + \lambda$

1-form

vector

- The structure group of the generalised tangent bundle is $O(d,d)$ the T-duality group of toroidal compactifications.

How do we insert fluxes?

- *O(d,d)* formalism encodes the 3-form flux $H = \mathrm{d}B$
  - The adjoint action naturally contains a 2-form
  - Define the twisted generalised vector

$$V = e^{-B}\tilde{V} = v + \lambda - \iota_v B$$

adjoint action of O(d,d)

This determines the topology of E

  - Patchings: on an overlapping of patches $U_\alpha \cap U_\beta$

$$V_\alpha = e^{-d\Lambda_{\alpha\beta}} V_\beta \quad \Longleftrightarrow \quad B_{(\alpha)} = B_{(\beta)} - \mathrm{d}\Lambda_{(\alpha\beta)}$$

2-form

Connection on a gerbe
This corresponds to gauge transformations of NSNS supergravity gauge potential.

Exceptional Generalised Geometry

[Hull; Pacheco, Waldram '08]

- One wants to include RR fields
  - T-duality group generalises to U-duality: define a generalised tangent bundle with a structure group given by the U-duality one.
- EGG depends on the theory: focus on IIA
  - Generalised tangent bundle

$$E \cong TM \oplus T^*M \oplus \Lambda^5 T^*M \oplus \Lambda^{\text{even}} T^*M \oplus \left(TM \otimes \Lambda^6 T^*M\right)$$

$\tilde{V} = \left(v, \lambda, \tilde{\lambda}, \omega, \tau\right)$ generalised vector

charges of wrapped strings and branes

Structure group $E_{d+1(d+1)}$

- Potentials live in the adjoint bundle

$$\operatorname{ad} F \cong \mathbb{R} \oplus (TM \otimes T^*M) \oplus \Lambda^2 T^*M \oplus \Lambda^2 TM$$
$$\oplus \Lambda^6 TM \oplus \Lambda^6 T^*M \oplus \Lambda^{\text{odd}} TM \oplus \Lambda^{\text{odd}} T^*M$$

$$\mathcal{A} = \left(\ldots, B, \ldots, \tilde{B}, \ldots, C_{\text{odd}}\right)$$

- E has a fibered structure

Adjoint rep

$$V = e^{\tilde{B}} e^{-B} e^{C_\pm} \tilde{V} \qquad R = e^{\tilde{B}} e^{-B} e^{C_\pm} \tilde{R} e^{-C_\pm} e^{B} e^{-\tilde{B}}$$

- Patching conditions give IIA gauge transformation

$$B_{(\alpha)} = B_{(\beta)} + \mathrm{d}\Lambda_{(\alpha\beta)}$$
$$C_{(\alpha)} = C_{(\beta)} + e^{B_{(\beta)} + \mathrm{d}\Lambda_{(\alpha\beta)}} \wedge \mathrm{d}\Omega_{(\alpha\beta)}$$
$$\ldots$$

Differential structure

- Ordinary Lie derivative generates diffeomorphisms

$$\mathcal{L}_v w^\mu = v^\nu \partial_\nu w^\mu - w^\nu \partial_\nu v^\mu = v^\nu \partial_\nu w^\mu - (\partial \otimes_{\mathrm{ad}} v)^\mu{}_\nu \, w^\nu$$

$gl(d, \mathbb{R})$

- Dorfman Derivative *[Pacheco, Waldram]*

$$L_V V' = V \cdot \partial V' - (\partial \otimes_{\mathrm{ad}} V) \cdot V'$$

L_V generates generalised diffeomorphisms = *diffeos + gauge*

$$\delta g = \mathcal{L}_v g \qquad \delta C_\pm = \mathcal{L}_v C_\pm + \mathrm{d}\omega^\mp + \ldots$$

$$\delta B = \mathcal{L}_v B + \mathrm{d}\lambda \qquad \delta \tilde{B} = \mathcal{L}_v \tilde{B} + \mathrm{d}\tilde{\lambda} + \ldots$$

- Gauge algebra $\quad [\delta_V, \delta'_V] = \delta_{L_V V'}$

Generalised Metric

- One can put the analogous of the Riemaniann metric on *E*
- Defined in terms of the generalised frame

$$\{\tilde{E}_A\} = \{\hat{e}_a\} \cup \{e^a\} \cup \{e^{a_1 \dots a_5}\} \cup \{e^{a_{2k}}\} \cup \{e^{a, a_1 \dots a_5}\}$$

$$E_A = e^{\tilde{B}} e^{-B} e^{C} e^{\Delta} e^{\phi} \cdot \tilde{E}_A$$

- Generalised Metric

$$\mathcal{G}^{-1} = \delta^{AB} E_A \otimes E_B$$

- It parametrises a coset $E_{d(d)}/H_d \longrightarrow$ Reduced structure
- It contains the metric, the B-field and all RR potentials

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- Generalised Metric

$$\mathcal{G}^{-1} = \delta^{AB} E_A \otimes E_B$$

For $E \cong T \oplus T^*$

$$\mathcal{G} = \begin{pmatrix} g - Bg^{-1}B & Bg^{-1} \\ -g^{-1}B & g^{-1} \end{pmatrix}$$

- It parametrises a coset $E_{d(d)}/H_d \longrightarrow$ Reduced structure

- It contains the metric, the B-field and all RR potentials

Generalised Scherk-Schwarz reductions

Goal: generalise Scherk-Schwarz reduction to Exceptional Generalised Geometry.

- Basic ingredients:
    - Generalised Parallelisability
    - Generalised frames
    - Generalised ansatz

As the ordinary ones, these reductions preserve all the SUSY.

Generalised Leibniz parallelisation

Extend to EGG the notion of parallelisability *[Lee, Strickland-Constable, Waldram '14]*

- Topological condition
  - On M_d there exists a frame $\{E_A\}\,,\;\; A = 1, \ldots, d$

    s. t. $\forall p \in M\,,\; \{E_A|_p\}$ is a basis for the gen. tangent bundle

- Differential condition

  GLP condition
  - The frame satisfies $$L_{E_A} E_B = X_{AB}{}^C E_C$$

    where $X_{AB}{}^C$ are constants and $[X_A, X_B] = -X_{AB}{}^C X_C$
  - $X_{AB}{}^C$ are related to the embedding tensor of the lower dim sugra

$$X_{AB}{}^C = \Theta_A{}^\alpha (t_\alpha)_B{}^C$$

- GLP implies the manifold is a coset $M \cong G/H$

Generalised frame and metric

Given the generalised tangent bundle

$$E \cong TM \oplus T^*M \oplus \Lambda^5 T^*M \oplus \Lambda^{\pm} T^*M \oplus \left(TM \otimes \Lambda^6 T^*M\right)$$

- Define the conformal split frame as a twist

$$\{\tilde{E}_A\} = \{\hat{e}_a\} \cup \{e^a\} \cup \{e^{a_1 \dots a_5}\} \cup \{e^{a_{2k}}\} \cup \{e^{a, a_1 \dots a_5}\}$$

$$E_A = e^{\tilde{B}} e^{-B} e^{C} e^{\Delta} e^{\phi} \cdot \tilde{E}_A$$

- Define the inverse generalised metric

$$G^{-1} = \delta^{AB} E_A \otimes E_B$$

Generalised Scherk-Schwarz ansatz

- Scalar ansatz
  - Twist the frame by an element of $E_{d+1(d+1)}$

$$E'_A{}^M(x,y) = U_A{}^B(x)E_B(y)$$

  - Compare with the generalised metric

$$\begin{aligned} G^{MN}(x,y) &= \delta^{AB}E'_A{}^M(x,y)E'_B{}^N(x,y) \\ &= \mathcal{M}^{AB}(x)E_A{}^M(y)E_B{}^N(y) \end{aligned}$$

$\mathcal{M}^{AB}$ contains all the scalar degrees of freedom of the truncated theory.

Generalised Scherk-Schwarz ansatz

- Vector ansatz
  - Take into account all fields with one external leg

Generalised vector

$$\mathcal{A}_\mu \stackrel{*}{=} h_\mu + B_\mu + \tilde{B}_\mu + C_{\mu,0} + C_{\mu,2} + C_{\mu,4} + C_{\mu,6}$$

  - Expand it on the parallelisation frame

$$\mathcal{A}_\mu{}^M(x,y) = \mathcal{A}_\mu{}^A(x)\hat{E}_A{}^M(y)$$

- A similar construction works for higher rank forms

Comments

- Generalised Scherk-Schwarz reduction reproduces the correct gauge transformations in lower dimensional supergravity.
  - Gauge group contains the isometry group of M_d
  - If $M_d = G$ it reduces to ordinary Scherk-Schwarz.
  - In addition, restricting to NSNS one can truncate to a $G \times G$ gauged sugra *[Baguet, Pope, Samtleben '14]*

- Generalised parallelisability guarantees the truncation to be consistent

Summary and Conclusions

- Generalised Geometry can describe geometrically the fields of supergravity
- One can construct consistent truncations using the extended symmetries of the theory
- How to find non maximally supersymmetric truncations?
    - Use generalised structures to define the invariant modes.
    - Applications to AdS/CFT: Finding truncations including marginal deformations.
    - Massive truncations on spheres with less supersymmetry.


Thank You