Electromagnetic duality anomaly

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What is this symmetry? Brief introduction

- The source-free Maxwell equations and energy-momentum tensor in 4 dimensions are manifestly invariant under the exchange of the electric and magnetic fields, $\vec{E} \longleftrightarrow \vec{B}$.

$$\nabla_\mu F^{\mu\nu} = 0$$
$$\nabla_\mu {}^\star F^{\mu\nu} = 0$$

$$T_{\mu\nu} = \frac{1}{2}\left[F_{\mu\sigma}F^{\sigma}{}_{\nu} + {}^\star F_{\mu\sigma}{}^\star F^{\sigma}{}_{\nu}\right]$$

$$F_{\mu\nu} \to F'_{\mu\nu} = F_{\mu\nu}\cos\theta + {}^\star F_{\mu\nu}\sin\theta$$
$${}^\star F_{\mu\nu} \to {}^\star F'_{\mu\nu} = {}^\star F_{\mu\nu}\cos\theta - F_{\mu\nu}\sin\theta$$

A duality transformation takes one solution of Maxwell equations and produces another one

What is this symmetry? Brief introduction

- This duality transformation is a symmetry of the Maxwell action, at the level of the basic dynamical variables A_i and for an arbitrary space-time background $(M, g_{\mu\nu})$ [*Deser, Teitelboim (1976)*].

$$S_M[A] = -\frac{1}{4}\int d^4x\sqrt{-g}F_{\mu\nu}F^{\mu\nu}\ , \qquad F = dA$$

- Noether's Theorem:

$$j_D^\mu = \frac{1}{2}\left[A_\nu {}^\star F^{\mu\nu} - 2F^{\mu\nu}Z_\nu - {}^\star G^{\mu\nu}Z_\nu\right]\ , \qquad \nabla_\mu j_D^\mu \approx 0$$

(Z_i is a non-local functional of A_i, and $G = dZ$)

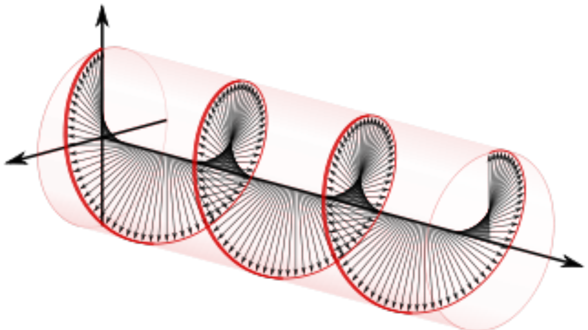
What is this symmetry? Brief introduction

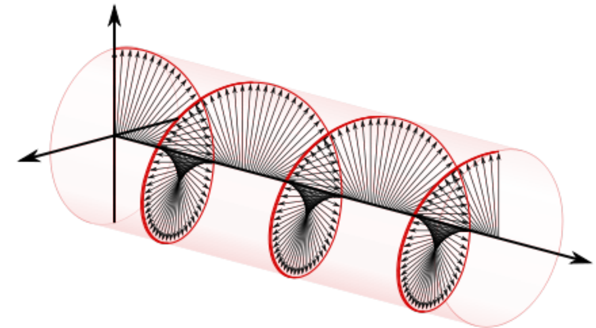
- The symmetry is generated by a conserved charge.

$$\delta H = \theta\{Q_D, H\} \approx 0\,, \qquad Q_D = \frac{1}{2}\int d^3x\sqrt{h}\,(A^i\,B_i - E_i Z^i)$$

Physically, it measures (in Minkowski) the net difference among right-handed and left-handed circularly polarized photons [*Calkin (1965)*].

$$Q_D = 2\int d^3k\left[a_R^\dagger(\vec{k})a_R(\vec{k}) - a_L^\dagger(\vec{k})a_L(\vec{k})\right]$$



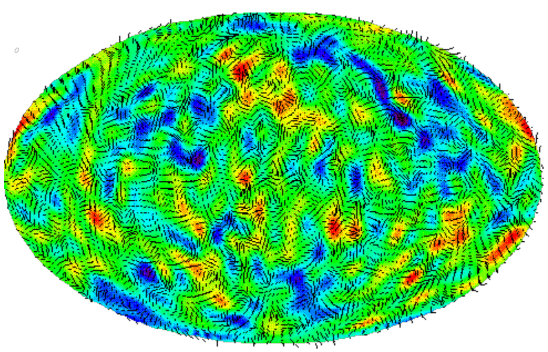


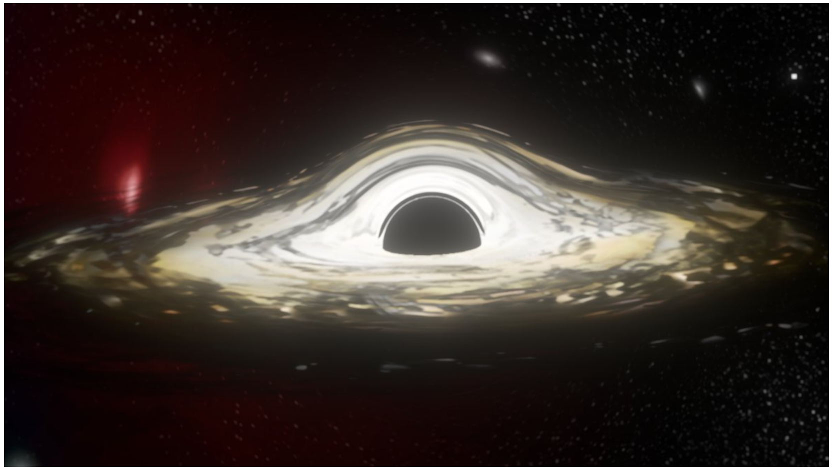
Main goal of the presentation

- Discuss the quantum breaking of the classical electromagnetic duality symmetry due to spacetime curvature.

$$\langle \nabla_\mu j^\mu_D \rangle \ = 0 \ ??$$

- Give some ideas about phenomenological applications in strong gravitational backgrounds.





Why do we expect a duality anomaly?

- **Quantization** of the **electromagnetic theory**.

Physical observables are generally given by (ill-defined) composite operators of the field. Need of renormalization.

$$\langle \nabla_\mu j^\mu_D \rangle = -\langle Z_\nu \nabla_\mu F^{\mu\nu} \rangle$$

Renormalization subtractions do not necessarily respect the equations of motion. Some examples are

$$\left[\Box + \frac{1}{6}R\right]\phi(x) = 0 \qquad \langle \phi \left[\Box + \frac{1}{6}R\right]\phi \rangle \propto \Box R - R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2$$

$$i\gamma^\mu(x)\nabla_\mu\psi(x) = 0 \qquad \langle \bar{\psi}\gamma_5 i\gamma^\mu \nabla_\mu \psi \rangle \propto R_{\mu\nu\lambda\sigma}{}^\star R^{\mu\nu\lambda\sigma}$$

Why do we expect a duality anomaly?

- The **emergence** of **anomalies** in quantum field theory is actually **not new.**

Conformal anomaly:

$$\langle T^\mu_\mu \rangle = \langle \phi \left[\Box + \frac{1}{6} R \right] \phi \rangle = \frac{1}{2880\pi^2} \left[\Box R - R_{\mu\nu} R^{\mu\nu} + \frac{1}{3} R^2 \right]$$

Chiral anomaly:

$$\langle \nabla_\mu j^\mu_5 \rangle = 2 \langle \bar{\psi} i \gamma^\mu \gamma_5 \nabla_\mu \psi \rangle = \frac{1}{192\pi^2} R_{\mu\nu\lambda\sigma}{}^\star R^{\mu\nu\lambda\sigma}$$

Why do we expect a duality anomaly?

- Some other works found **unexpected results.**

Dolgov *et al* (1987):

$$\langle F_{\mu\nu}{}^{\star}F^{\mu\nu}\rangle = 4\langle \vec{E}\cdot\vec{B}\rangle = \frac{1}{48\pi^2}R_{\alpha\beta\lambda\sigma}{}^{\star}R^{\alpha\beta\lambda\sigma}$$

Agullo, Landete, Navarro-Salas (2014) [in a spatially flat FLRW scenario]:

$$\langle F_{\mu\nu}F^{\mu\nu}\rangle = 2\left[\vec{B}^2(x) - \vec{E}^2(x)\right] = \frac{1}{480\pi^2}\left[-9R_{\alpha\beta}R^{\alpha\beta} + \frac{23}{6}R^2 + 4\Box R\right]$$

If the symmetry exists and leaves the vacuum state invariant, these values should be invariant under the exchange of *E* and *B*, but they are not.

How can we derive the duality anomaly?

- To clarify the issue we certainly need to find out the value of $\langle \nabla_\mu j^\mu_D \rangle$

$$\langle \nabla_\mu j^\mu_D \rangle = \lim_{m\to 0} m^2 \langle Z_i A^i \rangle = \lim_{m\to 0} im^2 \langle \vec{A}^2_+ - \vec{A}^2_- \rangle = \lim_{m\to 0} im^2 \left\langle \bar{\Psi} \beta_5 \Psi \right\rangle$$

Circular polarization variables: $A_{\pm,i} = \frac{1}{2}\left[A_i \pm i Z_i\right]$

This resembles the corresponding expression for the spin ½ chiral current. It suggests dealing with a similar formalism.

$$\langle \nabla_\mu j^\mu_5 \rangle = \lim_{m\to 0} 2im \langle \bar{\psi} \gamma_5 \psi \rangle$$

How can we derive the duality anomaly?

- Chiral spin ½ anomaly. Some background.

A massless Dirac field is described in terms of two (decoupled) fundamental spinors satisfying Weyl equations

$$\psi = \begin{pmatrix} u_+ \\ u_- \end{pmatrix} \qquad \begin{aligned} i\sigma^\mu \nabla_\mu u_+ &= 0 \\ i\bar{\sigma}^\mu \nabla_\mu u_- &= 0 \end{aligned} \qquad \begin{aligned} u_+ &\sim (1/2, 0) \\ u_- &\sim (0, 1/2) \end{aligned}$$

The action of a massless Dirac field inmersed in either an electromagnetic or gravitational background remains invariant under an infinitesimal chiral rotation:

$$S[\psi] = \int d^4x \sqrt{-g}\; i\bar{\psi}\gamma^\mu \nabla_\mu \psi\,, \qquad \psi \to e^{i\theta\gamma_5}\psi = \begin{pmatrix} e^{i\theta} u_+ \\ e^{-i\theta} u_- \end{pmatrix}$$

Noether's Thm leads to the conservation of the net difference between right and left chiral particles

$$Q = \int d^3x \sqrt{h} \left[u_+^\dagger u_+ - u_-^\dagger u_- \right]$$

How can we derive the duality anomaly?

- Chiral spin ½ anomaly. Some background.

At the quantum level, however, this is no longer conserved:

$$\langle \nabla_\mu j^\mu \rangle \propto e^2 F_{\mu\nu}{}^\star F^{\mu\nu}$$

[Adler, Bell, Jackiw (1969)]

$$\langle \nabla_\mu j^\mu \rangle \propto R_{\mu\nu\sigma\rho}{}^\star R^{\mu\nu\sigma\rho}$$

[Kimura (1969)]

From the mathematical point of view, the anomaly is understood as a local realization of the so-called index theorems.

$$\int d^4x \sqrt{-g}\, \langle \nabla_\mu j^\mu \rangle = 2[n(1/2,0) - n(0,1/2)]$$

[Eguchi et al (1980); Christensen, Duff (1978)]

How can we derive the duality anomaly?

- First-order formalism. Weyl-type equations of motion.

In absence of sources Maxwell EOM decouple in terms of 2 spinors: $\vec{H}_\pm \equiv \frac{1}{2}[\vec{E} \pm i\vec{B}]$

$$\left.\begin{array}{l} i\dfrac{\partial}{\partial t}\vec{H}_\pm = \pm\vec{\nabla}\times\vec{H}_\pm \\ \vec{\nabla}\cdot\vec{H}_\pm = 0 \end{array}\right\} \quad \begin{array}{l} (\alpha^a)^b{}_i\partial_a H^i_+ = 0 \\ (\bar{\alpha}^a)^b{}_i\partial_a H^i_- = 0 \end{array} \qquad \begin{array}{l} H_+ \sim (1,0) \\ H_- \sim (0,1) \end{array}$$

Introduce complex potentials, and fix the radiation gauge: **identical EOM**

$$\left.\begin{array}{l} \vec{H}_\pm \equiv i\,\vec{\nabla}\times\vec{A}_\pm \\ \vec{\nabla}\cdot\vec{A}_\pm = 0 \end{array}\right\} \quad \begin{array}{l} (\alpha^a)^b{}_i\partial_a A^i_+ = 0 \\ (\bar{\alpha}^a)^b{}_i\partial_a A^i_- = 0 \end{array} \qquad \alpha^{ab}{}_i \leftrightarrow \sigma^\mu_{A'A}$$

Generalize to a general spaceime by taking the connection-compatibility condition:

$$\nabla_\beta(\alpha^\mu)^\nu{}_i(x) = 0$$

How can we derive the duality anomaly?

- First-order formalism.

In this language, a duality transformation resembles a conventional chiral rotation

$$\beta^\mu \nabla_\mu \Psi(x) = 0 \ , \qquad \beta^\mu \equiv i \begin{pmatrix} 0 & \bar{\alpha}^\mu \\ -\alpha^\mu & 0 \end{pmatrix}$$

$$\Psi \equiv \begin{pmatrix} A_+^{\ i} \\ iA_{-\ i} \end{pmatrix} \ , \qquad \begin{pmatrix} A_+^i \\ iA_{-\ i} \end{pmatrix} \to e^{i\theta\beta_5} \begin{pmatrix} A_+^i \\ iA_{-\ i} \end{pmatrix} = \begin{pmatrix} e^{-i\theta} A_+^i \\ e^{i\theta} iA_{-\ i} \end{pmatrix}$$

These variables describe right / left handed (circularly polarized) radiation.

How can we derive the duality anomaly?

- Fujikawa's method: evaluate the symmetry transformation on the quantum effective action W

$$e^{iW} = \int d\mu[A] e^{iS[A]}$$

$S[A]$ → invariant $\quad S[A'] = S[A] - \int d^4x\sqrt{-g}\theta(x)\nabla_\mu j_D^\mu$

$d\mu[A]$ → not necessarily invariant!!

$$d\mu[A] = \prod_x (-g)^{1/2} \det[D_\mu D^\mu]^{1/2} D\bar{\Psi}(x) D\Psi(x) D\omega(x) DA_0$$

Gauge fixing: $\delta\omega$

How can we derive the duality anomaly?

- Eigenvalue problem to analyze the jacobian: $D\bar{\Psi}'(x)D\Psi'(x) = J D\bar{\Psi}(x)D\Psi(x)$

$$\beta^\mu \nabla_\mu \Psi_n(x) = \lambda_n \Psi_n(x) \longrightarrow \langle \nabla_\mu j_D^\mu \rangle = 2\ell^{-2} \sum_{n=0}^{\infty} (\Psi_n^\dagger \beta_5 \Psi_n)$$

UV-divergent

- Heat kernel to regularization:

$$K(\tau; x, x') \equiv \sum_{n=0}^{\infty} e^{-i\tau\lambda_n^2} \Psi_n(x)\, \Psi_n^\dagger(x') \longrightarrow \langle \nabla_\mu j_D^\mu \rangle = 2\ell^{-2} \lim_{\tau \to 0} \mathrm{Tr}[\beta_5 K(\tau; x, x)]$$

How can we derive the duality anomaly?

- Make use of his well-known asymptotic behaviour.

$$K(\tau; x, x) \sim \frac{i\ell^2}{16\pi^2} \sum_{k=0}^{\infty} (i\tau)^{k-2} E_k(x) \qquad\qquad (\Box + \mathcal{Q})\Psi(x) = 0$$

$$E_0(x) = \mathbb{I},$$

$$E_1(x) = \frac{1}{6} R\,\mathbb{I} - \mathcal{Q},$$

$$E_2(x) = \left[\frac{1}{72}R^2 - \frac{1}{180}R_{\mu\nu}R^{\mu\nu} + \frac{1}{180}R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}\right]\mathbb{I} - \frac{1}{30}\Box R + \frac{1}{12}W_{\mu\nu}W^{\mu\nu} + \frac{1}{2}\mathcal{Q}^2 - \frac{1}{6}R\mathcal{Q} + \frac{1}{6}\Box\mathcal{Q},$$

...

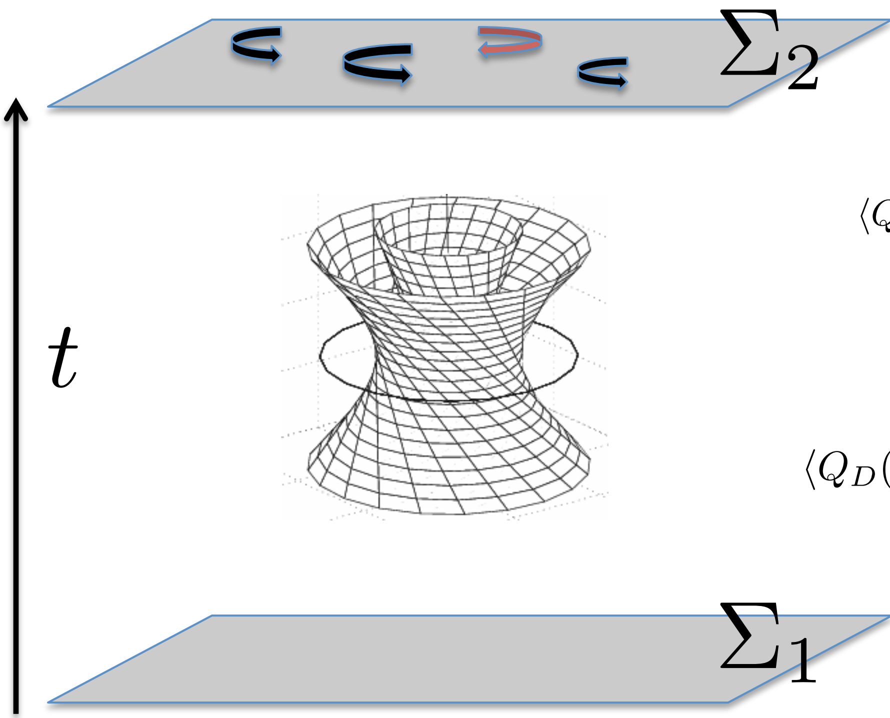
Final result:

$$\langle \nabla_\mu j^\mu_D \rangle = \frac{1}{24\pi^2} R_{\mu\nu\lambda\sigma}{}^\star R^{\mu\nu\lambda\sigma}$$

Possible phenomenological applications

- Heuristic vision

$$\langle Q(\Sigma_2)\rangle - \langle Q(\Sigma_1)\rangle = \frac{1}{24\pi^2}\int_{t_1}^{t_2} dt \int d^3\vec{x}\sqrt{-g}R_{\mu\nu\alpha\beta}{}^{\star}R^{\mu\nu\alpha\beta}$$



$$M = \mathbb{R}\times\Sigma_t$$

Gravity distinguishes right and left photons: **different** Bogoloubov coefficients

$$\langle Q_D(\Sigma_1)\rangle \sim 2\int d^3k\left[\left\langle a_R^\dagger(\vec{k})a_R(\vec{k})\right\rangle - \left\langle a_L^\dagger(\vec{k})a_L(\vec{k})\right\rangle\right]\approx 0$$

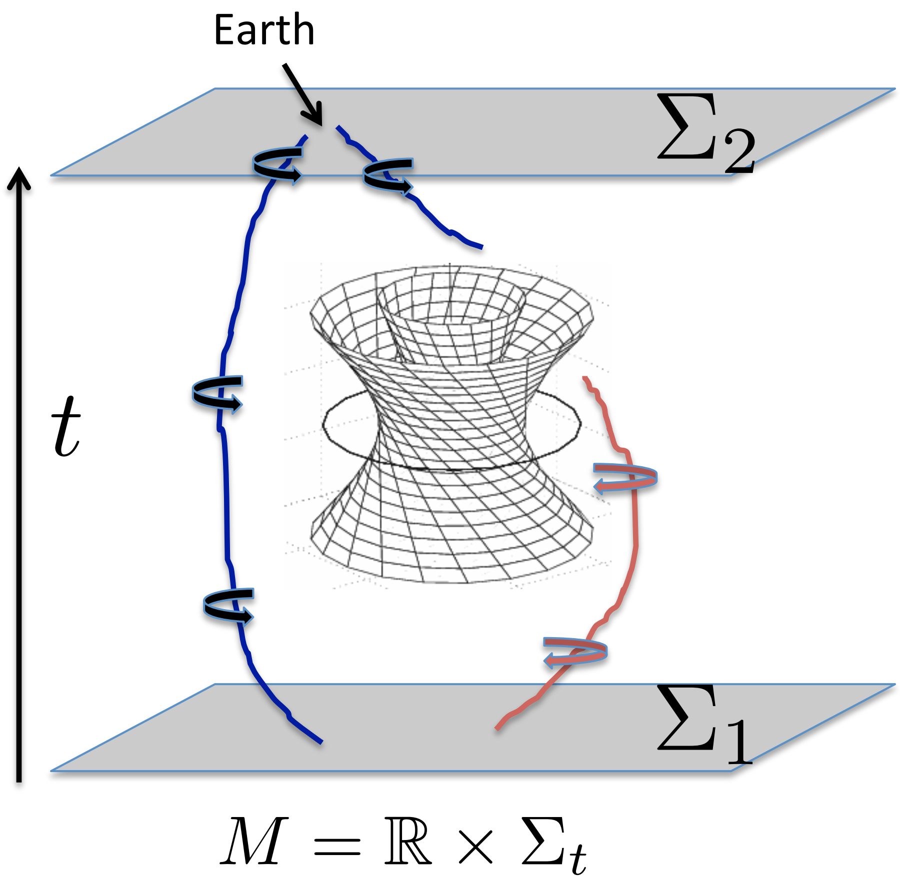
$$\downarrow$$

$$\langle Q_D(\Sigma_2)\rangle \sim 2\int d^3k\left[\left\langle A_R^\dagger(\vec{k})A_R(\vec{k})\right\rangle - \left\langle A_L^\dagger(\vec{k})A_L(\vec{k})\right\rangle\right]$$

$$= 2\int d^3k\left[|\beta_R|^2 - |\beta_L|^2\right]\neq 0$$

Possible phenomenological applications

- Heuristic vision $$\langle Q(\Sigma_2)\rangle - \langle Q(\Sigma_1)\rangle = \frac{1}{24\pi^2}\int_{t_1}^{t_2} dt \int d^3\vec{x}\sqrt{-g}R_{\mu\nu\alpha\beta}{}^{\star}R^{\mu\nu\alpha\beta}$$

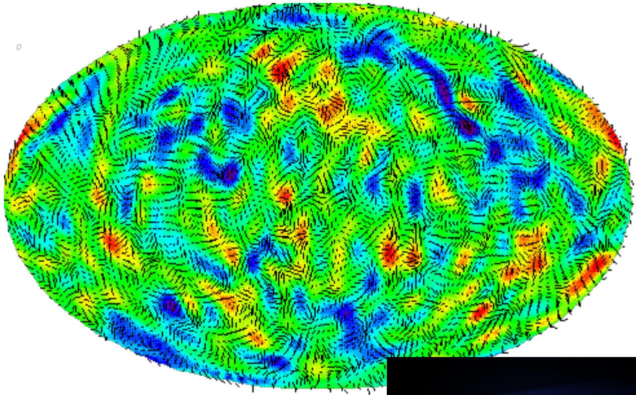


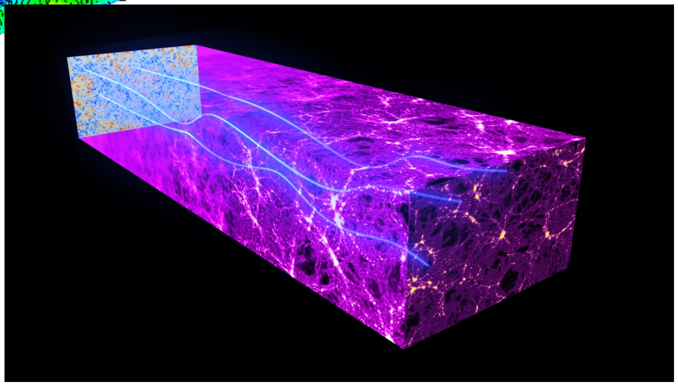
The **presence of photons** initially may probably **stimulate** this effect!

Possible phenomenological applications

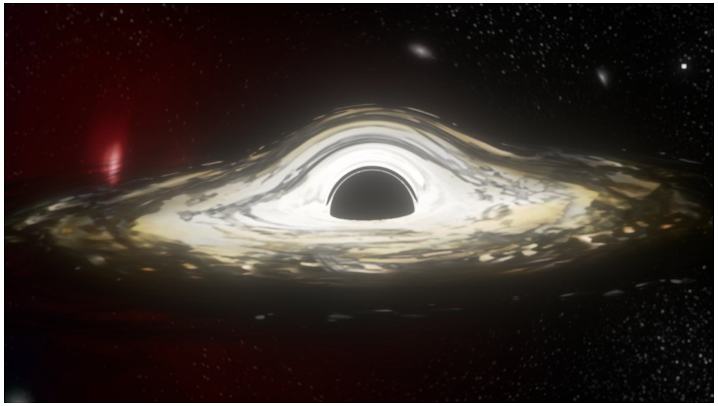
- Need very strong gravitational backgrounds to compensate $\hbar$

Cosmology. CMB circular polarization and lensing





Rotating **black holes**



Conclusions

- We find a **quantum anomaly** in the classical **electromagnetic duality symmetry** when a non-trivial spacetime is considered.

- Phenomenologically tiny effect ($\hbar, G$), but some ideas may be developed to measure **net polarization induced by gravitational dynamics.**

- The anomaly can be understood as a local version of the Hirzeburch signature theorem.

$$\int d^4x\sqrt{-g}\,\langle\nabla_\mu j_D^\mu\rangle = 2[n(1,0)-n(0,1)]$$

Thanks for your attention!