3-Forms, Axions and D-brane Instantons

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Motivation & Outline

- Motivation
  - **Axions** have very flat potentials $\Rightarrow$ Good for Inflation.
  - Axions with non-perturbative (instantons) potential can always be described as a **3-Form eating up the 2-Form** dual to the axion.
  - There is no candidate 3-Form for stringy instantons. We will look for it in the **geometry deformed by the instanton**.
- Outline
  1. Axions, Monodromy and 3-Forms.
  2. String Theory, D-Branes and Instantons.
  3. Backreacting Instantons.

Axions and Monodromy.

Axions are **periodic scalar fields**.

- This shift symmetry can be **broken to a discrete symmetry** by non-perturbative effects $\rightarrow$ Very flat potential, good for inflation.
- The discrete shift symmetry gives periodic potentials, but non-periodic ones can be used by endowing them with a **monodromy** structure. For instance:

$$|\phi|^2 + \mu^2\phi^2 \tag{1}$$

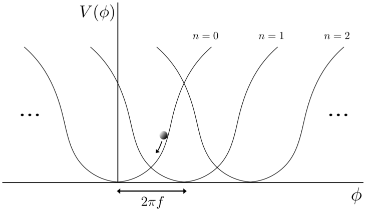
- Superplanckian field excursions in Quantum Gravity are under pressure due to the Weak Gravity Conjecture (Arkani-Hamed *et al.*, 2007). Monodromy may provide a workaround (Silverstein & Westphal, 2008; Marchesano *et al.*, 2014).
- This mechanism can be easily realized in string theory.

Kaloper-Sorbo Monodromy

- A quadratic potential with monodromy can be described by a Kaloper-Sorbo lagrangian (Kaloper & Sorbo, 2009),

$$|\,\mathrm{d}\phi|^2 + n\phi F_4 + |F_4|^2 \tag{2}$$

Where the monodromy is given by the vev of F_4, different fluxes correspond to different branches of the potential.



(Ibanez *et al.*, 2016)

Kaloper-Sorbo Monodromy

- Monodromy structure visible in potential solving E.O.M,

$$V_0 \sim (n\phi - q)^2, \quad q \in \mathbb{Z} \tag{3}$$

So there is a discrete shift symmetry,

$$\phi \to \phi + \phi_0, \quad q \to q + n\phi_0 \tag{4}$$

- A dual description can be found using the hodge dual of ϕ, b_2,

$$|\,\mathrm{d}b_2 + nc_3|^2 + |F_4|^2\,, \quad F_4 = \mathrm{d}c_3 \tag{5}$$

The flatness of the axion potential is protected by gauge invariance,

$$c_3 \to c_3 + \mathrm{d}\Lambda_2\,, \quad b_2 \to b_2 - n\Lambda_2 \tag{6}$$

We will look for a three form c_3 coupling to the axion as $\sim \phi F_4$

- This description has the periodicity built in. Whenever the potential breaks it, there will be monodromy in the UV.

Example: Peccei-Quinn mechanism in 3-Form language.

- **Strong CP problem**: QCD has an anomalous $\mathrm{U}(1)_A$ symmetry producing a *physical* θ-term.

$$\mathcal{L} \sim \frac{g^2\theta}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{a\mu\nu} \tag{7}$$

  Where θ classifies topologically inequivalent vacua and is typically ~ 1. This term breaks CP and experimental data constrain $\theta \lesssim 10^{-10} \rightarrow$ **Fine Tuning.**

- **Solution**: PQ Mechanism. New anomalous spontaneously broken $\mathrm{U}(1)_{PQ}$. So θ is promoted to pseudo-Goldstone boson, the axion. Non-perturbative effects give it a potential and fix $\theta = 0$.

Example: Peccei-Quinn mechanism in 3-Form language.

- The term can be rewritten as (Dvali, 2005),

$$\theta F\tilde{F} \sim \theta F_4 = \theta\, \mathrm{d}C_3, \quad C_{\alpha\beta\gamma} = \frac{g^2}{8\pi^2}\mathrm{Tr}\left(A_{[\alpha}A_{\beta}A_{\gamma]} - \frac{3}{2}A_{[\alpha}\partial_{\beta}A_{\gamma]}\right) \tag{8}$$

  So, making θ small = How can I make the F_4 electric field small?

- **Solution**: Screen a field $\Rightarrow$ Higgs mechanism. Let C_3 eat a 2-Form b_2! $\Rightarrow$ b_2 *is* Hodge dual of axion!

  Potential for the axion $\Longleftrightarrow$ 3-Form eating up a 2-form.

String Theory

- String theory *unifies* gravity and quantum mechanics.
- String theory aims to describe *everything* as vibrations of tiny strings (see IFT youtube).

  $\rightarrow$ Each particle is a different vibration state of the string

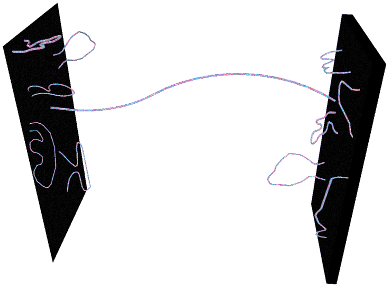
  $\Rightarrow$ So string theory is **awesome**!
- Caveats: 10 dimensions, infinite (or very big) landscape of vacua, only pertubatively defined... But *who cares?*

$\rightarrow$ Gravity is described by closed strings and gauge interactions by open strings.

$\rightarrow$ 6 dimensions are compact: $\mathcal{M}^4 \times X^6$

D-Branes

- Open Strings end on non-perturbative, dynamical objects called D-Branes,



$\rightarrow$ Rich physcis inside the Brane.

D-Brane Instantons

Dp-Branes completely wrapped around euclidean p cycles are D-brane instantons. There are two kinds:

- Dp-Brane instanton inside D($p+4$)-Brane $\rightarrow$ Gauge Instanton.
- Dp-Brane alone $\rightarrow$ Stringy Instanton.

$\Rightarrow$ We study non-perturbative potentials for axions coupling to stringy instantons.

- There are 5 string theories, all related through dualities. We will focus on type IIB. It has odd Dp-branes and odd RR Forms: $F_1, F_3, F_5 ...$

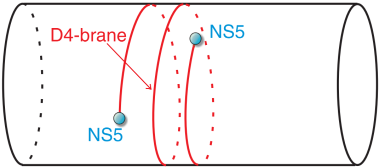
Axions in String Theory.

- Axions are ubiquitous in string theory. For instance upon compactification,

$$a(x) = \int_{\Sigma^p} C_p \tag{9}$$

  Where the shift symmetry arises from the gauge invariance of the RR form.

- Monodromy can be realised in many ways in string theory. For instance, as unwinding of a brane (Silverstein & Westphal, 2008),



The puzzle

Axion potentials come from non-perturbative effects and can be described by a 3-form eating up a 2-form. For instance, for gauge D-brane instantons it is the CS 3-Form (Dvali, 2005).

- For stringy instantons $\rightarrow$ **No candidate 3-Form!**
  $\Rightarrow$ Solution: look for the 3-Form in the *backreacted* geometry (Koerber & Martucci, 2007).
- In the backreacted geometry the **instanton disappears** and its open-string degrees of freedom are encoded into the geometric (closed strings) degrees of freedom. Both descriptions are related in an *"holographic"* way.

SU(3) $\times$ SU(3) structure manifolds.

- The backreacted geometry will, generically, be a non CY, SU(3) $\times$ SU(3) structure manifold. This structure has two globally well defined spinors $\eta_+^{(1)}, \eta_+^{(2)}$, with c.c. $\eta_-^{(1)}, \eta_-^{(2)}$.
  Define two polyforms (assume type IIB):

$$\Psi_\pm = -\frac{i}{||\eta^{(1)}||^2} \sum_l \frac{1}{l!} \eta_\pm^{(2)\dagger} \gamma_{m_1 \ldots m_l} \eta_+^{(1)} \, dy^{m_l} \wedge \ldots \wedge dy^{m_1} \tag{10}$$

- Organize 10d fields in holomorphic polyforms,

$$\mathcal{Z} \equiv e^{3A-\Phi} \Psi_2, \quad \mathcal{T} \equiv e^{-\Phi} \mathrm{Re}\, \Psi_1 + i \Delta C \tag{11}$$

- For an SU(3) structure manifold, $\eta^{(1)} \sim \eta^{(2)}$ and one recovers $\mathcal{Z} \sim \Omega$, $\mathcal{T} \sim e^{iJ}$

Backreacting a D3-instanton

- Consider type IIB String Theory with a D3-instanton wrapping a 4-cycle Σ_4 in a CY_3.
- One can show from the SUSY conditions for the D3-instanton that the **backreaction is encoded in a contribution to $\mathcal{Z}$** (Koerber & Martucci, 2007):

$$\mathrm{d}(\delta\mathcal{Z}) \sim \mathcal{W}_{\mathrm{np}}\delta_2(\Sigma_4) \tag{12}$$

$\Rightarrow$ So, the backreaction produces a 1-form $\mathcal{Z}_1$ that didn't exist in the original geometry.

- Note that $\mathcal{Z}_1$ is not closed and thus not harmonic.

The 3-form and the KS coupling

- Let us define $\alpha_1 \equiv \mathcal{Z}_1$ and $\beta_2 \equiv \mathrm{d}\alpha_1$.
- In type IIB string theory there is a RR 4-form C_4. We may expand it as,

$$C_4 = \alpha_1(y) \wedge c_3(x) + \beta_2(y) \wedge b_2(x) + \ldots \tag{13}$$

$\Rightarrow$ So we have a 3-form and a 2-form dual to an scalar.

- We see that,

$$F_5 = \mathrm{d}C_4 = \beta_2 \wedge (c_3 + \mathrm{d}b_2) - \alpha_1 \wedge F_4 \tag{14}$$

Which describes a 3-Form eating up a 2-Form, as we wanted! Furthermore,

$$\int_{10d} F_5 \wedge * F_5 = -\int_{10d} C_4 \wedge \mathrm{d}F_5 \to \int_{10d} C_4 \wedge \beta_2 \wedge F_4 = \int_{4d} \phi F_4 \tag{15}$$

Which is the KS coupling we were looking for.

Toroidal examples.

- For a 4-cycle defined by the equation $f = 0$ the 1-form is $\mathcal{Z}_{(1)} \sim \mathrm{d}f\,\tilde{W}_{np}$.
- **Example 1**: Factorisable 6-torus, $\mathcal{M}^4 \times \mathbf{T}^2 \times \mathbf{T}^2 \times \mathbf{T}^2$ with local coordinates z_1, z_2, z_3 and take the cycle $f = z_3 = 0$,

$$\mathcal{Z}_{(1)} \sim e^{-T}\,\mathrm{d}z_3 \tag{16}$$

  $\Rightarrow$ already in the original geometry, because $\mathbf{T}^6$ is non-CY.

- **Example 2**: Orbifold $\mathbf{T}^6/(\mathbf{Z}_2 \times \mathbf{Z}_2)$ with action,

$$\theta : (z_1, z_2, z_3) \to (-z_1, z_2, -z_3), \quad \omega : (z_1, z_2, z_3) \to (z_1, -z_2, -z_3) \tag{17}$$

  Local coordinates $u_i = z_i^2$ and cycle $f \sim u_3 = 0$ give backreaction,

$$\mathcal{Z}_{(1)} \sim \mathrm{d}u_3 \sim z_3\,\mathrm{d}z_3 \tag{18}$$

D-brane gauge Instanton backreaction

- D3-brane instantons wrapping the same 4-cycle as N spacetime-filling D7-branes are gauge instantons from the SU(N) point of view.
- In the gauge (open string) description a CS 3-Form is available to couple to the axion.
- We can backreact the D7's together with their non-perturbative effects to find a dual description in terms of closed string degrees of freedom. **Again a 1-Form arises** when taking the non-perturbative effects into account,

$$\mathrm{d}\mathcal{Z} = i l_s \langle S \rangle \, \delta_2(\Sigma) \tag{19}$$

Where S is the gaugino condensate describing the non-perturbative effects.

Generalization

- Supersymmetry equations in type IIA are trickier, but we can use mirror symmetry.
- **Mirror symmetry** is a symmetry between IIA and IIB living in mirror CY_3's. In the large complex structure limit it equal to 3 T-dualities.
- The mirror dual of our setup consists on a D2-brane wrapping a 3-cycle. The backreaction gives rise to,

$$\delta\mathcal{T} = \mathcal{T}_{(2)} + \mathcal{T}_{(4)} \tag{20}$$

$\Rightarrow$ so we can obtain the 3-Form in two ways,

$$C_7 = \mathcal{T}_{(4)} \wedge c_3, \quad C_5 = \mathcal{T}_{(2)} \wedge c_3 \tag{21}$$

Conclusions & Outlook

- **Axions** are a useful tool for inflation and are easily realised in String Theory.
- Axion **Monodromy** may help avoid the Weak Gravity Conjecture.
- Axion physics can be described in a dual language where a **3-form eats up a 2-form.**
- For stringy instantons **no known 3-form** to couple to the 2-form was known. We have showed that it only appears when the **"backreaction"** is taken into account.
- The geometry ceases to be CY and new forms (and cycles) that were not there arise.
- **Further work:**
  - Use this mechanism as a self consistency test for the Weak Gravity Conjecture for axions and 3-Forms.
  - Study the backreaction of particular instantons in specific setups $\Rightarrow$ $\Rightarrow$ Geometric transitions.

Thank You!

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