Anomaly Corrected Heterotic Horizons

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Black holes in higher dimensions

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  Black ring, BH with horizon topology $S^1 \times S^2$, discovered in
  - Einstein gravity [Emparan, Reall],
  - $\mathcal{N} = 2$ minimal supergravity [Elvang, Emparan, Mateos, Reall].

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- The full BH solution is in general difficult to find out

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- Spinor bilinears generate a global $\mathfrak{sl}(2,\mathbb{R})$, symmetry of the full solution (*symmetry enhancement*).
- The isometry group $SL(2,\mathbb{R})$ plays the essential role of conformal group in the dual CFT picture.

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Aim of this work:

- We shall investigate the effect of higher order corrections to $D = 10$ near-horizon geometries.
- We choose the Heterotic supergravity

Outline

- Gaussian Null Co-ordinates
- Heterotic near-horizon geometries
- Supersymmetry enhancement?
- Lichnerowicz Theorem
- Conclusions

Gaussian Null Co-ordinates

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- spacetime contains an (extremal) Killing horizon, i.e. a null-hypersurface $\mathcal{H}$ associated with the Killing vector V.

One can introduce a Gaussian Null Co-ordinate system $\{u, r, y^I\}$, such that $V = \frac{\partial}{\partial u}$, the horizon $\mathcal{H}$ is located at $r = 0$, and the metric is

$$ds^2 = 2drdu + 2rh_I dudy^I - r^2 \Delta dudu + \gamma_{IJ} dy^I dy^J$$

[Isenberg, Moncrief]

where Δ, h_I and γ_{IJ} are analytic in r, u-independent scalar, 1-form and metric of the 8-dim horizon spatial cross section $\mathcal{S}$, which we shall assume **smooth** and **compact without boundary**.

Then we perform the **near-horizon limit**

$$r \to \epsilon r \qquad u \to \frac{u}{\epsilon} \qquad y^I \to y^I \qquad \epsilon \to 0$$

the metric remains invariant in form, and the near-horizon data $\{\Delta, h_I, \gamma_{IJ}\} = \{\Delta(y), h_I(y), \gamma_{IJ}(y)\}$.

In light-cone basis:

$$\mathbf{e}^+ = du \qquad \mathbf{e}^- = dr + rh - \frac{1}{2}r^2\Delta du \qquad \mathbf{e}^i = e^i{}_J dy^J$$

$$ds^2 = 2\mathbf{e}^+\mathbf{e}^- + \delta_{ij}\mathbf{e}^i\mathbf{e}^j$$

The near-horizon limit only exists for *extremal* black holes.

Heterotic near-horizon geometries

The bosonic fields of heterotic supergravity are the metric g, a real scalar dilaton field Φ, a real 3-form H, and a non-abelian 2-form field F.

They must be well-defined and regular in the near-horizon limit $\epsilon \to 0$.

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dilaton	$\Phi = \Phi(y)$
3-form	$H = \mathbf{e}^+ \wedge \mathbf{e}^- \wedge N + r\mathbf{e}^+ \wedge Y + W$
2-form	$A = r\mathcal{P}\mathbf{e}^+ + \mathcal{B}\ , \qquad F = dA + A \wedge A$

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The Green-Schwarz anomaly cancellation mechanism requires that

$$dH = -\frac{\alpha'}{4}\Big(tr(R^{(-)} \wedge R^{(-)}) - tr(F \wedge F)\Big) + \mathcal{O}(\alpha'^2)$$

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We further assume that the solution is *supersymmetric*, i.e. there exists a Majorana-Weyl Killing spinor ϵ, well defined on $\mathcal{H}$, satisfying the KSE:

$$\nabla^{(+)}_M \epsilon \equiv \big(\nabla_M - \frac{1}{8} H_{M N_1 N_2} \Gamma^{N_1 N_2}\big)\epsilon = \mathcal{O}(\alpha'^2) \qquad \text{gravitino}$$

$$\big(\Gamma^M \nabla_M \Phi - \frac{1}{12} H_{N_1 N_2 N_3} \Gamma^{N_1 N_2 N_3}\big)\epsilon = \mathcal{O}(\alpha'^2) \qquad \text{dilatino}$$

$$F_{MN}\Gamma^{MN}\epsilon = \mathcal{O}(\alpha') \qquad \text{gaugino}$$

[Bergshoeff, de Roo]

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We shall integrate the gravitino KSE along the $\mathbf{e}^+$ and $\mathbf{e}^-$ directions (u, r dependence of all bosonic fields is known).

Split the Killing spinors into positive and negative light-cone chiralities

$$\epsilon = \epsilon_+ + \epsilon_- \ , \qquad \Gamma_\pm \epsilon_\pm = 0$$

Integrating the gravitino KSE along $\mathbf{e}^+$ and $\mathbf{e}^-$

$$\begin{aligned}
\epsilon_+ &= \eta_+ + \frac{1}{4} u (h+N)_i \Gamma^i \Gamma_+ \eta_- + \mathcal{O}(\alpha'^2) \\
\epsilon_- &= \eta_- + \frac{1}{4} r (h-N)_i \Gamma^i \Gamma_- \eta_+ + \frac{1}{8} r u (h-N)_i (h+N)_j \Gamma^i \Gamma^j \eta_- + \mathcal{O}(\alpha'^2)
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Supersymmetry requires that $\eta_\pm$ satisfy a number of differential and algebraic conditions (*reduced KSE*).

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Theorem 1 (Completes AdS classification):
No AdS_2 solutions in heterotic supergravity, up to $\mathcal{O}(\alpha'^2)$, for which $\mathcal{S}$ is smooth and compact without boundary, and all fields are smooth.

Supersymmetry enhancement

We simplify the reduced KSE to the necessary and sufficient conditions

$$\tilde{\nabla}_i^{(+)}\eta_\pm \equiv \left(\tilde{\nabla}_i - \frac{1}{8}W_{ijk}\Gamma^{jk}\right)\eta_\pm = \mathcal{O}(\alpha'^2)$$

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η_+ satisfies "$+$" $\Longrightarrow$ $\eta_- = \Gamma_-\Gamma^i h_i \eta_+$ satisfies "$-$"

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- Truncating at $\mathcal{O}(\alpha')$, Doubling of susy if $\eta_-^{[0]} \neq 0$.

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Define the modified connection with torsion:

$$\hat{\nabla}_i \equiv \tilde{\nabla}^{(+)} + \kappa \Gamma_i \mathcal{A}$$

and the modified near-horizon Dirac operator:

$$\mathcal{D} \equiv \Gamma^i \tilde{\nabla}_i^{(+)} + q\mathcal{A}$$

where $\kappa, q \in \mathbb{R}$, and $\mathcal{A} = W_i jk \Gamma^{ijk} - 12\tilde{\nabla}_i \Phi \Gamma^i - 6h_i \Gamma^i$.

Consider the functional

$$\mathcal{I} \equiv \int_{\mathcal{S}} e^{c\Phi} \Big(\langle \hat{\nabla}_i \eta_\pm, \hat{\nabla}^i \eta_\pm \rangle - \langle \mathcal{D}\eta_\pm, \mathcal{D}\eta_\pm \rangle \Big) \quad , \qquad c \in \mathbb{R}$$

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$$\begin{aligned} \mathcal{I} \quad &= \quad \left(8\kappa^2 - \frac{1}{6}\kappa\right) \int_{\mathcal{S}} e^{-2\Phi} \parallel \mathcal{A}\, \eta_\pm \parallel^2 + \int_{\mathcal{S}} e^{-2\Phi} \langle \eta_\pm, \Psi \mathcal{D}\eta_\pm \rangle \\ &- \quad \frac{\alpha'}{64} \int_{\mathcal{S}} e^{-2\Phi} \left(2 \parallel d\!\!\!/ h\, \eta_\pm \parallel^2 + \parallel \tilde{F\!\!\!/} \eta_\pm \parallel^2 - \parallel \tilde{R}^{(-)}_{\ell_1\ell_2,\, ij} \Gamma^{\ell_1\ell_2} \eta_\pm \parallel^2 \right) + \mathcal{O}(\alpha'^2) \end{aligned}$$

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Theorem 2 (Lichnerowicz): If $0 < \kappa < \frac{1}{48}$, then

$$\mathcal{D}\eta_\pm = \mathcal{O}(\alpha'^2) \qquad \Longrightarrow \qquad \tilde{\nabla}^{(+)}\eta_\pm = \mathcal{O}(\alpha')\ , \quad \mathcal{A}\eta_\pm = \mathcal{O}(\alpha')\ .$$

and the extra conditions

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However for higher order horizons, Lichnerowicz Theorem is not enough to establish susy enhancement to $\mathcal{O}(\alpha'^2)$.

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- Found sufficient conditions for susy enhancement.

$+$ lightcone chirality	$-$ lightcone chirality	Susy enhancement
$\eta_+^{[0]} \equiv 0$	$\eta_-^{[0]} \not\equiv 0$	same analysis as uncorrected horizon
$\eta_+^{[0]} \not\equiv 0$	all $\eta_-^{[0]} \equiv 0$	unknown
	at least one $\eta_-^{[0]} \not\equiv 0$	$\tilde{\nabla}^{(+)} h = \mathcal{O}(\alpha'^2)$ holds, susy enh. holds

Table: Status of the supersymmetry enhancement.

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  Finiteness of the moduli space of transverse deformations?

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  - Einstein theory [Li, Lucietti]
  - minimal $D = 5$ sugra [Dunajski, Gutowski, Sabra]
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  - Einstein-Maxwell-Dilaton theory and $D = 11$ sugra [AF, Gutowski]
- Apply results of studying the moduli space of the Strominger system to the near-horizon geometries? (see also R. Sisca talk)