

# High energy effects in multi-jet production at LHC

DAVID GORDO GÓMEZ



david.gordo@csic.es



Instituto de Física Teórica UAM/CSIC  
Madrid, Spain

in collaboration with

**F. Caporale, F. Celiberto, G. Chachamis, A. Sabio Vera**

based on

**Nuclear Physics B 910 (2016) 374-386**

[arXiv:1606.00574](https://arxiv.org/abs/1606.00574)

*V Postgraduate Meeting On Theoretical Physics*

November 17<sup>th</sup> - 18<sup>th</sup>, 2016  
Oviedo, Spain

# Outline

## 1 Introduction

- Motivation
- BFKL
- Mueller Navelet jets

## 2 Multi-jet production

- A new way to probe BFKL
- Three-jet at partonic level
- Three-jet at hadronic level

## 3 Conclusions & Outlook

# High energy limit

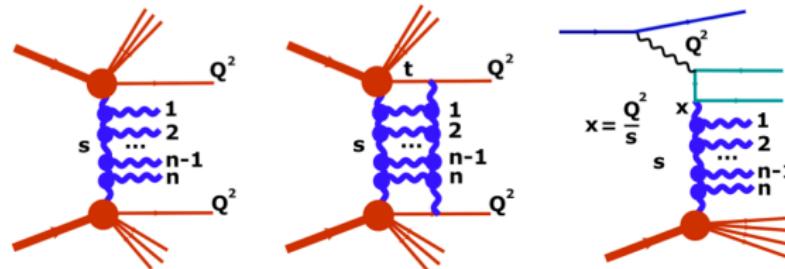
The high energy limit studies a limited part of the phase space, but allow us to compute things otherwise impractical

## Purely theoretical

- ◊ CFT's
- ◊ AdS/CFT
- ◊ Special Functions
- ◊ Integrability Methods
- ◊ Spin Chains

# High energy limit

## Phenomenology



- ◊ Mueller-Navelet jets
- ◊ Muellet-Tang jets
- ◊ DIS at small  $x$

With the advent of LHC we have access to higher energies:  
opportunity to test **pQCD in the high-energy limit** and the  
applicability of **BFKL resummation**.

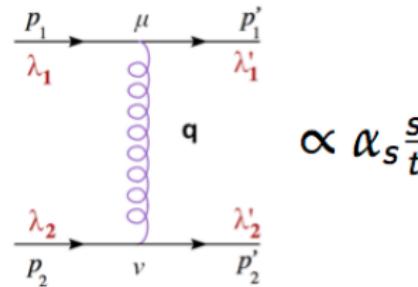
# BFKL

BFKL does not cover all high energy energy scattering, but it is essential to understand some of its aspects.

Consider quark-quark scattering in the **Regge Limit**.

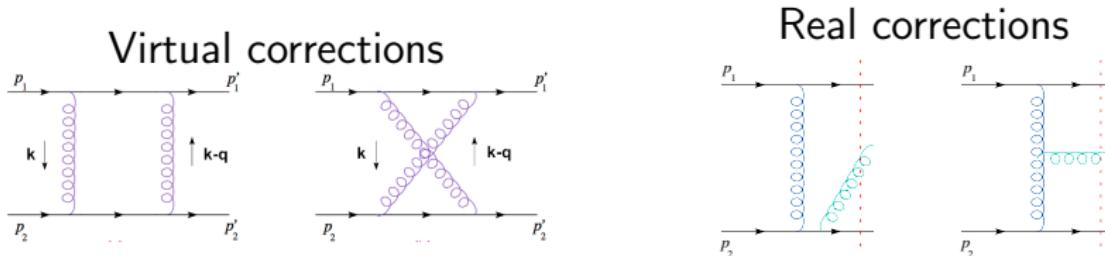
$$s \gg |t| \sim Q^2 \gg \Lambda_{QCD}^2$$

The amplitude at LO in  $\alpha_s$  is



# BFKL

If we go to NLO large logarithms appear  $\mathcal{A}^{(1)} \propto \mathcal{A}^{(0)} \alpha_s \log \frac{s}{Q^2}$

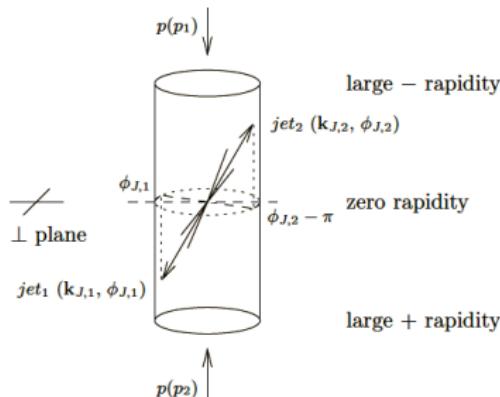


At arbitrary order, we will have terms proportional to  $(\alpha_s)^p (\alpha_s \log \frac{s}{Q^2})^q$  that are not negligible in the Regge limit.

- LLA BFKL:  $(\alpha_s \log \frac{s}{Q^2})^q$  terms
- NLLA BFKL:  $\alpha_s (\alpha_s \log \frac{s}{Q^2})^q$  terms

**All orders result in perturbation theory!**

# Rapidity variable



Picture from  
[D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)]

$$\tanh y = \frac{P_{||}}{E}$$

For  $m=0$  it coincides with the pseudo-rapidity

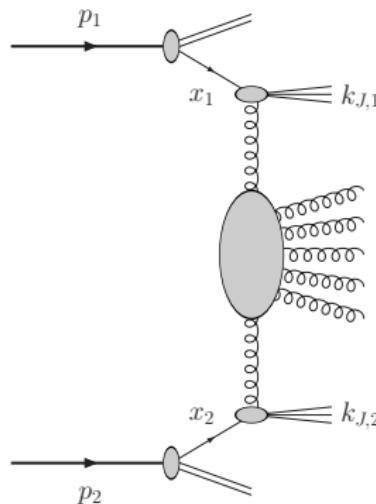
$$\eta = y(m=0) = -\log \tan \frac{\theta}{2}$$

Related to the angle of the momentum with the beam axis

...  $2 \rightarrow 2$  elastic scattering at high energies  $\Rightarrow Y \equiv y_1 - y_2 = \log \frac{s}{|t|}$

... Muller-Navelet jets  $\Rightarrow Y = \ln \frac{x_1 x_2 s}{|\vec{k}_{J,1}| |\vec{k}_{J,2}|}$

# Warming up: Mueller–Navelet jets



It has been the playground for BFKL tests since it was proposed in

[ A. H. Mueller, H. Navelet (1987) ]

- ◊ At  $Y=0$ , no minijet radiation in the rapidity interval. Exact correlation  $d\sigma \sim \delta^2(\vec{k}_{J,1} - \vec{k}_{J,2})$
- ◊ At large  $Y$  the BFKL approach predicts decorrelations (minijets)

Key observable: correlation in the azimuthal angle of the 2 tagged jets.

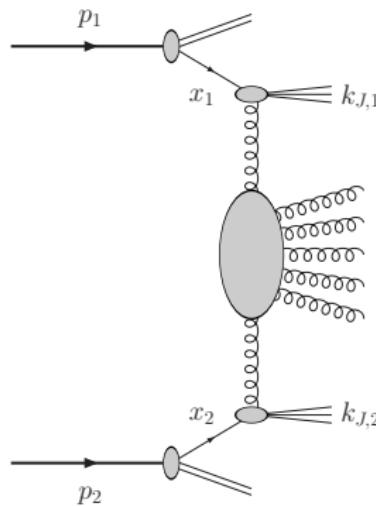
...large jet transverse momenta:  $\vec{k}_{J,1}^2 \sim \vec{k}_{J,2}^2 \gg \Lambda_{\text{QCD}}^2$   
DGLAP evolution. pQCD applicable.

...large rapidity interval between jets:  $Y = \ln \frac{x_1 x_2 s}{|\vec{k}_{J,1}| |\vec{k}_{J,2}|}$   
BFKL resummation effects  $\alpha Y \sim 1$

# Warming up: Mueller–Navelet jets

It has been the playground for BFKL tests since it was proposed in

[ A. H. Mueller, H. Navelet (1987) ]

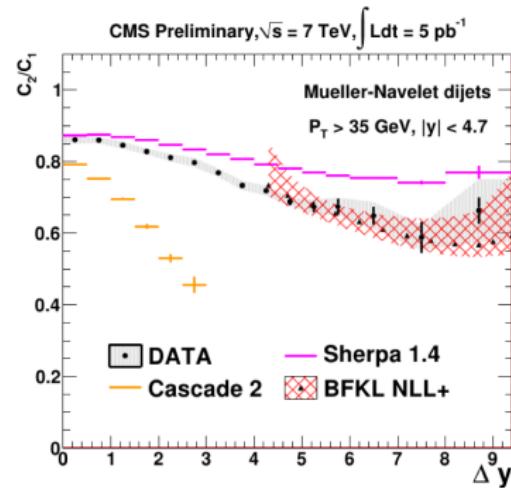
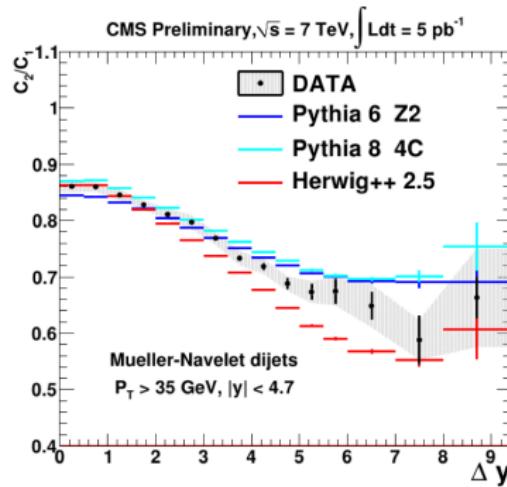


- ◊ At  $Y=0$ , no minijet radiation in the rapidity interval. Exact correlation  $d\sigma \sim \delta^2(\vec{k}_{J,1} - \vec{k}_{J,2})$
- ◊ At large  $Y$  the BFKL approach predicts decorrelations (minijets)

Key observable: correlation in the azimuthal angle of the 2 tagged jets.

$$\frac{d\sigma}{dx_1 dx_2 d|\vec{k}_{J,1}| d|\vec{k}_{J,2}| d\theta_1 d\theta_2} = \frac{1}{(2\pi)^2} \left[ C_0 + \sum_{n=1}^{\infty} 2 \cos(n\theta) C_n \right]$$

# Mueller–Navelet jets



NLLA predictions against LHC data quite successful for large rapidities.

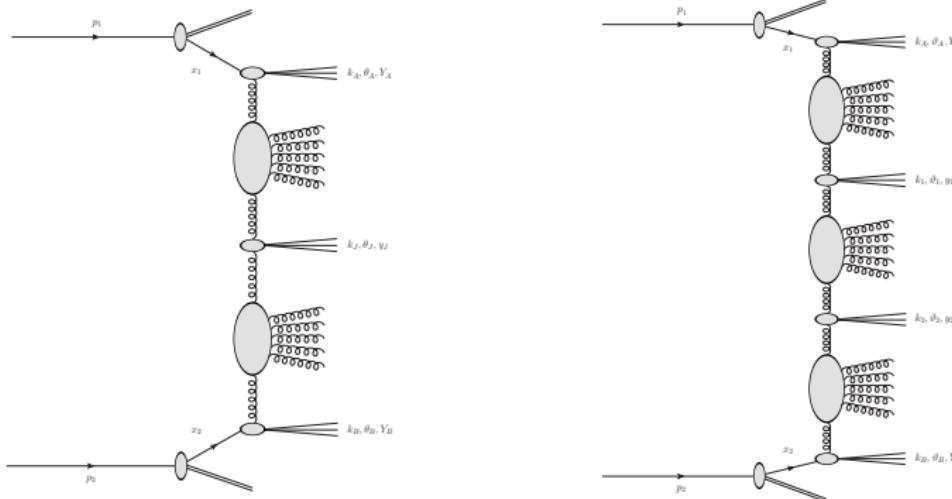
# Mueller–Navelet jets

- ◊ Big dependence on high order corrections in  $\mathcal{C}_0$  due to collinear contamination, better to define ratios.
- ◊ Focusing in azimuthal angle correlations is more fruitful than the usual "growth with energy" behaviour.
- ◊ Including more jets allow us to study azimuthal correlations and its dependence on transverse momentum.  
Less inclusive observables!

**Multi-jet production!**

A new way to probe BFKL

# Three- and four-jet production



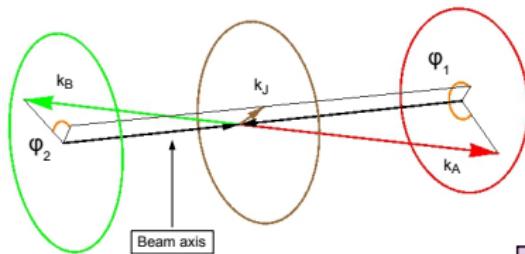
[F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2015)]

[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (2016)]

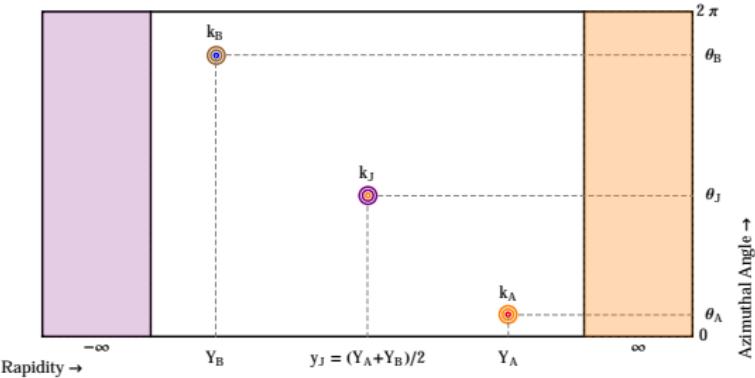
[F. Caporale, F.G. Celiberto., G. Chachamis, A. Sabio Vera (2016)]

[F. Caporale, F.G. Celiberto, G. Chachamis, D. G.G., A. Sabio Vera (2016)]

# An event with three tagged jets



$$Y_B < y_J < Y_A$$

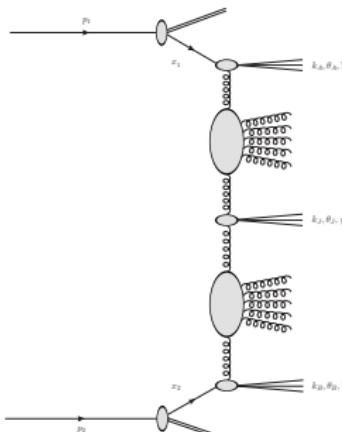


Three-jet at partonic level

# The three-jet partonic cross section

Starting point: differential partonic cross-section (no PDFs)

$$\frac{d^3\hat{\sigma}^{\text{3-jet}}}{dk_J d\theta_J dy_J} = \frac{\bar{\alpha}_s}{\pi k_J} \int d^2 \vec{p}_A \int d^2 \vec{p}_B \delta^{(2)}(\vec{p}_A + \vec{k}_J - \vec{p}_B) \times \\ \times \varphi(\vec{k}_A, \vec{p}_A, Y_A - y_J) \varphi(\vec{p}_B, \vec{k}_B, y_J - Y_B)$$



- *Multi-Regge kinematics* rapidity ordering:  $Y_B < y_J < Y_A$
- $k_J$  lie above the experimental resolution scale
- $\varphi$  is the BFKL gluon Green function (LLA or NLLA)
- $\bar{\alpha}_s = \alpha_s N_c / \pi$

# Generalized azimuthal correlations - partonic level

Prescription: integrate over all angles after using the projections on the two azimuthal angle differences indicated below...

→ ...to define:

$$\begin{aligned} & \int_0^{2\pi} d\theta_A \int_0^{2\pi} d\theta_B \int_0^{2\pi} d\theta_J \cos(M(\theta_A - \theta_J - \pi)) \cos(N(\theta_J - \theta_B - \pi)) \frac{d^3 \hat{\sigma}^{\text{3-jet}}}{dk_J d\theta_J dy_J} \\ &= \bar{\alpha}_s \sum_{L=0}^N \binom{N}{L} (k_J^2)^{\frac{L-1}{2}} \int_0^\infty dp^2 (p^2)^{\frac{N-L}{2}} \int_0^{2\pi} d\theta \frac{(-1)^{M+N} \cos(M\theta) \cos((N-L)\theta)}{\sqrt{(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos\theta)^N}} \\ & \times \phi_M(k_A^2, p^2, Y_A - y_J) \phi_N(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos\theta, k_B^2, y_J - Y_B) \end{aligned}$$

Main observables: **generalized azimuthal correlation ratios** (w/o the 0 component)

$$\mathcal{R}_{PQ}^{MN} = \frac{\mathcal{C}_{MN}}{\mathcal{C}_{PR}} = \frac{\langle \cos(M(\theta_A - \theta_J - \pi)) \cos(N(\theta_J - \theta_B - \pi)) \rangle}{\langle \cos(P(\theta_A - \theta_J - \pi)) \cos(Q(\theta_J - \theta_B - \pi)) \rangle}$$

# Next step: hadronic level predictions

- Introduce PDFs and running of the strong coupling:

$$\begin{aligned} \frac{d\sigma^{3\text{-jet}}}{dk_A dY_A d\theta_A dk_B dY_B d\theta_B dk_J dy_J d\theta_J} = \\ \frac{8\pi^3 C_F \bar{\alpha}_s(\mu_R)^3}{N_C^3} \frac{x_{J_A} x_{J_B}}{k_A k_B k_J} \int d^2 \vec{p}_A \int d^2 \vec{p}_B \delta^{(2)}(\vec{p}_A + \vec{k}_J - \vec{p}_B) \\ \times \left( \frac{N_C}{C_F} f_g(x_{J_A}, \mu_F) + \sum_{r=q,\bar{q}} f_r(x_{J_A}, \mu_F) \right) \\ \times \left( \frac{N_C}{C_F} f_g(x_{J_B}, \mu_F) + \sum_{s=q,\bar{q}} f_s(x_{J_B}, \mu_F) \right) \\ \times \varphi(\vec{k}_A, \vec{p}_A, Y_A - y_J) \varphi(\vec{p}_B, \vec{k}_B, y_J - Y_B) \end{aligned}$$

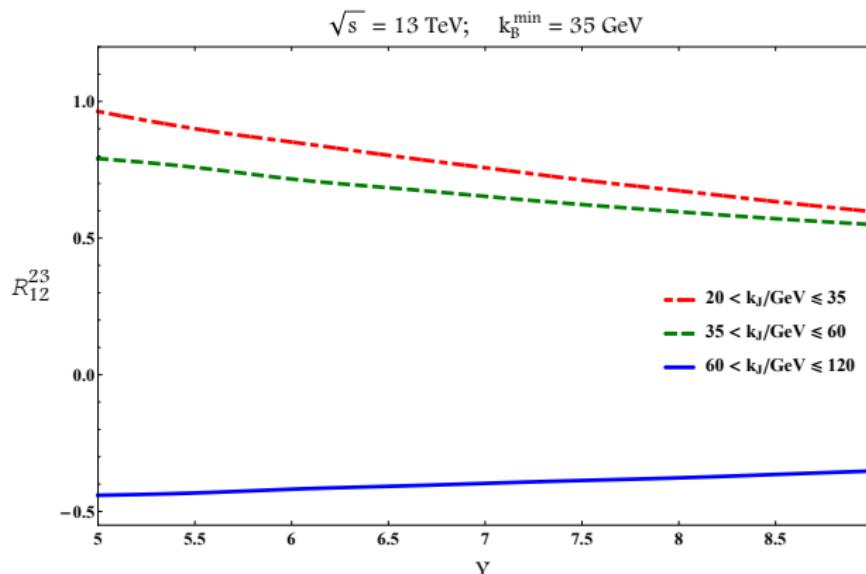
- Match the LHC kinematical cuts (integrate  $d\sigma^{3\text{-jet}}$  on  $k_T$  and rapidities  $Y_A, Y_B$ ):

- ◇ 1.  $35 \text{ GeV} \leq k_A \leq 60 \text{ GeV}; \quad 35 \text{ GeV} \leq k_B \leq 60 \text{ GeV}; \quad$  symmetric cuts  
2.  $35 \text{ GeV} \leq k_A \leq 60 \text{ GeV}; \quad 50 \text{ GeV} \leq k_B \leq 60 \text{ GeV}; \quad$  asymmetric cuts
- ◇  $Y = Y_A - Y_B$  fixed;  
 $y_J = (Y_A + Y_B)/2$
- ◇  $\sqrt{s} = 7, 13 \text{ TeV}$

Three-jet at hadronic level

# $R_{12}^{23}$ vs $Y$ for three different $k_J$ bins

$k_A^{\min} = 35 \text{ GeV}$ ,  $k_B^{\min} = 35 \text{ GeV}$ ,  $k_A^{\max} = k_B^{\max} = 60 \text{ GeV}$  (symmetric)



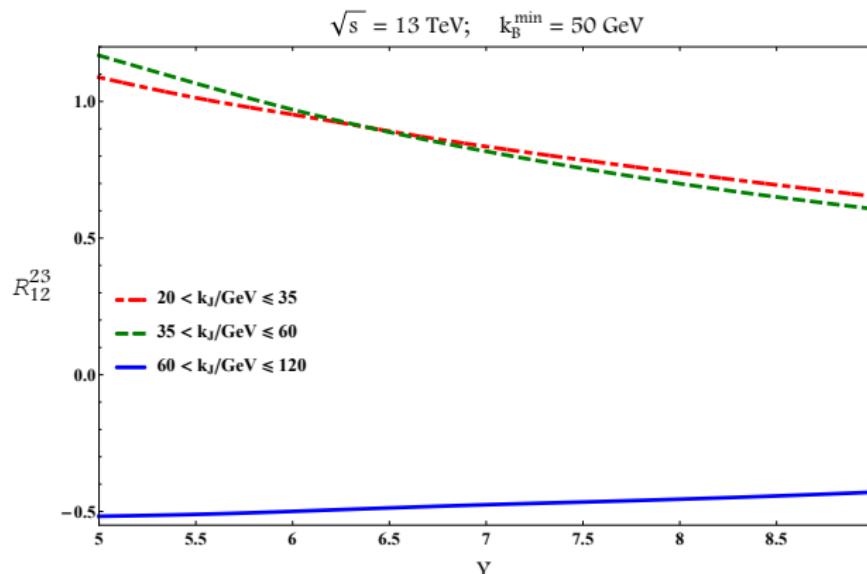
[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (2016)]

$Y$  is the rapidity difference between the most forward/backward jet;  $y_J = \frac{Y_A + Y_B}{2}$ .

Three-jet at hadronic level

# $R_{12}^{23}$ vs $Y$ for three different $k_J$ bins

$k_A^{\min} = 35 \text{ GeV}$ ,  $k_B^{\min} = 50 \text{ GeV}$ ,  $k_A^{\max} = k_B^{\max} = 60 \text{ GeV}$  (asymmetric)



[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (2016)]

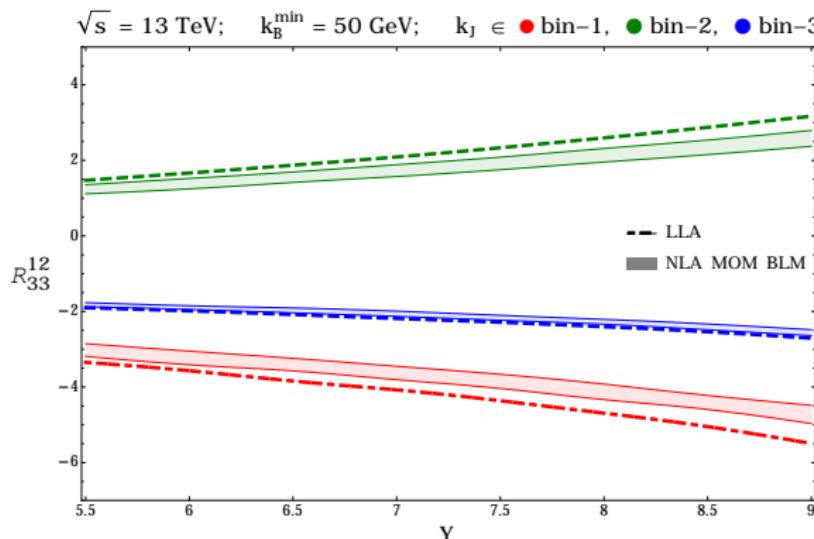
$Y$  is the rapidity difference between the most forward/backward jet;  $y_J = \frac{Y_A + Y_B}{2}$ .

Three-jet at hadronic level

# $R_{33}^{12}$ vs $\Upsilon$ at 13 TeV - NLLA

## Preliminary results

$k_A^{\min} = 35 \text{ GeV}$ ,  $k_B^{\min} = 50 \text{ GeV}$ ,  $k_A^{\max} = k_B^{\max} = 60 \text{ GeV}$  (asymmetric)

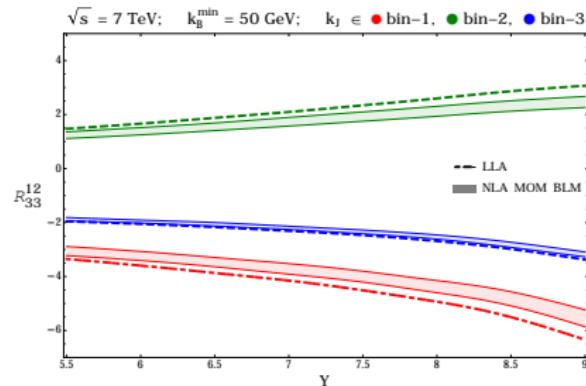
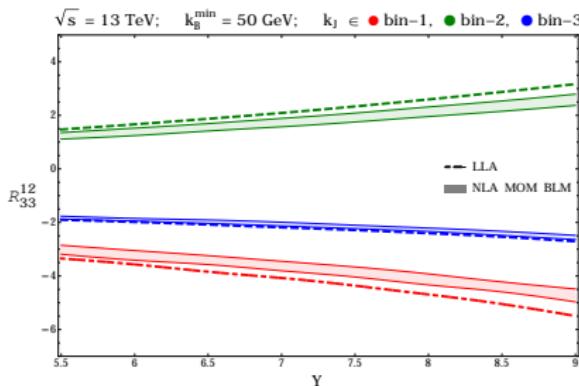


[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (in progress)]

$\Upsilon$  is the rapidity difference between the most forward/backward jet;  $y_J = \frac{Y_A + Y_B}{2}$ .

# $R_{33}^{12}$ vs $\Upsilon$ at 13 and 7 TeV - NLLA

## Preliminary results



We have entered in an asymptotic regime!

[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (in progress)]

# Conclusions

- Study of processes with **three** and **four** tagged jets to propose and **predict** new, more exclusive, BFKL observables: **generalized azimuthal correlation** with dependence on the transverse momenta of extra jets.
- Ratios of correlation functions used to minimize the influence of higher order corrections
- Comparison with other approaches such as fixed order calculations and Monte Carlo simulations are needed to determine if the observable is a genuine BFKL signal.
- Comparison with experimental data suggested and needed to know the window of applicability of the BFKL framework at LHC.

# Outlook

- ◊ Three- and four-jets in the NLLA accuracy: improved kernel(s), scale optimization
  - [F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (in progress)]
- ◊ Dependence on rapidity bins (asymmetric configurations for the central jet(s))
  - [F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (in progress)]
- ◊ Comparison with analyses where the four-jet predictions stem from two independent gluon ladders (double parton scattering)
  - [R. Maciula, A. Szczurek (2014, 2015)]
  - [K. Kutak, R. Maciula, M. Serino, A. Szczurek, A. van Hameren (2016, 2016)]

Thanks for your  
attention!!

# **BACKUP slides**

## Motivation

So far, search for BFKL effects had these general drawbacks:

- ◊ too low  $\sqrt{s}$  or rapidity intervals among tagged particles in the final state
- ◊ too inclusive observables, other approaches can fit them

Advent of LHC:

- higher energies  $\leftrightarrow$  larger rapidity gaps
- unique opportunity to **test pQCD in the high-energy limit**
- disentangle applicability region of energy-log resummation (**BFKL approach**)
  - [V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977)]
  - [Y.Y. Balitskii, L.N. Lipatov (1978)]

Last years:

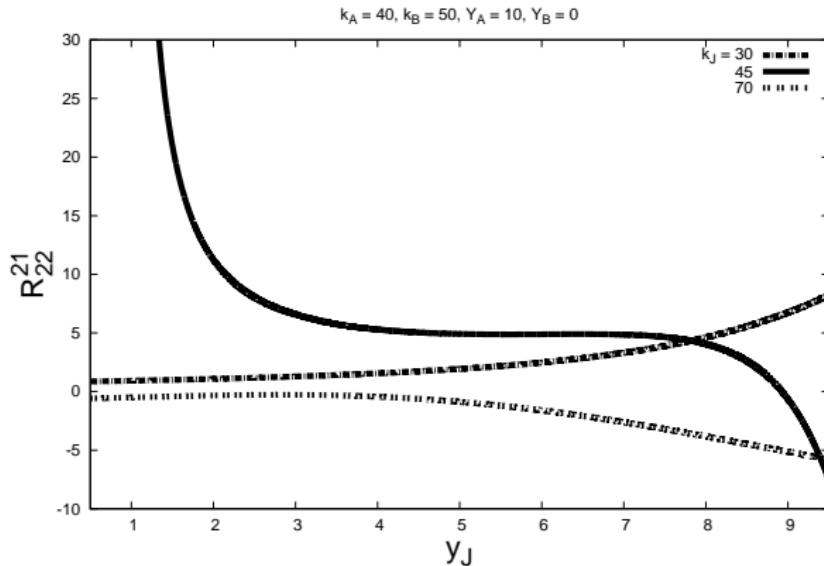
hadroproduction of two jets featuring high transverse momenta and well separated in rapidity, so called **Mueller–Navelet jets**...

- ◊ ...possibility to define *infrared-safe* observables...
- ◊ ...and constrain the PDFs...
- ◊ ...theory vs experiment

[B. Ducloué, L. Szymanowski, S. Wallon (2014)]  
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]

# BACKUP slides

## Partonic prediction of $\mathcal{R}_{22}^{21}$ for $k_J = 30, 45, 70$ GeV

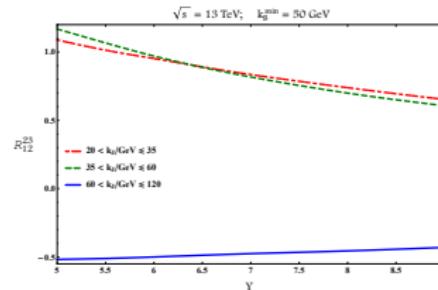
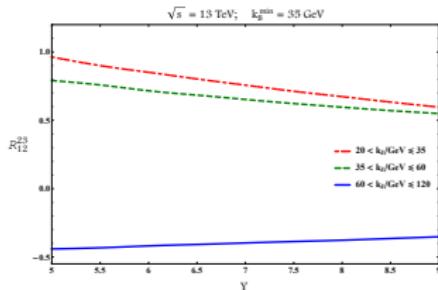
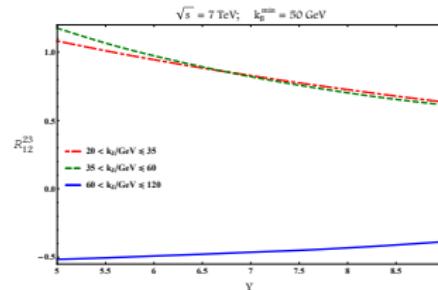
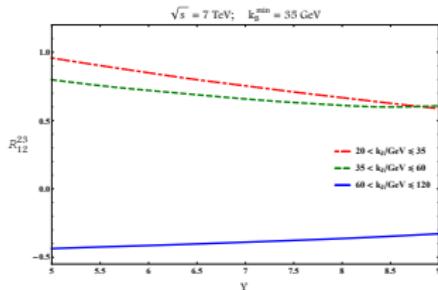


[F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2015)]

$Y_A - Y_B$  is fixed to 10;  $y_J$  varies between 0.5 and 9.5.

# BACKUP slides

$R_{12}^{23}$  vs  $Y = Y_A - Y_B$ ,  $\sqrt{s}$  and  $k_B^{\min}$  for three  $k_T$  bins

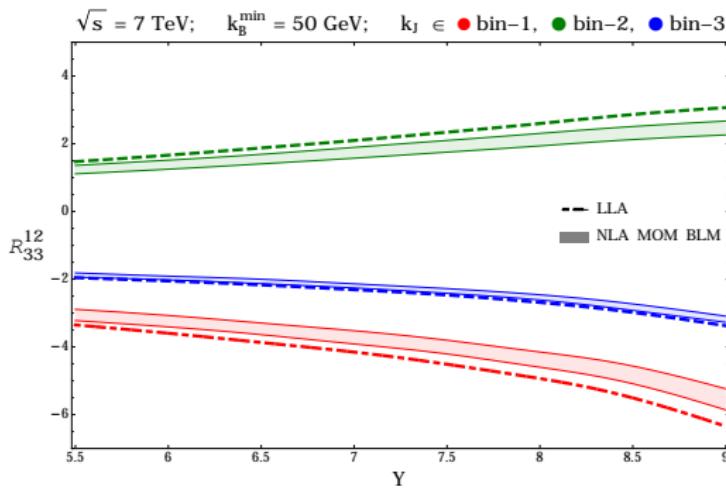


[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (in progress)]

# BACKUP slides

## $R_{33}^{12}$ vs $Y$ at 7 TeV - NLLA preliminary results

$k_A^{\min} = 35$  GeV,  $k_B^{\min} = 50$  GeV,  $k_A^{\max} = k_B^{\max} = 60$  GeV (asymmetric)

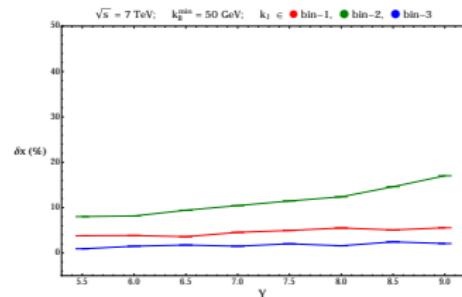
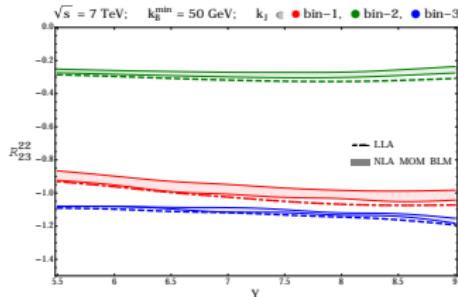
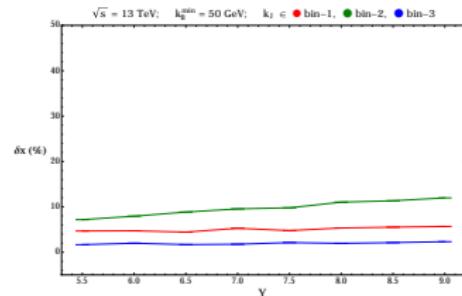
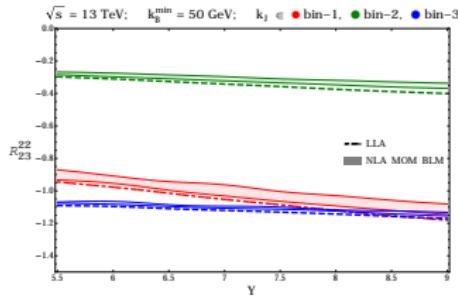


[F. Caporale, F.G. Celiberto, G. Chachamis, D. G.G., A. Sabio Vera (in progress)]

$Y$  is the rapidity difference between the most forward/backward jet;  $y_J = \frac{Y_A + Y_B}{2}$ .

# BACKUP slides

## $R_{23}^{22}$ vs $Y$ at 13 and 7 TeV - NLLA preliminary results



[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (in progress)]

## Four-jets: generalized azimuthal coefficients - partonic level

$$\begin{aligned}
 \mathcal{C}_{MNL} &= \int_0^{2\pi} d\vartheta_A \int_0^{2\pi} d\vartheta_B \int_0^{2\pi} d\vartheta_1 \int_0^{2\pi} d\vartheta_2 \cos(M(\vartheta_A - \vartheta_1 - \pi)) \\
 &\quad \cos(N(\vartheta_1 - \vartheta_2 - \pi)) \cos(L(\vartheta_2 - \vartheta_B - \pi)) \frac{d^6 \sigma^{\text{4-jet}}(\vec{k}_A, \vec{k}_B, Y_A - Y_B)}{dk_1 dy_1 d\vartheta_1 dk_2 d\vartheta_2 dy_2} \\
 &= \frac{2\pi^2 \bar{\alpha}_s (\mu_R)^2}{k_1 k_2} (-1)^{M+N+L} (\bar{\Omega}_{M,N,L} + \bar{\Omega}_{M,N,-L} + \bar{\Omega}_{M,-N,L} \\
 &\quad + \bar{\Omega}_{M,-N,-L} + \bar{\Omega}_{-M,N,L} + \bar{\Omega}_{-M,N,-L} + \bar{\Omega}_{-M,-N,L} + \bar{\Omega}_{-M,-N,-L})
 \end{aligned}$$

with

$$\begin{aligned}
 \bar{\Omega}_{m,n,l} &= \int_0^{+\infty} dp_A p_A \int_0^{+\infty} dp_B p_B \int_0^{2\pi} d\phi_A \int_0^{2\pi} d\phi_B \\
 &\quad \frac{e^{-im\phi_A} e^{il\phi_B} (p_A e^{i\phi_A} + k_1)^n (p_B e^{-i\phi_B} - k_2)^n}{\sqrt{(p_A^2 + k_1^2 + 2p_A k_1 \cos \phi_A)^n} \sqrt{(p_B^2 + k_2^2 - 2p_B k_2 \cos \phi_B)^n}} \\
 &\quad \varphi_m(|\vec{k}_A|, |\vec{p}_A|, Y_A - y_1) \varphi_l(|\vec{p}_B|, |\vec{k}_B|, y_2 - Y_B) \\
 &\quad \varphi_n\left(\sqrt{p_A^2 + k_1^2 + 2p_A k_1 \cos \phi_A}, \sqrt{p_B^2 + k_2^2 - 2p_B k_2 \cos \phi_B}, y_1 - y_2\right)
 \end{aligned}$$

## Four-jets: generalized azimuthal coefficients - partonic level

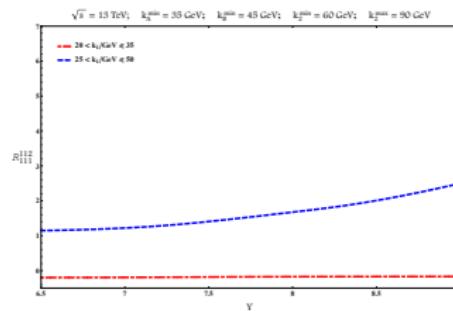
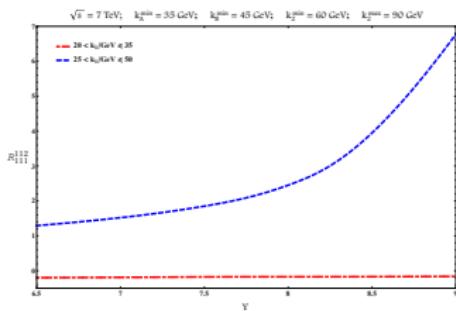
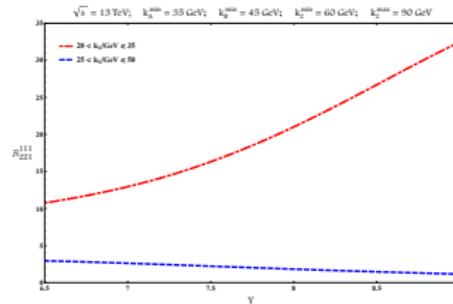
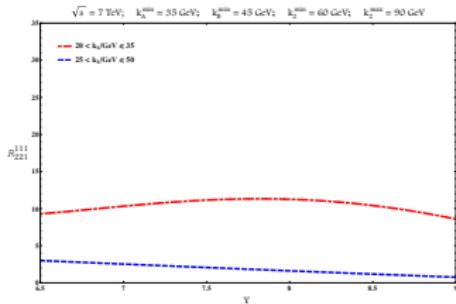
$$\mathcal{C}_{MNL} = \int_0^{2\pi} d\vartheta_A \int_0^{2\pi} d\vartheta_B \int_0^{2\pi} d\vartheta_1 \int_0^{2\pi} d\vartheta_2 \cos(M(\vartheta_A - \vartheta_1 - \pi)) \cos(N(\vartheta_1 - \vartheta_2 - \pi)) \cos(L(\vartheta_2 - \vartheta_B - \pi)) \frac{d^6 \sigma^{\text{4-jet}}(\vec{k}_A, \vec{k}_B, Y_A - Y_B)}{dk_1 dy_1 d\vartheta_1 dk_2 d\vartheta_2 dy_2}$$

Main observables: **generalized azimuthal correlations**

$$\mathcal{R}_{PQR}^{MNL} = \frac{\mathcal{C}_{MNL}}{\mathcal{C}_{PRQ}} = \frac{\langle \cos(M(\vartheta_A - \vartheta_1 - \pi)) \cos(N(\vartheta_1 - \vartheta_2 - \pi)) \cos(L(\vartheta_2 - \vartheta_B - \pi)) \rangle}{\langle \cos(P(\vartheta_A - \vartheta_1 - \pi)) \cos(Q(\vartheta_1 - \vartheta_2 - \pi)) \cos(R(\vartheta_2 - \vartheta_B - \pi)) \rangle}$$

# BACKUP slides

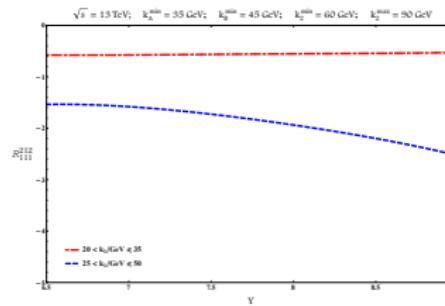
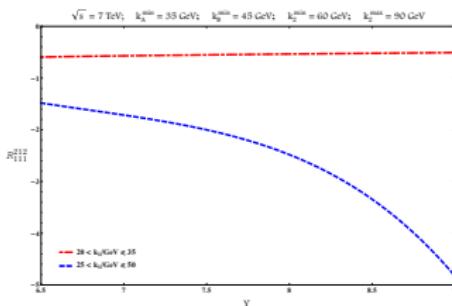
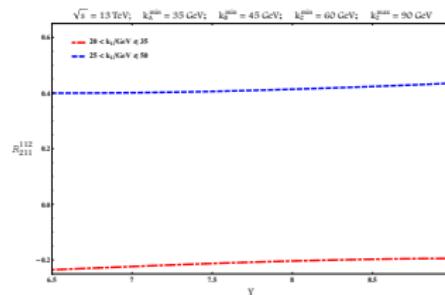
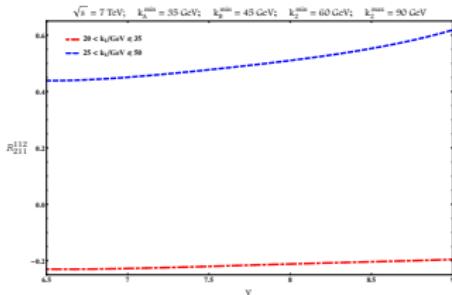
$R_{221}^{111}$  and  $R_{111}^{112}$  vs  $Y = Y_A - Y_B$  and  $\sqrt{s}$  for two  $k_1$  bins



[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (in progress)]

# BACKUP slides

$R_{211}^{112}$  and  $R_{111}^{212}$  vs  $Y = Y_A - Y_B$  and  $\sqrt{s}$  for two  $k_1$  bins



[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (in progress)]