Entanglement equilibrium in higher curvature gravity

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Entanglement Equilibrium

First Law of Diamond Mechanics

Extension to Higher Curvature Gravity

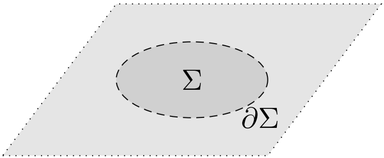
First Law of Thermodynamics

Entropy = Gravity

- Black holes obey thermodynamic laws. Hawking, Bekenstein, …
- Assuming thermodynamic laws implies gravity. Jacobson
- Bulk reconstruction using entanglement entropy in AdS/CFT van Raamsdonk, Ryu, Takayanagi, …
- AdS is a tensor networks Swingle, …
- Emergent gravity Verlinde
- Entanglement equilibrium implies the Einstein equation. Jacobson

Entanglement entropy of a geodesic ball

- We start in the vacuum of a QFT_d.
- Consider the S_{EE} of a geodesic ball.
- Claim: S_{EE} is maximal for the vacuum.



Maximal vacuum entanglement hypothesis

The Maximal Vacuum Entanglement Hypothesis (MVEH) for Einstein gravity states Jacobson

$$\delta S_{EE}|_V = 0\,.$$

The variation splits in a UV and IR part

$$\delta S_{EE} = \delta S_{\mathrm{UV}} + \delta S_{\mathrm{IR}}.$$

One can show that for Einstein gravity

$$\delta S_{\mathrm{UV}}|_V \propto -\frac{G_{ab}}{4G}\,, \qquad \delta S_{\mathrm{IR}}|_V \propto 2\pi T_{ab}\,.$$

Thus the MVEH implies the Einstein equation

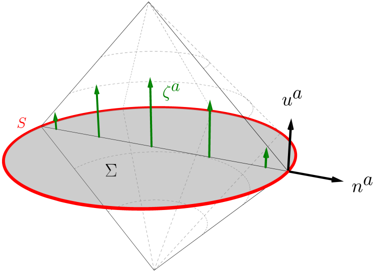
$$\boxed{G_{ab} = 8\pi G T_{ab}}$$

Why should one believe in the MVEH?

MVEH is a microscopic interpretation of the First Law of Diamond Mechanics (FLDM), the classical first order identity:

Iyer,Wald

$$\delta H_\zeta = \delta \oint_{\partial\Sigma} Q_\zeta \,,$$



where H_ζ is the Hamiltonian generating evolution along the flow of the conformal killing vector ζ and Q_ζ is the Noether charge $(d-2)$-form.

δH_ζ receives contributions from changes in the state $|\psi\rangle$, and the geometry g_{ab}, such that the FLDM reads

$$\boxed{\delta H^\psi_\zeta + \delta H^g_\zeta - \delta \oint_{\partial\Sigma} Q_\zeta = 0\,.}$$

Why keep the volume fixed?

For pure Einstein gravity the expressions in the FLDM read

$$\delta H_\zeta^g = -\frac{1}{8\pi G}\frac{d-2}{\ell}\delta V = -\frac{1}{8\pi G}\frac{\partial A}{\partial V}\delta V\,,$$
$$\delta \oint_{\partial\Sigma} Q_\zeta = -\frac{1}{8\pi G}\delta A\,.$$

The FLDM then reads

$$\delta H_\zeta^\psi + \delta H_\zeta^g - \delta \oint_{\partial\Sigma} Q_\zeta = \delta H_\zeta^\psi + \frac{1}{8\pi G}\,\delta A|_V = 0\,.$$

Identifying $2\pi\delta H_\zeta^\psi = \delta S_{\text{IR}}$ and $\frac{\delta A}{4G} = \delta S_{\text{UV}}$, we find the MVEH.

$$\boxed{\delta S_{EE}|_V = 0}$$

Higher curvature gravity

What would the generalization to higher curvature gravity look like?

$$\delta S_{EE}|_V = 0 \Rightarrow \delta X|_Y = 0$$

A natural guess for the entropy is the Wald entropy

$$\delta S_{\text{Wald}}|_Y = 0\,.$$

But what is the generalization of volume?

Back to First Law of Diamond Mechanics

What would the generalization to higher curvature gravity look like?

$$\delta H_\zeta^\psi + \delta H_\zeta^g - \delta \oint_{\partial\Sigma} Q_\zeta = 0\,.$$

Wald's formalism provides the extension to higher curvature gravity

$$\oint_{\partial\Sigma} Q_\zeta = -\frac{1}{2\pi} S_{\text{Wald}}$$

$$S_{\text{Wald}} = -2\pi \int_{\partial\Sigma} dA\, E^{abcd} n_{ab} n_{cd}\,, \quad E^{abcd} \equiv \frac{\partial \mathcal{L}}{\partial R_{abcd}}\,,$$

$$\delta H_\zeta^g = -\frac{4}{\ell}\delta \int_\Sigma dV \left(E^{abcd} u_a u_d h_{bc} - E_0\right) \equiv -\frac{d-2}{8\pi G \ell}\delta W$$

The generalization of volume is

$$\boxed{W = \frac{1}{d-2}\int_\Sigma dV \frac{1}{E_0}(E^{abcd} u_a u_d h_{bc} - E_0),}$$

Back to Entanglement Equilibrium

Can we still interpret the FLDM as a generalized MVEH?

$$\delta H_\zeta^\psi + \delta H_\zeta^g - \oint_{\partial\Sigma} Q_\zeta =$$
$$\delta H_\zeta^\psi + \frac{1}{2\pi}\left(\delta S_{\text{Wald}} - \frac{\partial S_{\text{Wald}}}{\partial W}\delta W\right) = 0\,.$$

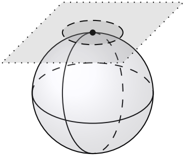
The generalized MVEH reads

$$\boxed{\delta S|_W = 0}$$

RIEMANN NORMAL COORDINATES

We can extract the equations of motion from the FLDM using Riemann Normal Coordinates (RNC)

$$g_{ab}(x) = \eta_{ab} + \frac{1}{6}R_{acbd}(0)x^c x^d + \mathcal{O}\left(x^3\right)$$



Therefore a small variation around flat space is

$$\delta g_{ab}(x) = \frac{1}{6}R_{acbd}(0)x^c x^d + \mathcal{O}\left(x^3\right)$$

In a ball of radius $\ell < L_c$ every spacetime looks locally like a variation around flat space.

Extracting the equations of motion

Evaluating the FLDM with RNC in small geodesic balls leads to

$$\delta H_\zeta^\psi + \delta H_\zeta^g - \oint_{\partial\Sigma} Q_\zeta \propto u_a u_b \delta\mathcal{E}^{ab}(0) + \mathcal{O}\big(\ell^2\big)\,.$$

We can extract the *linear* equations of motion for higher curvature gravity. Note that Jacobson found the *non-linear* Einstein equations in pure Einstein gravity.

Linearizing equations of motion with RNC

Remember that the metric perturbation in RNC reads

$$\delta g_{ab}(x) = \frac{1}{6} R_{acbd}(0) x^c x^d + \mathcal{O}\left(x^3\right)$$

However, curvatures do not vanish for small balls

$$\delta R(x) = R(0) + \mathcal{O}\left(x\right)$$

Linearizing the Einstein tensor around flat space reads

$$\begin{aligned}\delta G_{ab}(x) &= \left(R_{ab}[0] - \frac{1}{2}\eta_{ab} R[0]\right) + \mathcal{O}(x) \\ &= G_{ab}[0] + \mathcal{O}(x)\,,\end{aligned}$$

while linearization of higher curvature terms vanishes

$$\delta\left(R^2\right) = 0 + \mathcal{O}(x) \ \Rightarrow \delta\mathcal{E}_{ab}(x) \neq \mathcal{E}_{ab}[0] + \mathcal{O}(x)\,.$$

First Law of Thermodynamics

The first law of thermodynamics reads

$$dU = -PdV + TdS\,,$$

where

$$T \equiv \left(\frac{\partial U}{\partial S}\right)\Big|_V\,, \quad P \equiv -\left(\frac{\partial U}{\partial V}\right)\Big|_S = T\left(\frac{\partial S}{\partial V}\right)\Big|_U\,.$$

Thus the first law of thermodynamics can also be rewritten as

$$dU = T\,dS|_V\,,$$

which should remind you of the FLDM

$$\delta H_\zeta^\psi + \frac{1}{2\pi}\,\delta S_{\text{Wald}}|_W = 0\,.$$

The FLDM can be interpreted as a first law, identifying

$$H_\zeta^\psi = -U\,, \quad \frac{1}{2\pi} = T\,, \quad S_{\text{Wald}} = S\,, \quad W = V\,.$$

CONCLUSION

- The Maximal Vacuum Entanglement Hypothesis (MVEH) provides new insights into the emergence of gravity.
- The MVEH is an interpretation of the First Law of Diamond Mechanics (FLDM).
- The FLDM and MVEH can be generalized to higher curvature gravity.
- Extracting the non-linear e.o.m. from the MVEH is special to pure Einstein gravity.
- The FLDM can alternatively be interpreted as a first law of thermodynamics.
- We propose a generalized volume for higher curvature gravity.

OUTLOOK

- Can our generalized volume be applied to Complexity/Fidelity Susceptibility Susskind, Brown/ Miyajia, Numasawaa, Shiba
- Can we include non-conformal matter?
- What is the role of the cosmological constant?
- Include higher order corrections. Can this lead to the non-linear e.o.m. for higher curvature gravity?