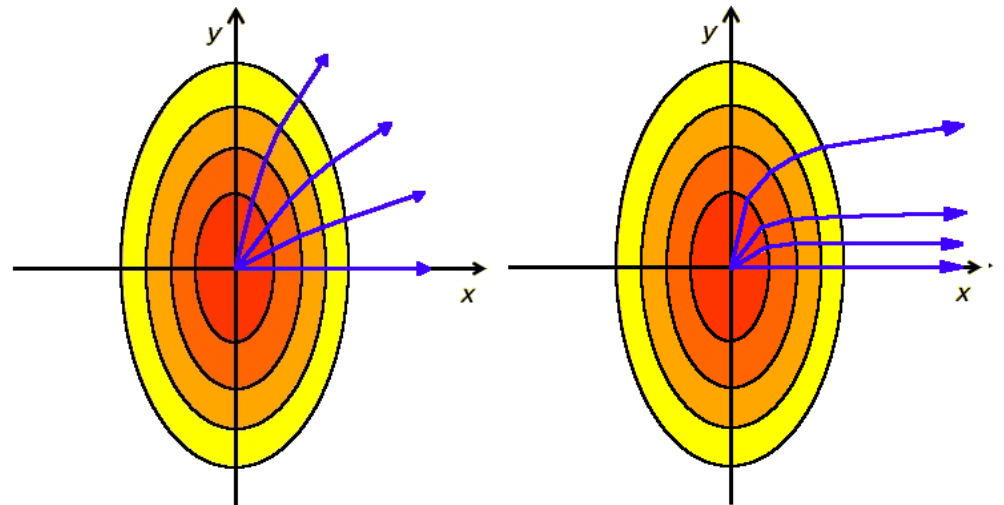
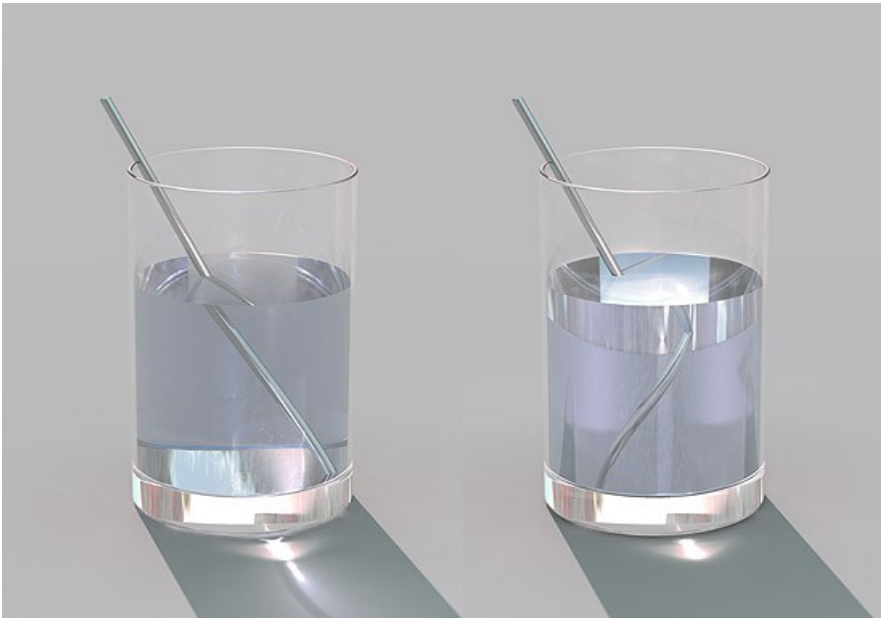


# QGP optical properties from gauge/gravity correspondence?



***Daniele Musso***



*based on:*

Forcella, Mezzalira, Musso hep-th/1404.4048

# Outline

- Optical properties of a **strongly coupled plasma** through *gauge/gravity* techniques
- **Medium-induced** optical properties for *direct photons* and anisotropic photon flows in Heavy Ion Collisions
- **Negative refraction**

# Refraction index

$$n(\omega) = \frac{q(\omega)}{\omega}$$

$$e^{-i(\omega t - qx)} = e^{-i\omega(t - nx)}$$

$$v_{flow} \sim \text{Im}[n]$$

$$v_{phase} = \frac{1}{\text{Re}[n]}$$

$$\text{Re}[n] \text{ Im}[n] < 0$$

**NEGATIVE REFRACTION**

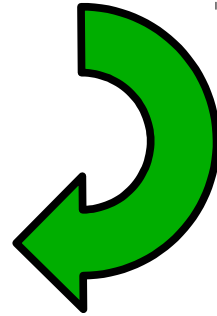
**Phase velocity and energy flux are directed oppositely!**

# Negative Refraction

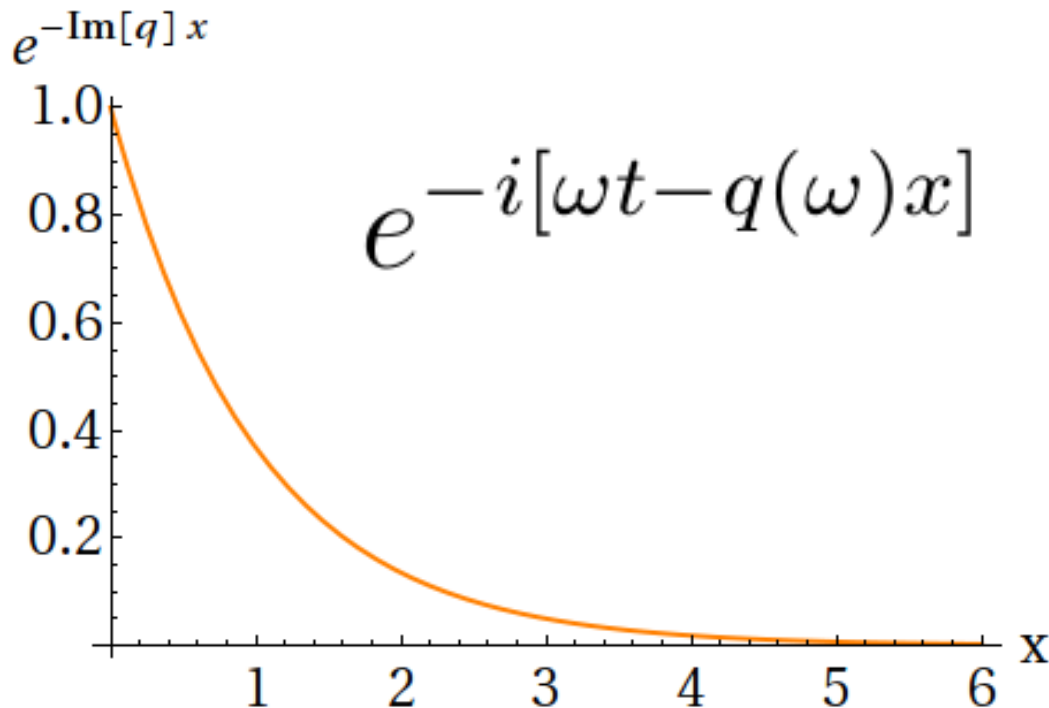
McCall, Lakhtakia, Weiglhofer class-ph/0204067

$$\operatorname{Re}[n] \operatorname{Im}[n] < 0$$

$$\operatorname{Re}[q] \operatorname{Im}[q] < 0$$



***real frequency  
formalism***



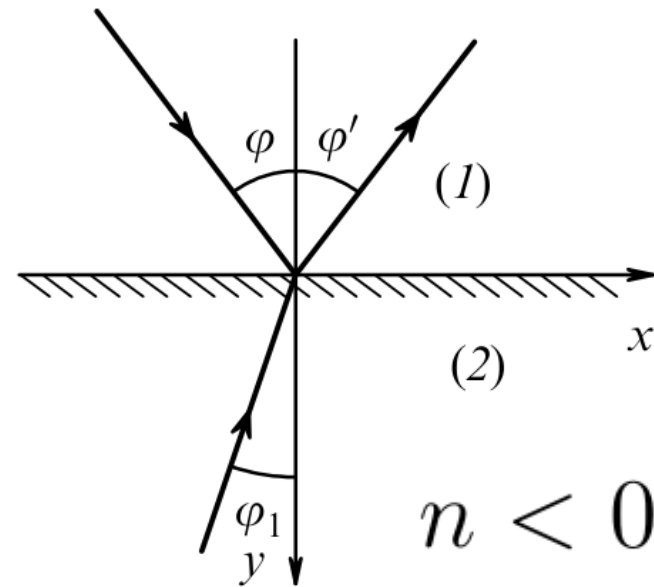
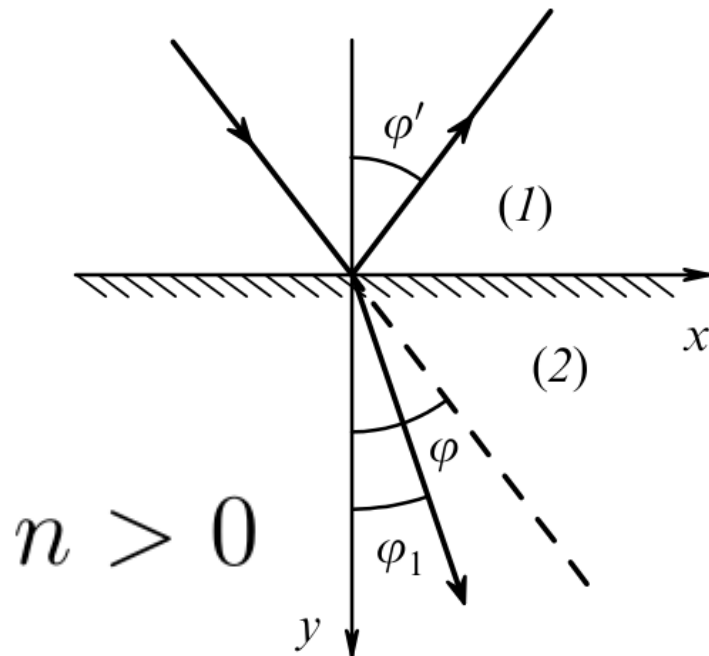
- Attenuation of the wave (*damping*)
- Passivity of the medium (*loss*)



*Jump ahead... we are not modeling  
the mechanisms of photo-  
production*

# Exotic optical properties

- Propagation of multiple waves
- Negative refraction  
(*negative Doppler, negative Snell,...*)



[Mandel'shtam; Agranovich, Gartstein]

# Linear response & spatial dispersion

**Permittivity tensor**  $\longrightarrow D_i = \epsilon_{ij}(\omega, \mathbf{q}) E_j$

Landau-Lifshitz

Decomposition in a transverse and longitudinal parts

$$\epsilon_{ij}(\omega, \mathbf{q}) = \epsilon_T(\omega, \mathbf{q}) \left[ \delta_{ij} - \frac{q_i q_j}{q^2} \right] + \epsilon_L(\omega, \mathbf{q}) \frac{q_i q_j}{q^2}$$

Relation with the usual *permittivity* and *permeability* formalism

$$\epsilon_T(\omega, \mathbf{q}) = \epsilon(\omega) + \frac{q^2}{\omega^2} \left( 1 - \frac{1}{\mu(\omega)} \right) + \dots$$

# Optics and Green functions

- Linear response and “**polarizability**”

$$J_\mu = e^2 G_{\mu\nu} A^\nu \quad \epsilon(\omega, q) = 1 - \frac{4\pi e^2}{\omega^2} G(\omega, q)$$

- The light-wave equations (*from Maxwell*)

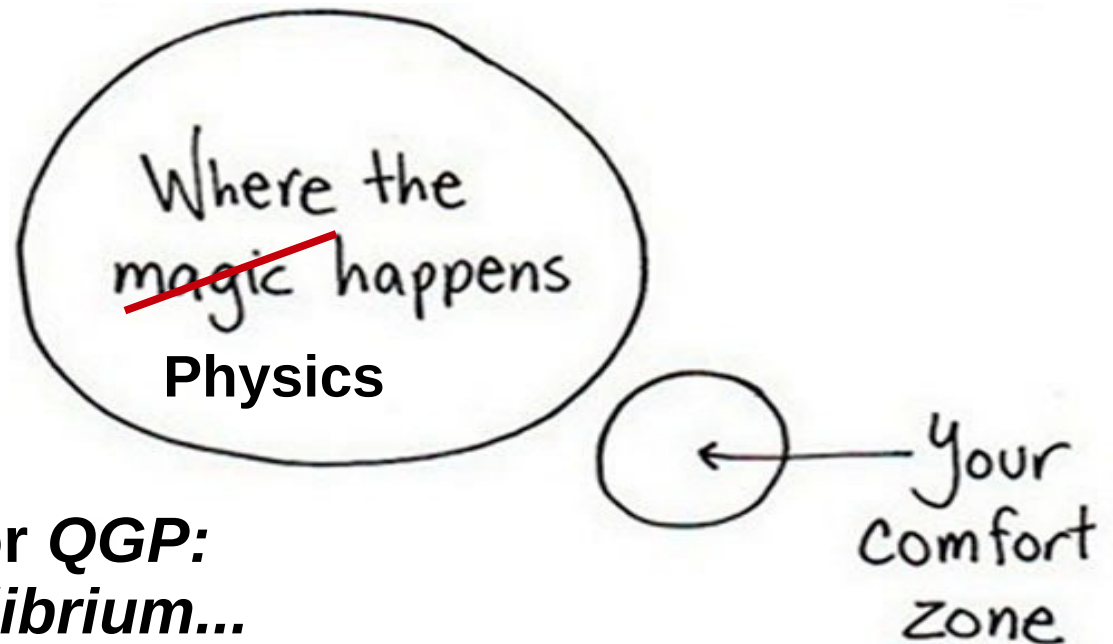
$$\epsilon_T(\omega, q) = \frac{q^2}{\omega^2} \quad \epsilon_L(\omega, q) = 0$$

- Light-wave solutions: Dispersion relations and refraction indexes (*note the plural!*)

$$n = \frac{q}{\omega}$$

# Out of the comfort zone(s)

- E.M. in the presence of strong dispersion (*temporal* and *spatial*) and dissipation
- Strong coupling, finite density, real time, no perturbation theory (*not even the lattice!*)



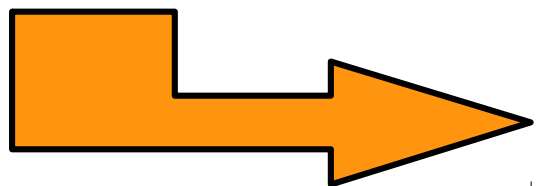
**Additional complications for QGP:**  
*many-body and out of equilibrium...*



# Optical properties from holography

- Schematization: light **weakly coupled** with a **strongly coupled** plasma
- Linear response
- Light-wave mode analysis (*no QNM!*)

$$G(\omega, q) \sim \frac{\delta^2 S_{\text{ren}}^{(\text{bulk})}}{(\delta A_0)^2}$$



$$\epsilon(\omega, q) = 1 - \frac{4\pi e^2}{\omega^2} G(\omega, q)$$

# Light-wave equation

- Solving the light-wave equation

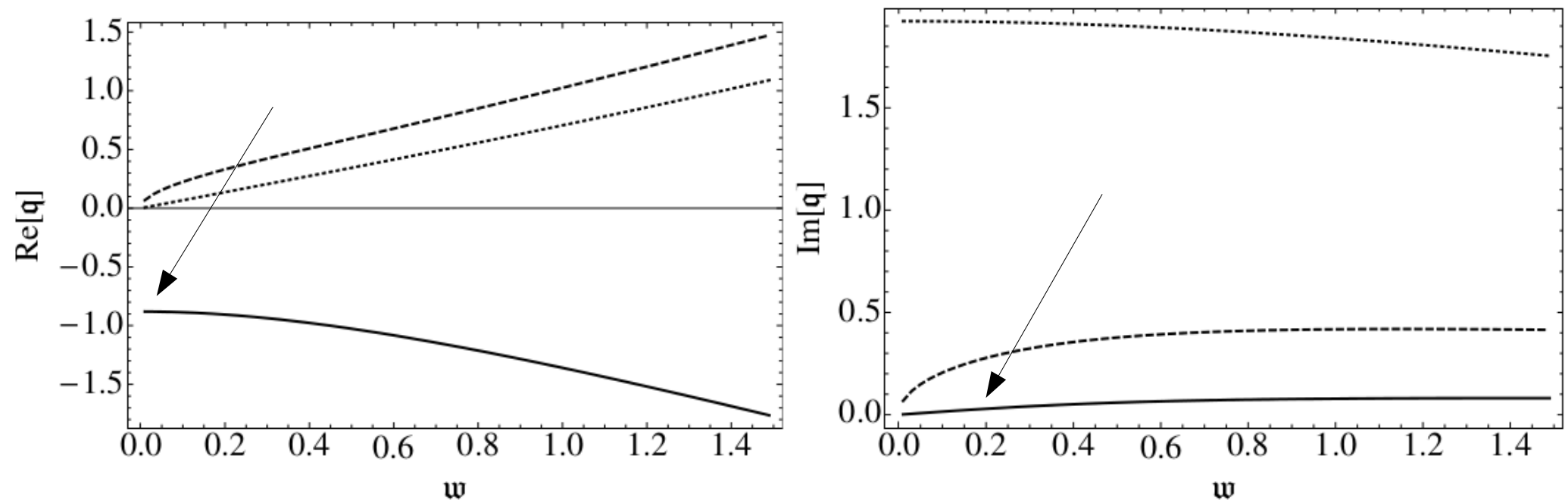
$$\frac{q^2}{\omega^2} = 1 - \frac{4\pi e^2}{\omega^2} G(\omega, q)$$

- Characterization of the light-wave modes

The exotic optical properties we are interested in are intimately related to additional light waves modes and the spatially **NON-LOCAL** response of the medium. Technically this is encoded in a non-trivial  $q$  dependence of the Green function describing the response of the medium

# Transverse light-wave modes

$$\text{Re}[q] \text{ Im}[q] < 0 \longrightarrow \textbf{NEGATIVE REFRACTION!}$$



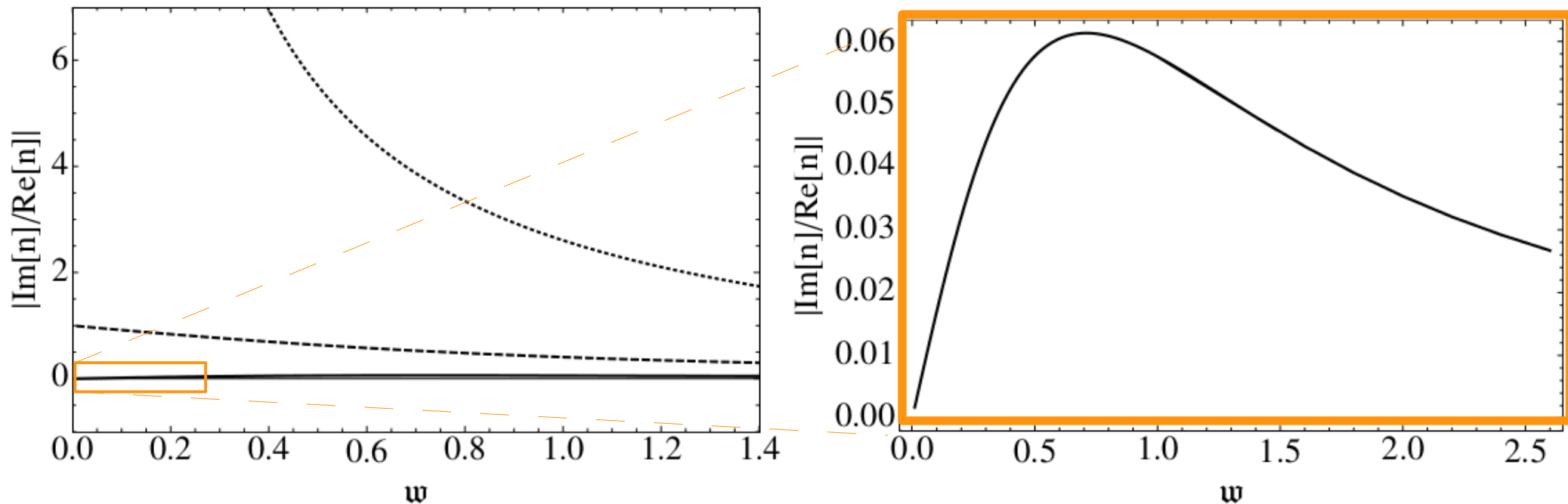
Negative refraction in the entire frequency range...

The presence of a finite value of  $q$  at null frequency (regardless of its real, imaginary or complex character) is due to the coupling of the electromagnetic waves to non-hydrodynamical quasi-normal modes of the plasma.

# Absorption/propagation

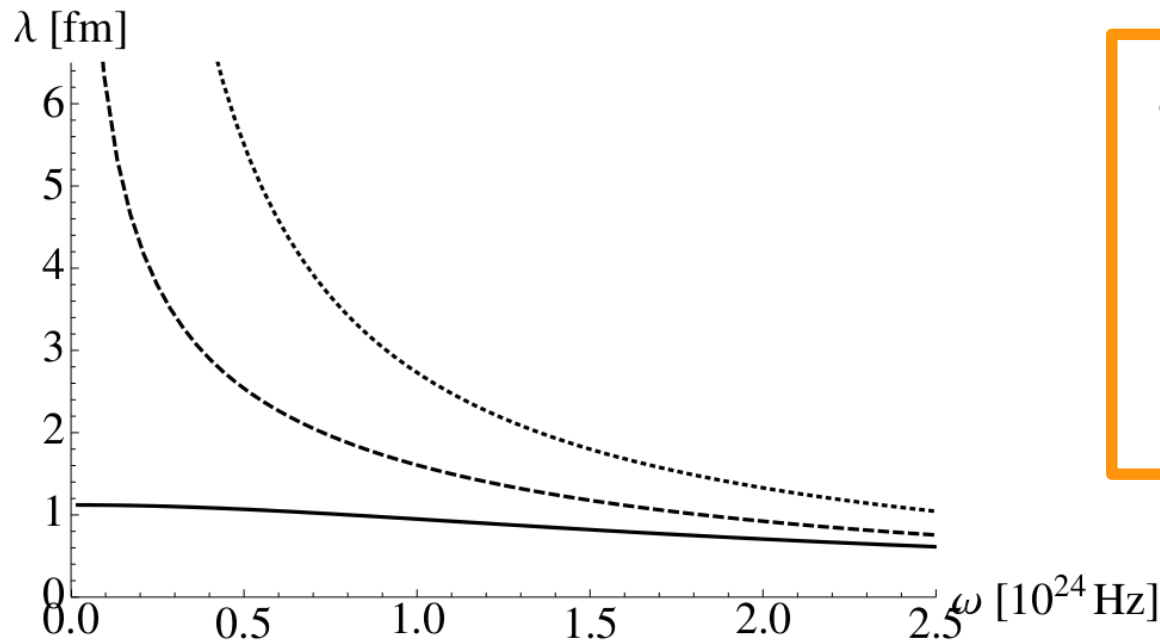
$$\text{Im}[q]/\text{Re}[q]$$

comparison between the absorption and propagation characteristic lengths



At higher frequency more modes take part to the propagating response

# Wave lengths



$$T = 200 \text{ MeV}$$

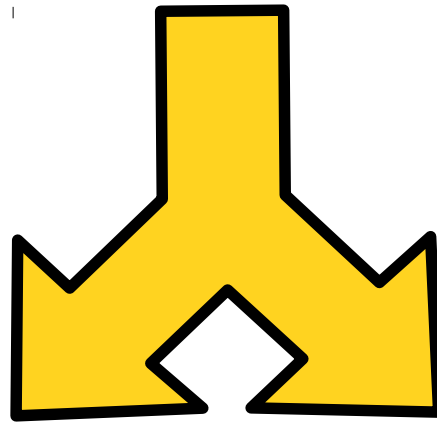
$$\lambda = \frac{\hbar c}{T \text{ Re}[\mathbf{q}]} \sim 1 \text{ fm}$$

**No order of magnitude  
obstruction to compare  
with actual QGP  
samples!**

- temperature dominated regime,  $T/\mu \approx 10$  (but not difficult to introduce finite density)
- Quantum corrections?
- caveat (finite size)

# Direct photons

Directly produced in scattering processes as opposed to those coming from decays (*e.g. neutral pions*)



## ***Prompt photons***

Hard scattering processes

Info on PDF's, QCD

## ***Thermal photons***

Thermal and hydrodynamic state

QGP

# Photon harmonics puzzle

***Anisotropic flow of direct photons*** should be smaller than the flow of hadrons since a large portion of direct photons are produced in **early stages** with **small momentum anisotropy** (*i.e. before hydrodynamization*)

Usually tackled acting on the ***production mechanism(s)***

Direct photons from holography convoluted with the medium evolution (hydro) and inclusion of other sources (prompt photons, thermal photons from hadron gas)

# Transparency

Assumption of perfect transparency based on *integrated intensity* measures

EM probes carry only *local* information

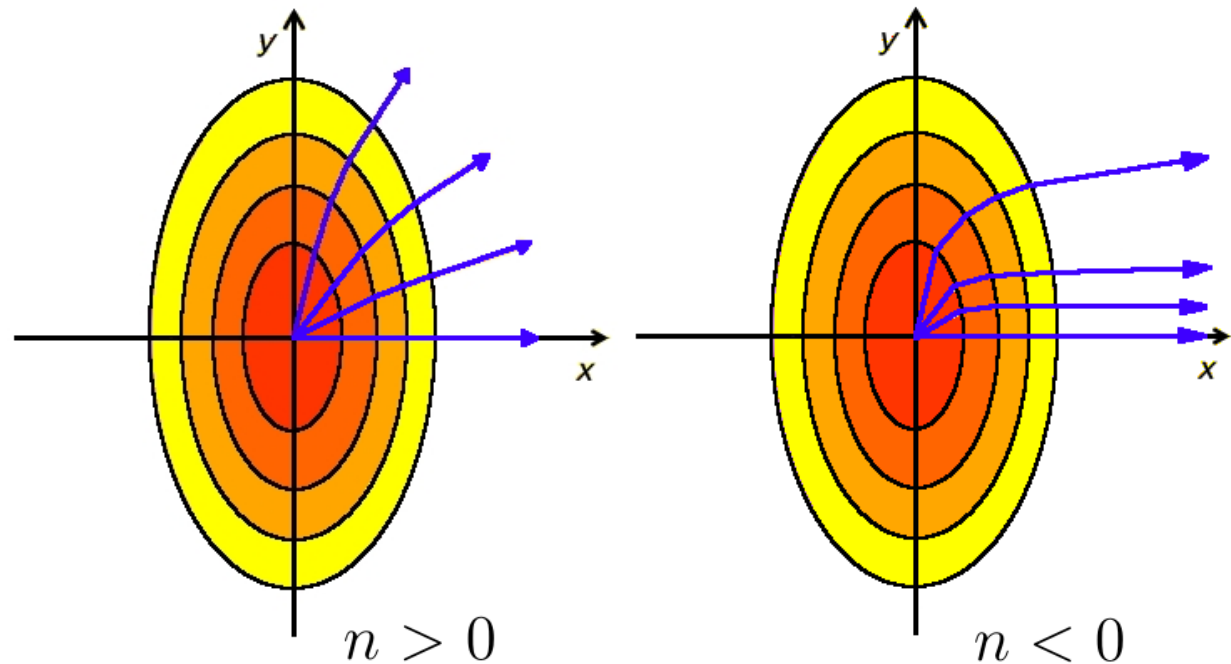
***Should we relax the perfect QGP transparency paradigm?***

It is still true that the direct photons are sensitive to the nature of the QCD matter during its time evolution ***as well as to the initial stage***

*Usually it is said that the MFP of a photon through the QGP is large compared to the fireball, however are we sure of basing our MFP estimates on a complete knowledge of the possible collective modes of the plasma which can couple to the photon?*



# Harmonics



Not only non-trivial refraction leads to important effects on angular distributions, negative refraction could affect even more significantly the photon harmonics

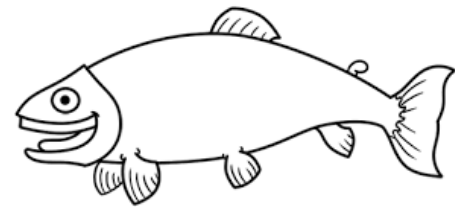
# Conclusions

- Direct ***photon angular distribution*** in QGP experiments is yet not fully understood
- From a theoretical standpoint there seems to be room (*and possibly suitable holographic tools*) for accounting photon interactions in the QGP
- General theoretical expectations predict ***negative refraction*** in strongly coupled plasmas
- ***Potential data (re)analysis assuming non-trivial optical properties of the QGP samples?***

# Open problems and future prospect

- Finite *density*
- Finite (*strong*) *magnetic field*. Relevant for CM applications too
- *Finite size effects and actual integrated Snell computation (character of the interfaces)*
- *Quantum corrections*? Some EFT scheme? (*quantum optics*?)
- *Non-linearities*? Back-reaction of the photon on the medium (*semi-holography*). Unlikely for the present purposes, but interesting in general

***THANKS!***



# Equilibrium, thermodynamics

## Holographic dictionary

- sources  $\leftrightarrow$  boundary conditions
- VEV of operators (observables)  $\leftrightarrow$  Asymptotic behavior of the fields
- temperature  $\leftrightarrow$  temperature (e.g. *black hole*)
- Identification of the thermodynamic potentials!

# Linear response, transport

- small source perturbation  $\leftrightarrow$  small variations of the boundary condition
  - Linear response  $\leftrightarrow$  linearized fluctuations
  - Dissipation  $\leftrightarrow$  “flow” into the black hole
- 

two-point correlators  $\rightarrow$  transport properties (Kubo formulae)

- charge (s)  $\rightarrow$  conductivity
- momentum  $\rightarrow$  viscosity
- heat  $\rightarrow$  thermal conductivity
- spin  $\rightarrow$  spin conductivity
- ...
- and mixed properties (thermo-electric, spintronics, Hall,...)

# Take it seriously!

Far more than a book keeping device:

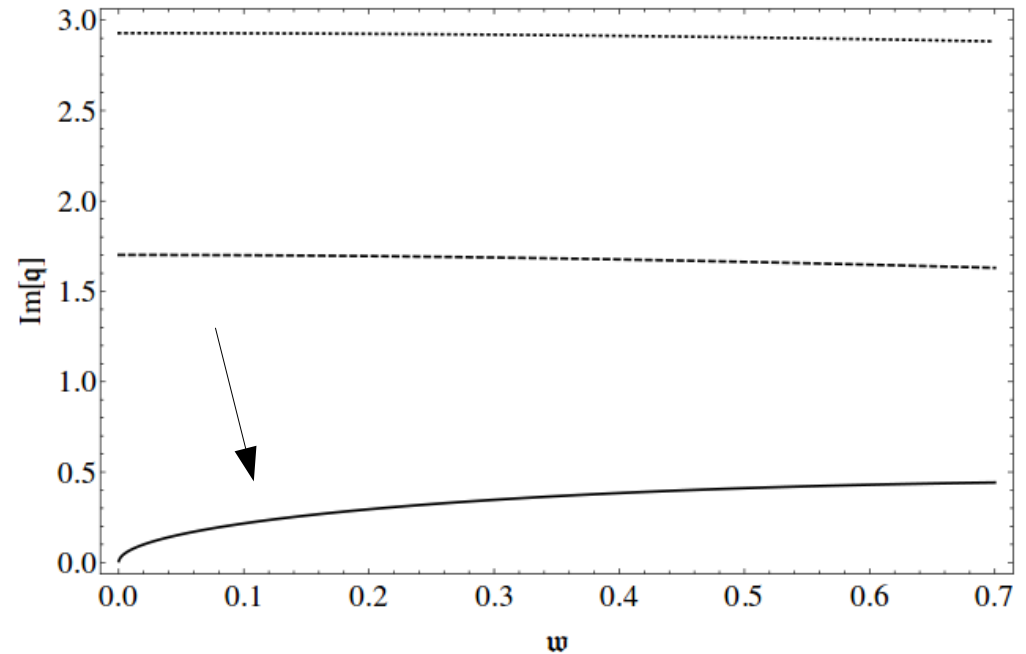
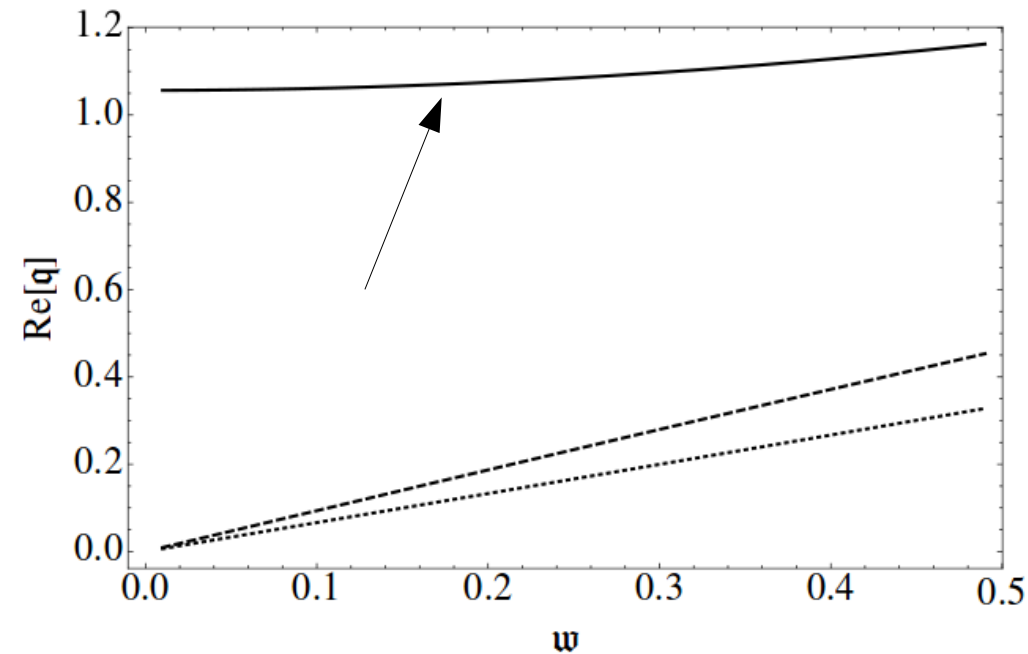
- dual interpretation and intuition
- correspondence on solid bases

---

*Example: quantum field theory at finite temperature and the Schwinger-Keldish formalism*

# Internal excitations of a plasma

- Probing the collective response (*e.g. excitons, plasmons*) gives rise to Additional Light Waves





# Photon production in holography

hep-th/0607237

- QFT in thermal equilibrium
- Interactions of photon with matter encoded in
- Assuming small coupling, no re-scattering no thermalization

$$\longrightarrow e J_{\mu}^{(\text{EM})} A^{\mu}$$

$$d\Gamma_{\gamma} = \frac{d^3 k}{(2\pi)^3} \frac{e^2}{2|\mathbf{k}|} \eta^{\mu\nu} C_{\mu\nu}^{<}(K) \big|_{k^0=|\mathbf{k}|}$$

$$C_{\mu\nu}^{<}(K) = n_{\text{BE}}(k^0) \chi_{\mu\nu}(K)$$

$$n_b(k^0) = \frac{1}{e^{\beta k^0} - 1}$$

$$\chi_{\mu\nu}(K) = -2 \text{Im} C_{\mu\nu}^{\text{ret}}(K)$$



# Plane waves

$$\mathbf{E}(z) = A \exp(ik_0nz) \mathbf{u}_x ,$$

$\exp(-i\omega t)$  time-dependence

$$\mathbf{B}(z) = \frac{1}{i\omega} \nabla \times \mathbf{E}(z) = \frac{nk_0}{\omega} A \exp(ik_0nz) \mathbf{u}_y ,$$

$$\mathbf{H}(z) = \frac{n}{\mu_r \eta_0} A \exp(ik_0nz) \mathbf{u}_y ,$$

$$n^2 = \epsilon_r \mu_r$$

$$P_z(n) = \frac{1}{2} \mathbf{u}_z \cdot \text{Re}[\mathbf{E}(z) \times \mathbf{H}^*(z)] = \text{Re} \left[ \frac{n}{\mu_r} \right] \frac{|A|^2}{2\eta_0} \exp(-2k_0 \text{Im}[n]z)$$

***Poyinting vector***

*They go hand in hand, see next slide...*

Causality of the material response imposes:

$$\text{Im}[\epsilon_r] \geq 0$$

$$\text{Im}[\mu_r] \geq 0$$

# Demystifying contra *double negative*

McCall, Lakhtakia, Weiglhofer class-ph/0204067

Now  $n_{\pm}$  may be written as

$$n_{\pm} = \pm n_0 \exp i\phi_n ,$$

where

$$n_0 = +\sqrt{|\epsilon_r||\mu_r|} , \quad \phi_n = \frac{\phi_{\epsilon} + \phi_{\mu}}{2} .$$

Here  $\phi_{\epsilon}$  and  $\phi_{\mu}$ , representing the arguments of  $\epsilon_r$  and  $\mu_r$  respectively, must obey the conditions  $0 \leq \phi_{\epsilon, \mu} \leq \pi$ . Consequently,  $0 \leq \phi_n \leq \pi$ . We then always have

$$\operatorname{Re} \left[ \frac{n_+}{\mu_r} \right] > 0 \quad \text{i.e. } P_z(n_+) > 0$$

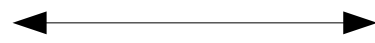
and also

$$\operatorname{Re} \left[ \frac{n_-}{\mu_r} \right] < 0 \quad \text{i.e. } P_z(n_-) < 0 .$$

Thus the choice  $n_+$  *always* relates to power flow in the  $+z$  direction, whilst  $n_-$  *always* relates to power flow in the  $-z$  direction. Since necessarily  $\operatorname{Im}[n_+] > 0$  and  $\operatorname{Im}[n_-] < 0$ , power flow is always in the direction of exponential decrease of the fields' amplitudes.

We can now identify when the phase velocity is opposite to the direction of power flow. This occurs whenever  $\operatorname{Re}[n_+] < 0$  (and consequently  $\operatorname{Re}[n_-] > 0$ , also).

$$\text{Re}[n_+] < 0$$



$$\phi_n > \frac{\pi}{2}$$

$$\phi_\epsilon > \frac{\pi}{2} \quad \phi_\mu > \frac{\pi}{2}$$

In order to have negative refraction the **double negative condition** (i.e. the real part of both the relative permeability and permittivity to be negative) is a **SUFFICIENT** but **NOT NECESSARY** condition!

# Shear viscosity

$$F = \eta A \frac{\partial u_x}{\partial y} \quad \Rightarrow \quad \left. \begin{aligned} [\eta] &= \frac{Js}{m^3} \\ [s] &= \frac{J}{Km^3} \end{aligned} \right\} \quad \left[ \frac{\eta}{s} \right] = Ks \rightarrow \frac{\hbar}{k_B}$$

Relation to transverse diffusion of momentum, the shear modes which are transverse fluctuations of the momentum density have the following dispersion relation

$$\omega = -\frac{i\eta}{\epsilon + P} q^2 \quad \rightarrow \quad D \sim \frac{\eta}{s} \frac{1}{T}$$

# Strong Coupling Tools

Although AdS/CFT is stronger in principle, is possibly not quantitatively stringent

**But qualitative control can nevertheless suggest some (potentially) very relevant pheno hints do be developed also outside the gauge/gravity framework !!**

# The gauge/gravity approach

The dynamics of membranes in String Theory admits two *dual* descriptions

*...at low energy:*

- Quantum Field Theory in flat space-time
- Gravity (+ fields) in the ambient space

- *Strong/weak* duality
- Amenable to explicit computations (*correlation functions*)

# Formulation of the duality

- Identification of the two partition functions

$$\mathcal{Z}_{CFT}^{(\mathcal{N}=4, Y.M.)}[\phi_0] = \mathcal{Z}_{AdS_5 \times S^5}^{(\text{Type IIB})}[\phi_0]$$

- Generalization to a class of dualities (*bottom-up*)

$$\mathcal{Z}_{QFT, \text{ boundary}}[\phi_0] = \mathcal{Z}_{\text{Gravity, bulk}}[\phi_0]$$

- Approximations

$$\mathcal{Z}_{QFT, \text{ boundary}}[\phi_0] \sim e^{-S_{\text{o.s.}}[\phi_0]}$$



# User Manual

A tool to compute correlation functions

$$\mathcal{Z}_{QFT, \text{boundary}}[\phi_0] \sim e^{-S_{\text{o.s.}}[\phi_0]}$$

**Sources of the quantum field theory  $\leftrightarrow$  Boundary condition for the gravity fields**

$$\langle \mathcal{O}_\phi \rangle_{\phi_0=0} = \left. \frac{\delta \mathcal{Z}_{QFT}}{\delta \phi_0} \right|_{\phi_0=0} \sim \left. \frac{\delta}{\delta \phi_0} e^{-S_{\text{o.s.}}} \right|_{\phi_0=0}$$

# Hydrodynamics

- **Finite temperature** (scale  $T$  setting the mean free path  $l$ )
- **Low frequency and momenta** (long wave-length w.r.t.  $l$ )
- Paradigmatic example: (diffusivity) **bounds**

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

[Son, Starinets, Kovtun, Policastro,...]

*Related to transverse  
**momentum diffusion** in  
a strongly coupled fluid*

*Momentum anisotropies of  
particle yields informative of  
the **collective properties** of  
the plasma*



Hadronic **elliptic flow** found to  
be **large** when compared with  
the geometrical anisotropy of the  
overlapping region of the  
colliding ions



**“Strongly coupled perfect  
fluid behavior”**

# Order of magnitude obstruction

	$\sim$ AGS	$\sim$ SPS	$\sim$ RHIC
$T$	125 MeV	148 MeV	177 MeV
$\mu = \frac{\mu_B}{2}$	270 MeV	200 MeV	14.5 MeV
$\rho/e$	$0.67/fm^3$	$0.66/fm^3$	$0.066/fm^3$
$\epsilon$	$1.32 GeV/fm^3$	$2.29 GeV/fm^3$	$4.28 GeV/fm^3$
$[w_0, w_c]$	$[4.02, 15.5] \times 10^{21} Hz$	$[2.07, 7.46] \times 10^{21} Hz$	$[9.79, 33.6] \times 10^{18} Hz$
$\lambda_0$	168 fm	225 fm	3100 fm

Taken from Amariti, Forcella, Mariotti hep-th/1107.1240